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## NETWORK MODELS

### 1. Introduction

Network loading models are needed to evaluate the efficiency of a variable bit rate coding scheme from a network operator's point of view.

This paper presents the three network models which have been used by the COST211ter simulation group to assess the impact of VBR coding on an ATM network. All of them have been presented in CCITT SGXV. The first model is based on the peak and mean bit rate of the source and is termed "The Method of Large Deviations". The other two models use the mean bit rate and variance of the source. One uses a Gaussian distribution and the other is based on the "Method of Equivalent Bursts". The first model was chosen as reference model in CCITT. All three network loading models, which are currently under study in Europe, are described in more detail in paragraph two.

In the third paragraph the models are compared and in the fourth the discrepancy between PSAT and CLR is highlighted. The fifth paragraph gives an example.

### 2. Network models

#### 2.1 The Method of Large Deviations

The source is assumed to be alternately active, transmitting at a peak rate of P, and silent. The relative durations of the on and off periods is chosen such that the mean rate of the model matches the mean rate of the source. No account is made of the variance of the source.

The equation of the model is

$$PSAT = \exp(-n \cdot k)$$

where PSAT is the probability of saturation  
n is the number of simulations calls

$$k = a \cdot \ln \frac{a}{p} + (1 - a) \cdot \ln \frac{1 - a}{1 - p}$$

where  $p = \frac{MVBR}{PVBR}$

and PVBR is the peak variable bit rate component of the codec  
MVBR is the mean variable bit rate component of the codec

where  $a = \frac{c - n.CBR}{n.PVBR}$

and c is the capacity of the network  
CBR is the constant bit rate component of the codec

The cell loss ratio (CLR) is given by the equation

$$CLR = \frac{PSAT}{n.p.\ln\left(\frac{a.(1-p)}{p.(1-a)}\right)}$$

Note  $\ln = \log$  to the base 'e'.

## 2.2 Gaussian (Normal Distribution) Model

A Gaussian model for the superposition of all sources takes into account the variance of the signals. The mean and variance characterizing the Gaussian model are obtained by direct summation of individual source means and variances.

Let  $m_i$  and  $\sigma_i^2$  be the mean bit rate and variance of source  $i$ . Then for  $n$  source the mean ( $M$ ) and the variance ( $\sigma^2$ ) are given by :

$$M = \sum m_i$$

$$\sigma^2 = \sum \sigma_i^2$$

Let  $R_i$  be the rate contribution of source  $i$ , then the total rate is given by the expression

$$R = \sum R_i \sim N(n.M, \sigma.\sqrt{n})$$

If the sum of the source is distributed normally with mean  $M$  and variance  $\sigma^2$  (shorthand  $N(M, \sigma^2)$ ), then the probability that the sum of the sources exceeds the network capacity  $c$  is given by

$$PSAT = \text{Prob}(R \geq c)$$

where  $PSAT$  is the probability of saturation

$$PSAT = Q\left(\frac{c - n.(CBR + MVBR)}{\sigma.\sqrt{n}}\right)$$

where  $\sigma$  is the standard deviation of the source  
 $CBR$  is the mean bit rate of the CBR component  
 $MVBR$  is the mean bit rate of the VBR component  
 $c$  is the network capacity  
 $n$  is the number of simultaneous calls  
 $Q(.)$  is the Gaussian distribution

Used values are in Table 1 below

PSAT	Solution $Q(x)$
$10^{-1}$	1.28155
$10^{-2}$	2.32635
$10^{-3}$	3.09023
$10^{-4}$	3.71920
$10^{-5}$	4.26489
$10^{-6}$	4.75342
$10^{-7}$	5.19934
$10^{-8}$	5.61200
$10^{-9}$	5.99781

Table 1. Useful values of  $kPSAT$

Writing  $k_{PSAT}$  for the solution in  $x$  of  $Q(x)$ , we obtain the expression of  $n$  for a given saturation probability by solving a second order equation.

