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Title : The use of a N-DCT device to perform a N/2 - DCT

Let be $Y_0, Y_1, \ldots.., Y_{N-1}$ a one dimensional N-length sequence (N = even number) and $F_0, F_1, \ldots.., F_{N-1}$ the N-DCT components (F_0 is the DC component and F_{N-1} the highest frequency).

Let be $C_0, C_1, \ldots.., C_{\frac{N}{2}-1}$ a N/2 - length sequence.

In order to have the same spatial blocksize for luminance (Y_n) or chrominance (C_n), or to allow compatibility between different bit rates by changing the blocksize, we may envisage to achieve a N/2 - DCT by using a N - DCT device.

To reduce the hardware, one can suggest to define an N - length sequence (A) as follows :

$$(A_0, A_1, \ldots., A_{N-1}) = (C_0, C_0, C_1, C_1, \ldots, C_{\frac{N}{2}-1}, C_{\frac{N}{2}-1})$$

$$A_n = C_{\frac{n}{2}} \quad \text{if } n = 2p \qquad 0 \leqslant p \leqslant N/2-1$$

$$A_n = C_{\frac{n-1}{2}} \quad \text{if } n = 2p+1$$

or an N-length sequence (A') :

$$(A'_0, A'_1, \ldots, A'_{N-1}) = (C_0, 0, C_1, 0, \ldots, C_{\frac{N}{2}}, 0)$$

$$\begin{cases} A'_n = C_{\frac{n}{2}} & \text{if } n = 2p \\ A'_n = C_{\frac{n}{2}} & \text{if } n = 2p+1 \end{cases} \qquad 0 \leqslant p \leqslant N/2-1$$

One can also suggest to define a N-length sequence B :

$$(B_0, B_1, \ldots, B_{N-1}) = (C_0, C_1, \ldots, C_{\frac{N}{2}-1}, C_{\frac{N}{2}-1}, \ldots, C_1, C_0)$$

$$\begin{cases} B_n = C_n & \text{if } 0 \leqslant n \leqslant N/2-1 \\ B_n = C_{N-1-n} & \text{if } N/2 \leqslant n \leqslant N-1 \end{cases}$$

Or a N-length sequence B':

$$(B'_0, B'_1, \ldots, B'_{N-1}) = (C_0, C_1, \ldots, C_{\frac{N}{2}-1}, 0, \ldots, 0)$$

$$\begin{cases} B'_n = C_n & \text{if } 0 \leqslant n \leqslant N/2-1 \\ B'_n = 0 & \text{if } N/2 \leqslant n \leqslant N-1 \end{cases}$$

(A_n), (A'_n), (B_n), (B'_n) have N-DCTs : (F_k^A), $(F_k^{A'})$, (F_k^B), $(F_k^{B'})$

with $0 \leqslant n \leqslant N-1$ and $0 \leqslant k \leqslant N-1$

(C_n) has a N/2-DCT : (F_k^C) with $0 \leqslant n \leqslant N/2-1$ and $0 \leqslant k \leqslant N/2-1$.

We shall examine the advantages and drawbacks when using either (A_n), (A_n'), (B_n) or (B_n') instead of (C_n) from a hardware point of view, in the pixel and the transform domains.

SITUATION A

$$F_k^A = 1/N \sum_{n=0}^{N-1} A_n \cos \frac{2\pi(2n+1)k}{4N}$$

A_{2p} and A_{2p+1} can be combined, giving :

$$F_k^A = 1/N \sum_{p=0}^{N/2-1} C_p \left[\cos \frac{2\pi(4p+1)k}{4N} + \cos \frac{2\pi(4p+3)k}{4N} \right]$$

$$= 1/N \sum_{p=0}^{N/2-1} 2\, C_p \cos \frac{\pi k}{2N} \cdot \cos \frac{\pi(2p+1)k}{N}$$

The N (F_k^A) values are not independent because there are only N/2 (C_p) independent data.

