Cost Models

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Contents

1. Introduction

Network Planning aims at producing plans which satisfy the increasing future demand at the least possible cost. Here, network planning is primarily concerned with the following problems:

- the evaluation and comparison of different approaches to provide a given telecommunication service;
- how to make decisions on capacity expansions which involve the sizes of facilities to be added and when they should be added;
- how to decide whether or not to introduce a new exchange, and to determine its location and boundaries;
- how to make decisions on the introduction of new technology, such as digital technology in switching and transmission.

In order to solve the above problems properly, both the cost structure of various parts of the plant need to be known and the relevant cost needs to be expressed as a function of certain parameters, e.g., the size or the number of plant components. For this purpose, the network can be divided into certain units called **plant units** . For example, a typical plant unit is 1 km of 100-pairs buried cable containing, in addition to the cable itself, other materials such as protective tiler, jointing materials, etc. Examples of other plant units are ducts including manholes per km, pole-lines per km, terminals, telephone sets, etc.

2. Cost Models Used in Telecommunications

2.1 *General*

Economies of scale occur in almost all telecommunication plants, i.e., the cost per unit decreases as we purchase more units. The most commonly encountered case is the model where cost is a linear function of units:

$$C = A + B \cdot x \qquad (1)$$

A is the basic cost or start-up cost
B is the incremental or marginal cost
x is the number of units

The above relationship is generally valid for costs of cables, open lines, exchanges, line concentrators, multiplexers, etc. Another cost model with limited use in telecommunications is the exponential model:

$$C = k \cdot x^a \qquad (2)$$

In the relationship, k and a are constants and x represents the number of units of the plant.

2.2 *Cable cost model*

As we have mentioned it in the previous paragraph, the cost of cables as a function of number of pairs is linear Eq (1). When the length of cable is taken into account, Eq (1) becomes:

$$C = (a + b \cdot x) \cdot \lambda \qquad (3)$$

λ is the length of cable.

The constants a and b are connected to A and B through:

$$A = a \cdot \lambda \qquad B = b \cdot \lambda$$

The relationship (3) is illustrated in Figure 1 below:

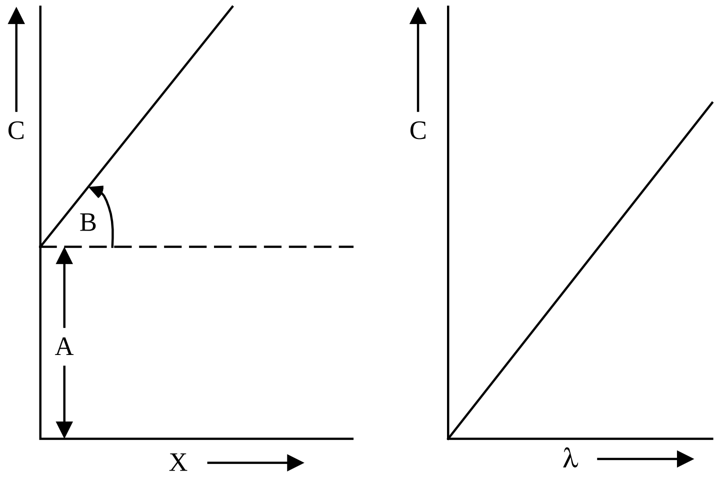


Figure 1

a) provides cost as a function of pairs with λ as parameter
b) provides cost as a function of length λ with *x* as parameter

For an installed cable, the overall basic cost **A** consists of the basic purchasing, digging, placement and transmission measurement cost. The constant **B** consists of the purchasing cost, joint cost per pair, cost of connecting one pair to the main distribution frame, etc. Making use of Eq (1), we can find for the cost of one pair

$$C = \frac{A}{x} + B \qquad (4)$$

Figure 2 illustrates the above relationship:

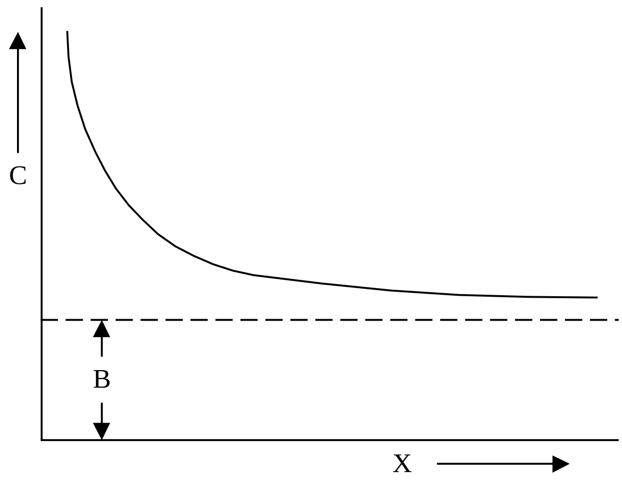


Figure 2 : Pair cost as a function of the capacity X of the cable

The curve is a hyperbola with asymptote **C = B**. When the capacity of the cable is large enough, the cost of one pair comes close to **B**. This hyperbola provides the economy of scale character of cables. From the above observation, we come to the conclusion that the larger the capacity of the plant, the cheaper the cost per pair.

Expression (4) can be used for cost comparison and capacity expansion problems.

When problems arise concerning optimal locations and exchange boundaries, it is necessary to express the the cost of a pair independently of the cable capacity. In this case, the cost is calculated on the average cable capacity used, $\bar{x}$.

$$C = A / \bar{x} + B = B_0 = b_0 \cdot \lambda$$

The constant $\mathbf{b_0}$ is expressed in monetary units per pair per kilometer, and gives the average cost per kilometer. For capacity expansion problems, the relationship (3) is taken into account.

2.3 *Cost of pair gain systems*

Pair gain systems reduce the need for subscriber cable pairs and, therefore, the obvious application of pair gain is an alternative to additional cable.

The most common pair gain systems used are:

- Line concentrators
- Carrier systems
- PCM systems

The cost of a pair gain system is expressed as:

$$C = (a + b \cdot x) + k \cdot c \cdot \lambda \qquad (5)$$

where

a is the basic cost of the pair gain system
b is the subscriber dependent cost, i.e., the cost of one additional susbscriber to be connected to the system
c is the cost of a link pair
x is the capacity of the system
k is the number of link pairs

In exchange location and boundary applications, the cost of one pair is expressed as:

$$C = p + s \cdot \lambda \qquad (6)$$

where **p** is the average cost of one additional subscriber

$$p = a / \bar{x} + b$$

and **s** is the average cost of links per subscriber per km.

$$s = k \cdot c / \bar{x}$$

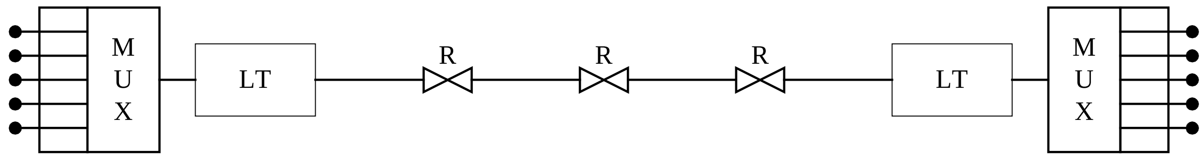
To deal with cost comparison and capacity expansion problems, the relationship (5) is used, while for problems concerning optimal locations and exchange boundaries, the relationship (6) will be sufficient. It is worthwhile to note that the parameters in the cost function must include, besides the purchase cost, the cost of installations, testing, measurements, operation, and maintenance.

2.4 *Cost of transmission systems*

Frequency Division Multiplex (FDM) and Time Division Multiplex (TDM) are two categories of multiplexing used in transmission.

The primary objective of this chapter is to provide guidance in the assessment of transmission cost. The parameters presented will be useful as a background information to assist the planner in formulating the cost of transmission systems as a function of number of circuits.

Figure 3 shows a layout of transmission system for the formulation of the cost function.



MUX = MUltipleXor / deMUltipleXor
LT = Line Terminal
R = Repeater / Regenerator

Figure 3 : An FDM or TDM transmission system

The cost function is expressed as

$$C = (a + b \cdot x) + k \cdot s \cdot \lambda \qquad (7)$$

The first term $a + b \cdot x$ provides the cost of multiplex equipment and the second term, $k \cdot s \cdot \lambda$ is the cost of transmission media.

a is the basic cost which consists of:

- the cost of multiplexers of higher order and line terminating equipment, namely for:

  FDM systems: primary, secondary, tertiary, etc. group modems and terminal repeaters, and for:

  PCM systems: the multiplexors of second, third, etc. order as well as the line terminating equipment;
- installation, measurement and alignment cost;
- operation and maintenance cost.

b is the cost of providing one additional channel which includes the cost of channel equipment and the corresponding cost for installation, measurement, maintenance, etc.

k is the number of pairs needed for the transmission media of the systems, namely cable, fiber optics or radio links.

s is the cost per pair per kilometer of the media and the corresponding line equipment. This includes:

- the purchase cost of transmission media
- the placement of transmission media
- the cost of line equipment (repeater equipment) per kilometer
- the cost for measurement, alignment, maintenance and operation.

