Measurement of

Traffic Distribution Matrix

in Alternative Routing Networks

(Exercise included)

Mr. H. Leijon, ITU

UNION INTERNATIONALE DES TELECOMMUNICATIONS
INTERNATIONAL TELECOMMUNICATION UNION
UNION INTERNACIONAL DE TELECOMUNICACIONES

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CONTENTS

1. INTRODUCTION

For planning of multi-exchange networks the traffic distribution in the network must be known i.e. a traffic matrix must be established. This can be done by measurement in the existing network, but for networks with alternative routing, bad grade of service and repeated attempts form the subscribers it is a difficult job.

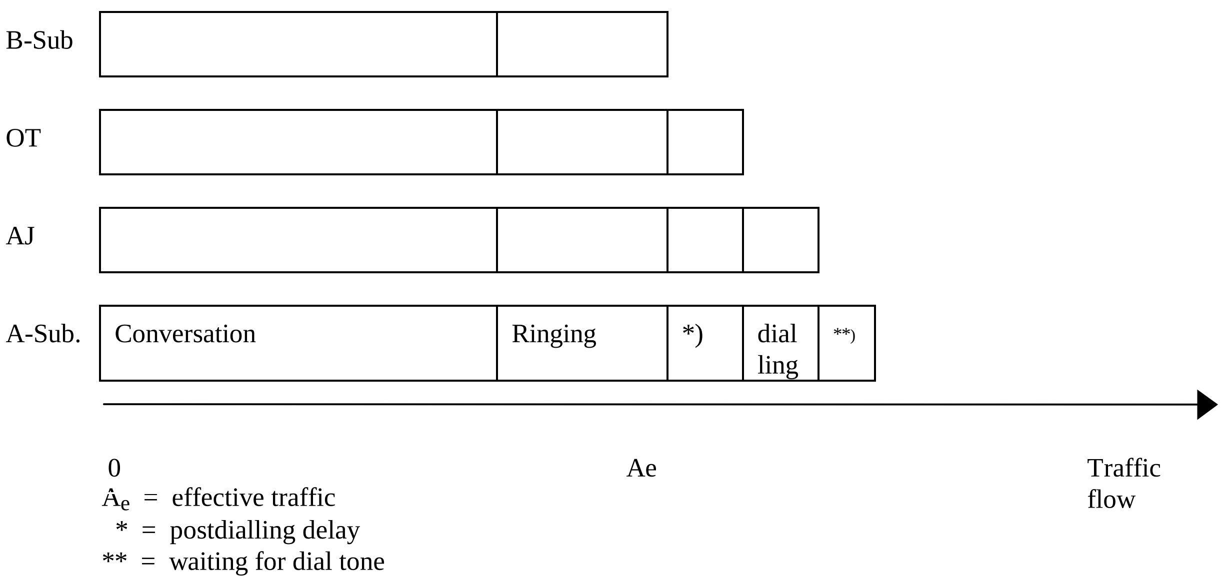
In practice the traffic figures in a matrix are often measured in different ways, with different equipments and with different precisions.

The scope of this paper is not tot give a list of measurements which must be done, it is merely to show the optimal use of available measurement information giving more weight to the more precise measurements.

