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Title: A Note on the Distribution of Motion-Compensated Interframe Errors and Motion Vectors

1. Introduction

This document describes statistical properties of motion-compensated (MC) interframe error signals obtained from the Miss America test sequence. It is shown that the distribution of the MC interframe errors in the spatial domain is well approximated by a Gaussian distribution function, and those of the transform coefficients of the errors, including the DC component, by Laplacian distribution functions. In addition, the distribution of motion vectors, which have been estimated using the 3-step search algorithm, is presented.

It is our opinion that the statistical parameters presented here will be helpful to get some insights on the MC interframe errors and motion vectors, and useful to evaluate coded image frames.

2. Simulation

To get statistical values of mean, variance, and entropy, 50 frames of the Miss America sequence after 2:1 temporal subsampling have been used. To estimate motion vectors, the 3-step search algorithm with the block size of 8x8 was adopted. To test goodness of fit of a distribution, the Kolmogorov-Smirnov (KS) test is performed. The KS test statistic t is defined as a maximum difference between the cumulative sample distribution function and the given cumulative distribution function:

$$t = \max | Ps(x) - Pg(x) |.$$

The distribution that yields the smallest KS test statistic is considered the best fit for the sample distribution function in this test. For simplicity, only the Gaussian and Laplacian distributions are considered here, and their mathematical forms are as follows:

Gaussian : $p(x) = 1/\sqrt{2\pi s^2} * \exp(-x^2/(2s^2))$

Laplacian: $p(x) = 1/s * \exp(-|x|/s)$

where s is the same as the variance of the sample distribution.

3. Simulation Results

Table I shows the statistical parameters about the original image frames, which tells the well-known fact that the DC component is well approximated by a Gaussian distribution, and non-DC coefficients, by Laplacian distributions. Also shown in Table II is the variance map of the transform coefficients F(k,l).

Table III shows the KS test results on the MC interframe errors, from which one can know that all distributions of the transform coefficients of the MC interframe errors can be well approximated by Laplacian distributions. Comparing Table II and IV which show variance maps of the transform coefficients of the original image and MC interframe errors, respectively, one can easily find that the transform coefficients of the interframe errors are less concentrated in the low frequency region than those of the original image, and therefore the effect of the DCT is not so remarkable in encoding MC interframe errors.

Finally shown in Table V is the distribution of the motion vectors of the Miss America test sequences. Almost half the motion vectors are zeros and the majority of motion vectors are located near the 9 points which are searched first in the 3-step motion estimation algorithm. These facts will be helpful in encding the motion vectors to have the bit rate as low as possible.

4. Conclusion

In this document, we presented some useful computer simulation results on the distribution of MC interframe errors and motion vectors. We also showed that the transform coefficients of the MC interframe errors, including the DC component, are well approximated by Laplacian distribution functions. We suggest to estimate the statistics shown in this document for all CCITT standard image sequences and make a comparison with statistics obtained from coded image frames for the performance evaluation of proposed video coding algorithms.

Table I. Statistical parameters of the Miss America image sequences.

spatial domain	Mean	64.58	
	Variance	905.18	
	Entrophy	5.70 bpp	
	KS test statistic	Gaussian	Laplacian
		0.28	0.33
transform domain	Mean	DC: 516.65, non-DC: 0	
	Variance	See Table II.	
	KS test statistic	Gaussian	Laplacian
	F(0,0)	0.29	0.35
	F(0,1)	0.28	0.25
	F(1,0)	0.27	0.23
	F(1,1)	0.26	0.21

Table II. A variance map of the transform coefficients of the original image frame.