λ is the length in kilometers of the systems.

The cost of one circuit must be known for certain applications. This cost, of course, is a function of the system fill **x** (utilisation), namely the number of circuits with which the system is equipped. There are two ways to find this cost:

<u>either</u> to consider the cost of one circuit at fill **x**

$$C = \left(b + \frac{a}{x}\right) + \frac{k \cdot s \cdot \lambda}{x} \qquad (8)$$

<u>or</u> to consider the cost of one circuit when the system is fully equipped :

$$C = \left(b + \frac{a}{x_N}\right) + \frac{k \cdot s \cdot \lambda}{x_N} \qquad (9)$$

where x_N is the capacity of the transmission system.

2.5 *Switching cost*

2.5.1 *Conventional (electromechanical) switching systems*

In Figure 4, an example of basic groups in a local exchange is shown. The cost pertinent to the system is a linear function of:

- the number of lines
- the number of incoming and outgoing circuits.

The capacity of the group selector stages depend, to a great extent, on the traffic to be handled.

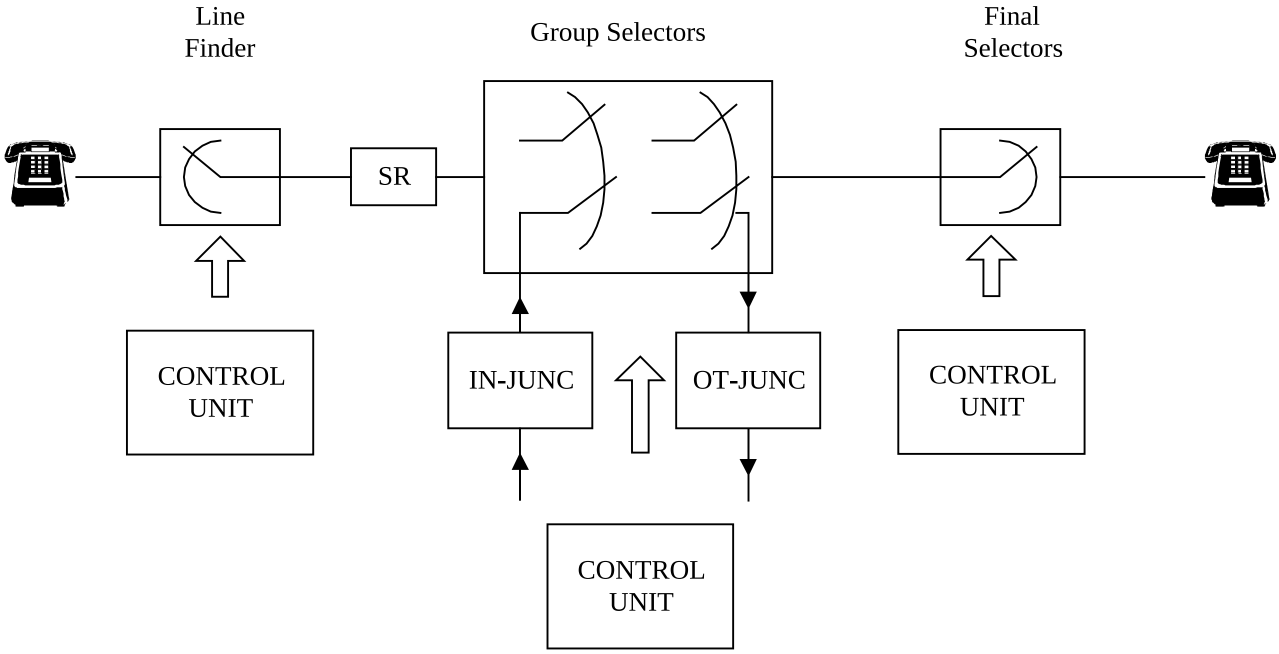


Figure 4 : Example of α switch groupings in the local exchange

SR - Supervising circuits for current-feeding of microphones
IN-JUNC - Incoming Junctor
OT-JUNC - Outgoing Junctor