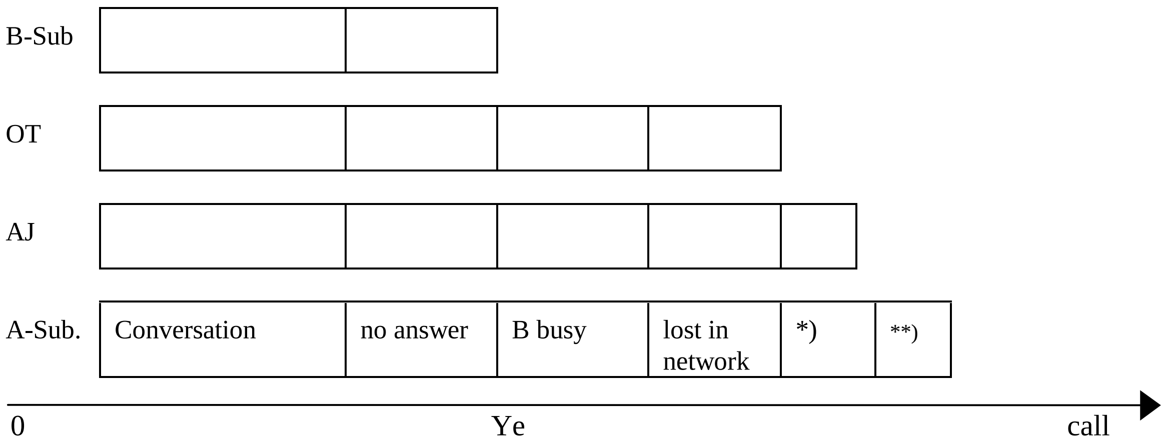
The traffic-matrix treated in the following is a matrix for carried, “effective” traffic which is common to A-and B-subscriber. Register traffic and PDD-traffic, “signalling traffic”, is excluded. See Figure 1.1. Using this convention, the originating and terminating traffic in the network are equal, as they must be in a matrix.

When using the matrix for design of network and exchanges the signalling traffic must be added separately.

TRAFFIC FLOW



CALL FLOW



Y_e = effective calls
*) = incomplete dialling
**) = register congestion

Fig. 1.1

This paper covers the use of ordinary equipment for erlang-measurement, more or less advanced number analysing equipment and to some extent of traffic route tester.

All measurements are assumed to be done simultaneously during busy hours in busy seasons.

The tandem network can have arbitrary design in several levels and combined local and tandem exchanges are permitted, though not used in this paper.

Transit exchanges for LD-traffic can be treated as ordinary local exchanges by considering the outside world analogous to the subscribers of a local exchange (traffic source and drain).

The methods used are illustrated by examples and the use of mathematics is kept at a minimum.

Fig 1.2. shows the basic traffic model for a local exchange without transit traffic.

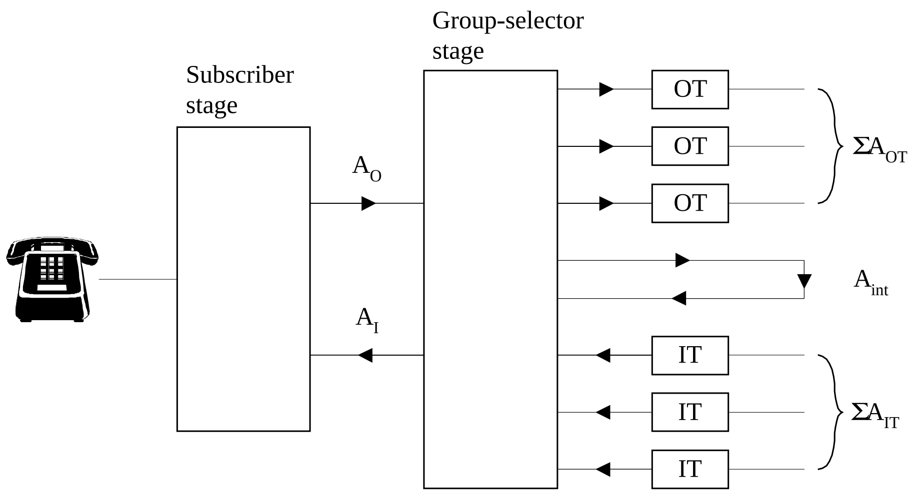


Fig. 1.2

A_O = originating traffic

A_I = terminating traffic

$$A_O = \Sigma A_{OT} + A_{int} \qquad (1.1)$$

$$A_I = \Sigma A_{IT} + A_{int} \qquad (1.2)$$

Register traffic is excluded.

For all local and transit exchanges together we have

$$\Sigma A_{OT} = \Sigma A_{IT} \qquad (1.3)$$

and

$$\Sigma A_O = \Sigma A_I \qquad (1.4)$$

For the purpose in this document it is necessary that eqs. (1.1)-(1.4) are fulfilled exactly. In practice the measurement results will have small deviations which must be corrected before starting the calculations.

