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International Telecommunication Union

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TELECOMMUNICATION
STANDARDIZATION SECTOR
OF ITU

O.182
Amendment 1
(01/2009)

SERIES O: SPECIFICATIONS OF MEASURING EQUIPMENT

Equipment for the measurement of digital and analogue/digital parameters

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Equipment to assess error performance on Optical Transport Network interfaces

Amendment 1: An additional evaluation procedure

Recommendation ITU-T O.182 (2007) – Amendment 1

ITU-T O-SERIES RECOMMENDATIONS

SPECIFICATIONS OF MEASURING EQUIPMENT

For further details, please refer to the list of ITU-T Recommendations.

Recommendation ITU-T O.182

Equipment to assess error performance on Optical Transport Network interfaces

Amendment 1

An additional evaluation procedure

Summary

Amendment 1 to Recommendation ITU-T O.182 adds an additional evaluation procedure using the goodness of the fit to the exponential distribution for the random error generator (clause C.6) and the detailed description of this evaluation method (Appendix II).

Source

Amendment 1 to Recommendation ITU-T O.182 (2007) was approved on 13 January 2009 by ITU-T Study Group 15 (2009-2012) under Recommendation ITU-T A.8 procedures.

FOREWORD

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Recommendation ITU-T O.182

Equipment to assess error performance on Optical Transport Network interfaces

Amendment 1

An additional evaluation procedure

1) Clause 10.2, Error generation

Change the first paragraph of clause 10.2 to:

Figure 10-1 presents the error generator. To measure the FEC performance, ME sender inserts symbol errors after FEC computation. Errors are inserted following a Poisson process law. Annex C defines the parameters and the procedure for testing the goodness of fit of a Poisson process.

2) Annex C

Change the title of Annex C to:

Procedure of goodness of fit for Poisson process by χ^2 test

3) Clause C.1

Change clause C.1 to:

C.1 Introduction

A "Poisson error generator" used for performance tests of digital communications systems should generate random errors satisfying the Poisson process. However, the distribution of the random errors generated from such equipment may not necessarily fit a Poisson process. Therefore, an objective method to evaluate the distribution characteristic of the random errors is needed. Although there are many methods for testing goodness of fit for Poisson process, this annex describes a method using the χ^2 test. Both Annexes C.5 and C.6 explain the concrete test procedure. Refer to Appendices I and II for the detailed explanation of this method.

4) Clause C.5

Change the title of clause C.5 to:

C.5 Procedure for test of goodness of fit for Poisson distribution by χ^2 test

5) Clause C.6

Insert the following new clause:

C.6 Procedure for test of goodness of fit for exponential distribution by χ^2 test

The test of goodness of fit for exponential distribution by χ^2 test is performed by the following steps. Refer to Appendix II for the detailed explanation of this method.

1) Measure the nearest neighbouring error intervals, $t_n = i_{n+1} - i_n - 1$, $n = 1, ..., N$.

2) Find t_{max} for sample $\{t_n\}^N,\ _{n=1}$.

3) Determine divisor M for interval [0, t_{max}].

Recommended $5 \le M \le 50$

Example $M = 30$

4) Determine interval width when creating histogram ΔT as $\Delta T = [t_{max}/M]$. Where $[x]$ is the minimum integer value of x.

5) Using $\{t_n\}^N,\ _{n=1}$, determine the sample size (namely observation frequency f_i, $i = 0, ..., M - 1$) for interval $[i\Delta T, (i+1)\Delta T]$.