The capacity of the control units depend, to a great extent, on call intensities, and the capacity of the final selector stage depends, of course, on terminating traffic. A practical mathematical relationship which can provide the cost of the system is the following:

$$C = a + b \cdot x + \sum_{j=1}^{k} S_0(i,j) \cdot n_0(j) + \sum_{j=1}^{k} S_i(i,j) \cdot n_i(j)$$

The parameters in the above relationship are:

a : reflects the basic cost of the system which consists of racks frames, fields, power supply units, control equipment, etc.

b : the incremental cost of subscriber lines

S_i(I,J) : the incremental cost of incoming circuits

S_0(I,J) : the incremental cost of outgoing circuits

K : determines the number of types of exchanges to which the switching system is connected

I, J : determine respectively the types of originating and terminating exchanges respectively.

Moreover,

x stands for the number of lines, and $\mathbf{n_0(j)}$, $\mathbf{n_i(j)}$ stands for the number of outgoing and incoming circuits respectively. The parameter a does not remain constant over $\mathbf{x}$, $\mathbf{n_i}$ and $\mathbf{n_0}$ but increases in steps from one level to a higher one. When x exceeds a specific number of lines, the value of basic cost jumps.
Figure 5 shows the cost function of a switching system.

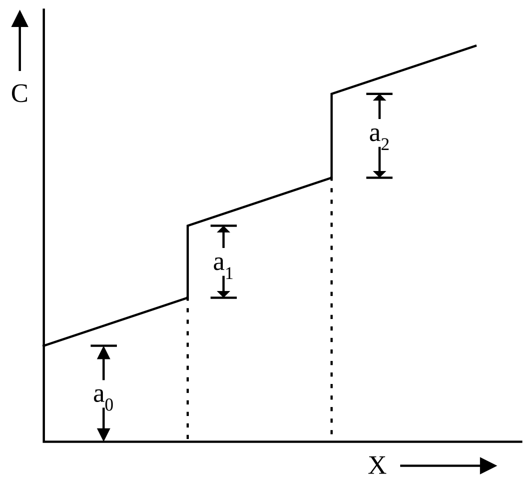


Figure 5 : Local switching system cost as a function of number of lines

The cost function is a linear step-function. The parameter a depends on the basic cost of the system $\mathbf{a_0}$ and $\mathbf{a_1}$, $\mathbf{a_2}$, etc. depend on the cost of additional common equipment needed when the number of lines of the systems exceeds certain levels ($\mathbf{x_1}$, $\mathbf{x_2}$). The slope, which shows the incremental cost of lines, remains constant over the range of **x**. The cost of junctors depends on the type of exchange (EMD, Pentaconta, Strowger, etc.) and the type of transmission system **L**. Figure 6 shows a junction circuit between two exchanges (local or long distance).

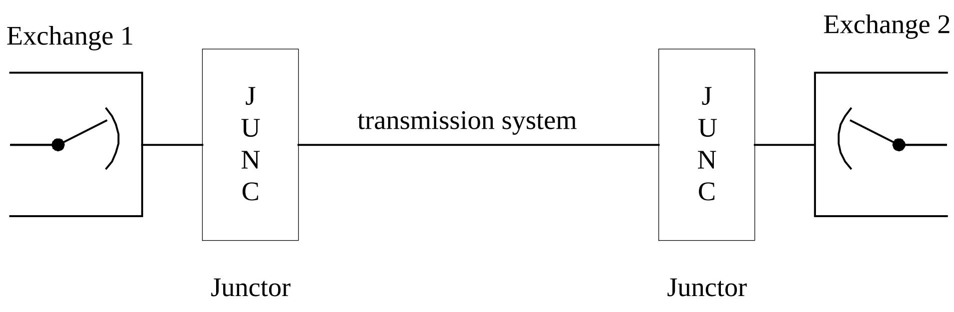


Figure 6 : A circuit connecting two exchanges

$\mathbf{S_0}$ **(I, J)** and $\mathbf{S_i}$ **(I,J)** can be written down as follows:

$$\begin{aligned} S_0(I,J) &= RS_0(I,J,L) + SW_0(I) \\ S_i(I,J) &= RS_i(I,J,L) + SW_i(I) \end{aligned} \qquad (11)$$

L provides the type of transmission media used.

$\mathbf{SW_0}$ **(I)** and $\mathbf{SW_i}$ **(I)** are the part of the cost of switching equipment needed for one connecting trunk, and

$\mathbf{RS_0}$ **(I, J, L)** and $\mathbf{RS_i}$ **(I, J, L)** are the part of the cost of relay sets for one outgoing and incoming trunk, respectively.

