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**ITU-T**

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OF ITU

**E.524**

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SERIES E: OVERALL NETWORK OPERATION,  
TELEPHONE SERVICE, SERVICE OPERATION AND  
HUMAN FACTORS

Quality of service, network management and traffic  
engineering – Traffic engineering – Determination of the  
number of circuits in automatic and semi-automatic  
operation

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**Overflow approximations for non-random inputs**

ITU-T Recommendation E.524

(Previously CCITT Recommendation)

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**OVERALL NETWORK OPERATION, TELEPHONE SERVICE, SERVICE OPERATION AND HUMAN FACTORS**

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**OVERFLOW APPROXIMATIONS FOR NON-RANDOM INPUTS**

**Summary**

To ensure accurate network design and planning in the presence of multiple streams (arising, e.g. from different services on the same network), it is imperative to determine the individual means and variances of the overflows of the different traffic streams offered to a common trunk group arrangement. The exact calculation of these parameters for the individual overflow streams is a very hard problem. Thus, this Recommendation introduces different approximate methods for the calculation of the individual overflows. These methods are evaluated in terms of accuracy, processing time, memory requirements and programming effort. This Recommendation also provides sample numerical results based on exact calculations to which the results based on these methods may be compared.

**Source**

ITU-T Recommendation E.524 was revised by ITU-T Study Group 2 (1997-2000) and was approved under the WTSC Resolution No. 1 procedure on the 10th of May 1999.

## FOREWORD

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The World Telecommunication Standardization Conference (WTSC), which meets every four years, establishes the topics for study by the ITU-T Study Groups which, in their turn, produce Recommendations on these topics.

The approval of Recommendations by the Members of the ITU-T is covered by the procedure laid down in WTSC Resolution No. 1.

In some areas of information technology which fall within ITU-T's purview, the necessary standards are prepared on a collaborative basis with ISO and IEC.

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## **Recommendation E.524**

### **OVERFLOW APPROXIMATIONS FOR NON-RANDOM INPUTS**

*(revised in 1999)*

#### **1 Scope**

This Recommendation introduces approximate methods for the calculation of means and variances of overflows for individual (non-random) traffic streams offered to a circuit group arrangement.

#### **2 References**

The following ITU-T Recommendations and other references contain provisions which, through reference in this text, constitute provisions of this Recommendation. At the time of publication, the editions indicated were valid. All Recommendations and other references are subject to revision; all users of this Recommendation are therefore encouraged to investigate the possibility of applying the most recent edition of the Recommendations and other references listed below. A list of the currently valid ITU-T Recommendations is regularly published.

- CCITT Recommendation E.521 (1988), *Calculation of the number of circuits in a group carrying overflow traffic.*
- ITU-T Recommendation E.600 (1993), *Terms and definitions of traffic engineering.*

#### **3 Terms and definitions**

General terms and definitions are given in Recommendation E.600; notations used in the description of the proposed methods appear with the methods themselves in clause 6.

#### **4 Abbreviations**

This Recommendation uses the following abbreviations:

|     |                                 |
|-----|---------------------------------|
| AWW | approximate Wilkinson Wallström |
| EC  | equivalent capacity             |
| EPS | equivalent Poisson stream       |
| ERT | equivalent random traffic       |
| IPP | interrupted Poisson process     |

#### **5 Introduction**

This Recommendation provides approximate methods for the calculation of means and variances of overflows for individual (non-random) traffic streams in a circuit group arrangement.

The considered methods are necessary complements to those included in the existing Recommendation E.521 where it is required to take into account concepts such as cluster engineering with service equalization, service protection and end-to-end grade of service. Recommendation E.521 is then insufficient as it is concerned with the grade of service for only one non-random traffic stream in a circuit group.

Design methods concerning the above-mentioned areas are subject to further study and this Recommendation will serve as a reference when, in the future, Recommendation E.521 is complemented or replaced.

In this Recommendation the proposed methods are evaluated in terms of accuracy, processing time, memory requirements and programming effort. Other criteria may be relevant and added in the future.

The proposed methods are described briefly in clause 6. Clause 7 defines a set of examples of circuit group arrangements with exactly calculated (exact resolution of equations of state) individual means and variances, to which the result of the methods can be compared. This leads to Table 2, where for each method the important criteria are listed. The publications cited in the bibliography section at the end contain detailed information about the mathematical background of each of the methods.

## **6 Proposed methods**

The following methods are considered:

- a) IPP method;
- b) EC method;
- c) AWW method;
- d) EPS method.

