



INTERNATIONAL TELECOMMUNICATION UNION

**CCITT**

THE INTERNATIONAL  
TELEGRAPH AND TELEPHONE  
CONSULTATIVE COMMITTEE

**X.135**

(11/1988)

SERIES X: DATA COMMUNICATION NETWORKS:  
TRANSMISSION, SIGNALLING AND SWITCHING,  
NETWORK ASPECTS, MAINTENANCE AND  
ADMINISTRATIVE ARRANGEMENTS

Data communication networks – Network aspects

---

**SPEED OF SERVICE (DELAY AND THROUGHPUT)  
PERFORMANCE VALUES FOR PUBLIC DATA  
NETWORKS WHEN PROVIDING INTERNATIONAL  
PACKET-SWITCHED SERVICES**

Reedition of CCITT Recommendation X.135 published in  
the Blue Book, Fascicle VIII.3 (1988)

---

## NOTES

- 1 CCITT Recommendation X.135 was published in Fascicle VIII.3 of the *Blue Book*. This file is an extract from the *Blue Book*. While the presentation and layout of the text might be slightly different from the *Blue Book* version, the contents of the file are identical to the *Blue Book* version and copyright conditions remain unchanged (see below).
- 2 In this Recommendation, the expression “Administration” is used for conciseness to indicate both a telecommunication administration and a recognized operating agency.

## Recommendation X.135

### SPEED OF SERVICE (DELAY AND THROUGHPUT) PERFORMANCE VALUES FOR PUBLIC DATA NETWORKS WHEN PROVIDING INTERNATIONAL PACKET-SWITCHED SERVICES

(Malaga-Torremolinos, 1984; amended at Melbourne, 1988)

The CCITT,

*considering*

- (a) that Recommendation X.1 specifies the international user classes of service in public data networks;
- (b) that Recommendation X.2 specifies the international data transmission services and optional user facilities in public data networks;
- (c) that Recommendation X.25 specifies the DTE/DCE interface for packet mode terminals connected to public data networks by dedicated circuit;
- (d) that Recommendation X.75 specifies the packet switched signalling system between public networks providing data transmission services;
- (e) that Recommendation X.323 specifies general arrangements for interworking between packet-switched public data networks;
- (f) that Recommendation X.96 specifies call progress signals in public data networks;
- (g) that Recommendation X.110 specifies the international routing principles and routing plan for public data networks;
- (h) that Recommendation X.213 defines the OSI Network Layer service;
- (i) that Recommendation X.140 defines general quality of service parameters for communication via public data networks;
- (j) that Recommendation X.134 specifies portion boundaries and packet layer reference events for defining packet-switched performance parameters;
- (k) that Recommendation X.136 specifies accuracy and dependability (including blocking) performance values for public data networks when providing international packet-switched service;
- (l) that Recommendation X.137 specifies availability performance values for public data networks when providing international packet-switched service,

*unanimously declares*

- (1) that the speed of service parameters defined in this Recommendation shall be used in the planning and operation of international packet-switched data communication services provided in accordance with Recommendations X.25 and X.75;
- (2) that in such services, the performance values specified in this Recommendation shall be taken as worst-case limits under the conditions specified herein.

## 1 Introduction

1.1 This Recommendation is the second in a series of four CCITT Recommendations (X.134-X.137) that define performance parameters and values for international packet-switched data communication services. Figure 1/X.135 illustrates the scope of these four Recommendations and the relationships among them.

1.2 Recommendation X.134 divides a virtual connection into basic sections whose boundaries are associated with X.25 and X.75 interfaces; defines particular collections of basic sections, called virtual connection portions, for which performance values will be specified; and defines a set of packet layer reference events (PEs) which provide a basis for performance parameter definition. The basic sections consist of network sections and circuit sections. They are delimited, in each case, by physical data terminal equipment (DTE) or data switching equipment (DSE) interfaces.

Virtual connection portions are identified either as national portions or international portions. Each PE is defined to occur when a packet crossing a section boundary changes the state of the packet layer interface.

1.3 For comparability and completeness, packet-switched network performance is considered in the context of the  $3 \times 3$  performance matrix defined in Recommendation X.140. Three protocol-independent data communication functions are identified in the matrix: access, user information transfer, and disengagement. These general functions correspond to call set-up, data (and interrupt) transfer, and call clearing in packet-switched virtual call services conforming to the X.25 and X.75 Recommendations. Each function is considered with respect to three general performance concerns (or "performance criteria"): speed, accuracy, and dependability. These express, respectively, the delay or rate, degree of correctness, and degree of certainty with which the function is performed.

1.4 This Recommendation defines protocol-specific speed of service parameters and values associated with each of the three data communication functions. Recommendation X.136 defines protocol-specific accuracy and dependability parameters and values associated with each function. The Recommendation X.135 and Recommendation X.136 parameters are called "primary parameters" to emphasize their direct derivation from packet layer reference events.

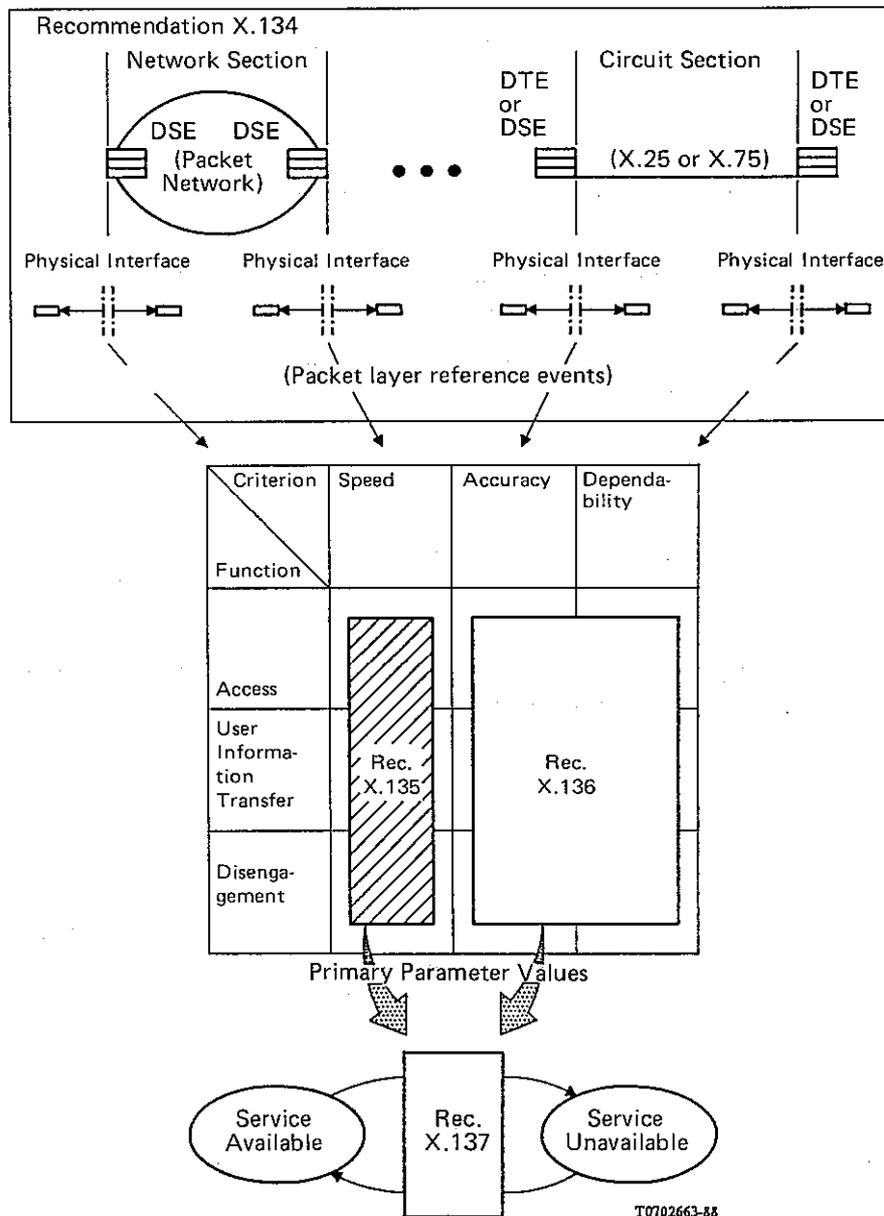


FIGURE 1/X.135

Packet-switched service performance description framework

1.5 An associated two-state model provides a basis for describing overall service availability. A specified availability function compares the values for a subset of the primary parameters with corresponding outage thresholds to classify the service as “available” (no service outage) or “unavailable” (service outage) during scheduled service time. Recommendation X.137 specifies the availability function and defines the availability parameters and values that characterize the resulting binary random process.