A traffic matrix is a square lattice with one row and one column per exchange. An exchange is defined as a traffic source and/or traffic drain, thus tandem exchanges are not represented in the matrix. The traffic value in row *i* and column *j* is the traffic <u>from</u> exchange *i* <u>to</u> exchange *j* . The value is independent of the routing i.e. whether the traffic goes over tandem or not.

The sum of traffics in row *i* is the total originating traffic A_0 in exchange nr *i* and the sum of traffics in column *j* is the total terminating traffic A_I in exchange nr *j* .

Also routing information can be given in the matrix. *D* means direct low loss route, *H* means high usage routing with overflow over tandem and *T* means pure tandem routing i.e. the two exchanges have no direct circuits between them.

Now the building-up of a traffic matrix shall be described in short terms.

1. The number of exchanges, the routing between them and the carried traffics on all routes are known. Then the matrix-lattice can be drawn, the routing letters put in the routing matrix and the row and column sums put into the traffic matrix.

   Also all the *D*-traffics (direct low loss routing) can be put into the traffic matrix which is now partly filled in.

2. For the *H*-cases the carried traffic A_h on the high-usage route is known and the overflow-traffic A_t is in principle found from the calculated or measured congestion on the *H*-route. See section 2. Thus the total traffic $A_H = A_h + A_t$ for the *H*-cases can be filled into the matrix.

3. The traffics in the remaining *T*-cases are calculated row by row using the given row-sums and the already calculated *D*- and *H*-traffics. In a row the total *T*-traffic ΣA_T is the traffic <u>not</u> belonging to *D*- and *H*- cases. i.e.

$$\sum A_T = A_0 - \sum A_D - \sum A_H$$

   This traffic sum is then distributed on the appropriate terminating exchanges in proportion to e.g. the calling rates to these exchanges. See section 3. This procedure is repeated for every row containing *T*-traffics.

4. Now the matrix is completely filled in, but probably the content disagrees more or less with the prescribed (measured with good precision) row- and column sums. Obviously the matrix is rather approximate, corrections must be done and these corrections shall preferably be done on the more uncertain traffic data. Here we assume the A_T-values to be “worst” and the A_D-values to be “best”. Section 4 describes how this “weighted” correction can be done by using a modification of the Kruithof method.

2 HIGH USAGE TRAFFIC

2.1 Basic ideas

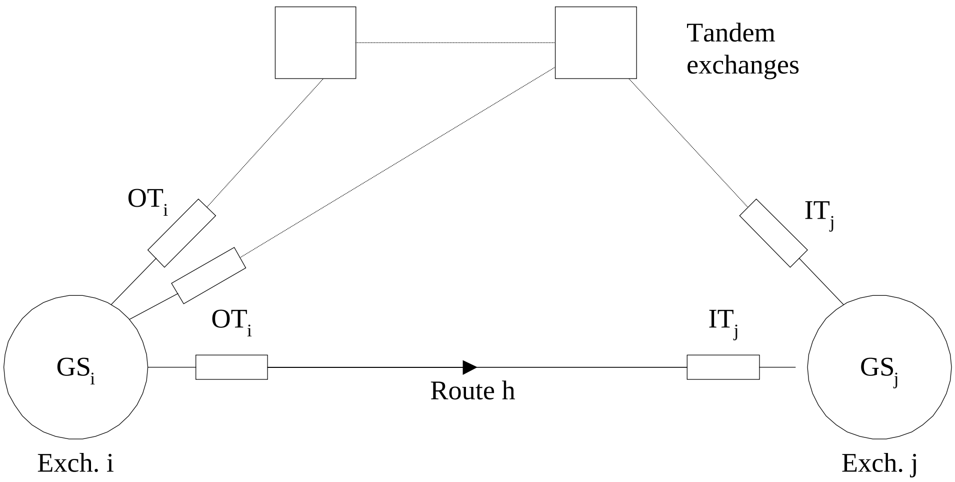


Fig. 2.1

Fig 2.1 shows a typical alternative routing arrangement. Calls to exchange *j* are offered first to route *h* (high usage) and eventually overflowed to the tandem network.