### **6.1 IPP method**

IPP method is a Poisson process interrupted by a random switch. The on-/off-duration of the random switch has a negative exponential distribution. Overflow traffic from a circuit group can be accurately approximated by an IPP, since IPP can represent bulk characteristics of overflow traffic. IPP has three parameters, namely, on-period intensity and mean on-/off-period durations. To approximate overflow traffic by an IPP, those three parameters are determined so that some moments of overflow traffic will coincide with those of IPP.

The following two kinds of moment match methods are considered in this Recommendation:

- three-moment match method [1] where IPP parameters are determined so that the first three moments of IPP will coincide with those of overflow traffic;
- four-moment ratio match method [2] where IPP parameters are determined so that the first moment and the ratios of the 2nd/3rd and 7th/8th binomial moments of IPP will coincide with those of overflow traffic.

To analyse a circuit group where multiple Poisson and overflow traffic streams are simultaneously offered, each overflow stream is approximated by an IPP. The IPP method is well suited to computer calculation. State transition equations of the circuit group with IPP inputs can be solved directly and no introduction of equivalent models is necessary. Characteristics of overflow traffic can be obtained from the solution of state transition equations. The main feature of the IPP method is that the individual means and variances of the overflow traffic can be solved.



## 6.2 EC method

The EC method [3] does not use the traffic-moments but the transitional behaviour of the primary traffic, by introducing a certain function  $\rho(n)$  versus the equivalent capacity ( $n$ ) of the partial overflow traffic, as defined by the recurrent process:

$$\begin{cases} \rho(0) = Em(\alpha) \quad [\text{Erlang loss formula}] \\ \frac{n}{\rho(n)} = (m+n-a) + \alpha \cdot \rho(n-1) \end{cases} \quad (6-1)$$

if  $n$  is a positive integer, and approximated by linear interpolation if not.

A practical approximation, considering the predominant overflow congestion states only, leads to the equations:

$$\frac{n_i}{n} = \frac{a_i \rho_i(n_i) / D_i(n_i + 1)}{\sum_{k=1}^x a_k \rho_k(n_k) / D_k(n_k + 1)} \quad (6-2)$$

with:

$$D_i(n) = 1 + a_i [\rho_i(n) - \rho_i(n-1)] \quad (6-3)$$

defining the equivalent capacity ( $n_i$ ) of the partial overflow traffic labelled  $i$ , and influenced by the mutual dependency between the partial overflow traffic streams.

The mean value of the partial second overflow is:

$$O_i = a_i \pi \rho_i(n_i) \quad (6-4)$$

where  $\pi$  is the time congestion of the overflow group.

The partial GOS (grade of service) equalization is fulfilled if:

$$\rho_i(n_i) = C \quad (6-5)$$

$C$  being a constant to be chosen.

## 6.3 AWW method

The AWW method uses an approximate ERT model based on an improvement of Rapp's approximation. The total overflow traffic is split up in the individual parts by a simple expression, see equations (6-7) and (6-9). To calculate the total overflow traffic, any method can be used. An approximate Erlang formula calculation for which the speed is independent of the size of the calculated circuit group is given in [4].

The following notations are used:

|                      |   |
|----------------------|---|
| $M$                  | mean of total offered traffic                             |
| $V$                  | variance of total offered traffic                         |
| $Z$                  | $V/M$   |
| $B$                  | mean blocking of the studied group                        |
| $m_i, v_i, z_i, b_i$ | corresponding quantities for an individual traffic stream |
| $\sim$               | is used for overflow quantities.                          |

### 6.3.1 Blocking of overflow traffic

For overflow calculations, an approximate ERT model is used. By numerical investigations, a considerable improvement has been found to Rapp's classical approximation for the fictitious traffic. The error added by the approximation is small compared to the error of the ERT model. It is known that ERT underestimates low blockings when mixing traffic of diverse peakedness [2]. The formula, which was given in [4] (although with one printing error), is for  $Z > 1$ :

$$A^* \approx V + Z(Z - 1) (2 + \gamma^\beta)$$

where:

$$\gamma = (2.36 Z - 2.17) \log \left\{ 1 + \frac{Z - 1}{M(Z + 1.5)} \right\}$$

and:

$$\beta = Z / (1.5 M + 2 Z - 1.3) \quad (6-6)$$

### 6.3.2 Wallström formula for individual blocking

There has been much interest in finding a simple and accurate formula for the individual blocked traffic  $\tilde{m}_i$ . Already in 1967, Katz [5] proposed a formula of the type:

$$\tilde{m}_i = m_i B (1 - w + w z_i / Z) \quad (6-7)$$

with  $w$  being a suitable expression. Wallström proposed a very simple one but with reasonable results [6] and [2]:

$$w = 1 - B \quad (6-8)$$

One practical problem is, however, that a small peaked substream could have a blocking  $b_i > 1$  with this formula. To avoid such unreasonable results, a modification is used in this case. Let  $z_{\max}$  be the largest individual  $z_i$ . Then the value used is:

$$w = \begin{cases} 1 - B & \text{if } z_{\max} < Z(1 + B) \\ \frac{Z(1 - B)}{B(z_{\max} - Z)} & \text{otherwise} \end{cases} \quad (6-9)$$