1.6 Four speed of service parameters are defined in this Recommendation: one access parameter (call set-up delay), two user information transfer parameters (data packet transfer delay and throughput capacity), and one disengagement parameter (clear indication delay). Each parameter can be applied to any basic section or portion of a virtual connection. This generality makes the parameters useful in performance allocation and concatenation.

1.7 This Recommendation specifies delay and throughput values for national portions and international portions of two types (Table 1/X.135). Performance values for data terminal equipment are not specified, but the parameters defined in this Recommendation may be employed in such specification to assist users in establishing quantitative relationships between network performance and quality of service (see Recommendation X.140).

TABLE 1/X.135

**Virtual connection portion types for which performance values are specified<sup>a)</sup>**

Portion type	Typical characteristics
National A	Terrestrial connection via an access network section
National B	Connection via an access network section with one satellite circuit; or via an access network section and one or more transit network sections
International A	Connection via a direct terrestrial internetwork circuit section
International B	Connection via two satellite circuits and one transit network section; or via one satellite circuit and two or more transit network sections

<sup>a)</sup> The values specified for Type B portions also apply to virtual connection portions not explicitly identified as Type A or Type B.

1.8 Worst-case mean and 95% probability values for call set-up delay, data packet transfer delay, throughput capacity, and clear indication delay are specified for each virtual connection portion type identified in Table 1/X.135. The term “worst case” means that these values should be met during the normal busy hour in the worst-performing virtual connection portion used in providing international packet-switched service. The performance of a virtual connection portion will normally be much better than the worst-case values specified in this Recommendation.<sup>1)</sup> Design objectives that take into account more demanding user applications and network performance and connectivity enhancements are for further study.

Numerical methods for combining individual portion performance values to estimate end-to-end performance are also provided in this Recommendation. DTE to DTE values for two particular hypothetical reference connections are derived using these methods in Annex C.

<sup>1)</sup> Supplement 1 presents delay and throughput values measured on particular connections at particular times and is for illustrative purposes only.

## 2 Call set-up delay

Call set-up delay applies only to the virtual call capability of packet-switched networks.

Call set-up delay observed at a single section boundary,  $B_i$ , is defined first and then call set-up delay between a pair of section boundaries ( $B_i, B_j$ ) is defined based on the former definition. In the former case, the call set-up delay includes the delay for all virtual connection sections on the called user side of  $B_i$  and the called user response time. In the latter case, the call set-up delay includes only the delays between  $B_i$  and  $B_j$ . Values are specified for call set-up delay observed between section boundaries.

### 2.1 Definition of call set-up delay at a single section boundary

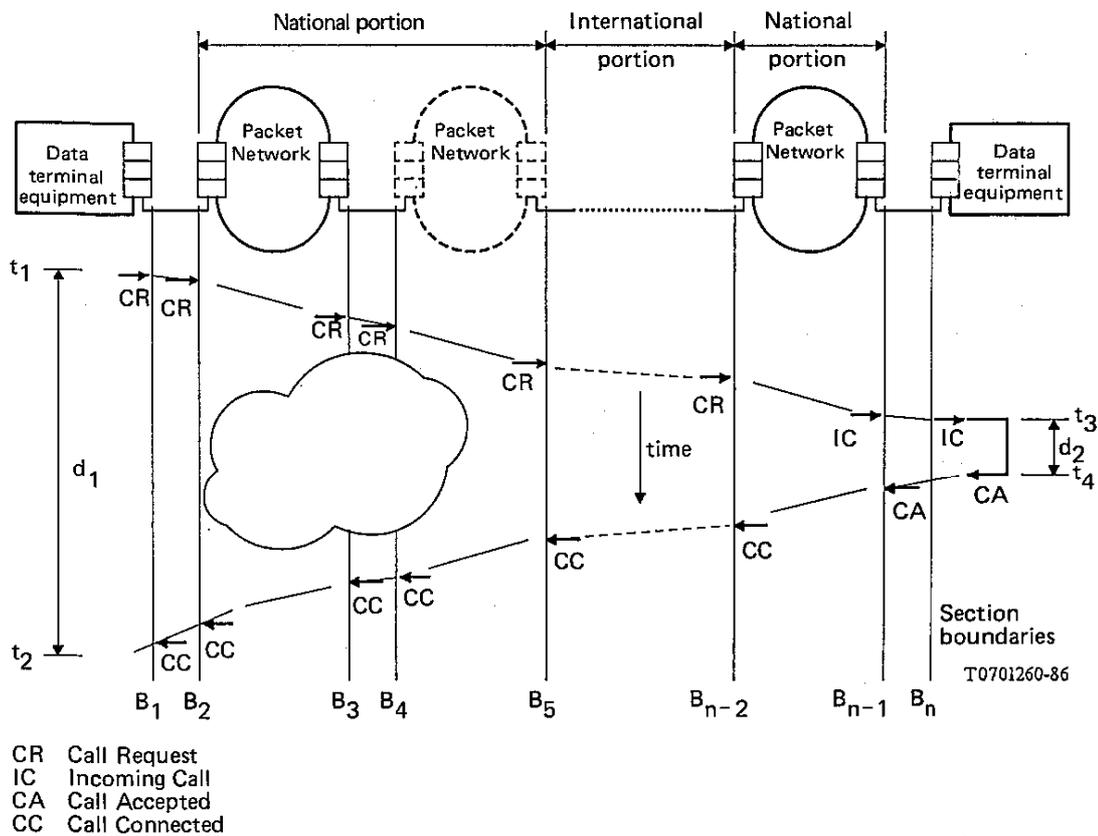
Call set-up delay at a section boundary,  $B_i$ , is defined using two Recommendation X.134 packet layer reference events (PEs). It is the period of time that starts when either a call request or an incoming call packet creates a PE at  $B_i$ , and ends when the corresponding call connected or call accepted packet, accepting the virtual call, returns and creates its PE at  $B_i$ .

Call set-up delay at a section boundary =  $\{t_2 - t_1\}$  where

$t_1$  = Time of occurrence for the first PE.

$t_2$  = Time of occurrence for the second PE.

The two PEs can occur at any single section boundary within a virtual connection. The identities of the packets depend on the boundary of interest, as shown in Figure 2/X.135. The first packet is the call request packet and the second packet is the corresponding call connected packet at every boundary except the two boundaries that delimit the access circuit section associated with the called DTE. The first packet is the incoming call packet and the second packet is the call accepted packet at the latter two boundaries. The specific X.134 PEs used in measuring call set-up delay at each section boundary are identified in Table 2/X.135.



Note - ( $t_1, t_2$ ) and ( $t_3, t_4$ ) may be observed on the calling side and called side of any virtual connection portion.

