# Series G 

SERIES G: TRANSMISSION SYSTEMS AND MEDIA, DIGITAL SYSTEMS AND NETWORKS

## Variable bit rate calculations for ITU-T Recommendation G. 767 Digital Circuit Multiplication Equipment (DCME)

Supplement 38 to
ITU-T G-series Recommendations
(Previously CCITT Recommendation)

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## SUPPLEMENT 38 TO ITU-T G-SERIES RECOMMENDATIONS

VARIABLE BIT RATE CALCULATIONS FOR ITU-T RECOMMENDATION G. 767 DIGITAL CIRCUIT MULTIPLICATION EQUIPMENT (DCME)

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## Supplement 38 to G-series Recommendations

## VARIABLE BIT RATE CALCULATIONS FOR ITU-T RECOMMENDATION G. 767 DIGITAL CIRCUIT MULTIPLICATION EQUIPMENT (DCME)

(Geneva, 1998)

This Supplement explains how the VBR equations were derived. The same notations of Recommendation G. 767 are used here. The notation (X, Y, Z) shall be used to note that $N_{16}=\mathrm{X}$, $N_{12.8}=\mathrm{Y}$ and $N_{9.6}=\mathrm{Z}$, i.e. X QBs of the bearer carry traffic at $16 \mathrm{kbit} / \mathrm{s}, \mathrm{Y} \mathrm{QBs}$ at $12.8 \mathrm{kbit} / \mathrm{s}$ and Z QBs at 9.6 kbit/s.

In this combination, $\mathrm{X}+\mathrm{Y}+\mathrm{Z}$ QBs are used to support traffic from up to $X+\frac{5}{4} Y+\frac{5}{3} Z$ BCs.
If $N_{V}$ is greater than $N_{Q B}$, then VBR is required to create the additional overload channels. Creation of the extra channels can be done using several combinations of $N_{16}, N_{12.8}$ and $N_{9.6}$.
If, for example, there are 19 BCs that need bearer resources and 15 available QBs , then several combinations exist that use the available resources and support this traffic. Some combinations create more BCs than required, like $(2,4,9)$ that creates 22 BCs or $(1,8,6)$ that creates 21 BCs . These combinations support all traffic from all BCs but the average bit rate is not optimal since additional unused BCs are created. Obviously, a combination that creates too few BCs is not acceptable. Other combinations do not use all available QBs but still support all traffic, like $(4,4,6)$ that uses only 14 QBs of the list to create 19 BCs , but, again, such combinations lower the bit rate as the bearer resources are not optimally utilized.

One exceptional case needs special attention: If $N_{Q B}$ is a multiplier of 3 and also $N_{V}=\frac{5}{3} N_{Q B}-1$, there is no combination that creates exactly $N_{V} \mathrm{BCs}$ and an additional unused BC must be created. For example, if $N_{Q B}=3$, it is possible to create 3 channels (each QB carries one voice channel at $16 \mathrm{kbit} / \mathrm{s}$ ) or 5 channels (the 3 QBs carry five $9.6 \mathrm{kbit} / \mathrm{s}$ channels), but it is not possible to create exactly 4 channels.

To this point, an optimized combination is one that provides the highest bit rate and highest utilization supporting traffic from all BCs in the voice list. This means that no unnecessary additional BCs are created and all the QBs of the list are used.

Given $N_{V}$ and $N_{Q B}$ there still may be more than one combination that meets these requirements. Advancing the above example, both $(9,0,6)$ and $(4,8,3)$ use all 15 QBs, create no unused BCs and provide the same bit rate -1.58 bit/sample.

So another criterion is required to choose a unique combination. The criterion will be: Prefer creation of as few $9.6 \mathrm{kbit} / \mathrm{s}$ channels as possible. In the given example, the second combination is favourable since it uses less $9.6 \mathrm{kbit} / \mathrm{s}$ channels, even though it also uses less $16 \mathrm{kbit} / \mathrm{s}$ than the first combination. The motivation to use this criterion is that linear decrease of the bit rate might result in a steeper deterioration of speech quality so, when more $9.6 \mathrm{kbit} / \mathrm{s}$ channels are used, even when the average bit rate is maintained, the total quality decreases.
To sum, the criteria for choosing one unique combination of $N_{16}, N_{12.8}$ and $N_{9.6}$ are:
a) Maximize instantaneous average bit rate of the traffic while using available bearer resources.
b) Create enough channels for all BCs in the voice list.
c) Minimize the number of $9.6 \mathrm{kbit} / \mathrm{s}$ channels.

