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TELEPHONE SERVICE, SERVICE OPERATION AND
HUMAN FACTORS

Traffic engineering – Determination of the number of
circuits in automatic and semi-automatic operation

**OVERFLOW APPROXIMATIONS FOR NON-
RANDOM INPUTS**

Reedition of CCITT Recommendation E.524 published in
the Blue Book, Fascicle II.3 (1988)

NOTES

1 CCITT Recommendation E.524 was published in Fascicle II.3 of the *Blue Book*. This file is an extract from the *Blue Book*. While the presentation and layout of the text might be slightly different from the *Blue Book* version, the contents of the file are identical to the *Blue Book* version and copyright conditions remain unchanged (see below).

2 In this Recommendation, the expression “Administration” is used for conciseness to indicate both a telecommunication administration and a recognized operating agency.

OVERFLOW APPROXIMATIONS FOR NON-RANDOM INPUTS

1 Introduction

This Recommendation introduces approximate methods for the calculation of blocking probabilities for individual traffic streams in a circuit group arrangement. It is based on contributions submitted in the Study Period 1984-1988 and is expected to be amended and expanded in the future (by adding the latest developments of methods).

The considered methods are necessary complements to those included in the existing Recommendation E.521 when it is required to take into account concepts such as cluster engineering with service equalization, service protection and end-to-end grade of service. Recommendation E.521 is then insufficient as it is concerned with the grade of service for only one non-random traffic stream in a circuit group.

Design methods concerning the above-mentioned areas are subject to further study and this Recommendation will serve as a reference when, in the future, Recommendation E.521 is complemented or replaced.

In this Recommendation the proposed methods are evaluated in terms of accuracy, processing time, memory requirements and programming effort. Other criteria may be relevant and added in the future.

The proposed methods are described briefly in § 2. Section 3 defines a set of examples of circuit group arrangements with exactly calculated (exact resolution of equations of state) individual blocking probabilities, to which the result of the methods can be compared. This leads to a table in § 4, where for each method the important criteria are listed. The publications cited in the reference section at the end contain detailed information about the mathematical background of each of the methods.

2 Proposed methods

The following methods are considered:

- a) Interrupted Poisson Process (IPP) method,
- b) Equivalent Capacity (EC) method,
- c) Approximative Wilkinson Wallstrom (AWW) method.

2.1 IPP method

IPP (Interrupted Poisson Process) is a Poisson process interrupted by a random switch. The on-/off-duration of the random switch has a negative exponential distribution. Overflow traffic from a circuit group can be accurately approximated by an IPP, since IPP can represent bulk characteristics of overflow traffic. IPP has three parameters, namely, on-period intensity and mean on-/off-period durations. To approximate overflow traffic by an IPP, those three parameters are determined so that some moments of overflow traffic will coincide with those of IPP.

The following two kinds of moment match methods are considered in this Recommendation:

- three-moment match method [1] – where IPP parameters are determined so that the first three moments of IPP will coincide with those of overflow traffic;
- four-moment ratio match method [2] where IPP parameters are determined so that the first moment and the ratios of the 2nd/3rd and 7th/8th binomial moments of IPP will coincide with those of overflow traffic.

To analyze a circuit group where multiple Poisson and overflow traffic streams are simultaneously offered, each overflow stream is approximated by an IPP. The IPP method is well suited to computer calculation. State transition equations of the circuit group with IPP inputs can be solved directly and no introduction of equivalent models is necessary. Characteristics of overflow traffic can be obtained from the solution of state transition equations. The main feature of the IPP method is that the individual means and variances of the overflow traffic can be solved.

2.2 EC method

The EC (Equivalent Capacity) method [3] does not use the traffic-moments but the transitional behaviour of the primary traffic, by introducing a certain function $\rho(n)$ versus the equivalent capacity (n) of the partial overflow traffic, as defined by the recurrent process:

$$\left[\begin{array}{l} \rho(o) = Em(\alpha) \quad [\text{Erlang loss formula}] \\ \frac{n}{\rho(n)} = (m + n - a) + \alpha \cdot \rho(n - 1) \end{array} \right. \quad (2-1)$$

if n is a positive integer and approximated by linear interpolation, if not.

A practical approximation, considering the predominant overflow congestion states only, leads to the equations:

$$\frac{n_i}{n} = \frac{a_i \rho_i(n_i) / D_i(n_i + 1)}{\sum_{k=1}^x a_k \rho_k(n_k) / D_k(n_k + 1)} \quad (2-2)$$

with

$$D_i(n) = 1 + a_i [\rho_i(n) - \rho_i(n - 1)] \quad (2-3)$$

defining the equivalent capacity (n_i) of the partial overflow traffic labelled i , and influenced by the mutual dependency between the partial overflow traffic streams.