### 6.3.3 Handling of overflow variances

Covariances are not separately calculated within the AWW method. Rather, their effect is included in the individual overflow parameters  $\tilde{v}_i$  so that they sum up to the total variance. The quantities  $\tilde{v}_i$  are obtained from the total overflow variance  $\tilde{V}$  by a simple splitting formula:

$$\tilde{v}_i = \tilde{V} v_i / V \quad (6-10)$$

One can prove that Wallström's splitting formula (6-8) and formula (6-10) together with the ERT model satisfies a certain consistency requirement. One will obtain the same values for the individual blocked traffic when calculating a circuit group of  $N_1 + N_2$  circuits as when calculating first the  $N_1$  circuits and then offering the overflow to the  $N_2$  circuits.

Since the individual variances are treated in this manner, they are not comparable with the results reported in Table 1.

## 6.4 EPS method

The EPS method [7] consists in transforming the individual peaked streams into equivalent Poisson streams (using the standard ERT model) and determining the conditions under which the equivalence is valid (peakedness = variance/mean).

The following notations are used:

|             |   |
|-------------|---|
| $a_i$       | Mean of traffic of input stream $i$                         |
| $z_i$       | Peakedness of traffic of input stream $i$                   |
| $a$         | Mean of total offered traffic                               |
| $v$         | Variance of total offered traffic                           |
| $z$         | Peakedness of total offered traffic                         |
| $O$         | Mean of total overflow traffic                              |
| $Z$         | Peakedness of total overflow traffic                        |
| $V$         | Variance of total overflow traffic                          |
| $O_i$       | Mean of overflow of traffic stream $i$                      |
| $V_i$       | Variance of overflow of traffic stream $i$                  |
| $Z_i$       | Peakedness of overflow of traffic stream $i$                |
| $C_{(i,j)}$ | Covariance between the overflow traffic streams $i$ and $j$ |

The conditions for the validity of the EPS method are:

$$z_i \ll a_i \quad (6-11a)$$

$$(z_i - 1) \ll a_i / 3 \quad (6-11b)$$

These conditions are met in today's large traffic networks, where the traffic is of the order of hundreds of erlangs and peakedness is of order unity.

The method yields the following results:

$$O_i = f_i O \quad (6-12a)$$

$$V_i = (f_i)^2 V + f_i(1 - f_i) O \quad (6-12b)$$

where the stream fractions  $f_i$  are:

$$f_i = a_e(i) / \sum a_e(i) \quad (6-12c)$$

$a_e(i)$ , mean of equivalent Poisson traffic of stream  $i$ , is given by:

$$a_e(i) = a_i z_i + 3z_i (z_i - 1) \quad (6-13)$$

The mean total overflow  $O = \sum O_i$  and the corresponding variance  $V$  are:

$$O = a_e B(s + s_e, a_e) \quad (6-14a)$$

$$V = O (1 - O + a_e / (s + s_e + 1 - a_e + O)) \quad (6-14b)$$

where  $a_e$  and  $s_e$  are given by:

$$a_e = v + 3z (z - 1) \quad (6-15a)$$

$$s_e = a_e (a + z) / (a + z - 1) - a - 1 \quad (6-15b)$$

$B$  denotes the Erlang B formula. Peakedness  $Z$  of the total overflow, equal to  $V/O$ , is:

$$Z = 1 - O + a_e / (s + s_e + 1 - a_e + O) \quad (6-16)$$

The covariance between overflow streams  $i$  and  $j$  is:

$$C(i, j) = f_i f_j (V - O) \quad (6-17)$$

Similarly,

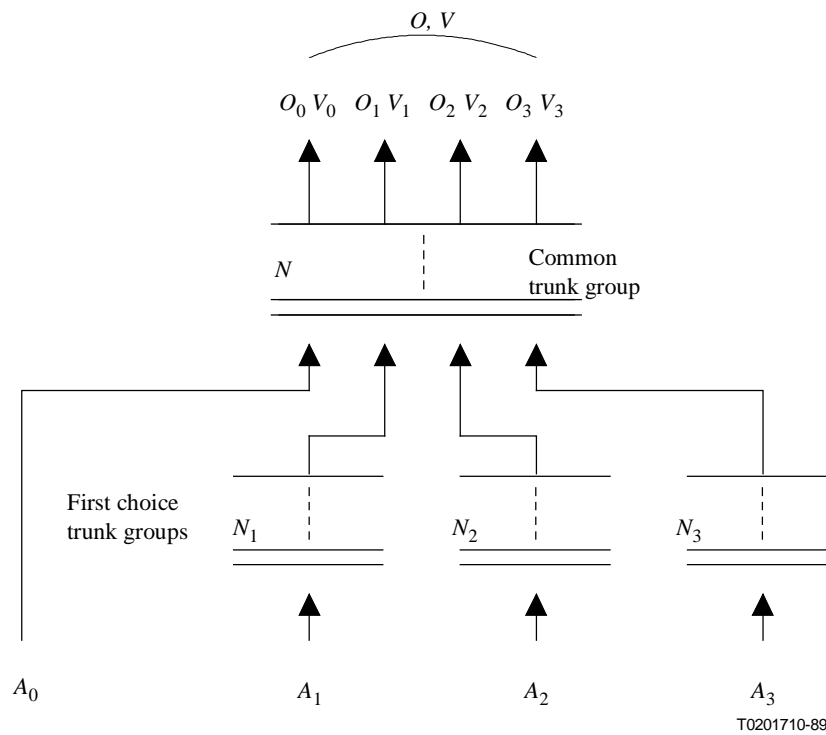
$$Z_i - 1 = f_i (Z - 1) \quad (6-18)$$

Clearly, as the input peaked streams approach Poisson in character,  $a_e(i) \rightarrow a_i$ , since  $z_i \rightarrow 1$ . As a result,  $f_i = a_i / \sum a_i$ , and the above equations appropriately reduce to those for Poisson input streams [7]. Note also that under the conditions defined by equations (6-11a) and (6-11b), equation (6-12c) reduces to  $f_i \approx a_i z_i / \sum a_i z_i$ .

## 7 Examples and criteria for comparison

The defined methods are tested by calculating the examples given in Table 1.

The calculation model is given in Figure 1.



- $A_i$  Offered Poisson traffic volume
- $N_i$  Number of first choice trunk group
- $N$  Number of common trunk group
- $O_i$  Mean of individual overflow traffic from common trunk group
- $V_i$  Variance of individual overflow traffic from common trunk group
- $O$  Mean of total overflow traffic from common trunk group
- $V$  Variance of total overflow traffic from common trunk group

**Figure 1/E.524 – Calculation model**

For comparison, the following criteria are established:

i) *Overflow traffic error*

- *accuracy of the individual overflow traffic mean and variance*
  - *Mean error*

$$\varepsilon_O^I = \frac{\sum_{\text{all examples}} \sum_{i=1}^M \delta_O^i}{\{\text{number of streams}\} \{\text{number of examples}\}}$$

$$\varepsilon_V^I = \frac{\sum_{\text{all examples}} \sum_{i=1}^M \delta_V^i}{\{\text{number of streams}\} \{\text{number of examples}\}}$$

- *Standard deviation of error*

$$sd_O^I = \frac{\left\{ \sum_{\text{all examples}} \sum_{i=1}^M (\delta_O^i - \varepsilon_O^I)^2 \right\}^{1/2}}{\{\text{number of streams}\} \{\text{number of examples}\}}$$

$$sd_V^I = \frac{\left\{ \sum_{\text{all examples}} \sum_{i=1}^M (\delta_V^i - \varepsilon_V^I)^2 \right\}^{1/2}}{\{\text{number of streams}\} \{\text{number of examples}\}}$$

where:

$$\delta_O^i = (O_i - \bar{O}_i) / \bar{O}_i$$

$$\delta_V^i = (V_i - \bar{V}_i) / \bar{V}_i$$

$O_i, V_i$  Calculated individual mean and variance of approximate method.

$\bar{O}_i, \bar{V}_i$  Exactly calculated individual mean and variance.

$M$  Number of input streams to common trunk group.

- *accuracy of the total overflow traffic mean and variance*

- *Mean error*

$$\varepsilon_O^T = \frac{\sum_{\text{all examples}} \delta_O}{\{\text{number of examples}\}}$$

$$\varepsilon_V^T = \frac{\sum_{\text{all examples}} \delta_V}{\{\text{number of examples}\}}$$

- *Standard deviation of error*

$$sd_O^T = \frac{\left\{ \sum_{\text{all examples}} (\delta_O - \varepsilon_O^T)^2 \right\}^{1/2}}{\{\text{number of examples}\}}$$

$$sd_V^T = \frac{\left\{ \sum_{\text{all examples}} (\delta_V - \varepsilon_V^T)^2 \right\}^{1/2}}{\{\text{number of examples}\}}$$

where:

$$\delta_o = (O - \bar{O}) / \bar{O}$$

$$\delta_v = (V - \bar{V}) / \bar{V}$$

$O, V$  Calculated total mean and variance of approximate method.