I, J determines the types of originating and terminating exchanges respectively

L is the kind of transmission system used (2 wire voice frequency, 4 wire voice frequency, FDM, PCM, etc.).

So RS is determined through three indices (I, J, L). In other words, it is a three-dimension matrix which can be broken into L two-dimension matrices.

2.5.2 *SPC Systems*

The cost structure of SPC switching systems is, in essence, the same as that of conventional systems. All previous blocks into which the systems were divided can be distinguished here. Junctors or interfaces (signalling units) are not needed between SPC digital exchanges when PCM transmission is used since the signals processed and switched by the system are also PCM. This should be taken into account when constructing the cost of switching systems.

There are some differences between conventional line concentrators and those SPC digital systems which are called remote subscriber units (RSU). The most important is that RSU are remotely located subscriber stages. This implies that the subscriber stages are removed from the parent exchange and placed into the RSU. Another point that is important to mention here is that the RSU links should be exclusively PCM, although an FDM connection could, in principle, be considered; that would, however, be economically prohibitive.

Figure 7 shows a RSU connected to its parent exchange.

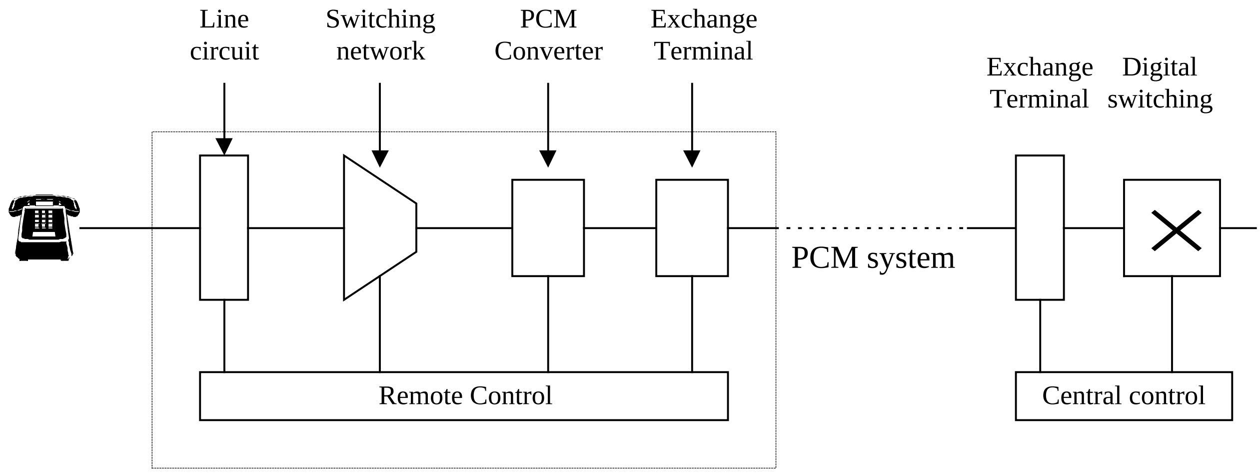


Figure 7 : Concept of Remote Subscriber Unit (RSU)

2.6 *Cost of buildings*

The cost of new buildings is a linear function of the area of a building and can be formulated as

$$C = A + B(q) \qquad (12)$$

where

A is the basic cost;
B is the incremental cost with respect to the area **q** of the building;
q is the area of the building..

The parameter A in the above relationship jumps when the area exceeds certain levels $\mathbf{q_1}$, $\mathbf{q_2}$. On the other hand, **B** remains constant but its value is extremely low compared to **A**. For practical applications, we can accept

$$B = O$$

which leads to the cost of building

$$C = A$$

In Fig. 8, the step-function of C is shown.

