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Title: PROGRAMMABLE TRANSFORMER

1. Introduction

In this contribution, we shall discuss a programmble transformer and specific parameters for DCT are presented. Operation order, accuracy of operation and truncation methods at each stage are proposed with respect to compatibility issue.

2. Transformer

As agreed upon in the last Tokyo meeting, the 2-D transform is implemented as the equivalent of two independent 1-D transforms. Furthermore, all coefficients in the forward and inverse transforms are programmable.

(1) Forward Transform

Following the agreement, the 2-D transform is implemented in three steps i.e. first (horizontal) transform, second (vertical) transform and lastly normalization.

Definition:

$A_C(u,j)$......second transform
$f(j,k)$.......pel amplitude at (j,k)
$A_R(v,k)$......first transform
NORM.........Normalization Factor (Constant)
$F(u,v)$.......transform coefficient for (u,v) component
$B_C(u,j)$......second inverse-transform
$B_R(v,k)$......first inverse-transform

Using this definition forward transform is described as follows in general.

$$F(u,v) = \mathrm{NORM}\{\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} A_C(u,j)f(j,k)A_R(v,k)\}$$

This equation can be expressed in the following matrix representation.

$$[F(u,v)] = \mathrm{NORM}\,[A_C(u,j)][f(j,k)][A_R(v,k)]^T$$

For flexible hardware implementation, coefficient matrices $A_C(u,j)$ and $A_R(v,k)$ are selected such that the absolute value of each component is less than unity.

(2) Inverse Transform

Similarly, inverse transform is defined in the following matrix representation.

$$[f(j,k)] = NORM\ [B_C(u,j)]^T\ [F(u,v)][B_R(v,k)]$$

The normalization factor NORM in the inverse-transform may be different from that in forward-transform if a different transform from DCT is employed.

3. Accuracy

(1) Matrix Elements

Accuracy of matrix elements should be defined.

Eg. $\overbrace{\text{s.xxxxxxxxxxx}}^{12}$ 12-bit accuracy including a sign bit (s=0/1, x=0/1)

Accuracy of normalization should also be defined.

Eg. 0.xxxxxxxxxxx or shift number

(2) Accumulation

In the matrix computation, approximation error is accumulated. Approximation method should be defined in conjunction with the order of operation described in the next section.

4. Order of Operation

The order of operation affects compatibility when approximation is carried in each multiply-and-add oreration in the inverse transform.

(1) Order of Matrix Operation

Order	Forward	Inverse
1	$[X]=[f(j,k)][A_R(v,k)]^T$	$[X]=\{F(u,v)][B_R(v,k)]$
2	$[Y]=[A_C(u,j)][X]$	$[Y]=[B_C(u,j)][X]$
3	[F(u,v)]=NORM [Y]	[f(j,k)]=NORM [Y]

(2) Order of MULTIPLY-and-ADD Calculation

The calculation order should be rigorously defined in the inverse transform since
truncation is carried out for hardware simplification.

Here, it is recommended that the calculation be defined in the order where MULTIPLY-and-ADD is carried out at first for component designated by the smallest subscript and then for larger (higher order) subscript components. (Compatibility can be assured even if the order is defined in the reverse way.)

(3) Approximation

Approximation method and accuracy should be defined.
As an approximation method, round-off is recommended for a little bit higher accuracy than truncation though the latter is simpler. The bit place where round-off is done in each row or column computation (accumulation) should be clarified.

5. Example for DCT

Since DCT is used as an orthogonal transform at present, specification of DCT is exemplified.

(1) DCT Matrix

Assume that N=8.

a) Forward Transform

F(u,v)=

$$\frac{1}{4}\sum_{j=0}^{7}\sum_{k=0}^{7} C(c)C(v)f(j,k)\cos\left[\frac{\pi}{8}\{u(j+1/2)\}\right]\cos\left[\frac{\pi}{8}\{v(k+1/2)\}\right]$$

$$A_C(u,j)=C(u)\cos\left[\frac{\pi}{8}\{u(j+1/2)\}\right],$$

$$A_R(v,k)=C(v)\cos\left[\frac{\pi}{8}\{v(k+1/2)\}\right].$$

Therefore,

$$F(u,v)=\frac{1}{4}\sum_{j=0}^{7}\sum_{k=0}^{7} A_C(u,j)f(j,k)A_R(v,k).$$

This computation can be expressed in the following matrix representation.

$$[F(u,v)] = \frac{1}{4}\,[A_C(u,j)][f(j,k)][A_R(v,k)]^T$$

Absolute values of the elements in these matrices are limited to be less than unity so that the transform remains programmable and easy to change.

b) Inverse Transform

$$f(j,k)=$$

$$\frac{1}{4}\sum_{u=0}^{7}\sum_{v=0}^{7} C(u)C(v)F(u,v)\cos[\frac{\pi}{8}\{u(j+1/2)\}]\cos[\frac{\pi}{8}\{v(k+1/2)\}]$$

In matrix representation,

$$[f(j,k)]=\frac{1}{4}\,[A_C(u,j)]^T\,[F(u,v)][A_R(v,k)].$$

(2) Order of Operation

Order	Forward	Inverse
1	$[X]=[f(j,k)][A_R(v,k)]^T$	$[X]=[F(u,v)][A_R(v,k)]$
2	$[Y]=[A_C(u,j)][X]$	$[Y]=[A_C(u,j)][X]$
3	$[F(u,v)]=\frac{1}{4}[Y]$	$[f(j,k)]=\frac{1}{4}[Y]$

[Note]

Numerical data for A(i,j) is described in Annex to Document #116.