FIGURE 2/X.135  
Call set-up delay events

TABLE 2/X.135

**Packet layer reference events (PEs) used  
in measuring call set-up delay<sup>a)</sup>**

X.134 packet layer reference event	Starting PE	Ending PE
Circuit section		
Calling DTE access circuit section	2 (X.25)	3 (X.25)
Called DTE access circuit section	1 (X.25)	4 (X.25)
Internetwork circuit section	1 (X.75)	2 (X.75)

<sup>a)</sup> The PE numbers in this table refer to Tables 1/X.134 and 2/X.134 in Recommendation X.134.

## 2.2 Definition of call set-up delay between two section boundaries

For a particular virtual call, call set-up delay can be measured at one boundary,  $B_i$ , and measured at another boundary,  $B_j$ , further from the calling DTE. The difference in the values obtained is the call set-up delay contributed by the virtual connection section(s) between the two boundaries.

Call set-up delay between two section boundaries =  $\{d_1 - d_2\}$  where

$d_1$  = Call set-up delay measured at  $B_i$ .

$d_2$  = Call set-up delay measured at  $B_j$ .

The **end-to-end call set-up delay** is the call set-up delay between DTE boundaries, e.g.,  $B_1$  and  $B_n$  in Figure 2/X.135. This end-to-end delay excludes the called user response time. The **national portion call set-up delay** is the call set-up delay between the boundaries delimiting a national portion, e.g.,  $B_1$  and  $B_5$  in Figure 2/X.135. The **international portion call set-up delay** is the call set-up delay between the boundaries delimiting an international portion, e.g.,  $B_5$  and  $B_{n-2}$  in Figure 2/X.135.

## 2.3 Values

Table 3/X.135 defines worst-case call set-up delay values for each of the four virtual connection portion types identified in Table 1/X.135. DTE to DTE call set-up delay values for two hypothetical reference connections are calculated in Annex C. All values are based on (and only apply under) the following assumptions<sup>2)</sup>:

- 1) Normal busy hour load conditions for the observed virtual connection. The definition of "normal busy hour load" as a traffic description is for further study.
- 2) A basic call, in which none of the optional user facilities defined in Recommendation X.25 are used and no call user data is sent.
- 3) Data link layer windows of entities outside the portion being specified are open (not flow controlled).

<sup>2)</sup> Values for other conditions are for further study. In the case of extremely long access lines and/or excessive delays in the access circuit section transmission equipment, these values may be exceeded.

The defined values consist of mean and 95% probability values. The mean is the expected value of the call set-up delay distribution. The 95% probability value is the value below which 95% of the call set-up delay values lie. Call set-up attempts that are unsuccessful under the conditions of Recommendation X.136 are excluded and are addressed separately in that Recommendation.

TABLE 3/X.135

**Worst-case call set-up delay values for virtual connection portions**

Statistic	Virtual connection portion type			
	National		International	
	A	B	A	B
mean (ms)	1000 + X	1600 + X	250	1600
95% (ms)	1500 + X	2100 + X	250	1800

In Table 3/X.135, the value  $X$  depends on the signalling rate of the access circuit section that is included in the national portion. Table 4/X.135 presents the  $X$  values for user classes of service 8-11 in Recommendation X.1<sup>3)</sup>. The  $X$  values for other signalling rates may be computed using the formula

$$X = 400/R \text{ ms,}$$

where  $R$  is the signalling rate in kilobits per second.<sup>4)</sup>

TABLE 4/X.135

**X-values for Table 3/X.135**

X.1 user class of service	$R$ (kbit/s)	$X$ (Milliseconds)
8	2.4	167
9	4.8	84
10	9.6	42
11	48.0	9

<sup>3)</sup> These  $X$  values are not intended to represent the delay performance of the access circuit section, since these values do not include propagation delays, multiplexing delays, or the effects of retransmission.

<sup>4)</sup> The formula assumes that the transfer of each call set-up packet across an access circuit section involves the transmission of 25 octets: 5 octets of frame level overhead, a 5-octet packet header, and 15 octets of DTE address information

The call set-up delay values defined in Table 3/X.135 are intended to be used as worst-case limits in planning international packet-switched services. The actual delay performance achieved on a virtual connection portion will depend on many factors, including the traffic expected and actually offered, the internal network topology, and the signalling rates on the internetwork circuit sections. Variation away from the worst-case value for each factor can improve the performance.

The overall call set-up delay value for a set of concatenated virtual connection portions can be calculated directly by adding the individual portion means defined in Table 3/X.135. A method of calculating an overall 95% probability call set-up delay value for a set of concatenated virtual connection portions from the individual 95% probability values is described in Annex C.

### 3 Data packet transfer delay

This delay refers to successful transfer of data packets and applies to both the virtual call and the permanent virtual circuit capabilities of packet-switched networks. It is defined only between pairs of section boundaries.

#### 3.1 data packet transfer delay definition

Data packet transfer delay is the period of time that starts when a data packet creates a PE at a particular boundary,  $B_i$ , and ends when this same packet creates a later PE at another boundary,  $B_j$ . The specific X.134 PEs used in measuring data packet transfer delay at each section boundary are identified in Table 5/X.135.

Data packet transfer delay =  $\{t_2 - t_1\}$  where

$t_1$  = Time of occurrence for the first PE.

$t_2$  = Time of occurrence for the second PE.

TABLE 5/X.135

**Packet layer reference events (PEs) used in measuring data packet transfer delay**

X.134 packet layer reference event	Starting/Ending PE
Circuit section	
Source access circuit section	10a (X.25)
Destination access circuit section	9a (X.25)
Internetwork circuit section	5a (X.75)

The **end-to-end data packet transfer delay** is the one-way delay between DTE boundaries, e.g.,  $B_1$  and  $B_n$  in Figure 2/X.135. The **national portion data packet transfer delay** is the delay between the boundaries delimiting a national portion, e.g.,  $B_1$  and  $B_5$  in Figure 2/X.135. The **international portion data packet transfer delay** is the delay between the boundaries delimiting an international portion, e.g.,  $B_5$  and  $B_{n-2}$  in Figure 2/X.135.

3.2 Values

Table 6/X.135 defines worst case data packet transfer delay values for each of the four virtual connection portion types identified in Table 1/X.135. DTE to DTE data packet transfer delay values for two hypothetical reference connections are calculated in Annex C. All values are based on (and only apply under) the following assumptions<sup>5)</sup>:

- 1) Normal busy hour load conditions for the observed virtual connection. The definition of “normal busy hour load” as a traffic description is for further study.
- 2) A user data field length of 128 octets.
- 3) Data link and packet layer windows on the receiving DTE side of the portion being specified are open.

The defined values consist of mean and 95% probability values. The mean is the expected value of the data packet transfer delay distribution, excluding values that exceed a specified maximum data packet transfer delay. The 95% probability value is the value below which 95% of the data packet transfer delay values lie. Data packet transfer attempts that are unsuccessful under the conditions of Recommendation X.136 are excluded and are addressed separately in that Recommendation.

TABLE 6/X.135

**Worst-case data packet transfer delay values for virtual connection portions**

Statistic	Virtual connection portion type			
	National		International	
	A	B	A	B
mean (ms)	350 + Y	650 + Y	215	950
95% (ms)	525 + Y	825 + Y	215	1125

In Table 6/X.135, the value *Y* depends on the signalling rate of the access circuit section that is included in the national portion. Table 7/X.135 presents the *Y* values for user classes of service 8-11 in Recommendation X.1<sup>6)</sup>. The *Y* values for other signalling rates may be computed using the formula

$$Y = 1088/R \text{ ms,}$$

where *R* is the signalling rate in kilobits per second<sup>7)</sup>.

<sup>5)</sup> Values for other conditions are for further study. In the case of extremely long access lines and/or excessive delays in the access circuit section transmission equipment, these values may be exceeded.

<sup>6)</sup> These *Y* values are not intended to represent the delay performance of the access circuit section, since these values do not include propagation delays, multiplexing delays, or the effects of retransmission.

<sup>7)</sup> The formula assumes that the transfer of a data packet across an access circuit section involves the transmission of 136 octets: 5 octets of frame level overhead, a 3-octet packet header, and 128 octets of user data.