Consider one hour of measurement. During this hour Y calls are offered to route h and Y_h calls lead to seizure.

$$Y_h = Y\,(1\text{-}B_h)$$

B_h is the blocking on route h. The congested $Y \cdot B_h$ calls go to the tandem network and with a blocking B_t in this network the number

$$Y \cdot B_h \cdot (1 - B_t)$$

arrive to IT_j.

Totally Y_H calls arrive to IT_j from OT_1

$$Y_H = Y \cdot (1 - B_h) + Y \cdot B_h \cdot (1 - B_t) \text{ or } Y_H = Y \cdot (1 - B_h) \cdot \frac{1 - B_h \cdot B_t}{1 - B_h} \qquad (2.1)$$

All these calls arrive to IT_j and their holding times are determined exclusively by their fate in exchange j (and by exchange i) but <u>not</u> by their routing between i and j . Using eq (2.1) and the holding times we find a carried traffic

$$A_H = s \cdot Y_H = s \cdot Y \cdot (1 - B_h) \cdot \frac{1 - B_h \cdot B_t}{1 - B_h} \text{ or } A_H = A_h \cdot \frac{1 - B_h \cdot B_t}{1 - B_h} \qquad (2.2)$$

where A_h is the traffic carried on route H and $B_h \cdot B_t$ is the resulting congestion from GV-inlet in exchange i to GS-inlet (IT_j) in exchange j.

Note that eq (2.2) only requires measurement of traffic flow (TKT) and of rejections while the holding time disappeared in the calculations.

For later use we will write the total carried traffic between the exchanges i and j as a sum of high usage traffic A_h and tandem traffic A_t i.e.

$$A_H = A_h + A_t \qquad (2.3)$$

In the above calculations the holding times for blocked calls and the post dialling delays both are assumed to be zero.

3. PROPORTIONAL TRAFFIC DISTRIBUTION

A traffic matrix is formed from a number of rows (vectors) each row giving the distribution of originated traffic for an individual exchange. Some of the elements A_D and A_H of the row are found by direct measurement and the others A_T by making a proportional distribution of their sum

$$A_T = A_0 - \Sigma A_D - \Sigma A_H$$

Several approaches can be used. The simplest is to use terminating traffics as proportional factors.

Suppose that exchange no. 1 has total originating traffic A_O = 500 erl and of this 300 erl is D - or H-traffic i.e. A_T = 500-300 = 200 erl. These 200 erl shall be divided between the exchanges 2,5 and 8 which are the only exchanges to which no direct circuits exist. Their terminating traffics are

$$A_{I2} = 200 \ erl$$

$$A_{I5} = 600 \ erl$$

$$A_{I8} = 1100 \ erl$$

and the traffics from exchange 1 are calculated as

$$A_{12} = 200 \cdot \frac{200}{200 + 600 + 1100} = 21 \ \ erl$$

$$A_{15} = 600 \cdot \frac{200}{200 + 600 + 1100} = 63 \ \ erl$$

$$A_{18} = 1100 \cdot \frac{200}{200 + 600 + 1100} = 116 \ \ erl$$

Another method is to use the calling rates as proportional factors. With the same example as before we have e.g.

$$Y_{12} = 7000 \ \ c/h \qquad A_{12} = 7000 \cdot \frac{200}{7000 + 18000 + 35000} = 23 \ \ erl$$

$$Y_{15} = 18000 \ \ c/h \qquad A_{15} = 18000 \cdot \frac{200}{7000 + 18000 + 35000} = 60 \ \ erl$$

$$Y_{18} = 35000 \ \ c/h \qquad A_{18} = 35000 \cdot \frac{200}{7000 + 18000 + 35000} = 117 \ \ erl$$

If the mean holding times h in different directions are much different it will pay to estimate h and use the proportional values

$Y_{12}\, h_{12}$, $Y_{15}\, h_{15}$ and $Y_{18}\, h_{18}$

instead of Y_{12} , Y_{15} and Y_{18}

The choice of method is dependent on the available measuring equipment.