If we consider the N/2 lowest frequencies (F_k^A) $0 \leqslant k \leqslant N/2-1$ as independent, the N/2 highest frequencies ($F^A{}_k$) $0 \leqslant k \leqslant N-1$ are related to the previous ones as follows :

$$F^A_{2N-k} = \sum_{p=0}^{N-1} 2\, C_p \sin \frac{\pi k}{2N} .(-1). \cos \frac{\pi(2p+1)k}{N} \qquad 0 \leqslant k \leqslant N/2-1$$

[1] $$\boxed{F^A_{N-k} = -\, tg \frac{\pi k}{2N} F_k^A \qquad 0 \leqslant k \leqslant N/2-1}$$

The situation in the transform domain is not pleasant : instead of having N/2 compoments there we have N non-independent coefficients. Of course, it is not necessary to transmit the N/2 highest frequencies, but it is necessary to reconstruct them by using [1] before applying the inverse N-DCT. If not we may replace them by zero, but then perform a very strong low pass filter on the (A_n) sequence and blocking effects may appear. From a hardware point of view [1] is not very simple. Moreover, the F_k^A values are not simply related to the N/2-1 DCT values F_k^C.

Fig. 2 shows the 2 dimensional situation

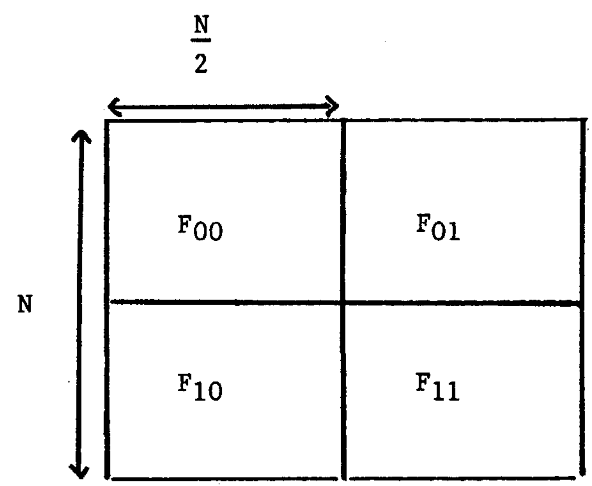

Fig. 2 : Transform block

F_{00} components can be considered as independent and F_{01}, F_{10} and F_{11} components are related to F_{00} ones by a 2 dimensional version of [1].

If one wants to ignore it is a N/2xN/2 block, he has to transmit F_{00}, F_{01}, F_{10}, F_{11} : it is not realistic ! On the other hand one may transmit F_{00} components only, but must reconstruct F_{01}, F_{10}, F_{11} before applying the 2-dimensional inverse N-DCT which is not also realistic from a hardware point of view.

Situation A'

$$F_k^{A'} = 1/N \sum_{n=0}^{N-1} A'_n \cos \frac{2\pi(2n+1)k}{4N} \qquad A'_{2p} = C_p$$

$$= 1/N \sum_{p=0}^{N/2-1} C_p \cos \frac{2\pi(4p+1)k}{4N} \qquad A'_{2p+1} = 0$$

The N ($F_k^{A'}$) values are not independent for the same reason as in the situation A. But this case is worse, because the relationship between, for example, $F_k^{A'}$ and $F_{N-k}^{A'}$ requires the use of the sine transform of the sequence A'. In the transform domain, this situation is completely unpracticable, and the $F_k^{A'}$ values are not simply related to the N/2-DCT values F_k^C.

Situation B

The N-DCT coefficients are given by :

$$F_k^B = 1/N \sum_{n=0}^{N-1} B_n \cos \frac{2\pi(2n+1)k}{4N}$$

B_n and B_{N-n-1} can be combined, giving :

$$F_k^B = 1/N \sum_{n=0}^{N/2-1} C_n \left[\cos \frac{2\pi(2n+1)k}{4N} + \cos \frac{2\pi(2N-2n-1)k}{4N} \right]$$

$$= 1/N \sum_{n=0}^{N/2-1} C_n \left[\cos \frac{2\pi(2n+1)k}{4N} + (-1)^k \cos \frac{2\pi(2n+1)k}{4N} \right]$$

$$F_{2l+1}^B = 0 \qquad k=2l+1$$

$$F_{2C}^B = 1/N \sum_{n=0}^{N/2-1} 2\, C_n \cos \frac{2\pi(2n+1)l}{4N}$$

$$= 1/(N/2) \sum_{n=0}^{N/2-1} C_n \cos \frac{2\pi(2n+1)l}{4N/2}$$

In this case, two sets of interleaved coefficients are obtained :