6) Find the average error interval, $\bar{t}$, using the following equation:

$$\bar{t} = \frac{1}{N}\sum_{n=1}^{N} t_n$$

7) Find the maximum likelihood estimator of p_e, $\tilde{p}_e$, using the following equation:

$$\tilde{p}_e = \frac{1}{1+\bar{t}},\ \tilde{q} = 1 - \tilde{p}_e$$

8) Find the theoretical frequency e_i, $i = 0, ..., M - 1$ for $[i\Delta T, (i+1)\Delta T]$ using the following equation:

$$e_i = N\tilde{q}^{i\Delta T}(1-\tilde{q}^{\Delta T})\left(= N\tilde{p}_e \sum_{x=1i\Delta T}^{(i+1)\Delta T-1} \tilde{q}^x \right)$$

9) Using the observed frequency $\{f_i\}^{M-1},\ _{i=0}$ and the theoretical frequency $\{e_i\}^{M-1},\ _{i=0}$, check the χ^2 goodness of fit as described in items 7 to 11 of clause C.5.

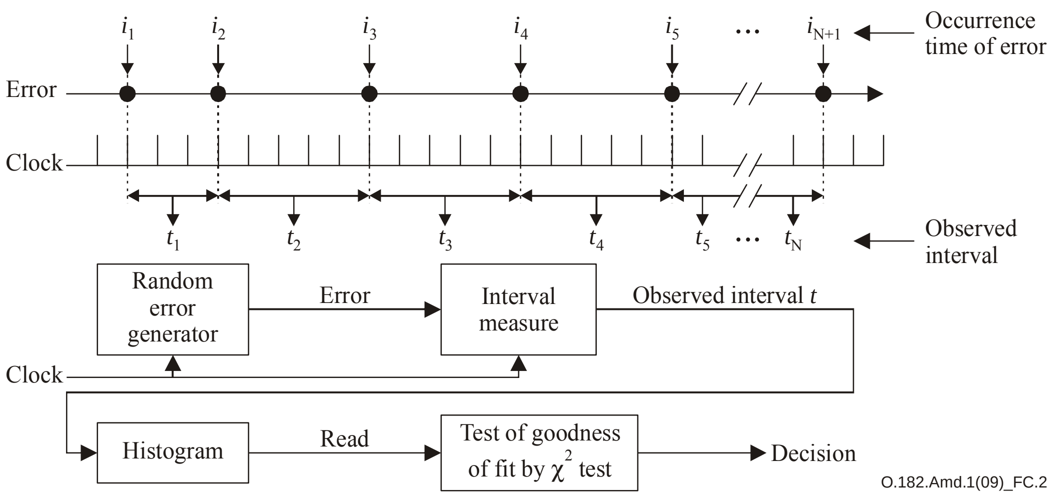

Figure C.2 – Error interval property block diagram

6) New Appendix II

Add the following new appendix.

Appendix II

Test of goodness of fit for exponential distribution by χ^2 test

(This appendix does not form an integral part of this Recommendation)

This appendix describes the detail of the evaluation method for exponential distribution described in clause C.6. Furthermore, this appendix describes why this method is used.

II.1 Introduction

An ideal error generator with error rate p_e is a device that repeats independently the Bernoulli process at some given period with the parameter p_e. Consequently, the error generator performance can be evaluated by using the following two statistical methods:

1) Binomial distribution of errors occurring within some observation interval: Error frequency property.
2) Time interval error distribution: Error interval property.

Appendix I describes an evaluation method based on the error frequency property in method 1 above; it assumes that when the observation interval n is sufficiently long and p_e is sufficiently small, the binomial distribution becomes asymptotic and approaches the Poisson distribution where parameter $\lambda = np_e$. On the other hand, this appendix describes a χ^2 goodness of fit evaluation method for the error interval property in method 2 above, where the error generation interval (error interval below) follows the exponential distribution.

Clause II.2 describes that the nearest neighbouring error interval follows an exponential distribution and describes the theoretical frequency required for performing the χ^2 goodness of fit test. Clause II.3 describes a χ^2 goodness of fit test using actual measured data.

II.2 Nearest neighbouring error interval distribution and theoretical values

A sample Bernoulli procedure with population parameter p_e is represented as:

$$x_i \in \{0,1\},\ i = 1,2,....$$

where error occurrence time which becomes "$x_i = 1$" is represented by $i_1, i_2, ..., i_{N+1}$.