4 KRUITHOF METHODS

4.1 Classical method

By forming traffic matrices the problem often arises that the content of the matrix, i.e. the individual elements do not agree with prescribed row-and columnsums i.e. total originating and terminating traffics. Fig. 4.1 shows an example.

	i ↗ j	1	2	3	sum is	sum shall be
	1	5	8	10	23	25
	2	2	25	15	42	35
	3	6	3	2	11	15
sum	is	13	36	27	76	—
	shall be	18	30	27	—	75

Fig 4.1

The engineers Kruithof and Furness have proposed a simple solution to this problem.

By alternate multiplication of rows and colums by suitable factors the row- and columnsums are alternatively made correct. After a few steps both row and colum sums are correct.

In the example fig 4.1 the three rows are multiplied by factors 25/23, 35/42, 15/11 respectively thus giving a matrix like fig. 4.2.

	i ↗ j	1	2	3	sum is	sum shall be
	1	5.4	8.7	10.9	25.0	25
	2	1.7	20.8	12.5	35.0	35
	3	8.2	4.1	2.7	15.0	15
sum	is	15.3	33.6	26.1	75.0	—
	shall be	18	30	27	—	75

Fig 4.2

Next time the columns are multiplied by 18/15 • 3 , 30/33 • 6 and 27/26 • 1 thus giving the matrix fig 4.3.

	i ↗ j	1	2	3	sum is	sum shall be
	1	6.4	7.8	11.3	25.5	25
	2	2.0	18.6	12.9	33.5	35
	3	9.6	3.7	2.8	16.1	15
sum	is	18	30.1	27.0	75.1	—
	shall be	18	30	27	—	75

Fig 4.3

After three more iterations the result is like fig 4.4. Which is a matrix rather similar to the original one, but with correct row- and column sums.

	i → j	1	2	3	sum is	sum shall be
	1	6.5	7.5	11.0	25.0	25
	2	2.2	19.3	13.5	35.0	35
	3	9.2	3.3	2.6	15.0	15
sum	is	17.8	30.1	27.1	75.0	—
	shall be	18	30	27	—	75

Fig 4.4

Generally the kruithof-method has the property of removing physical impossibilities and systematic errors from the matrix while random errors in big matrices are almost unaffected.

4.2 Modified Kruithof Method

In the above described operations all elements of the matrix have been changed more or less. In practive this may be undesirable if some elements are known to be rather correct from the beginning.

Generally the matrix in fig 4.1 is a result of more or less accurate measurements and with knowledge of the precision of each element it is possible to state a lower limit for the value of the element. E.g. for H-cases the element is $A_H = A_h + A_t$ and its lower limit could be put to $A_{Hm} = A_h + 0.3\ A_t$ assuming that the real overflow traffic is not less than 30% of the estimated A. For D-cases a limit of $A_{Dm} = 0.95\ A_D$ could be used and for T-cases the limit could be $A_{Tm} = 0.6\ A_T$.

Then the Kruithof adaption should be made with the additional condition that not element should be changed to a value below its preassigned minimum value.

This condition can be fulfilled by removing the minimum values from the matrix before "Kruithof-adaption" and bringing them back again after the adaption. An example shows the principle. The matrix fig 4.1 is rewritten with both assumed and minimum values given. See fig 4.5.

	i → j	1	2	3	sum is	sum shall be
	1	5/4.9	8/7.8	10/5.0	23/17.7	25/17.7
	2	2/1.8	25/20.0	15/12.0	42/33.8	35/33.8
	3	6/4.0	3/2.0	2/1.2	11/7.2	15/7.2
sum	is	13/10.7	36/29.8	27/18.2	76/58.7	———
	shall be	18/10.7	30/29.8	27/18.2	———	75/58.7