F(k,l)

k \ l	0	1	2	3	4	5	6	7
0	53786	1344	401	195	109	60	26	10
1	1310	238	82	28	12	6	4	3
2	202	86	36	15	6	3	2	1
3	54	28	15	8	4	2	3	2
4	27	14	10	6	4	2	2	1
5	19	11	8	5	4	3	5	3
6	12	8	6	4	2	2	2	1
7	16	13	9	7	7	4	24	13

Table III. Statistical parameters of the motion-compensated interframe errors obtained from the Miss America image sequences.

spatial domain	Mean	0.0	
	Variance	8.17	
	Entrophy	3.37 bpp	
	KS test statistic	Gaussian	Laplacian
		0.105	0.052
trans-form domain	Mean	0.0	
	Variance	See Table IV.	
	KS test statistic	Gaussian	Laplacian
	F(0,0)	0.14	0.075
	F(0,1)	0.14	0.078
	F(1,0)	0.14	0.073
	F(1,1)	0.15	0.088
	F(0,2)	0.15	0.089
	F(2,0)	0.11	0.052

Table IV. A variance map of the transform coefficients of the motion-compensated interframe errors.

F(k,l)

k \ l	0	1	2	3	4	5	6	7
0	50.4	34.8	33.7	22.4	12.6	8.8	8.0	5.8
1	27.3	20.8	14.5	9.5	5.4	3.5	3.1	2.8
2	18.0	11.7	8.3	6.0	4.1	2.8	2.1	1.8
3	13.4	8.6	7.3	5.3	4.0	2.8	2.1	1.7
4	13.6	8.8	7.5	5.9	4.3	3.1	2.3	1.8
5	13.7	10.6	8.6	6.4	4.6	3.1	2.7	2.2
6	13.0	10.8	8.6	5.8	3.5	2.3	1.8	1.5
7	17.8	13.2	10.7	6.3	3.6	1.9	3.8	3.1

Table V. Distribution of the motion vectors (in percentage)

y\x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
7	0.00	0.00	0.00	0.05	0.00	0.11	0.02	0.17	0.02	0.05	0.02	0.13	0.00	0.00	0.00
6	0.00	0.13	0.00	0.03	0.06	0.17	0.03	0.17	0.02	0.19	0.06	0.19	0.00	0.11	0.00
5	0.03	0.06	0.05	0.30	0.06	0.09	0.17	0.85	0.14	0.08	0.09	0.35	0.02	0.09	0.03
4	0.00	0.14	0.05	0.66	0.08	0.24	0.11	1.45	0.03	0.14	0.09	0.95	0.03	0.17	0.00
3	0.02	0.06	0.05	0.49	0.05	0.13	0.17	0.98	0.13	0.19	0.16	0.35	0.00	0.02	0.03
2	0.00	0.19	0.00	0.46	0.05	0.32	0.21	1.28	0.08	0.44	0.05	0.28	0.06	0.21	0.02
1	0.03	0.11	0.06	0.25	0.21	0.21	1.80	0.73	0.73	0.28	0.41	0.33	0.09	0.11	0.06
0	0.09	0.39	0.19	1.36	0.35	1.28	1.56	44.92	1.22	1.55	0.51	1.69	0.13	0.38	0.11
-1	0.03	0.00	0.05	0.38	0.09	0.21	1.80	1.77	2.79	0.88	0.38	0.33	0.13	0.06	0.09
-2	0.00	0.24	0.06	0.51	0.05	0.44	0.11	1.66	0.09	0.36	0.11	0.60	0.02	0.22	0.03
-3	0.00	0.14	0.02	0.41	0.11	0.13	0.14	1.03	0.19	0.16	0.14	0.41	0.08	0.13	0.03
-4	0.02	0.05	0.03	0.79	0.05	0.24	0.05	1.33	0.03	0.16	0.03	0.73	0.06	0.21	0.00
-5	0.02	0.06	0.08	0.46	0.08	0.14	0.02	0.71	0.02	0.06	0.05	0.30	0.02	0.05	0.00
-6	0.00	0.11	0.00	0.19	0.03	0.13	0.03	0.19	0.00	0.17	0.02	0.13	0.00	0.06	0.03
-7	0.02	0.05	0.00	0.03	0.02	0.06	0.00	0.17	0.03	0.08	0.02	0.09	0.03	0.03	0.00