TABLE 7/X.135

Y-values for Table 6/X.135

X.1 user class of service	R (kbit/s)	Y (Milliseconds)
8	2.4	453
9	4.8	227
10	9.6	113
11	48.0	23

The data packet transfer delay values defined in Table 6/X.135 are intended to be used as worst-case limits in planning international packet-switched services. The actual delay performance achieved on a virtual connection portion will depend on many factors, including the traffic expected and actually offered, the internal network topology, and the signalling rates on the internetwork circuit sections. Variation away from the worst-case value for each factor can improve the performance.

The overall mean data packet transfer delay value for a set of concatenated virtual connection portions can be calculated directly by adding the individual portion means defined in Table 6/X.135. A method of calculating an overall 95% probability data packet transfer delay value for a set of concatenated virtual connection portions from the individual 95% probability values is described in Annex C.

#### 4 Throughput parameters

This section defines three throughput parameters: throughput, steady-state throughput, and throughput capacity. Values are specified for throughput capacity.

##### 4.1 throughput definition

Throughput for a virtual connection section is the number of user data bits successfully transferred in one direction across that section per unit time<sup>8)</sup>. Successful transfer means that no user data bits are lost, added, or inverted in transfer.

Assume:

- 1) That data packet  $A_0$  is the final packet of a complete packet sequence crossing input boundary  $B_i$ .
- 2) That subsequently,  $k$  sequential data packets ( $A_1, A_2, \dots, A_k$ ) forming the next complete packet sequence cross the input boundary  $B_i$  immediately following  $A_0$ .
- 3) That data packet  $\hat{A}_0$  is the final packet of the first complete packet sequence when it crosses output boundary  $B_j$ .
- 4) That packets  $\hat{A}_1, \hat{A}_2, \dots, \hat{A}_m$  comprise the second complete packet sequence when it crosses output boundary  $B_j$ .

<sup>8)</sup> User data bits are the bits of the user data field in data packets of the X.25 or X.75 packet level (protocols and data above the packet level). Framing, routing, bit stuffing, error control, and other protocol fields introduced by all protocols at or below the packet level are excluded.

The X.134 PEs used in measuring throughput are the same as those used in measuring data packet transfer delay, as identified in Table 5/X.135.

Let:

$t_1$  = Time of occurrence for the PE created by  $A_0$  at  $B_i$ .

$t_2$  = Time of occurrence for the PE created by  $A_k$  at  $B_i$ .

$t_3$  = Time of occurrence for the PE created by  $\hat{A}_0$  at  $B_j$ .

$t_4$  = Time of occurrence for the PE created by  $\hat{A}_m$  at  $B_j$ .

$f(A_r)$  = Number of user data bits in packet  $A_r$ .

Then a throughput measurement of size  $k$  is defined as follows:

$$\text{Throughput measurement} = \frac{\sum_{r=1}^k f(A_r)}{\text{MAX} [(t_2 - t_1), (t_4 - t_3)]}$$

Recommendation X.136 defines conditions under which a transfer of consecutive data packets is considered to be unsuccessful. Only successful throughput measurements should be included in the assessment of throughput performance.

#### 4.2 steady-state throughput definition

The steady-state throughput for a virtual connection is the value to which a throughput measurement converges as the duration of the observation period increases with statistically constant load on the virtual connection. Assuming successful transfer, steady-state throughput is the same when measured at every pair of section boundaries of the virtual connection. Thus, assuming no user data bits are lost, added, or inverted in transfer, a steady-state throughput measurement can be made at any single section boundary within a virtual connection:

$$\text{Steady-state throughput measurement} = \frac{\sum_{r=1}^k f(A_r)}{(t_2 - t_1)}$$

where  $t_1$ ,  $t_2$  and  $f(A_r)$  are defined above<sup>9)</sup>.

Alternatively, the above equation can be used to calculate steady-state throughput with different definitions for  $t_1$  and  $t_2$ . Times  $t_1$  and  $t_2$  can be chosen in advance of the measurement. In this case, let  $(A_1, A_2, \dots, A_k)$  be the set of all virtual connection data packets crossing boundary  $B$  (creating PEs in one direction) at or following time  $t_1$  but before time  $t_2$ . Then the above equation still measures steady-state throughput.

#### 4.3 throughput capacity definition

Let  $B_i$  and  $B_j$  be two virtual connection section boundaries. Assume steady-state throughput is to be estimated with data packets flowing from  $B_i$  to  $B_j$ . Assume there is a statistically constant load,  $L$ , on the virtual connection section between  $B_i$  and  $B_j$ . Then the throughput capacity of that section under load  $L$  is defined as the steady-state throughput maximized over all offered combinations of virtual connection parameter settings and choices for the performance and loading outside  $B_i$  and  $B_j$ . Measurement of throughput capacity for a section between boundaries  $B_i$  and  $B_j$  is accomplished in the same way as measurement of steady-state throughput. However, measurement of throughput capacity requires that the components outside of  $B_i$  and  $B_j$  have significantly higher throughput capacity under their respective loads than the throughput capacity being measured.

For the given statistically constant load  $L$  between  $B_i$  and  $B_j$ , and for a given set of testing arrangements, any measured steady-state throughput is a lower bound for the throughput capacity. To improve the estimate, the experiment may be repeated with different testing arrangements outside of  $B_i$  and  $B_j$  (see Annex B).

---

<sup>9)</sup> Ancillary information on steady-state throughput measurement is provided in Annex B.

The end-to-end throughput capacity is the throughput capacity between DTE boundaries, e.g.,  $B_1$  and  $B_n$  in Figure 2/X.135. The national portion throughput capacity is the throughput capacity between the boundaries delimiting a national portion, e.g.,  $B_1$  and  $B_5$  in Figure 2/X.135. The international portion throughput capacity is the throughput capacity between the boundaries delimiting an international portion, e.g.,  $B_5$  and  $B_{n-2}$  in Figure 2/X.135.

#### 4.4 Values

Table 8/X.135 defines worst-case throughput capacity values for each of the four virtual connection portion types identified in Table 1/X.135. DTE to DTE throughput capacity values for two hypothetical reference connections are calculated in Annex C. All values are based on (and only apply under) the following assumptions<sup>10)</sup>:

- 1) Normal busy hour load conditions for the observed virtual connection. The definition of “normal busy hour load” as a traffic description is for further study. No other traffic on the access circuit sections.
- 2) 9600 bit/s signalling rates on the access circuit sections. Applicability of the specified throughput capacity values to lower access circuit section signalling rates is for further study.
- 3) A user data field length of 128 octets. Requested throughput class corresponding to 9600 bit/s. (Note that the throughput class finally applying to the call may be lower than the requested throughput class.)
- 4) Packet layer window sizes of 2 and data link layer window sizes of 7 on the access circuit sections.
- 5)  $D$  bit not used ( $D = 0$ ).
- 6) Values apply to either direction of transfer.
- 7) No unavailability (as defined in Recommendation X.137) during the observation period.
- 8) No resets or premature disconnects (as defined in Recommendation X.136) during the observation period.
- 9) Throughput capacity sample sizes of 200 packets (in the case of the first measurement technique specified in § 4.2) or 2 minutes (in the case of the alternative measurement technique specified in § 4.2).

TABLE 8/X.135

**Worst-case throughput capacity values for virtual connection portions**

Statistic	Virtual connection portion type			
	National		International	
	A	B	A	B
mean (bit/s)	3000	2400	2000	1800
95% (bit/s)	2400	2000	1800	1500

The defined values consist of mean and 95% probability values. The mean is the expected value of the throughput capacity distribution. The 95% probability value is the value above which 95% of the throughput capacity measurements lie.

---

<sup>10)</sup> Values for other conditions are for further study.