Fig 4.5

Now the minimum matrix is subtracted from the "assumed" matrix and the remaining "working" matrix, subject to Kruithof adaption, looks like fig 4.6

	i ↷ j	1	2	3	sum is	sum shall be
	1	0.1	0.2	5.0	5.3	7.3
	2	0.2	5.0	3.0	8.2	1.2
	3	2.0	1.0	0.8	3.8	7.8
sum	is	2.3	6.2	8.8	17.3	—
	shall be	7.3	0.2	8.8	—	16.3

Fig 4.6

After 13 steps of iteration * the matrix has been transformed to fig 4.7.

	i ↷ j	1	2	3	sum is	sum shall
	1	0.4	0.0	6.9	7.3	7.3
	2	0.2	0.1	0.9	1.2	1.2
	3	6.7	0.1	1.0	7.8	7.8
sum	is	7.3	0.2	8.8	16.3	—
	shall	7.3	0.2	8.8	—	16.3

Fig 4.7

and the working matrix and minimum matrix are added to form the required traffic matrix. See fig 4.8

	i ↷ j	1	2	3	sum is	sum shall
	1	5.3	7.8	11.9	25.0	25
	2	2.0	20.1	12.9	35.0	35
	3	10.7	2.1	2.2	15.0	15
sum	is	18.0	30,0	27.1	75.0	—
	shall	18	30	27	—	75