When the nearest neighbouring error interval t_n is defined as $t_n = i_{n+1} - i_n - 1$, $n = 1, ..., N$, the probability of $t_n = t$, $t = 0,1,...$ is the probability of 1 appearing after t continuous 0s, or, in other words, $q^t p_e$. Here, q is defined as $q = 1 - p_e$. Therefore, sample $\{t_n\}^N{}_{,\, n=1}$ is the sample extracted from the population according to the geometrical distribution of parameter p_e.

The theoretical frequency of the geometric distribution is calculated as follows, where the sample average $\bar{t}$ is defined as:

$$\bar{t} = \frac{1}{N}\sum_{n=1}^{N} t_n$$

By solving $\bar{t} = (1 - \tilde{p}_e)/\tilde{p}_e$ for the maximum likelihood estimator of p_e, $\tilde{p}_e$ is defined as:

$$\tilde{p}_e = \frac{1}{1 + \bar{t}}, \; \tilde{q} = 1 - \tilde{p}_e$$

The maximum value T for $\{t_n\}^N{}_{, n=1}$ is defined as $\Delta T = [T/M]$, using the appropriate divisor M. $[x]$ is the minimum integer value x. The theoretical frequency e_i, $i = 0, ..., M - 1$ for interval $[i\Delta T, (i+1)\Delta T]$ is given by the following equation, using the sum of the geometric progression:

$$e_i = N\tilde{p}_e \sum_{x=i\Delta T}^{(i+1)\Delta T - 1} \tilde{q}^x = N\tilde{p}^{i\Delta T}(1 - \tilde{q}^{\Delta T}). \qquad \text{(II-1)}$$

The theoretical frequency of equation (II-1) when $a^b = \exp(b \log a)$ has the following asymptotic form when p_e<<1 and the $\log q = \log(1 - p_e) \approx -p_e$ approximation is used.

$$e_i \approx N \exp(-i\tilde{p}_e \Delta T)\left(1 - \exp(-\tilde{p}_e \Delta T)\right) = N \int_{i\Delta T}^{(i+1)\Delta T} \tilde{p}_e \exp(-\tilde{p}_e t) dt \qquad \text{(II-2)}$$

The integer expression for e_i on the right side of equation (II-2) is the theoretical frequency for the relevant interval corresponding to the exponential distribution of parameter $\tilde{p}_e$. This corresponds to an asymptotic Poisson process when the Bernoulli process is p_e<<1.

II.3 Goodness of fit test results

The error generator operation period and the error rate, p_e, were set to 10 ns and 10^{-2}, respectively, and a goodness of fit test described in clause C.6 was performed on data obtained using three different types of generator.

Table II.1 shows the goodness of fit test results, and Figure II.1 shows the theoretical and observed frequency histograms with the number of errors on the left and the intervals between nearest neighbouring errors on the right. Although errors generated by the type A generation method are evaluated as not fitting the Poisson distribution for error frequency properties, neither is there a fit with the exponential distribution for the error interval property. Actually, from Figure II.1, it is clear that errors generated by the type A method are concentrated in a narrower interval than is ideal. Next, using the type B method unlike the fit of the error frequency property, the interval property does not fit the geometric distribution. However, type B is closer to the exponential distribution than type A, as shown by the χ^2 values (Table II.1) and the decrease in the difference between the observed and theoretical frequencies (Figure II.1). Last, the error generated by the type C generation method fits the distribution for both the error frequency and error interval properties.

Error generation method	**Error frequency property: Poisson distribution of goodness of fit**				**Error interval property: exponential distribution goodness of fit**			
	df	χ^2 value	$\chi^2_{M,0.05}$	Fit/No Fit	df	χ^2 value	$\chi^2_{M,0.05}$	Fit/No Fit
Type A[1)]	28	5330.8	41.3	No Fit	26	14341.6	38.9	No Fit
Type B[2)]	18	19.6	28.9	Fit	25	173.0	37.7	No Fit
Type C[3)]	18	18.7	28.9	Fit	25	18.2	37.7	Fit

1) Type A is the generation method used in the example 2, described in clause I.4.2.