$$n = \left[ \frac{-k_{PSAT} \cdot \sigma}{2 \cdot (CBR + MVBR)} + \sqrt{\left( \frac{k_{PSAT} \cdot \sigma}{2 \cdot (CBR + MVBR)} \right)^2 + \frac{c}{CBR + MVBR}} \right]^2$$

### 2.3 The Method of Equivalent Bursts

This is a method where the total stream is substituted with a superposition of substreams generated by infinitely many on/off sources. The height of the on-state for these sources is given by

$$h = \frac{\sigma^2}{m}$$

where  $\sigma^2$  is the variance of the source (VBR component)  
 $m$  is the mean of the source (VBR component)

We get the following approximation for PSAT

$$PSAT = P(X \geq n') \approx \frac{P(X = n')}{1 - \frac{\mu}{n'}}$$

where

$$n' = \frac{c - n \cdot CBR}{h}$$

$$X \sim Po(\mu)$$

$$\mu = \frac{n \cdot m}{h}$$

$n'$  is the the total number of fictive bursts with peakrate  $h$ , which can be transmitted on the available bandwidth after subtracting the CBR parts of the sources.

$X$  is the number of burst (fictive bursts with peakrate  $h$ ) going on at a certain time

$\mu$  is the mean number of fictive bursts

$n$  the number of simulataneous calls

CBR the constant bitrate component

The poisson function can be used, since the number of bursts is infinite.

When the number of bursts  $n'$  is not too small the following approximation can be used for expressing the cell loss ratio

$$CLR = \frac{P(X \geq n')}{n' - \mu} = \frac{PSAT}{n' - \mu}$$

### 3. A comparison of network models

The "Large Deviation Model" is more pessimistic with regards to the number of simultaneous sources that a defined network capacity will support compared to the other models based on mean and variance. Both models based on mean and variance give approximately the same result.

In paragraph 5 some examples are given for the Method of Large Deviation and for the Method of Equivalent Bursts in order to allow a comparison.

### 4. PSAT and CLR

PSAT evaluates the probability of the total bit rate for the number of sources exceeding the channel capacity (ie. congestion probability), whereas CLR evaluates the probability of a lost cell, on a particular call, when the total bit rate for the number of sources exceeds the channel capacity.

Two models evaluate both PSAT and CLR. In paragraph 5 some results are given, see Table 2 and 3 and Figures 1 to 4.

The figures show that the CLR is about two orders of magnitude less than the congestion probability. However when this discrepancy is represented as a difference in the number of simultaneous calls the difference is no more than 5 % for these examples.

### 5. Example

The examples were calculated at Bt Labs using a 2-layer codec data for two test sequences, Table Tennis and Jack-in-the-Box, for a network capacity of 599 Mbit/s.

Sequence Name	CBR Mean Bit Rate (kbit/s)	VBR Mean Bit Rate (kbit/s)	VBR Peak Bit Rate (kbit/s)	VBR Std. Dev (kbit/s)
Table Tennis	1856	1282	3341	1046
Jack-in-the-Box	320	234	1532	364

Table 2. Bit rate results for a 2-layer CBR / VBR codec

Sequence Name	$P_{\text{sat}} = 10^{-3}$		$\text{CLR} = 10^{-3}$	
	Large Dev.	Equiv. Bursts	Large Dev.	Equiv. Bursts
	No. of calls	No. of calls	No. of calls	No. of calls
Table Tennis	166	176	172	182
Jack-in-the-Box	880	933	913	958