$\bar{O}, \bar{V}$  Exactly calculated total mean and variance.

ii) *Computational effort*

- *Relative processor time*

$$C = \frac{\{\text{Total central processing unit (CPU) time for calculating all examples by using the approximate method}\}}{\{\text{Total CPU time for calculating all examples by Erlang formula}\}}$$

- *Memory requirements*

$$M = \frac{\text{The size of memory required for the execution of approximate method}}{\text{The size of memory required for execution of Erlang formula}}$$

- *Program size*

$$S = \frac{\text{Source program size for approximate method}}{\text{Source program size for Erlang formula}}$$

NOTE 1 –  $C, M$  and  $S$ , for a specific approximate method, should be based on the same processor, language and supporting algorithms.

NOTE 2 – Depending on the type of approximate method (direct calculating or recursive) different trade-offs between  $C, M$  and  $S$  may be reached, e.g. more memory versus less time, small program versus more time, etc.

**Table 1a/E.524 – Exactly calculated mean and variance of individual overflow traffic – Three first choice groups**

| Case | $A_1$  | $A_2$  | $A_3$  | $A_0$ | $O_0$    | $O_1$   | $O_2$   | $O_3$   | $O$    |
|------|--------|--------|--------|-------|----------|---------|---------|---------|--------|
|      | $N_1$  | $N_2$  | $N_3$  | $N$   | $V_0$    | $V_1$   | $V_2$   | $V_3$   | $V$    |
| 1    | 7.036  | 26.688 | 64.169 | –     | –        | 0.4337  | 0.7490  | 1.091   | 2.274  |
|      | 5      | 28     | 70     | 11    | –        | 0.7656  | 2.111   | 4.441   | 10.51  |
| 2    | 7.036  | 26.688 | 64.169 | –     | –        | 0.1449  | 0.2758  | 0.4944  | 0.9150 |
|      | 5      | 28     | 70     | 16    | –        | 0.2436  | 0.7328  | 1.911   | 4.293  |
| 3    | 7.036  | 26.688 | 64.169 | –     | –        | 0.01369 | 0.02846 | 0.06628 | 0.1084 |
|      | 5      | 28     | 70     | 25    | –        | 0.02041 | 0.06461 | 0.2205  | 0.4464 |
| 4    | 7.036  | 10.176 | 13.250 | –     | –        | 0.7459  | 1.262   | 1.785   | 3.792  |
|      | 5      | 6      | 7      | 14    | –        | 1.193   | 2.292   | 3.625   | 11.28  |
| 5    | 7.036  | 10.176 | 13.250 | –     | –        | 0.2888  | 0.4857  | 0.6832  | 1.458  |
|      | 5      | 6      | 7      | 19    | –        | 0.4636  | 0.9089  | 1.460   | 4.754  |
| 6    | 7.036  | 10.176 | 13.250 | –     | –        | 0.03570 | 0.05915 | 0.08237 | 0.1772 |
|      | 5      | 6      | 7      | 26    | –        | 0.05358 | 0.1026  | 0.1621  | 0.5249 |
| 7    | 7.036  | 32.395 | 77.617 | –     | –        | 0.4516  | 1.176   | 2.344   | 3.972  |
|      | 5      | 31     | 77     | 16    | –        | 0.7434  | 3.466   | 10.39   | 21.41  |