The mean value of the partial second overflow is:

$$O_i = a_i \pi \rho_i(n_i) \quad (2-4)$$

where π It is the time congestion of the overflow group.

The partial GOS (grade of service) equalization is fulfilled if:

$$\rho_i(n_i) = C \quad (2-5)$$

C being a constant to be chosen.

2.3 A WW method

The AWW (Approximative Wilkinson WallstrOm) method uses an approximate ERT (Equivalent Random Traffic) model based on an improvement of Rapp's approximation. The total overflow in traffic is split up in the individual parts by a simple expression, see Equations (2-7) and (2-9). To calculate the total overflow traffic, any method can be used. An approximate Erlang formula calculation for which the speed is independent of the size of the calculated circuit group is given in [4].

The following notations are used:

- M mean of total offered traffic;
- V variance of total offered traffic;
- Z V/M ;
- B mean blocking of the studied group;
- m_i, v_i, z_i, b_i corresponding quantities for an individual traffic stream;
- \sim is used for overflow quantities.

2.3.1 Blocking of overflow traffic

For overflow calculations, an approximate ERT-model is used. By numerical investigations, a considerable improvement has been found to Rapp's classical approximation for the fictitious traffic. The error added by the approximation is small compared to the error of the ERT-model. It is known that ERT underestimates low blockings when mixing traffic of diverse peakedness [2]. The formula, which was given in [4] (although with one printing error), is for $Z > 1$:

$$A^* \approx V + Z(Z - 1)(2 + \gamma^\beta)$$

where

$$\gamma = (2.36Z - 2.17) \log \{1 + (Z - 1)/[M(Z + 1.5)]\}$$

and

$$\beta = Z/(1.5M + 2Z - 1.3) \quad (2-6)$$

2.3.2 Wallstrom formula for individual blocking

There has been much interest in finding a simple and accurate formula for the individual blocked traffic \tilde{m}_i . Already in 1967, Katz [5] proposed a formula of the type

$$\tilde{m}_i = m_i B(1 - w + wz_i/Z) \quad (2-7)$$

with w being a suitable expression. Wallström proposed a very simple one but with reasonable results [6], [2]:

$$w = 1 - B \quad (2-8)$$

One practical problem is, however, that a small peaked substream could have a blocking $b_i > 1$ with this formula. To avoid such unreasonable results a modification is used in this case. Let z_{\max} be the largest individual z_i .

Then the value used is

$$w = \begin{cases} 1 - B & \text{if } z_{\max} < Z(1 + B)/B \\ Z(1 - B)/(B(z_{\max} - Z)) & \text{otherwise} \end{cases} \quad (2-9)$$

2.3.3 Handling of overflow variances

For the calculation of a large network it would be very cumbersome to keep track of all covariances. The normal case is that the overflow traffic from one trunk group is either lost or is offered to a secondary group without splitting up. Therefore it is practical to include covariances in the individual overflow parameters \tilde{v}_i so that they sum up to the total variance. The quantities v_i are obtained from the total overflow variance \tilde{V} by a simple splitting formula:

$$\tilde{v}_i = \tilde{V} v_i/V \quad (2-10)$$

One can prove that Wallstrom's splitting formula (2-8) and formula (2-10) together with the ERT-model satisfies a certain consistency requirement. One will obtain the same values for the individual blocked traffic when calculating a circuit group of $N_1 + N_2$ circuits as when calculating first the N_1 circuits and then offering the overflow to the N_2 circuits.

Since the individual variances are treated in this manner, they are not comparable with the results reported in Table 2/E.524.

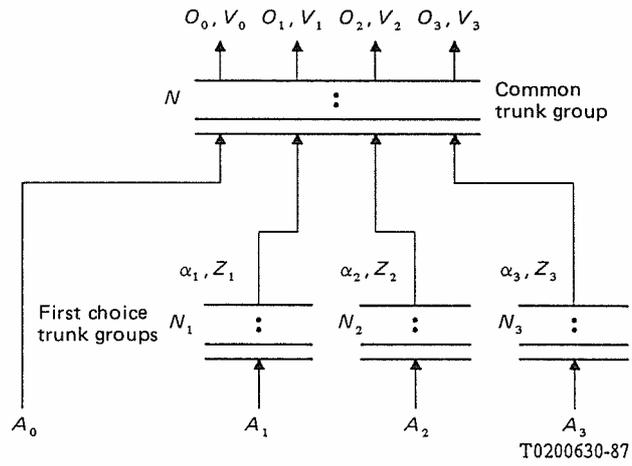
3 Examples and criteria for comparison

The defined methods are tested by calculating the examples given in Table 1/E.524.