Table 3. Comparison of PSAT and CLR

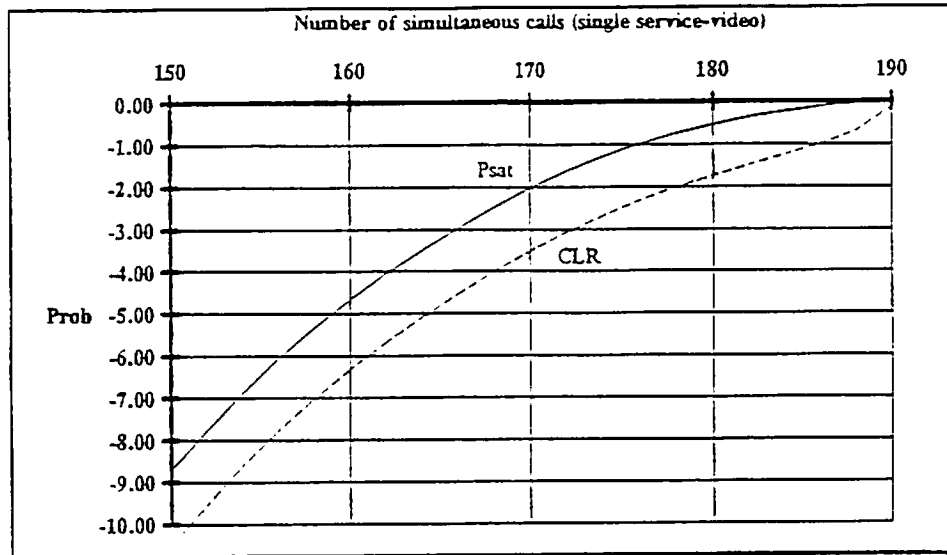


Figure 1: Plots of  $P_{sat}$  and CLR using the Method of Large Deviations  
(Sequence : Table Tennis, Network Capacity = 599 Mbit/s)

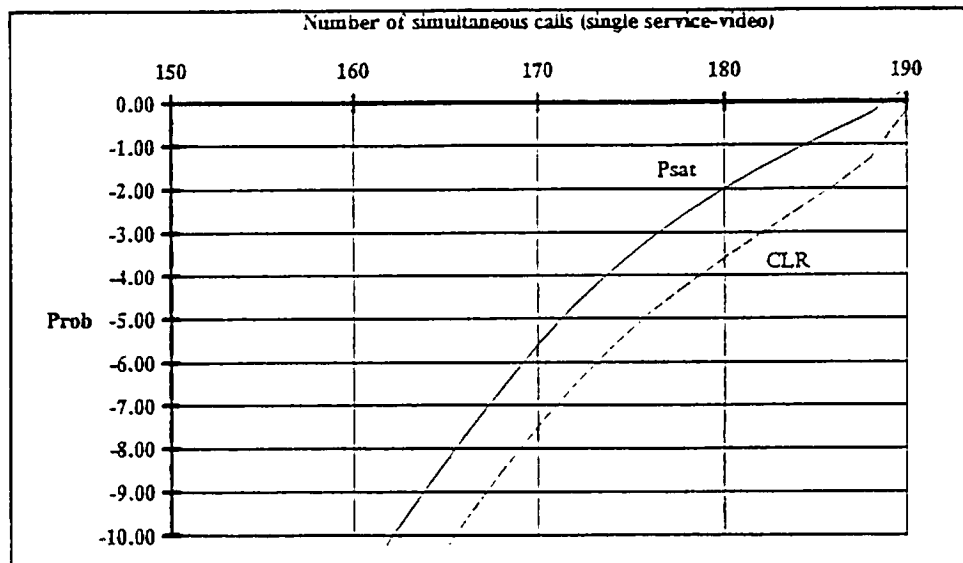


Figure 2: Plots of  $P_{sat}$  and CLR using the Method of Equivalent Bursts  
(Sequence : Table Tennis, Network Capacity = 599 Mbit/s)