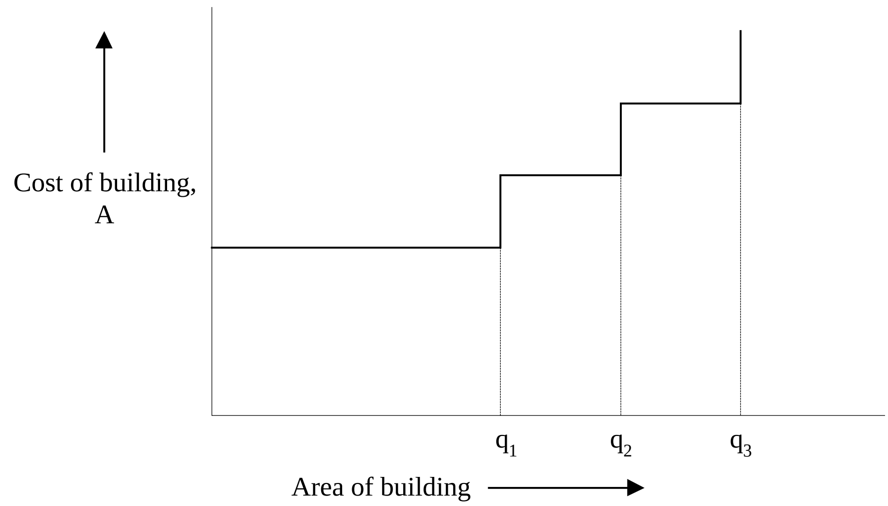


Figure 8 : Character of step-function of A

The jumps can be explained by the fact that when the size of a building exceeds a certain value, a larger type of building should be considered. To determe the cost model of a building, it is necessary to know the various jumps in cost over the size of the building. The initial jump is the highest of all because it includes the cost of land. The type of building needed depends primarily on the type of switching system and the maximum number of lines. This means that for every exchange, the mathematical relationship between the size of the building and number of lines should be known. This would be a linear step-function converting lines to area of the building.

Figure 9 shows a typical conversion curve. λ_1, λ_2, ... shows how the number of subscriber lines would affect the area needed.

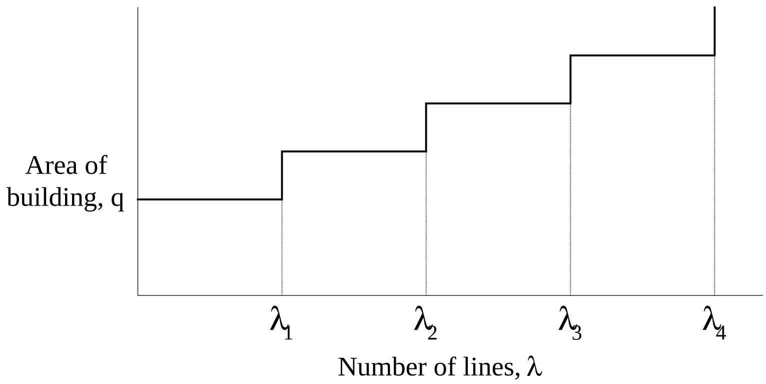


Figure 9

3. <u>Capital Cost</u>

3.1 *<u>Investment cost and its components</u>*

The term investment designates the amount of money required to build a new plant - the investment in plant. It is the total original implementation cost. The investment estimates used in the cost studies include all anticipated expenditures up to the time the project is completed and ready for use. The more important investment items are described below:

- **Material**: Purchasing of all material used in the construction of the plant, freight costs, sales taxes, and supply expenses.

- **Installation**: All direct labor costs as well as incidental expenses.

- **Miscellaneous**: Supervision, tool expenses, general expenses, social security taxes, and relief and pensions.

- **Engineering**: Cost of all engineering time and associated costs.

- **Costs occurring during construction**, which are added to the plant investments accounts. These costs include the interest during construction, value added taxes, and insurance where necessary.

3.2 *Operation and maintenance costs*

Operation and maintenance costs are those which are incurred because of the existence and use of a plant. These costs are dependent on the physical plant - the quantities of each of the various types of equipment, how it is assembled, where it is located, how it is used, how it is maintained and on the extent of re-arrangement and changes. A prominent characteristic of operation costs is that they are generally continuous or recurring for as long as the plant remains in service.

Components of operation and maintenance costs include:

- The cost of material and labor associated with the upkeep and re-arrangement of the plant (maintenance costs). This includes the cost for personnel training and for testing of equipment and services.
- The cost of labor associated with day-to-day operation of the plant, for example, the handling of long distance calls by an operator.
- Miscellaneous expenses, such as workshop repairs, tool expenses, caretaker, utilities, etc.
- Cost of procuring, handling and storing materials and spare parts.
- Supervision costs.
- Rent and taxes.

4. Economy Study Techniques

4.1 *General considerations*

The basic economic study methods are:

- The present worth method
- The annuity method
- The rate of return method

The choice of method to be employed for a certain study is rather arbitrary and the ease of calculation and the simplicity of presentation are factors which should always be considered when making the decision. In network planning optimization problems, the most convenient method is the present worth technique. The method of rate of return is seldom used in network planning applications and so it will not be described.