The calculation model is given in Figure 1/E.524.

For comparison, the following criteria are established:

- accuracy of the overflow traffic mean and variance (mean and standard deviation),
- computational criteria (processor time, memory requirements, programming effort).



- A_i : Offered Poisson traffic volume
- N_i : Number of first choice trunk group
- α_i : Mean of overflow traffic from first choice trunk group
- Z_i : Peakedness of overflow traffic from first choice trunk group
- N : Number of common trunk group
- O_i : Mean of overflow traffic from common trunk group
- V_i : Variance of overflow traffic from common trunk group

FIGURE 1/E.524

Calculation model

TABLE 1a/E.524

Exactly calculated mean and variance of individual overflow traffic – Three first choice circuit groups

Case	A_1	A_2	A_3	α_1	α_2	α_3	A_0	N	O_0	O_1	O_2	O_3
	N_1	N_2	N_3	Z_1	Z_2	Z_3			V_0	V_1	V_2	V_3
1	7.036	26.688	64.169	3.003	3.001	3.000	-	11	-	0.4337	0.7490	1.091
	5	28	70	1.573	3.022	4.527			-	0.7656	2.110	4.441
2	7.036	26.688	64.169	3.003	3.001	3.000	-	16	-	0.1149	0.2758	0.4944
	5	28	70	1.573	3.022	4.527			-	0.2436	0.7328	1.911
3	7.036	26.688	64.169	3.003	3.001	3.000	-	25	-	0.01369	0.02846	0.06627
	5	28	70	1.573	3.022	4.527			-	0.02041	0.06461	0.2205
4	7.036	10.176	13.250	3.003	5.003	7.002	-	14	-	0.7459	1.262	1.785
	5	6	7	1.573	1.567	1.559			-	1.193	2.292	3.624
5	7.036	10.176	13.250	3.003	5.003	7.002	-	19	-	0.2884	0.4857	0.6832
	5	6	7	1.573	1.567	1.559			-	0.4636	0.9089	1.460
6	7.036	10.176	13.250	3.003	5.003	7.002	-	26	-	0.03570	0.05915	0.08237
	5	6	7	1.573	1.567	1.559			-	0.05358	0.1026	0.1621
7	7.036	32.395	77.617	3.003	5.002	7.001	-	16	-	0.4516	1.176	2.344
	5	31	77	1.573	3.029	4.511			-	0.7434	3.466	10.39

TABLE 1a/E.524 (cont.)

Case	A_1	A_2	A_3	α_1	α_2	α_3	A_0	N	O_0	O_1	O_2	O_3
	N_1	N_2	N_3	Z_1	Z_2	Z_3			V_0	V_1	V_2	V_3
8	7.036	32.395	77.617	3.003	5.002	7.001	-	23	-	0.1538	0.4294	0.9739
	5	31	77	1.573	3.029	4.511	-		-	0.2427	1.200	4.219
9	7.036	32.395	77.617	3.003	5.002	7.001	-	35	-	0.01303	0.03984	0.1006
	5	31	77	1.573	3.029	4.511	-		-	0.1841	0.09378	0.3690
10	64.169	32.395	13.250	3.000	5.002	7.002	-	15	-	1.157	1.456	1.320
	70	31	7	4.527	3.029	1.559	-		-	4.442	4.256	2.850
11	64.169	32.395	13.250	3.000	5.002	7.002	-	21	-	0.5564	0.5849	0.4749
	70	31	7	4.527	3.029	1.559	-		-	2.026	1.675	1.023
12	64.169	32.395	13.250	3.000	5.002	7.002	-	32	-	0.06907	0.05265	0.03848
	70	31	7	4.527	3.029	1.559	-		-	0.2167	0.1295	0.07165
13	7.036	26.688	64.169	3.003	3.001	3.000	3.000	13	0.4064	0.5038	0.8274	1.160
	5	28	70	1.573	3.022	4.527	-		0.5578	0.8566	2.243	4.574
14	7.036	26.688	64.169	3.003	3.001	3.000	3.000	18	0.1460	0.1840	0.3384	0.5729
	5	28	70	1.573	3.022	4.527	-		0.1992	0.3043	0.8779	2.163

TABLE 1a/E.524 (cont.)