The formal representation of these criteria is:
a)

$$
\max \left(16 \times N_{16}+12.8 \times \frac{5}{4} N_{12.8}+9.6 \times \frac{5}{3} N_{9.6}\right)
$$

b)

$$
N_{16}+\frac{5}{4} N_{12.8}+\frac{5}{3} N_{9.6} \geq N_{V}
$$

c) $\quad \min \left(N_{9.6}\right)$

Another obvious requirement is:
d) $\quad N_{16}$ is an integer; $N_{12.8}$ is an integer multiplier of 4; and $N_{9.6}$ is an integer multiplier of 3 .

The first criterion is satisfied when all bearer resources are consumed, so that:
a) $\quad N_{16}+N_{12.8}+N_{9.6}=N_{Q B}$

The special case, when an unused BC must be created, is handled in the following manner: If $N_{Q B}$ is a multiplier of 3 and also $N_{V}=\frac{5}{3} N_{Q B}-1$, then $N_{V}$ is adjusted by adding 1 to its value. After handling this special case, the second criterion is satisfied when exactly the number of required channels are created, so that:
b)

$$
N_{16}+\frac{5}{4} N_{12.8}+\frac{5}{3} N_{9.6}=N_{V}
$$

In order to find the unique combination, the third criterion is handled first:
$N_{9.6}$ is an integral multiple of 3 , say $N_{9.6}=3 \times I_{1}$ where $I_{1}$ is an integer. Then exactly $5 \times I_{1} 9.6 \mathrm{kbit} / \mathrm{s}$ channels are created. The maximum number of additional channels that can be created will be achieved if as many of these additional channels will be at $12.8 \mathrm{kbit} / \mathrm{s}$ and the rest at $16 \mathrm{kbit} / \mathrm{s}$. The maximum number of channels at $12.8 \mathrm{kbit} / \mathrm{s}$ is $5 \times \operatorname{int}\left(\left(N_{Q B}-3 \times I_{1}\right) / 4\right)$ which leaves $\left(N_{Q B}-3 \times I_{1}\right)-4 \times \operatorname{int}\left(\left(N_{Q B}-3 \times I_{1}\right) / 4\right)$
channels at $16 \mathrm{kbit} / \mathrm{s}$. It is required that the sum of the $9.6 \mathrm{kbit} / \mathrm{s}, 12.8 \mathrm{kbit} / \mathrm{s}$ and $16 \mathrm{kbit} / \mathrm{s}$ channels will be at least $N_{v}$ :

$$
\begin{gathered}
5 \times I_{1}+5 \times \operatorname{int}\left(\left(N_{Q B}-3 \times I_{1}\right) / 4\right)+\left(N_{Q B}-3 \times I_{1}\right)-4 \times \operatorname{int}\left(\left(N_{Q B}-3 \times I_{1}\right) / 4\right) \geq N_{V} \\
\Rightarrow 2 \times I_{1}+N_{Q B}+\operatorname{int}\left(\left(N_{Q B}-3 \times I_{1}\right) / 4\right) \geq N_{V}
\end{gathered}
$$

$2 \times I_{1}$ is an integer. $N_{Q B}$ and $N_{V}$ are also integers so all three quantities can be moved inside the parentheses and the equation is still valid so we get:

$$
\operatorname{int}\left(\frac{5}{4} I_{1}+\frac{5}{4} N_{Q B}-N_{V}\right) \geq 0
$$