The throughput capacity values defined in Table 8/X.135 are intended to be used as worse-case limits in planning international packet-switched services. The actual throughput capacity achieved in a virtual connection portion will depend on many factors, including the traffic expected and actually offered, the internal network topology, and the signalling rates on the internetwork circuit sections. Variation away from the worse-case value for each factor can improve the performance. The throughput capacity values defined here will not necessarily be achieved concurrently with the delay values defined in Table 6/X.135.

An upper bound for the throughput capacity of a set of concatenated virtual connection portions can be derived from the individual portion throughput capacities as follows. If a portion between boundaries  $B_i$  and  $B_j$  has throughput capacity  $T_1$  under load  $L_1$ , and a portion between boundaries  $B_k$  and  $B_m$  has throughput capacity  $T_2$  under load  $L_2$ , and those portions are concatenated so that  $B_j = B_k$  with  $L_1$  and  $L_2$  unchanged, then the resulting portion has throughput capacity.

$$T \leq \text{MIN} [T_1, T_2]$$

Further information on estimating the throughput capacity of a set of concatenated virtual connection portions is provided in Annex C.

## 5 Clear indication delay

Clear indication delay applies only to the virtual call capability of packet-switched networks. It is defined only between a pair of section boundaries.

### 5.1 clear indication delay definition

Clear indication delay is the period of time that starts when either a clear request packet or a clear indication packet creates a PE at a boundary,  $B_i$ , and ends when the corresponding clear request or clear indication packet creates a later PE at another boundary,  $B_j$ . The specific X.134 PEs used in measuring clear indication delay at each section boundary are identified in Table 9/X.135.

Clear indication delay =  $\{t_2 - t_1\}$  where

$t_1$  = Time of occurrence for the first PE.

$t_2$  = Time of occurrence for the second PE.

TABLE 9/X.135

Packet layer reference events (PEs) used  
in measuring clear indication delay

Circuit section	X.134 packet layer reference event	Starting/Ending PE
Clearing DTE access circuit section		6 (X.25)
Cleared DTE access circuit section		5 (X.25)
Internetwork circuit section		3 (X.75)

The **end-to-end clear indication delay** is the one-way delay between DTE boundaries, e.g.,  $B_1$  and  $B_n$  in Figure 2/X.135. The **national portion clear indication delay** is the delay between the boundaries delimiting a national portion, e.g.,  $B_1$  and  $B_5$  in Figure 2/X.135. The **international portion clear indication delay** is the delay between the boundaries delimiting an international portion, e.g.,  $B_5$  and  $B_{n-2}$  in Figure 2/X.135.

## 5.2 Values

Table 10/X.135 defines worst case clear indication delay values for each of the four virtual connection portion types identified in Table 1/X.135. DTE to DTE clear indication delay values for two hypothetical reference connections are calculated in Annex C. All values are based on (and only apply under) the following assumptions<sup>11)</sup>:

- 1) Normal busy hour load conditions for the observed virtual connection. The definition of “normal busy hour load” as a traffic description is for further study.
- 2) Data link layer windows on the cleared DTE side of the portion being specified are open.
- 3) The extended format of the clear request packet is not used.

TABLE 10/X.135

**Worst-case clear indication delay values for virtual connection portions**

Statistic	Virtual connection portion type			
	National		International	
	A	B	A	B
mean (ms)	$500 + Z$	$800 + Z$	110	800
95% (ms)	$750 + Z$	$1050 + Z$	110	900

The defined values consist of mean and 95% probability values. The mean is the expected value of the clear indication delay distribution, excluding values that exceed a specified maximum clear indication delay. The 95% probability value is the value below which 95% of the clear indication delay values lie. Unsuccessful call clear attempts are excluded and are addressed separately in Recommendation X.136.

In Table 10/X.135, the value  $Z$  depends on the signalling rate of the access circuit section that is included in the national portion. Table 11/X.135 presents the  $Z$  values for user classes of service 8-11 in Recommendation X.1<sup>12)</sup>.

The  $Z$  values for other signalling rates may be computed using the formula

$$Z = 80/R \text{ ms}$$

where  $R$  is the signalling rate in kilobits per second<sup>13)</sup>.

<sup>11)</sup> Values for other conditions are for further study. In the case of extremely long access lines and/or excessive delays in the access circuit section transmission equipment, these values may be exceeded.

<sup>12)</sup> These  $Z$  values are not intended to represent the delay performance of the access circuit section, since these values do not include propagation delays, multiplexing delays, or the effects of retransmission.

<sup>13)</sup> The formula assumes that the transfer of each call clearing packet across an access circuit section involves the transmission of 10 octets: 5 octets of frame level overhead and 5 octets of packet header information.

TABLE 11/X.135

**Z-values for Table 10/X.135**

X.1 user class of service	R (kbit/s)	Z (Milliseconds)
8	2.4	34
9	4.8	17
10	9.6	9
11	48.0	2

The clear indication delay values defined in Table 10/X.135 are intended to be used as worst-case values in planning international packet-switched services. The actual delay performance achieved on a virtual connection portion will depend on many factors, including the traffic expected and actually offered, the internal network topology, and the signalling rates on the internetwork circuit sections. Variation away from the worst-case value for each factor can improve the performance.

The overall mean clear indication delay value for a set of concatenated virtual connection portions can be calculated directly by adding the individual portion means defined in Table 10/X.135. A method of calculating an overall 95% probability clear indication delay value for a set of concatenated virtual connection portions from the individual 95% probability values is described in Annex C.

## ANNEX A

(to Recommendation X.135)

### **Factors to be specified in reporting throughput performance**

Many factors affect the throughput capacity that can be obtained on a virtual connection section.

#### A.1 *Signalling rates*

The choice of signalling rates on circuit sections bounds throughput. In general, faster signalling rates improve throughput.

#### A.2 *Interface windows*

The choice of window size has an effect on throughput. In general, larger window sizes improve throughput. For maximum throughput, each user-controllable window size should be optimized with respect to delays and retransmission rates.

#### A.3 *Packet length*

The choice of packet length has an effect on throughput. In general, the use of larger packets improves throughput. For maximum throughput, packet sizes should be optimized with respect to the known error properties of the access links.

#### A.4 *Additional virtual connections*

Throughput of a tested virtual connection is dependent on the number of additional virtual connections and the loading in each direction on each connection. Throughput per virtual connection decreases as the number of additional virtual connections or the loading on the individual connections increases. When stating the throughput capacity of a virtual connection portion, the number of additional active virtual connections on the access circuit sections should be specified. Also, the total throughput in each direction on those virtual connections should be reported. For example:

“The throughput capacity of a virtual connection on this international portion is at least 1.2 kbit/s. There can be at most 4 additional virtual connections transmitting in the same direction between the same two portion boundaries at the same throughput.”

#### A.5 *Time-of-day*

When measuring throughput it is assumed that the loads on many connection components cannot be user controlled or observed. However, it is assumed that those loads are correlated with time-of-day, day-of-week, and holidays. Thus users can improve their throughput by transmitting at particular times.

#### A.6 *Direction*

If the direction of the measurement affects the throughput capacity, the direction should be specified when stating throughput capacity. Otherwise the capacities in the two directions will be assumed to be equal.

#### A.7 *Throughput class*

Network internal windows and acknowledgement schemes may or may not be a function of a virtual connection's requested or default throughput class. For maximum throughput and when measuring throughput capacity, the throughput class for the virtual connection should be set to the maximum allowed by the section being measured. Because the optimum throughput class is always the maximum allowable, a statement of throughput capacity need not explicitly specify the throughput class.

#### A.8 *D bit usage*

If the *D* bit is set to 1 during a throughput measurement, that fact should be reported. Otherwise, the *D* bit setting need not be reported.

#### A.9 *Delay*

Throughput and data packet transfer delay are related. If the throughput is specified under a delay constraint, then the delay should be reported.

