CCITT SGXV
Working Party XV/l
Experts Group for ATM Video Coding

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TITLE : ATM Network Models

PURPOSE: Information

## 1. Introduction

The models listed below have been used by the COST211ter simulation group to assess the impact of VBR coding on an ATM network.

All these models are for a single service, ie. video, on the ATM network.

The models evaluate the congestion probability  $(P_{sat})$ , probability of the rate exceeding the channel capacity and not the cell loss ratio (CLR). The two are related. Although  $P_{sat}$  is typically two orders of magnitude greater than the CLR it does give the same rank ordering.

## 2. The Method of Large Deviations

The source is assumed to be alternately active, transmitting at a peak rate of P, and silent. The relative durations of the on and off periods is chosen such that the mean rate of the model matches the mean rate of the source.

The equation of the model is

$$P_{sat} = exp(-n.k)$$

where  $P_{\text{sat}}$  is the probability of saturation n is the number of simultaneous calls

$$k = (a \ln(a/p)) + ((1-a) \ln((1-a)/(1-p)))$$

where p = MVBR/PVBR

and PVBR is the peak variable bit rate component of the codec MVBR is the mean variable bit rate component of the codec

$$a = AV_CAP / (n.PVBR)$$

where AV\_CAP is the network capacity available for the VBR component

$$AV_CAP = CAP - (n.CBR)$$

where CAP is the capacity of the network CBR is the constant bit rate component of the codec.

Note ln = log to the base 'e'.

## 3. Gaussian (Normal Distribution) Model

A Gaussian model for the superposition of all sources takes into account the variance of the signals. The mean and variance characterizing the Gaussian model are obtained by direct summation of individual source means and variances.

Let  $m_i$  and  $\sigma_i^2$  be the mean bit rate and variance of source i. Then for n sources the mean (M) and variance  $(\sigma^2)$  are given by :-

$$M - \sum_{\sigma^2 = \sum_{i} \sigma_i^2} m_i$$

If the sum of the sources is distributed normally with mean M and variance  $\sigma^2$  (shorthand N(M, $\sigma^2$ )) then the probability that the sum of the sources exceeds the network capacity c is given by

$$Prob(X>=c) = P_{sat}$$

where M is the sum of the sources and  $P_{\mbox{\scriptsize Sat}}$  is the probability of saturation.

This expands to

$$Q((c - n(cbr+mvbr))/(\sigma.(n)^{1/2})) = P_{sat}$$

where

 $\sigma$  is the standard deviation of the source cbr is the mean bit rate of the CBR component mvbr is the mean bit rate of the VBR component c is the network capacity n is the number of simultaneous calls. Q(.) is the Gaussian distribution.

Useful values are in table 1 below.

P <sub>sat</sub>	Solution Q(x)
10-1	1.28155
10-2	2.32635
10-3	3.09023
10-4	3.7192
10 <sup>-5</sup>	4.26489
10-6	4.75342
10-7	5.19934
10-8	5.61200
10 <sup>-9</sup>	5.99781

Table 1 Useful values of  $k_{Psat}$ .

Writing  $k_{PSAT}$  for the solution in x of Q(x), we obtain the expression of n for a given saturation probability by solving a second order equation.

$$n = \begin{bmatrix} -k_{p_{sat}} \cdot \sigma & k_{p_{sat}} \cdot \sigma \\ \frac{2(cbr+mvbr)}{2(cbr+mvbr)} + \sqrt{\frac{k_{p_{sat}} \cdot \sigma}{2(cbr+mvbr)}} + \sqrt{\frac{c}{cbr+mvbr}} \end{bmatrix}^{2}$$

## 4. The Method of Equivalent Bursts

This is a method where the total stream is substituted with a superposition of substreams generated by infinetely many on/off sources. The height of the on-state for these sources is given by

$$h = \sigma^2/m$$

where  $\sigma^2$  and m are the variance and mean of the source.

The probability of saturation is given by the Poisson distribution

$$P_{sat} = P(X >= n) = P(X = n)$$

$$\frac{1 - (a/n)}{1 - (a/n)}$$

where n = (c - k.cbr)/h

(n is the total number of substreams with height h which can be transmitted on the network capacity available for the VBR component)

and 
$$a = km/h$$

(a is the mean number of subtreams going on at a certain time).

The number of simultaneous calls is ,k, the capacity of the network is ,c, and cbr is the constant bit rate component of a two-layer coding scheme.

The above can be expanded out to

$$Lg(P_{sat}) = nLg(a) - aLg(e) - (0.5Ln(2\pi n) + n.Ln(n) - n)Lg(e) - Lg(1-(a/n))$$

Note Lg Log base 10 Ln Log base e