| 8    | 7.036  | 32.395 | 77.617 | –     | –        | 0.1538  | 0.4294  | 0.9739  | 1.557  |
|      | 5      | 31     | 77     | 23    | –        | 0.2427  | 1.200   | 4.219   | 8.558  |
| 9    | 7.036  | 32.395 | 77.617 | –     | –        | 0.01303 | 0.03984 | 0.1006  | 0.1535 |
|      | 5      | 31     | 77     | 35    | –        | 0.01841 | 0.09378 | 0.3690  | 0.7124 |
| 10   | 64.169 | 32.395 | 13.250 | –     | –        | 1.157   | 1.455   | 1.320   | 3.933  |
|      | 70     | 31     | 7      | 15    | –        | 4.442   | 4.256   | 2.849   | 18.28  |
| 11   | 64.169 | 32.395 | 13.250 | –     | –        | 0.5564  | 0.5849  | 0.4749  | 1.616  |
|      | 70     | 31     | 7      | 21    | –        | 2.026   | 1.675   | 1.023   | 8.008  |
| 12   | 64.169 | 32.395 | 13.250 | –     | –        | 0.06907 | 0.05265 | 0.03848 | 0.1602 |
|      | 70     | 31     | 7      | 32    | –        | 0.2167  | 0.1295  | 0.07165 | 0.7075 |
| 13   | 7.036  | 26.688 | 64.169 | 3.000 | 0.4064   | 0.5038  | 0.8274  | 1.160   | 2.897  |
|      | 5      | 28     | 70     | 13    | 0.5578   | 0.8566  | 2.243   | 4.574   | 13.16  |
| 14   | 7.036  | 26.688 | 64.169 | 3.000 | 0.1460   | 0.1840  | 0.3384  | 0.5729  | 1.241  |
|      | 5      | 28     | 70     | 18    | 0.1992   | 0.3043  | 0.8779  | 2.163   | 5.893  |
| 15   | 7.036  | 26.688 | 64.169 | 3.000 | 0.01170  | 0.01506 | 0.03086 | 0.07035 | 0.1280 |
|      | 5      | 28     | 70     | 28    | 0.01472  | 0.02218 | 0.06861 | 0.2287  | 0.5374 |
| 16   | 7.036  | 32.395 | 77.617 | 1.000 | 0.1253   | 0.4451  | 1.156   | 2.304   | 4.031  |
|      | 5      | 31     | 77     | 17    | 0.1392   | 0.7266  | 3.366   | 10.10   | 21.77  |
| 17   | 7.036  | 32.395 | 77.617 | 1.000 | 0.04250  | 0.1536  | 0.4275  | 0.9674  | 1.591  |
|      | 5      | 31     | 77     | 24    | 0.04696  | 0.2409  | 1.183   | 4.148   | 8.775  |
| 18   | 7.036  | 32.395 | 77.617 | 1.000 | 0.004542 | 0.01687 | 0.05106 | 0.1282  | 0.2006 |
|      | 5      | 31     | 77     | 35    | 0.004891 | 0.02398 | 0.1214  | 0.4751  | 0.9548 |
| 19   | 64.169 | 32.395 | 13.250 | 9.000 | 1.761    | 1.251   | 1.654   | 1.630   | 6.295  |
|      | 70     | 31     | 7      | 21    | 3.052    | 4.517   | 4.406   | 3.103   | 27.73  |
| 20   | 64.169 | 32.395 | 13.250 | 9.000 | 0.6761   | 0.6501  | 0.7389  | 0.6427  | 2.708  |
|      | 70     | 31     | 7      | 28    | 1.253    | 2.225   | 1.956   | 1.279   | 13.61  |
| 21   | 64.169 | 32.395 | 13.250 | 9.000 | 0.06219  | 0.09577 | 0.07978 | 0.06069 | 0.2984 |
|      | 70     | 31     | 7      | 40    | 0.1054   | 0.2884  | 0.1887  | 0.1099  | 1.409  |