#### A.10 *Reporting throughput capacity*

Throughput capacity reports should specify the values of the controllable factors that were in effect during the throughput capacity measurement. All factors listed in this Annex should be reported unless otherwise specified. A typical report might specify conditions as follows:

“For this connection the network throughput capacity is at least 4.1 kbit/s. The capacity was measured using two 9.6 kbit/s access circuit sections, data link layer window sizes of 7, packet layer window sizes of 2, and 128 octet user data fields. No additional virtual connections were present on either of the access circuit sections. The capacity was measured during the busiest hour of the weekday. The average data packet transfer delay during the measurement period was 500 milliseconds. The precision of the throughput measurement is plus or minus 0.1 kbit/s.”

With such statements, the throughput capacity is more easily verified and more easily matched to the throughput needs of potential users.

## ANNEX B

(to Recommendation X.135)

### **Ancillary information on throughput measurement and the application of throughput capacity values**

The following points should be noted with regard to throughput measurement:

- A measurement of steady-state throughput requires a measurement size of  $k = 200$  packets. An alternative is to specify a value for the measurement time period ( $t_2 - t_1$ ) of 2 minutes.
- When measuring steady-state throughput, data packets  $A_1$  through  $A_k$  need not constitute a single complete packet sequence.
- One way of verifying successful transfer of the test sequence in a steady-state throughput measurement is to transfer another complete packet sequence.
- Throughput-related measurements should not be conducted with user data sequences with high density of binary “ones” to avoid biasing the results by the effects of bit stuffing.

The following describes one way of applying the throughput capacity parameter. The discussion uses throughput capacity to design an international circuit section.

Assuming:

$m$  = the mean throughput per call (for the duration of the call)

$n$  = the total number of calls present at any time

$p$  = the number of those calls requiring the throughput capacity at any instant in time

$b$  = the bit rate of the international internetwork circuit section and

$T$  = the throughput capacity objective per call

Then the bit rate  $b$  should be:

$$b \geq (m * n) + p(T - m)$$

The actual  $m$ ,  $n$ , and  $p$  values may be network dependent and reflect basically the population of the access line speeds and their traffic characteristics. It is therefore recommended that the value of  $b$  is chosen considerably higher than the value of  $(m * n)$ . The number of logical channels assigned to international internetwork links should depend on the relationship of the values  $b$  and  $m$ .

## ANNEX C

(to Recommendation X.135)

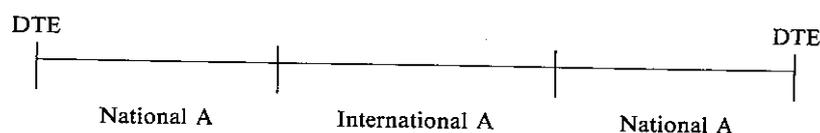
### Representative end-to-end speed of service performance

This Annex provides two examples to illustrate how end-to-end (DTE to DTE) speed of service performance can be estimated from the individual virtual connection portion performance values specified in X.135. Two example concatenations of Type A and Type B virtual connection portions are defined. The end-to-end call set-up delay, data packet transfer delay, throughput capacity, and clear indication delay are calculated for each example. Although alternative network models and statistical assumptions are possible, the methods presented in this Annex provide one practical way of estimating end-to-end performance from the performance of individual network portions.

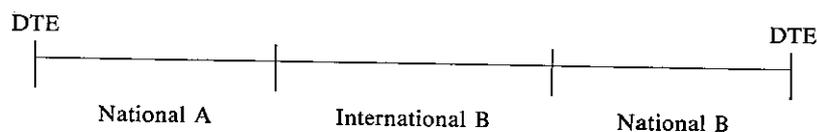
#### C.1 Definition of the example end-to-end connections

For ease of reference the two example end-to-end (i.e., DTE to DTE) connections presented in this Annex will be referred to as "Type 1" and "Type 2" configurations. These hypothetical, but representative, configurations use the portion boundaries and packet layer reference events described in X.134. Figure 2/X.135 shows the relevant network boundaries and Table 1/X.135 defines the virtual connection portion types.

The Type 1 configuration is defined to be:



The Type 2 configuration is defined to be:



C.2 *End-to-end speed of service performance for the Type 1 and Type 2 configuration examples*

End-to-end speed of service performance values have been calculated for the example Type 1 and Type 2 connection configurations and are reported below in Tables C-1/X.135 and C-2/X.135. These calculations have been made by applying the methods derived in § C.3 (below) to the individual network portions that, for convenience in defining these examples, are characterized by the worst-case speed of service performance values specified in X.135.

The end-to-end performance for the mean call set-up delay, data packet transfer delay, and clear indication delay are computed by simply summing the mean delays associated with the appropriate individual network portions.

*Example* – For the Type 1 configuration the end-to-end mean call set-up delay in milliseconds is computed by referring to Table 3/X.135 and adding the mean values for the National A and International A portion types:

$$(1000 + X) + (250) + (1000 + X) = 2250 + 2 * X$$

The end-to-end performance for the 95th percentile call set-up delay, data packet transfer delay, and clear indication delay can be determined by assuming (see § C.3) that the variance of the end-to-end delay is the sum of the variances of the individual network portion delays.

*Example* – For the Type 1 configuration, referring to Table 3/X.135 and § C.3, the 95th percentile value for the end-to-end call set-up delay in milliseconds is:

$$\begin{aligned} & (2250 + 2 * X) + [((1500 + X) - (1000 + X))^2 + ((250) - (250))^2 + ((1500 + X) - (1000 + X))^2]^{0.5} \\ & = 2957 + 2 * X \end{aligned}$$

The end-to-end performance for the mean and 95th percentile for throughput capacity are determined by assuming that:

- 1) the end-to-end throughput at any particular time is the minimum taken over all the individual network portions; and
- 2) the throughput of an individual network portion is an independent and normally distributed random variable. § C.3 derives formulas that combine the overlapping individual probability distributions to give the end-to-end throughput capacity distribution.

*Example* – Numerical computations of the end-to-end mean and 95th percentile throughput capacities for the Type 1 and Type 2 configurations are provided as examples in § C.3.2.

TABLE C-1/X.135

**End-to-end speed of service performance  
for the type 1 configuration example**

Statistic	Type 1 configuration	
	Mean	95%ile
Call set-up delay (ms)	$2250 + 2 * X$	$2957 + 2 * X$
Data packet transfer delay (ms)	$915 + 2 * Y$	$1162 + 2 * Y$
Throughput capacity (bit/s)	1999	1800
Clear indication delay (ms)	$1110 + 2 * Z$	$1464 + 2 * Z$

TABLE C-2/X.135

**End-to-end speed of service performance  
for the Type 2 configuration example**

Statistic	Type 2 configuration	
	Mean	95%ile
Call set-up delay (ms)	$4200 + 2 * X$	$4935 + 2 * X$
Data packet transfer delay (ms)	$1950 + 2 * Y$	$2284 + 2 * Y$
Throughput capacity (bit/s)	1797	1500
Clear indication delay (ms)	$2100 + 2 * Z$	$2467 + 2 * Z$

The parameters  $X$ ,  $Y$  and  $Z$  depend on the signalling rate of the access circuit section that is included in the national portion. Definitions, relevant assumptions, and values for  $X$ ,  $Y$ , and  $Z$  can be found in the appropriate sections of X.135. As noted in § 4.4 of X.135, a 9.6 kbit/s signalling rate for the access circuit sections is assumed for the worst-case throughput capacity performance values.

**C.3 Methods for calculating mean and 95% points of delays and throughputs of packet-switched services with two or more concatenated portions**

This section describes the methods used in calculating end-to-end speed of service performance from individual network portion performance values.