2) Type B is the generation method used in the example 1, described in clause I.4.1.

3) Type C is a new generation method improving randomness.

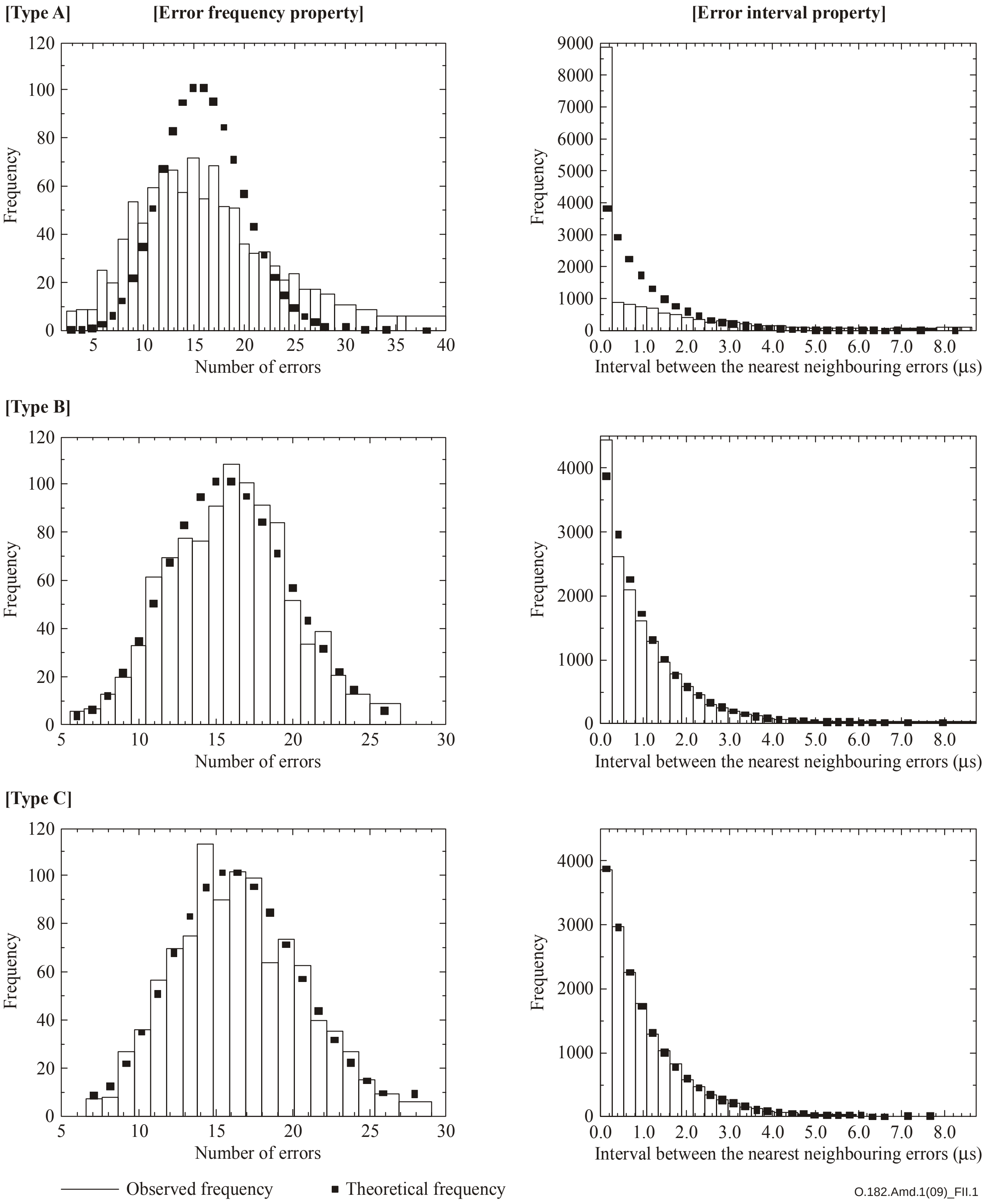

Figure II.1 – Observed and theoretical frequency histograms for three types of error generators

Tables II.2, II.3 and II.4 show the sample data for the error frequency property of types A, B and C, respectively.