**Table 1b/E.524 – Exactly calculated mean and variance of individual overflow traffic – Two first choice groups**

| Case | $A_1$  | $A_2$  | $A_3$ | $A_0$ | $O_0$ | $O_1$   | $O_2$   | $O_3$ | $O$     |
|------|--------|--------|-------|-------|-------|---------|---------|-------|---------|
|      | $N_1$  | $N_2$  | $N_3$ | $N$   | $V_0$ | $V_1$   | $V_2$   | $V_3$ | $V$     |
| 22   | 8.200  | 30.000 | –     | –     | –     | 0.6153  | 1.139   | –     | 1.755   |
|      | 5      | 30     | –     | 10    | –     | 1.179   | 3.473   | –     | 6.159   |
| 23   | 8.200  | 30.000 | –     | –     | –     | 1.807   | 2.465   | –     | 4.272   |
|      | 5      | 30     | –     | 5     | –     | 3.263   | 7.431   | –     | 13.00   |
| 24   | 8.200  | 30.000 | –     | –     | –     | 0.01866 | 0.04814 | –     | 0.06680 |
|      | 5      | 30     | –     | 21    | –     | 0.03026 | 0.1233  | –     | 0.1993  |
| 25   | 8.200  | 30.000 | –     | –     | –     | 0.2111  | 0.4629  | –     | 0.6740  |
|      | 5      | 30     | –     | 14    | –     | 0.3902  | 1.372   | –     | 2.355   |
| 26   | 8.200  | 14.300 | –     | –     | –     | 0.04699 | 0.09279 | –     | 0.1398  |
|      | 5      | 7      | –     | 22    | –     | 0.07700 | 0.1980  | –     | 0.3724  |
| 27   | 8.200  | 14.300 | –     | –     | –     | 0.3744  | 0.7547  | –     | 1.129   |
|      | 5      | 7      | –     | 16    | –     | 0.6603  | 1.763   | –     | 3.322   |
| 28   | 8.200  | 14.300 | –     | –     | –     | 0.9282  | 1.892   | –     | 2.820   |
|      | 5      | 7      | –     | 12    | –     | 1.614   | 4.212   | –     | 7.774   |
| 29   | 8.200  | 14.300 | –     | –     | –     | 2.002   | 4.095   | –     | 6.098   |
|      | 5      | 7      | –     | 7     | –     | 3.272   | 7.806   | –     | 13.64   |
| 30   | 8.200  | 42.000 | –     | –     | –     | 0.02324 | 0.09886 | –     | 0.1221  |
|      | 5      | 37     | –     | 27    | –     | 0.03602 | 0.3019  | –     | 0.4197  |
| 31   | 8.200  | 42.000 | –     | –     | –     | 0.2136  | 0.8353  | –     | 1.049   |
|      | 5      | 37     | –     | 19    | –     | 0.3682  | 2.945   | –     | 4.204   |
| 32   | 8.200  | 42.000 | –     | –     | –     | 1.499   | 4.437   | –     | 5.935   |
|      | 5      | 37     | –     | 8     | –     | 2.616   | 14.60   | –     | 21.00   |
| 33   | 8.200  | 42.000 | –     | –     | –     | 0.6940  | 2.416   | –     | 3.110   |
|      | 5      | 37     | –     | 13    | –     | 1.237   | 8.493   | –     | 12.30   |
| 34   | 30.000 | 14.300 | –     | –     | –     | 0.06570 | 0.05450 | –     | 0.1202  |
|      | 30     | 7      | –     | 25    | –     | 0.1628  | 0.1116  | –     | 0.3922  |
| 35   | 30.000 | 14.300 | –     | –     | –     | 0.4669  | 0.4662  | –     | 0.9331  |
|      | 30     | 7      | –     | 18    | –     | 1.300   | 1.088   | –     | 3.461   |
| 36   | 30.000 | 14.300 | –     | –     | –     | 1.374   | 1.739   | –     | 3.113   |
|      | 30     | 7      | –     | 12    | –     | 3.932   | 4.001   | –     | 10.91   |
| 37   | 30.000 | 14.300 | –     | –     | –     | 2.425   | 3.806   | –     | 6.231   |
|      | 30     | 7      | –     | 7     | –     | 6.994   | 7.628   | –     | 18.16   |
| 38   | 8.200  | 67.900 | –     | –     | –     | 0.01656 | 0.1007  | –     | 0.1173  |
|      | 5      | 65     | –     | 30    | –     | 0.02497 | 0.3667  | –     | 0.4658  |
| 39   | 8.200  | 67.900 | –     | –     | –     | 0.1835  | 0.9716  | –     | 1.155   |
|      | 5      | 65     | –     | 20    | –     | 0.3132  | 4.189   | –     | 5.488   |
| 40   | 8.200  | 67.900 | –     | –     | –     | 0.5393  | 2.449   | –     | 2.986   |
|      | 5      | 65     | –     | 14    | –     | 0.9685  | 10.73   | –     | 14.35   |
| 41   | 8.200  | 67.900 | –     | –     | –     | 1.361   | 4.707   | –     | 6.068   |
|      | 5      | 65     | –     | 8     | –     | 2.441   | 19.71   | –     | 26.55   |
| 42   | 51.500 | 14.300 | –     | –     | –     | 0.07517 | 0.04089 | –     | 0.1161  |
|      | 54     | 7      | –     | 27    | –     | 0.2290  | 0.08146 | –     | 0.4339  |
| 43   | 51.500 | 14.300 | –     | –     | –     | 0.6402  | 0.4689  | –     | 1.109   |
|      | 54     | 7      | –     | 18    | –     | 2.248   | 1.101   | –     | 4.799   |