4.2 *Present Worth (PW) method*

The most important method in optimization problems is the present worth method. This method refers to all the events in an economy, both incomes and expenditures , as one figure at one point in time.

When comparing different alternatives for a given revenue or cost saving, the alternative with the least present worth of all expenditures or annual charges should be selected. There are two methods of present worth:

- the present worth of expenditures (PWE), and
- the present worth of annual charges (PWAC).

4.2.1 *Present Worth of Expenditures (PWE)*

The present worth of expenditures (PWE) method measures how attractive an alternative is based on the comparative cost to an administration of undertaking each alternative. By finding the PWE of each alternative, we we are in a position to select that alternative with the lowest PWE for a given service to the subscribers. The PWE does not require any estimate of revenues; however, if a difference in revenues is anticipated, revenues must be taken into consideration in order to maintain comparable conditions.

Example 1

Assume two alternatives for which we have the following table:

Alternative	**A**	**B**
First cost	4000 MU	5000 MU
Gross salvage	400 MU	300 MU
Cost of removal	300 MU	300 MU
Operation + maintenance cost/year	1300 MU	700 MU
Service life	5 years	5 years
Interest rate	10 %	10 %

The objective is to find the most attractive alternative. A good test is to make a comparison of the present worth of expenditures (PWE).

PWE calculations

Alternative A:

- *PWE of operation and maintenance*

We use the formula (see appendix) which gives the present worth of an annuity (present worth of 1 monetary unit paid at the end of each year for **N** years).

$$\left(P / A\right)_N^{i\%} = \frac{(1+i)^N - 1}{i \cdot (1+i)^N}$$

Rearranging the formula, we calculate:

$$P = A \cdot \frac{(1+i)^N - 1}{i \cdot (1+i)^N} = 1300 \cdot \frac{(1+0.1)^5 - 1}{0.1 \cdot (1+0.1)^5} = 4928 \;\; MU$$

- *PWE of net salvage*

We use the formula for present worth **P** of a future amount (present value of 1 monetary unit paid at the end of the year **N**).

$$(P / F)_n^{i\,\%} = (1+i)^{-N}$$

We rearrange the formula and calculate:

$$P = F \cdot (P / F)_N^{i\,\%} = F \cdot (1+i)^{-N} = -(400 - 300) \cdot (1+0.1)^{-5} = -62 \;\; MU$$

Salvage is taken negative because it is a receipt.

- *PWE of first cost = 4000 MU*

Total PWE = 4000 + 4928 - 62 = 8866 MU

Alternative B:

- *PWE of operation and maintenance*

$$700\cdot(P/A)_5^{10\ \%}=700\cdot\frac{(1+i)^N-1}{i\cdot(1+i)^N}=700\cdot\frac{1{,}1^5-1}{0{,}1\cdot 1{,}1^5}=2654\ MU$$

- *PWE of net salvage*

$$-(500-300)\cdot(P/F)_5^{10\ \%}=-124\ MU$$

- *PWE of first cost = 5000 MU*

Total PWE = 5000 + 2654 - 124 = 7530 MU

Comparing the PWE of each alternative, we choose the alternative **B** since it has smaller PWE, despite the fact that its first cost is greater than alternative **A**.

4.2.2 *Present value factor*

When making a cost calculation, it is sometimes convenient to express the total present worth of expenditures in terms of its provision costs, including the costs of replacement, maintenance, and operation. This may be achieved by multiplying the first provision by the present worth factor:

$$\mu=1+\frac{1-s}{(1+i)^T-1}+\frac{u}{i} \qquad (13)$$

where

T	is the service life of a plant
s	is the scrap value of retired plant (reduced by the dismantling costs) in relation to provision costs
u	is the annual operating plus maintenance cost in relation to provision cost
i	is the interest rate (expressed in decimals)

In this expression, the first term is proportional to the provision costs, the second term represents the net replacement costs (for an infinite period of time) and the third to the maintenance plus operating costs (for an infinite period of time). In most cases, the scrap value of retired plant is almost absorbed by the dismantling costs so that **s = 0**. Assuming that **s = 0**, the present value factor can be rewritten:

$$\mu=\frac{(1+i)^T}{(1+i)^T-1}+\frac{u}{i} \qquad (14)$$

The full annual charges a of a scheme may be obtained from the provision cost by multiplying them by the present value factor and by the interest rate i .

$$a=c\cdot\mu\ \cdot i \qquad (15)$$

a	is the annual charge
c	is the provision cost
μ	is the present value factor
i	is the interest rate

The (PWE) is obtainable by multiplying the investment cost c by μ.