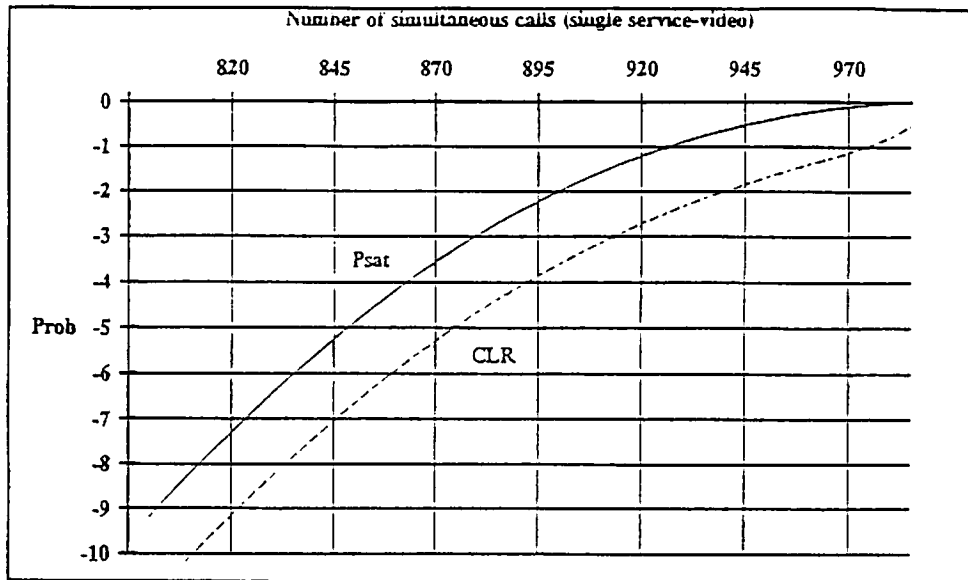


Figure 3: Plots of  $P_{sat}$  and CLR using the Method of Large Deviations  
(Sequence : Jack-in-the-Box, Network Capacity = 599 Mbit/s)

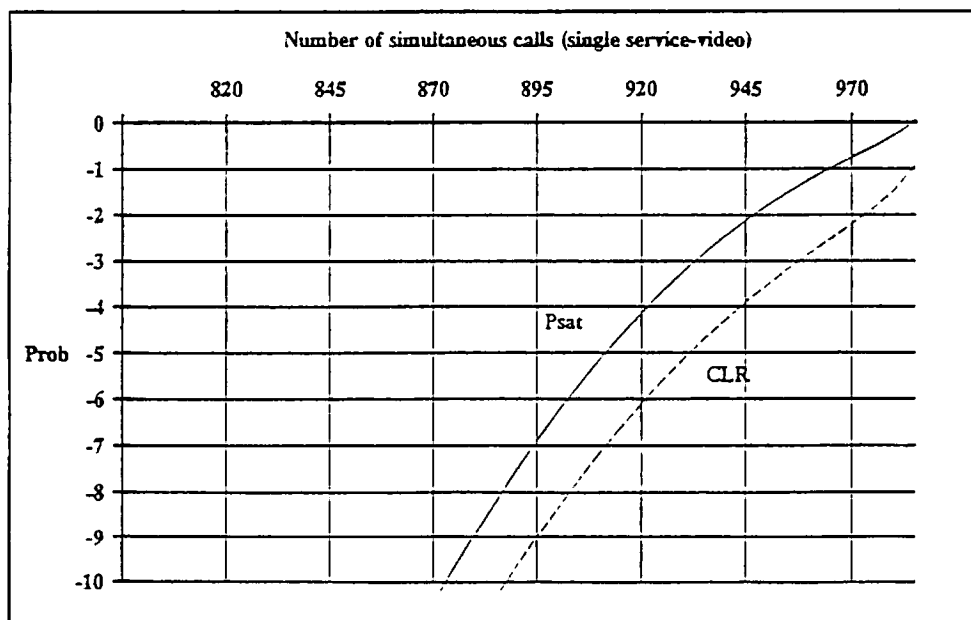


Figure 4: Plots of  $P_{sat}$  and CLR using the Method of Equivalent Bursts  
(Sequence : Jack-in-the-box, Network Capacity = 599 Mbit/s)