**C.3.1 Delays**

It is assumed that a packet-switched service has  $n$  portions with delays  $d_1, d_2, \dots, d_n$  varying randomly with means  $m_1, m_2, \dots, m_n$  and 95% points  $z_1, z_2, \dots, z_n$ . Then the total delay  $D = d_1 + d_2 + \dots + d_n$  has a distribution with mean

$$M = m_1 + m_2 + \dots + m_n$$

(with no further assumption). In order to obtain the 95% point of  $D$  it is assumed that the delays  $d_i$  are statistically independent and that  $z_i = m_i + k\sigma_i$  with the same  $k$  for all portions, where  $\sigma_i$  is the standard deviation of  $d_i$ . The like equality is also assumed for  $D$ , i.e.,  $Z = M + k\sigma_D$ , where  $Z$  is the 95% point of  $D$ . These equalities are true for normal distributions with  $k = 1.645$ . Then the variance of  $D$  is the sum of the variances of the  $d_i$ . It follows that the 95% point of  $D$  is given by

$$Z = M + [(z_1 - m_1)^2 + (z_2 - m_2)^2 + \dots + (z_n - m_n)^2]^{1/2}$$

The assumption of normality seems reasonable, but other assumptions are possible and could give substantially different answers.

**C.3.2 Throughputs**

It is assumed that a packet-switched service has  $n$  portions with throughputs  $T_1, T_2, \dots, T_n$  varying randomly and independently with means  $M_1, M_2, \dots, M_n$  and 5% points (points exceeded by 95% of the values)  $Z_1, Z_2, \dots, Z_n$ . The net throughput of the service is assumed to be  $V = \min(T_1, T_2, \dots, T_n)$ . The cumulative distribution function (cdf) of  $T_i$  is the probability that  $T_i$  is less than or equal to any value, say  $t$ , and is denoted by  $F_i(t)$ :

$$F_i(t) = \text{Prob } [T_i \leq t], i = 1, 2, \dots, n$$

The probability density function (pdf) of  $T_i$  is the derivative of  $F_i(t)$  and is denoted by  $f_i(t) = dF_i/dt$ .

In order to calculate the mean, say  $M_{Vn}$ , and the 5% point,  $V_{.05, n}$ , of the net throughput  $V$ , it is in general not sufficient to consider just the portion  $M_i$ 's and  $Z_i$ 's; it is necessary to combine the entire distributions  $F_i(t)$  (or  $f_i(t)$ ) to obtain the pdf of  $V$ , to be denoted by  $g_n(v)$ . However, in the important special case that the portion with the usually smallest throughput (the "slowest portion") has a distribution that is not overlapped at all by the distributions of the larger throughputs, then the net throughput distribution is identical with that of the slowest portion, having the same mean and 5% point in particular. If the overlap of any other distribution with the slowest portion's distribution is negligible, then the same conclusion can be drawn. Later examples will suggest how much overlap can be considered negligible.

The case of general distributions is now resumed, that with  $n = 2$  at first. Integration in the two dimensions of  $(T_1, T_2)$  shows that the pdf of  $V$  is given by

$$g_2(v) = f_1(v) [1 - F_2(v)] + f_2(v) [1 - F_1(v)] \quad (C-1)$$

The mean net throughput of the service is then

$$M_{V2} = \int_0^{\infty} v g_2(v) dv \quad (C-2)$$

and the 5% point is the value  $V_{.05, 2}$  such that

$$\int_0^{V_{.05, 2}} g_2(v) dv = 0.05 \quad (C-3)$$

If  $f_1(t) = f_2(t)$ , then

$$g_2(v) = 2f_1(v) [1 - F_1(v)] \quad (C-4)$$

It is now assumed that the portion throughput distributions are normal and that they are sufficiently concentrated that the tail of the fitted normal distribution to the left to zero is negligible (as is true for all the numerical values in X.135). The assumption is expressed in terms of the standard normal pdf  $\phi(u)$  and cdf  $\Phi(x)$ :

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}, \quad \Phi(x) = \int_{-\infty}^x \phi(u) du \quad (C-5)$$

Then

$$f_i(t) = \frac{1}{\sigma_i} \phi\left(\frac{t - M_i}{\sigma_i}\right), \quad F_i(t) = \int_{-\infty}^t f_i(y) dy \quad (C-6)$$

where the standard deviation  $\sigma_i = (M_i - Z_i)/1.64485$ . In the case  $f_1(t) = f_2(t)$ , then

$$g_2(v) = \frac{2}{\sigma_1} \phi\left(\frac{v - M_1}{\sigma_1}\right) \left[1 - \Phi\left(\frac{v - M_1}{\sigma_1}\right)\right] \quad (C-7)$$

The case  $n = 3$  is now considered. The pdf  $g_3(v)$  of  $V_3 = \min(T_1, T_2, T_3)$  can be obtained by iteration on the distribution of  $V_2 = \min(T_1, T_2)$  since  $V_3 = \min(V_2, T_3)$ . Hence

$$g_3(v) = g_2(v) [1 - F_3(v)] + f_3(v) [1 - G_2(v)] \quad (C-8)$$

where  $g_2(v)$  is given by (C-1) and  $G_2(v)$  is its indefinite integral,

$$G_2(v) = \int_0^v g_2(x) dx \quad (C-9)$$

If all three pdf's  $f_i(t)$  are identical, the  $g_3(v)$  simplifies to

$$g_3(v) = 3f_1(v) [1 - F_1(v)]^2 \quad (\text{C-10})$$

Normal as well as identical distributions are now assumed. Then, from (C-5), (C-6), and (C-10),

$$\begin{aligned} M_{V_3} &= \int_0^{\infty} v g_3(v) dv \\ &= M_1 + 3\sigma_1 \int_{-\infty}^{\infty} u \varphi(u) [1 - \Phi(u)]^2 du \\ &= M_1 - 3\sigma_1 \int_0^{\infty} u \varphi(u) [2\Phi(u) - 1] du \\ &= M_1 - \sigma_1 K_3, \end{aligned} \quad (\text{C-11})$$

where  $K_3 = 0.8463$  by Teichroew (1956). Likewise

$$V_{.05, 3} = M_1 + \sigma_1 U_{.05, 3} \quad (\text{C-12})$$

where

$$3 \int_{-\infty}^{U_{.05, 3}} \varphi(u) [1 - \Phi(u)]^2 du = 0.05 \quad (\text{C-13})$$

By integration

$$\Phi(-U_{.05, 3}) = 1 - 0.095^{1/3} = 0.016952 \quad (\text{C-14})$$

Hence from any cumulative normal distribution table,  $U_{.05, 3} = 2.121$ .

*Example 1* – Calculate the mean and 95th percentile net throughputs assuming there are three identical and normal portion distributions with  $M_1 = M_2 = M_3 = 2000$  bit/s and  $Z_1 = Z_2 = Z_3 = 1800$  bit/s. Then  $\sigma_1 = \sigma_2 = \sigma_3 = 2000/1.645 = 121.6$  bit/s. From (C-11):

$$M_{V_3} = 2000 - 121.6 \times 0.8463 = 1897 \text{ bit/s}$$

From (C-12) and (C-14)

$$V_{.05, 3} = 2000 - 121.6 \times 2.121 = 1742 \text{ bit/s}$$

*Example 2* – Consider the Type 1 configuration. From Table 8/X.135,  $M_1 = M_2 = 3000$  bit/s,  $M_3 = 2000$  bit/s,  $Z_1 = Z_2 = 2400$  bit/s,  $Z_3 = 1800$  bit/s. With normal distributions there is slight but probably negligible overlap of the larger throughputs with the smallest throughput; the probability of either national throughput being less than or equal to the upper 5% point of the international throughput, 2200 bit/s, is 0.014. Hence, at least approximately,  $M_{V_3} = M_3 = 2000$  bit/s,  $V_{.05, 3} = Z_3 = 1800$  bit/s.