Case	A_1	A_2	A_3	α_1	α_2	α_3	A_0	N	O_0	O_1	O_2	O_3
	N_1	N_2	N_3	Z_1	Z_2	Z_3			V_0	V_1	V_2	V_3
15	7.036	26.688	64.169	3.003	3.001	3.000	3.000	28	0.01170	0.01506	0.03086	0.07035
	5	28	70	1.573	3.022	4.527			0.01472	0.02218	0.06861	0.2287
16	7.036	32.395	77.617	3.003	5.002	7.001	1.000	17	0.1253	0.4451	1.156	2.304
	5	31	77	1.573	3.029	4.511			0.1392	0.7266	3.366	10.10
17	7.036	32.395	77.617	3.003	5.002	7.001	1.000	24	0.04250	0.1536	0.4275	0.9674
	5	31	77	1.573	3.029	4.511			0.04696	0.2409	1.183	4.148
18	7.036	32.395	77.617	3.003	5.002	7.001	1.000	35	0.004542	0.01687	0.05106	0.1282
	5	31	77	1.573	3.029	4.511			0.004891	0.02398	0.1214	0.4751
19	64.169	32.395	13.250	3.000	5.002	7.002	9.000	21	1.761	1.251	1.654	1.630
	70	31	7	4.527	3.029	1.559			3.052	4.517	4.406	3.103
20	64.169	32.395	13.250	3.000	5.002	7.002	9.000	28	0.6761	0.6501	0.7389	0.6427
	70	31	7	4.527	3.029	1.559			1.253	2.225	1.956	1.279
21	64.169	32.395	13.250	3.000	5.002	7.002	9.000	40	0.06219	0.09577	0.07978	0.06069
	70	31	7	4.527	3.029	1.559			0.1054	0.2884	0.1887	0.1099

TABLE 1b/E.524

Exactly calculated mean and variance of individual overflow traffic – Two first choice circuit groups

A_1	N_1	A_2	N_2	N	O_1	V_1	O_2	V_2		
8.2	5	30.0	30	10	0.6155	1.1791	1.1393	3.4723		
				5	1.8068	3.2634	2.4656	7.4312		
				21	0.0188	0.0304	0.0485	0.1240		
				14	0.2108	0.3898	0.4624	1.3701		
		14.3	7	42.0	37	22	0.0470	0.0771	0.0929	0.1983
						16	0.3743	0.6602	0.7546	1.7626
						12	0.9282	1.6137	1.8320	4.2120
						7	2.0023	3.2718	4.0953	7.8064
						27	0.0230	0.0354	0.0978	0.2984
						19	0.2136	0.3683	0.8356	2.9450
30.0	30	14.3	7	8	1.4984	2.6161	4.4363	14.6018		
				13	0.6940	1.2375	2.4148	8.4923		
				25	0.0653	0.1613	0.0541	0.1112		
				18	0.4664	1.2990	0.4662	1.0879		
8.2	5	67.9	65	12	1.3746	3.9321	1.7390	4.0015		
				7	2.4255	6.9941	3.8063	7.6277		
				14	0.5385	0.9676	2.4438	10.7208		
51.5	54	14.3	7	8	1.3598	1.4401	4.7035	19.7109		
				27	0.0735	0.2239	0.0399	0.0802		
				19	0.6404	1.2499	0.4699	1.1030		
				13	1.4033	5.0795	1.3609	3.2229		
				7	2.5873	9.6136	3.6744	7.5139		

TABLE 1c/E.524

Exactly calculated mean and variance of individual overflow traffic – One first choice circuit group

A_1	N_1	A_0		N	O_1	V_1	O_0	V_0
8.2	5	4.0		16	0.0499	0.0872	0.0331	0.0479
				11	0.4859	0.9154	0.3494	0.5382
				9	1.1692	2.1202	0.9011	1.3274
				5	2.1422	3.5883	1.8018	2.3694
30.0	30			20	0.0601	0.1565	0.0167	0.023
				13	0.5804	1.7427	0.1990	0.3062
				9	1.3997	4.2546	0.5988	0.9338
				5	2.5579	5.6196	1.5661	2.1991
51.5	54			22	0.9751	0.2497	0.0144	0.0197
				15	0.5141	1.8924	0.1209	0.1819
				10	1.8820	5.3004	0.4297	0.6790
				5	2.4294	3.2974	1.1450	1.7255

4 Summary of results

The available methods and the performance measures with respect to the criteria are listed in Table 2/E.524.

TABLE 2/E.524

Comparison of different approximation methods

Functions Method	Input	Output	Comparison							
	Required higher moments	Highest moments of overflow traffic	Overflow traffic error				Computational effort			
			Mean		Variance					
			Mean	Standard deviation	Mean	Standard deviation	Processor time	Memory requirements	Programming effort	
IPP method										
a) 3 moment match	3	3	-0.0045	0.0585	-0.0210	0.0922				
b) 4 moment ratio	8	∞	0.0008	0.0255	-0.0053	0.0373				
EC method	1	1	-0.0661	0.1527						
AWW method	2	2	-0.0448	0.1647						

References

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