Table II.2 – Sample data for the error frequency property (type A)

k	**Observed frequency** f_k	**Expected frequency** $e_k = np_k$	**Deviation** $(f_k - e_k)^2 / e_k$
3	8	0.093	668.636
4	9	0.315	239.538
5	9	1.008	63.395
6	25	2.687	185.303
7	20	6.141	31.277
8	38	12.281	53.860
9	54	21.832	47.398
10	45	34.929	2.904
11	60	50.802	1.665
12	69	67.732	0.024
13	67	83.358	3.210
14	58	95.260	14.574
15	72	101.605	8.626
16	55	101.599	21.373
17	69	95.617	7.409
18	52	84.987	12.804
19	51	71.564	5.909
20	36	57.248	7.886
21	32	43.615	3.093
22	33	31.718	0.052
23	27	22.063	1.105
24	21	14.708	2.692
25	24	9.412	22.608
26	17	5.792	21.689
27	17	3.432	53.638
28	15	1.961	86.695
30	11	1.659	52.600
32	9	0.447	163.795
34	6	0.106	327.269
38	6	0.011	3219.725
Total	$\sum f_k = 1015$	$\sum e_k = 1023.982$	$\chi^2 = \sum(f_k - e_k)^2 / e_k = 5330.752$

Table II.3 – Sample data for the error frequency property (type B)

k	**Observed frequency** f_k	**Expected frequency** $e_k = np_k$	**Deviation** $(f_k - e_k)^2 / e_k$
6	6	3.701	1.427
7	7	6.151	0.117
8	13	12.299	0.04
9	20	21.86	0.158
10	33	34.967	0.111
11	62	50.849	2.445
12	70	67.782	0.073
13	78	83.404	0.35
14	77	95.295	3.512
15	91	101.624	1.111
16	109	101.599	0.539
17	101	95.599	0.305
18	92	84.956	0.584
19	85	71.525	2.539
20	52	57.206	0.474
21	34	43.575	2.104
22	39	31.683	1.69
23	21	22.035	0.049
24	13	14.686	0.194
26	9	5.781	1.792
Total	$\sum f_k = 1012$	$\sum e_k = 1006.576$	$\chi^2 = \sum (f_k - e_k)^2 / e_k = 19.613$

Table II.4 – Sample data for the error frequency property (type C)

k	**Observed frequency** f_k	**Expected frequency** $e_k = np_k$	**Deviation** $(f_k - e_k)^2 / e_k$
7	7	8.843	0.384
8	8	12.299	1.503
9	27	21.86	1.209
10	36	34.967	0.031
11	57	50.849	0.744
12	70	67.782	0.073
13	75	83.404	0.847
14	114	95.295	3.671
15	90	101.624	1.329
16	102	101.599	0.002
17	99	95.599	0.121

Table II.4 – Sample data for the error frequency property (type C)

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
18	64	84.956	5.169
19	74	71.525	0.086
20	63	57.206	0.587
21	40	43.575	0.293
22	35	31.683	0.347
23	27	22.035	1.119
24	15	14.686	0.007
25	9	9.397	0.017
27	6	9.206	1.117
Total	$\Sigma f_k = 1018$	$\Sigma e_k = 1018.389$	$\chi^2 = \Sigma(f_k - e_k)^2 / e_k = 18.654$

Tables II.5, II.6 and II.7 show the sample data for the error interval property of types A, B and C, respectively.