The minimal $I_{1}$ that satisfies this equation is:

$$
I_{1}=\frac{4}{5} N_{V}-N_{Q B}
$$

Since $I_{1}$ is an integer, the solution is:

$$
I_{1}=\operatorname{int}\left(\frac{4}{5} N_{V}-N_{Q B}\right)+\alpha
$$

where $\alpha=0$ if $\frac{4}{5} N_{V}-N_{Q B}$ is an integer (because this way the value inside the parentheses is exactly zero) or $\alpha=1$ otherwise.
If $I_{1}$ is smaller than zero, then it is set to zero since the number of QBs cannot be negative.
After setting the value of $N_{9.6}$, the first two criteria are simply two linear equations with two variables, and their solution is:

$$
\begin{gathered}
N_{12.8}=4\left(N_{V}-N_{Q B}-\frac{2}{3} N_{9.6}\right) \\
N_{16}=N_{Q B}-N_{12.8}-N_{9.6}
\end{gathered}
$$

## Example \#1

Case of $\mathrm{N}_{Q^{B}}=10$ and $\mathrm{N}_{V}=13$
$I_{1}=\frac{4}{5} N_{V}-N_{Q B}=\frac{4}{5} 13-10=0.4$
0.4 is not an integer therefore $I_{1}=1.4$
$N_{9.6}=3 \times \operatorname{int}\left(I_{1}\right)=3$
$N_{12.8}=4\left(N_{V}-N_{Q B}-\frac{2}{3} N_{9.6}\right)=4\left(13-10-\frac{2}{3} 3\right)=4$
$N_{16}=N_{Q B}-N_{12.8}-N_{9.6}=10-4-3=3$
Assume $\mathrm{IT}=120$ and $\mathrm{BC}=54$. Then:

$$
P_{V}=(B C+I T) \bmod N_{V}=(120+54) \bmod 13=5
$$

And the VBR mapping is as follows:

| $N_{Q B}=10, N_{V}=13$ <br> $N_{9.6}=3, N_{12.8}=4, N_{16}=3$ <br> $\mathrm{IT}=120, \mathrm{BC}=54$ <br> $P_{V}=5$ | BCs number | mapped to QBs <br> number | at rate <br> $[\mathbf{k b i t / s} \mathbf{]}$ |
| :---: | :---: | :---: | :---: |
|  | 5 | 1 | 16 |
|  | 6 | 2 | 16 |
|  | $8,9,10,11$ and 12 | $4,5,6$ and 7 | 12.8 |
|  | $13,1,2,3$ and 4 | 8,9 and 10 | 9.6 |

## Example \#2

Case of $\mathrm{N}_{Q B}=15$ and $\mathrm{N}_{V}=18$
$I_{1}=\frac{4}{5} N_{V}-N_{Q B}=\frac{4}{5} 18-15=-0.6$
-0.6 is smaller than zero, therefore $I_{1}=0$
$N_{9.6}=3 \times \operatorname{int}\left(I_{1}\right)=0$ (creation of $9.6 \mathrm{kbit} / \mathrm{s}$ channels is not required)
$N_{12.8}=4\left(N_{V}-N_{Q B}-\frac{2}{3} N_{9.6}\right)=4\left(18-15-\frac{2}{3} 0\right)=12$
$N_{16}=N_{Q B}-N_{12.8}-N_{9.6}=15-12-0=3$
Assume $\mathrm{IT}=79$ and $\mathrm{BC}=136$. Then:

$$
P_{V}=(B C+I T) \bmod N_{V}=(79+136) \bmod 18=17
$$

And the VBR mapping is as follows:

| $\begin{gathered} N_{Q B}=15, N_{V}=18 \\ N_{9.6}=0, N_{12,8}=12, N_{16}=3 \end{gathered}$ | BCs number | mapped to QBs number | at rate [kbit/s] |
| :---: | :---: | :---: | :---: |
|  | 17 | 1 | 16 |
|  | 18 | 2 | 16 |
|  | 1 | 3 | 16 |
| $\begin{gathered} \mathrm{IT}=79, \mathrm{BC}=136 \\ P_{v}=17 \end{gathered}$ | 2, 3, 4, 5 and 6 | 4, 5, 6 and 7 | 12.8 |
|  | 7, 8, 9, 10 and 11 | 8, 9, 10 and 11 | 12.8 |
|  | 12, 13, 14, 15 and 16 | 12, 13, 14 and 15 | 12.8 |

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[^0]:    Source
    Supplement 38 to ITU-T G-series Recommendations, was prepared by ITU-T Study Group 15 (1997-2000) and was approved under the WTSC Resolution No. 5 procedure on the $13^{\text {th }}$ of October 1998.