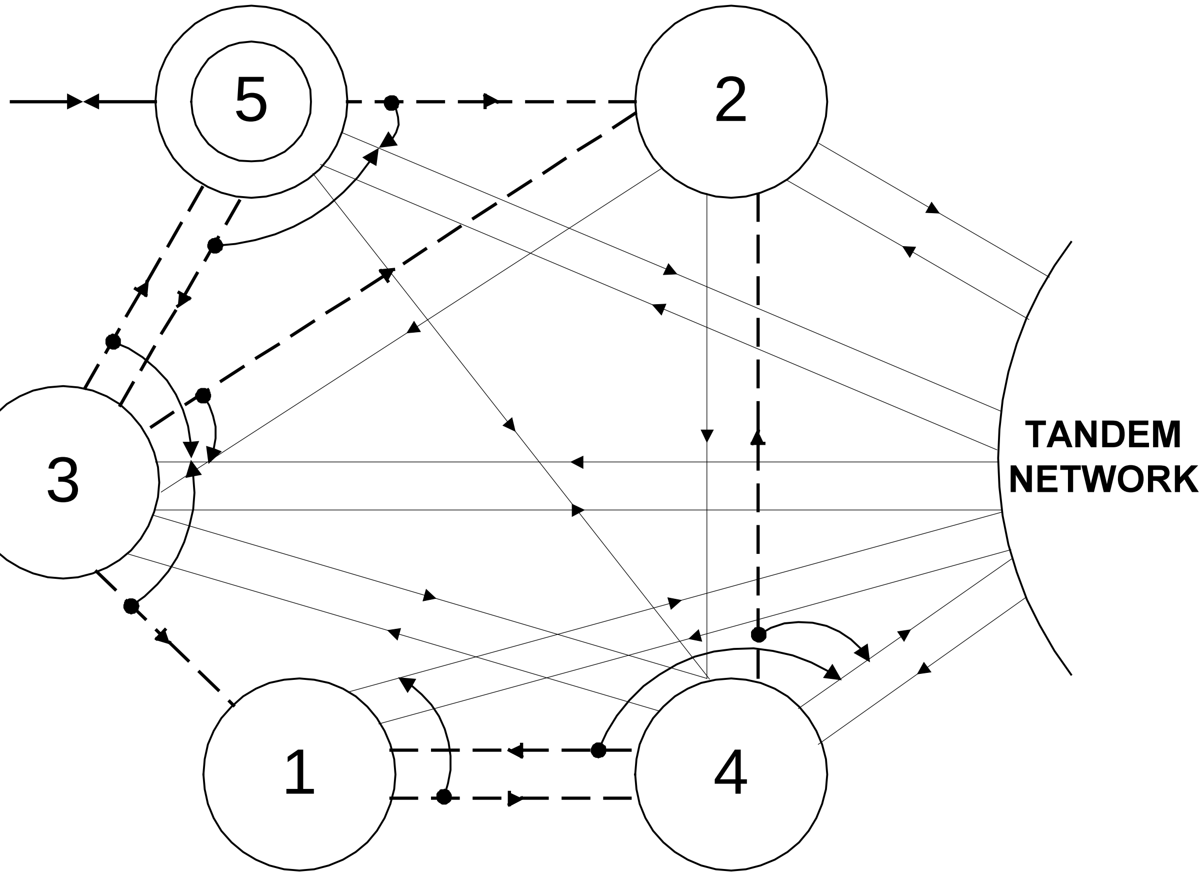
Fig 4.8

Compared to the matrix fig 4.4 it is seen that the values (i,j) = (1,2) and (2,2) now are kept on or above their prescribed minimum 7.8 and 20.0 respectively.

*) Many steps are required due to the bad accordance between “is” and “shall be” values.

5. EXAMPLE

For an alternative routing network (urban area) consisting of four local exchanges, one transit excahnge connected to the ordinary local network and an arbitrary number of tandemexchanges the traffic matrix shall be formed.



Routing matrix:

from i \ to j	1	2	3	4	5
1	(D)	T	T	H	T
2	T	(D)	D	D	T
3	H	H	(D)	D	H
4	H	H	D	(D)	D
5	T	H	H	D	——

Figure 5.1

Exch. no	Type of exch.	No. of main lines	Originating Traffic A_O	Terminating Traffic A_I	Internal traffic
1	XBAR	4000	292	221	35
2	SXS	5000	318	265	48
3	XBAR	7000	421	398	95
4	XBAR	10.000	496	690	186
5	XBAR (LD)	———	229	182	———

Recorded route traffics:

Route	Traffic	Route	Traffic
1 → 4	102	4 → 1	55
2 → 3	70	4 → 2	60
2 → 4	115	4 → 3	115
3 → 1	40	4 → 5	60
3 → 2	61	5 → 2	30
3 → 4	157	5 → 3	50
3 → 5	50	5 → 4	103

Exchange no.1 is provided with number	1386 c/h to exch. no 2
analysing equipement.	2299 c/h to exch no 3
The call intensities related to addresses have	1116 c/h to exch no 5
been estimated:	4801 c/h

The congestion on high-usage routes is measured. The congestion per traffic case is estimated, e.g. by use of traffic route testers, etc

Route no. resp. Traffic Case	Congestion in % High-usage route, B_h	Point-to-point, B_h-B_t	Route no. resp. Traffic Case	Congestion in % High-usage route, B_n	Point-to-point, $B_h \bullet B_t$
1 → 4	16.5	5	4 → 1	15.0	6
3 → 1	20.0	4	4 → 2	16.7	3
3 → 2	9.3	5	5 → 2	17.4	4
3 → 5	11.9	7	5 → 3	17.5	9

Figure 5.2

To From	1	2	3	4	5	A_O
1 Routing Estim. traffic Min. traffic						
2 Routing Estim. traffic Min. traffic						
3 Routing Estim. traffic Min. traffic						
4 Routing Estim. traffic Min. traffic						
5 Routing Estim. traffic Min. traffic						
Terminating traffic, A_I						

i \ j	1	2	3	4	5	SUM is	SUM SHALL BE
1							
2							
3							
4							
5							
SUM is							
SUM SHALL BE							

i \ j	1	2	3	4	5	SUM is	SUM SHALL BE
1							
2							
3							
4							
5							
SUM is							
SUM SHALL BE							

i \ j	1	2	3	4	5	SUM is	SUM SHALL BE
1							
2							
3							
4							
5							
SUM is							
SUM SHALL BE							

i \ j	1	2	3	4	5	SUM is	SUM SHALL BE
1							
2							
3							
4							
5							
SUM is							
SUM SHALL BE							

i \ j	1	2	3	4	5	$\mathbf{A_0 - \Sigma A_m}$
1						
2						
3						
4						
5						
$\mathbf{A_0 - \Sigma A_m}$						

i \ j	1	2	3	4	5	A_O
1						
2						
3						
4						
5						
A_I						