**Table 1c/E.524 – Exactly calculated mean and variance of individual overflow traffic – One first choice group**

| Case | $A_1$  | $A_2$  | $A_3$ | $A_0$ | $O_0$   | $O_1$   | $O_2$ | $O_3$ | $O$     |
|------|--------|--------|-------|-------|---------|---------|-------|-------|---------|
|      | $N_1$  | $N_2$  | $N_3$ | $N$   | $V_0$   | $V_1$   | $V_2$ | $V_3$ | $V$     |
| 44   | 51.500 | 14.300 | –     | –     | –       | 1.403   | 1.362 | –     | 2.765   |
|      | 54     | 7      | –     | 13    | –       | 5.079   | 3.224 | –     | 11.50   |
| 45   | 51.500 | 14.300 | –     | –     | –       | 2.586   | 3.675 | –     | 6.261   |
|      | 54     | 7      | –     | 7     | –       | 9.612   | 7.513 | –     | 21.11   |
| 46   | 8.200  | –      | –     | 4.000 | 0.03309 | 0.04990 | –     | –     | 0.08299 |
|      | 5      | –      | –     | 16    | 0.04785 | 0.08712 | –     | –     | 0.1789  |
| 47   | 8.200  | –      | –     | 4.000 | 0.3494  | 0.4859  | –     | –     | 0.8354  |
|      | 5      | –      | –     | 11    | 0.5382  | 0.9155  | –     | –     | 1.975   |
| 48   | 8.200  | –      | –     | 4.000 | 0.9011  | 1.169   | –     | –     | 2.070   |
|      | 5      | –      | –     | 8     | 1.327   | 2.120   | –     | –     | 4.554   |
| 49   | 8.200  | –      | –     | 4.000 | 1.802   | 2.142   | –     | –     | 3.944   |
|      | 5      | –      | –     | 5     | 2.369   | 3.588   | –     | –     | 7.333   |
| 50   | 30.000 | –      | –     | 4.000 | 0.01660 | 0.05973 | –     | –     | 0.07633 |
|      | 30     | –      | –     | 20    | 0.02296 | 0.1558  | –     | –     | 0.2228  |
| 51   | 30.000 | –      | –     | 4.000 | 0.1991  | 0.5806  | –     | –     | 0.7796  |
|      | 30     | –      | –     | 13    | 0.3062  | 1.743   | –     | –     | 2.656   |
| 52   | 30.000 | –      | –     | 4.000 | 0.5988  | 1.400   | –     | –     | 1.999   |
|      | 30     | –      | –     | 9     | 0.9338  | 4.255   | –     | –     | 6.730   |
| 53   | 30.000 | –      | –     | 4.000 | 1.560   | 2.558   | –     | –     | 4.118   |
|      | 30     | –      | –     | 5     | 2.199   | 7.620   | –     | –     | 12.01   |
| 54   | 51.500 | –      | –     | 4.000 | 0.01445 | 0.07537 | –     | –     | 0.08982 |
|      | 54     | –      | –     | 22    | 0.01966 | 0.2413  | –     | –     | 0.3131  |
| 55   | 51.500 | –      | –     | 4.000 | 0.1208  | 0.5143  | –     | –     | 0.6351  |
|      | 54     | –      | –     | 15    | 0.1819  | 1.893   | –     | –     | 2.575   |
| 56   | 51.500 | –      | –     | 4.000 | 0.4286  | 1.383   | –     | –     | 1.812   |
|      | 54     | –      | –     | 10    | 0.6788  | 5.300   | –     | –     | 7.549   |
| 57   | 51.500 | –      | –     | 4.000 | 1.145   | 2.429   | –     | –     | 3.574   |
|      | 54     | –      | –     | 6     | 1.726   | 9.299   | –     | –     | 13.55   |

## 8 Summary of results

The available methods and the performance measures with respect to the criteria are listed in Table 2.

**Table 2/E.524 –Comparison of different approximation methods**

| Functions      | Comparison           |                     |                    |
|----------------|----------------------|---------------------|--------------------|
|                | Computational effort |                     |                    |
| Method         | Processor time       | Memory requirements | Programming effort |
| IPP method     |                      |                     |                    |
| 3-moment match | 400                  | 5                   | 50                 |
| 4-moment ratio | 1000                 | 5                   | 50                 |
| EC method      | 40                   | 2                   | 3                  |
| AWW method     | 3                    | 1.6                 | 3                  |
| EPS method     | 3                    | 2                   | 3                  |

## 9 History

Recommendation E.524 – first issued in 1988; revised in 1992; second revision in 1999.

### Bibliography

- [1] MATSUMOTO (J.), WATANABE (Y.): Analysis of individual traffic characteristics for queuing systems with multiple Poisson and overflow inputs, *Proc. 10th ITC*, paper 5.3.1, Montreal 1983.
- [2] RENEBY (L.): On individual and overall losses in overflow systems, *Proc. 10th ITC*, paper 5.3.5, Montreal 1983.
- [3] LE GALL (P.): Overflow traffic combination and cluster engineering, *Proc. 11th ITC*, paper 2.2B-1, Kyoto 1985.
- [4] LINDBERG (P.), NIVERT (K.), SAGERHOLM (B.): Economy and service aspects of different designs of alternate routing networks, *Proc. 11th ITC*, Kyoto 1985.
- [5] KATZ (S.): Statistical performance analysis of a switched communications network, *Proc. 5th ITC*, New York 1967.
- [6] LINDBERGER (K.): Simple approximations of overflow system quantities for additional demands in the optimization, *Proc. 10th ITC*, Montreal 1983.
- [7] BHANDARI (R.): A model for stream overflows in circuit-switched communication networks, *IEICE Transactions on Communications: Special Issue on Telecommunications Network Planning and Design*, Vol. E80-B, No. 2, pp. 324-331 (1997).

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