- the odd components F_1^B, F_3^B, ..., F_{N-1}^B are zero values
- the even components F_0^B, F_2^B, ..., F_{N-2}^B are the N/2-DCT coefficients $F^C{}_k$ ($0 \leqslant k \leqslant N/2-1$).

The coder needs to know that it is a chrominance block (or a N/2xN/2 block) in order to only scan the even coefficients. The rest of the coding strategy can remain the same, everything being applied to the N/2-DCT coefficients instead of the N-DCT ones.

$$
\begin{bmatrix}
F^B_{00} & 0 & F^B_{01} & 0 & F^B_{02} & 0 & F^B_{03} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F^B_{10} & 0 & F^B_{11} & 0 & F^B_{12} & 0 & F^B_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F^B_{20} & 0 & F^B_{21} & 0 & F^B_{22} & 0 & F^B_{23} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F^B_{30} & 0 & F^B_{31} & 0 & F^B_{32} & 0 & F^B_{33} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Odd rows and columns are zero values (the first row is numbered by 0, the second one by 1, ...). The rest represents the N/2-DCT coefficients.

Fig. 3 : Example for N = 8

In the 2 dimensional case it is necessary to scan only even rows and the same coding scheme can be applied to the N/2-DCT coefficients for N/2xN/2 blocks as well as to the N-DCT coefficients for NxN blocks. Before performing the inverse N-DCT odd rows and colums of the NxN transform block must be set up to zero.

Situation B'

The N-DCT coefficients are given by :

$$F_k^{B'} = 1/N \sum_{n=0}^{N-1} B'_n \cos \frac{2\pi(2n+1)k}{4N} \qquad \begin{cases} B'_n = C_n & 0 \leq n \leq N/2-1 \\ B'_n = 0 & N/2 \leq n \leq N-1 \end{cases}$$

$$= 1/N \sum_{n=0}^{N/2-1} C_n \cos \frac{2\pi(2n+1)k}{4N}$$

$$F_{2l}^{B'} = 1/N \sum_{n=0}^{N/2-1} C_n \cos \frac{2\pi(2n+1)l}{4N/2} = 1/2\ F_l^C$$

The even components $F_0^{B'}, F_2^{B'}, \ldots, F_{N-1}^{B'}$ represent half the value of the N/2-DCT coefficients $F_0^C, F_1^C, \ldots, F_{N/2-1}^C$.

The odd components are not zero values, but represent nothing interesting.

So the coder needs to know that it is a N/2xN/2 block instead of a N x N block, in order to only scan the even coefficients. Before applying the inverse N x N transform, it is necessary to magnify the even coefficients and to replace the odd ones by zero.

The situation is then similar to the situation B.

Conclusion

Bearing in mind that the same N-DCT device could be applied for both NxN blocks and N/2xN/2 blocks, for question of compatibility or other :

- in the situation A, no entra hardware is required in the pel domain, but it upsets everything in the transform domain.

- in the situation A', it is similar is the pel domain but worse in the transform domain.

- in the situation B some entra hardware is required in the pel domain (by repeating the N/2 sequence in the inverse order). But in the transform domain, the even rows and columns represent the N/2xN/2 DCT.

- The situation B' is obviously the best one :

* The coder needs to know that it is a N/2xN/2 block instead of a NxN block.

* There is no extra hardware in the pixel domain, the N x N block is obtained by completing the N/2xN/2 block by zero values.

* The even coefficients (in the transform domain) must be magnified by 2 (which can be taken into account by the quantization strategy) to give the N/2xN/2-DCT components of the N/2xN/2-block.

* By only scanning the even components the same coding strategy can be applied.

* It is necessary to set up to zero the odd rows and columns of the NxN transform block before applying the inverse NxN-DCT.