Table II.5 – Sample data for the error interval property (type A)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
0	27	8857	3825.962	6615.681
27	54	910	2932.476	1394.865
54	81	850	2247.648	869.095
81	108	764	1722.749	533.566
108	135	684	1320.431	306.752
135	162	540	1012.068	220.191
162	189	490	775.717	105.237
189	216	434	594.562	43.36
216	243	389	455.712	9.766
243	270	285	349.289	11.833
270	297	325	267.719	12.256
297	324	267	205.198	18.614
324	351	213	157.277	19.742
351	378	190	120.548	40.014
378	405	162	92.396	52.434
405	432	141	70.819	69.55
432	459	107	54.28	51.204
459	486	104	41.604	93.579

Table II.5 – Sample data for the error interval property (type A)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
486	513	79	31.888	69.604
513	540	64	24.441	64.027
540	567	77	18.733	181.227
567	594	69	14.358	207.939
594	621	65	11.005	264.911
621	648	39	8.435	110.751
648	675	37	6.465	144.211
675	729	66	8.754	374.372
729	810	64	6.853	476.565
810	–	111	5.609	1980.296
Total		$\sum f_k = 16383$	$\sum e_k = 16382.996$	$\chi^2 = \sum (f_k - e_k)^2 / e_k = 14341.641$

Table II.6 – Sample data for the error interval property (type B)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
0	27	4427	3865.333	81.615
27	54	2614	2953.363	38.995
54	81	2099	2256.56	11.001
81	108	1609	1724.157	7.691
108	135	1301	1317.367	0.203
135	162	969	1006.553	1.401
162	189	774	769.072	0.032
189	216	589	587.62	0.003
216	243	468	448.98	0.806
243	270	324	343.049	1.058
270	297	261	262.112	0.005
297	324	218	200.27	1.57
324	351	157	153.019	0.104
351	378	133	116.917	2.212
378	405	103	89.332	2.091
405	432	66	68.255	0.075
432	459	62	52.151	1.86
459	486	52	39.847	3.706

Table II.6 – Sample data for the error interval property (type B)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
486	513	29	30.446	0.069
513	540	24	23.262	0.023
540	567	27	17.774	4.789
567	594	16	13.581	0.431
594	621	12	10.376	0.254
621	648	10	7.928	0.541
648	702	16	10.686	2.642
702	783	10	8.303	0.347
783	---	13	5.67	9.478
Total		Σf_k = 16383	Σe_k = 16381.984	$\chi^2 = \Sigma(f_k - e_k)^2 / e_k$ = 173.003

Table II.7 – Sample data for the error interval property (type C)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
0	27	3855	3873.101	0.085
27	54	2962	2957.462	0.007
54	81	2249	2258.289	0.038
81	108	1756	1724.408	0.579
108	135	1277	1316.741	1.199
135	162	1013	1005.451	0.057
162	189	808	767.752	2.11
189	216	569	586.248	0.507
216	243	473	447.653	1.435
243	270	331	341.824	0.343
270	297	257	261.013	0.062
297	324	199	199.307	0
324	351	149	152.189	0.067
351	378	110	116.21	0.332
378	405	88	88.737	0.006
405	432	65	67.759	0.112
432	459	52	51.74	0.001
459	486	37	39.508	0.159
486	513	33	30.168	0.266

Table II.7 – Sample data for the error interval property (type C)

Interval		Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2 / e_k$
Begin	End			
513	540	28	23.036	1.07
540	567	13	17.59	1.198
567	594	16	13.432	0.491
594	621	12	10.256	0.296
621	648	7	7.832	0.088
648	702	10	10.546	0.028
702	756	6	6.149	0.004
756	---	8	5.676	0.951
Total		$\sum f_k$ = 16383	$\sum e_k$ = 16380.076	$\chi^2 = \sum(f_k - e_k)^2 / e_k$ = 11.492

II.4 Conclusion

The χ^2 goodness of fit test is described as a method for evaluating errors generated by an error generator for the exponential distribution of the nearest neighbour error interval. The test examples are shown. Errors generated using the type B method were shown to fit using the error frequency property but did not fit using the error interval property, indicating the importance of evaluation using both methods.

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