This can be checked by numerical integration. Since this will come up in other applications, general formulas are given here. When  $f_1(v) = f_2(v)$ ,  $G_2(v)$  in (C-9) becomes

$$G_2(v) = 2F_1(v) - [F_1(v)]^2$$

When the distributions are also normal, it follows from (C-8) and (C-5) that

$$g_3(v) = \left[ 1 - \Phi \left( \frac{v - m_1}{\sigma_1} \right) \right] \left\{ \frac{2}{\sigma_1} \varphi \left( \frac{v - m_1}{\sigma_1} \right) \left[ 1 - \Phi \left( \frac{v - m_3}{\sigma_3} \right) \right] + \frac{1}{\sigma_3} \varphi \left( \frac{v - m_3}{\sigma_3} \right) \left[ 1 - \Phi \left( \frac{v - m_1}{\sigma_1} \right) \right] \right\} \quad (C-15)$$

Hence the mean throughput for a three-portion network with two portions identical is, with the change of variable  $u = (v - m_1)/\sigma_1$ ,

$$M_{V_3} = \int_{-\infty}^{\infty} (m_1 + \sigma_1 u) [1 - \Phi(u)] \left\{ Z\varphi(u) \left[ 1 - \Phi \left( \frac{m_1 - m_3 + \sigma_1 u}{\sigma_3} \right) \right] + \frac{\sigma_1}{\sigma_3} \varphi \left( \frac{m_1 - m_3 + \sigma_1 u}{\sigma_3} \right) [1 - \Phi(u)] \right\} du \quad (C-16)$$

This can be integrated numerically using a pocket calculator and the National Bureau of Standards *Tables of Normal Probability Functions*. Since these tables give the integral of  $\varphi(u)$  from  $-x$  to  $x$ , say  $S(x)$ , rather than  $\Phi(x)$ , the following substitution is made in (C-16) (in three places):

$$1 - \Phi(u) = \begin{cases} [1 - S(u)]/2 & \text{if } u \geq 0 \\ [1 + S(|u|)]/2 & \text{if } u < 0 \end{cases} \quad (C-17)$$

In the above Example 2, (C-16) becomes

$$\frac{2M_{V_3}}{\sigma_1} = \int_{-\infty}^{\infty} (8.225 + u) [1 \pm S(|u|)] \{ \varphi(u) [1 \pm S(|8.225 + 3u|)] + 1.5 \varphi(8.225 + 3u) [1 \pm S(|u|)] \} du$$

Numerical integration with  $\Delta u = 0.1$  and the Trapezoidal Rule yields  $M_{V_3} = 1999.09$  bit/s. With Simpson's Rule  $M_{V_3} = 1999.11$  bit/s. Hence the slight overlap of the distributions of the two larger throughputs with the smaller throughput distribution reduces the mean net throughput by less than 1 bit/s. The effect on the lower 5% point will be much less, so  $V_{0.5, 3} = 1800$  bit/s. However, comparison with Example 1 shows that *complete* overlap of three portion distributions does reduce the throughput substantially below that of an individual portion.

*Example 3* – Consider the Type 2 configuration. From Table 8/X.135,  $M_1 = 3000$ ,  $M_2 = 2400$ ,  $M_3 = 1800$ ,  $Z_1 = 2400$ ,  $Z_2 = 2000$ ,  $Z_3 = 1500$  (all bit/s). Three non-identical portions result in an integral substantially messier than (16). It could be programmed on a computer, but that is unnecessary because a tight bound can be obtained by replacing the fastest portion by one identical with the next faster portion and using (16). Doing so with  $\Delta u = 0.1$  and the Trapezoidal Rule gives  $M_{V_3} = 1794.4$  bit/s; the more accurate Simpson's Rule gives  $M_{V_3} = 1794.7$  bit/s. Since  $M_{V_3}$  must be less than or equal to  $M_3 = 1800$  bit/s, the mean throughput with the original three non-identical portions is bounded by 1795 and 1800 bit/s. It is estimated as 1797 bit/s with an error probably no more than 1 bit/s. The effect on the lower 5% point will be even less; numerical integration with  $\Delta u = 0.1$  gives  $V_{0.5, 3} = 1499.2$  bit/s when the fastest portion is replaced by one identical with the next faster portion, so it is estimated that the original network has  $V_{0.5, 3} = 1500$  bit/s to the nearest unit.

These examples suggest the following when the smallest throughput distribution is not greatly overlapped by others, and this applies no matter how many portions there are:

*General Rule* – If the mean throughput of the slowest portion is less than the mean of the next slowest portion by at least twice the difference between the mean and 95%ile of the slowest portion or of the next slowest portion, whichever difference is larger, then the mean and 95%ile of the throughput of the network are the same as those of the slowest portion (with negligible error). (This rule can probably be relaxed by replacing “twice” by “1.5 times” or deleting “twice” without incurring too much error in practice.)

The case of general  $n$  is considered similarly. With different distributions  $f_i(t)$  the pdf  $g_n(v)$  of  $V_n = \min(T_1, T_2, \dots, T_n)$  is obtainable by iteration from  $g_{n-1}(v)$ :

$$g_n(v) = g_{n-1}(v) [1 - F_n(v)] + f_n(v) [1 - G_{n-1}(v)]$$

If all  $f_i(t)$  are identical, then

$$g_n(v) = n f_i(v) [1 - F_i(v)]^{n-1}$$

If, in addition, normal distributions are assumed for the  $f_i(t)$ , then the mean net throughput is

$$\begin{aligned} M_{V_n} &= M_1 + n \sigma_1 \int_{-\infty}^{\infty} u \varphi(u) [1 - \Phi(u)]^{n-1} du, \\ &= M_1 - n \sigma_1 \int_0^{\infty} u \varphi(u) \{\Phi^{n-1}(u) - [1 - \Phi(u)]^{n-1}\} du \\ &= M_1 - K_n \sigma_1 \end{aligned} \tag{C-18}$$

and the 5% point of the net throughput is

$$V_{.05, n} = M_1 - \sigma_1 U_{.05, n} \tag{C-19}$$

where

$$\Phi(-V_{.05, n}) = 1 - 0.95^{1/n} \tag{C-20}$$

The values  $K_n$  and  $U_{.05, n}$  can be tabulated as a function of  $n$ :

$n$	1	2	3	4	5
$K_n$	0	0.5642	0.8463	1.0294	1.1630
$U_{.05, n}$	1.645	1.955	2.121	2.234	2.319

#### C.4 Notes on key assumptions, results, and implications

For further study.

##### Reference

- [1] TEICHROEW, D., Tables of expected values of order statistics and products of order statistics for samples of size twenty and less from the normal distribution, *Annals of Mathematical Statistics*, **27**, pp. 410-426, 1956.



## ITU-T RECOMMENDATIONS SERIES

Series A	Organization of the work of the ITU-T
Series B	Means of expression: definitions, symbols, classification
Series C	General telecommunication statistics
Series D	General tariff principles
Series E	Overall network operation, telephone service, service operation and human factors
Series F	Non-telephone telecommunication services
Series G	Transmission systems and media, digital systems and networks
Series H	Audiovisual and multimedia systems
Series I	Integrated services digital network
Series J	Transmission of television, sound programme and other multimedia signals
Series K	Protection against interference
Series L	Construction, installation and protection of cables and other elements of outside plant
Series M	TMN and network maintenance: international transmission systems, telephone circuits, telegraphy, facsimile and leased circuits
Series N	Maintenance: international sound programme and television transmission circuits
Series O	Specifications of measuring equipment
Series P	Telephone transmission quality, telephone installations, local line networks
Series Q	Switching and signalling
Series R	Telegraph transmission
Series S	Telegraph services terminal equipment
Series T	Terminals for telematic services
Series U	Telegraph switching
Series V	Data communication over the telephone network
<b>Series X</b>	<b>Data networks and open system communications</b>
Series Y	Global information infrastructure and Internet protocol aspects
Series Z	Languages and general software aspects for telecommunication systems