Some approximate values for service life, annual operating plus maintenance cost and the average present value factor for some of the most common transmission equipment and facilities are illustrated in Table I.

Interest rate = 10 %, Scrap value = 0			
Plant unit	*Service life*	*Maintenance plus operating cost (%)*	*Present value factor (PVF)*
Open wire line	15-20	3.0	1.53
Aerial cables	15-20	5.0	1.73
Buried cables	25-35	2.0	1.26
Underground cable	30-40	2.0	1.26
Conduits	40-60	1.0	1.11
Carrier systems	15-20	5.0	1.73
Electronic equipment	15-20	5.0	1.73
Radio equipment	15-20	5.0	1.73

Table I : Service life, maintenance plus operating cost and PVF

Example 2

We can provide facilities to subscribers using either of the two alternatives below:

Alternative A:

- Service life **T** = 40 years
- Investment cost **C** = 2500 MU (Monetary units)
- Maintenance plus operating cost **u** = 2 %
- Scrap value **s** = 0

or

Alternative B:

- Service life **T** = 15 years
- Cost of provision **CB** = 1800 MU
- Maintenance plus operating cost **u** = 5 %
- Scrap value **s** = 0
- Interest rate **i** = 10 %

We would like to find which alternative is the most economical.

Calculation of present value factor of alternative A:

$$\mu_A = 1 + \frac{1}{(1+0.1)^{40} - 1} + \frac{0.02}{0.1} = 1.223$$

The present worth of expenditures for alternative A is

$$PWE_A = \mu_A \cdot C_A = 1.223 \cdot 2500 = 3057 \ MU$$

Calculation of present value factor of alternative B

$$\mu_B = 1 + \frac{1}{(1+0.1)^{15} - 1} + \frac{0.05}{0.1} = 1.815$$

Present worth of expenditures for alternative B

$$PWE_B = 1.815 \cdot 1800 = 3267 \ MU$$

Comparing the two alternatives:

$$PWE_A = 3057 < 3267 = PWE_B$$

we find that alternative A is more economical than B.

We would be likely to make an erroneous decision if our comparison were based solely on the investment cost where alternative B looks more advantageous.

4.2.3 *Present Worth of Annual Cost (PWAC)*

The present worth of annual costs (PWAC) method is essentially the same as the PWE method, except that capital costs are converted to equivalent annual costs (AC) before their worth is found. Operation costs should be treated as they are expected to occur. In many instances, the results of a PWAC analysis will be exactly the same as those for a PWE analysis because the present worth of an annuity which is equivalent to an expenditure is the expenditure itself. Therefore, if the present worth of the annual costs is found over a period equal to the life of the plant, the PWE equals the PWAC. Of any number of mutually exclusive alternatives for accomplishing a specific job, choose, as the most economic alternative, the one with the lowest PWAC at a given cost of money. The PWAC is very popular because working with annual costs can simplify the treatment of non-coincidental equipment installations and retirements.

Many studies are not co-terminated. That is, some plant is expected to live longer in one plan than in another. The PWAC method is most advantageous in such studies.

4.3 *The annuity method*

Using this method, initial capital costs are converted to equivalent annual costs. Constant annual receipts and/or operating costs are then substracted and/or added to the annual capital costs. The residual value is then assumed to be zero.

The application of the annuity method is limited by the assumptions and conditions:

- all investments have to be made at one time, at the beginning of the calculation period;
- operating expenses and receipts have to remain constant during the calculation period;
- residual value is zero.

These difficulties can be avoided by using the present worth method. For each plant item, total annual costs are determined and are then converted to present values.

5. Appendix

Time value factors

5.1 *Future worth of a present amount (F/P)*

$$(F/P) = (1+i)$$

5.2 *Present worth of a future amount (P/F)*

$$(P/F) = 1/(1+i)$$

5.3 *Present worth of an annuity (P/A)*

$$(P/A)_N^i = \frac{(1+i)^N - 1}{i \cdot (1+i)^N}$$

5.4 *Future worth of an annuity (F/A)*

$$(F/A)_N^i = \frac{(1+i)^N - 1}{i}$$

5.5 *Present worth of a continuous annuity a(t)*

$$PW = \int_0^T a(t) \cdot e^{-r \cdot t} \cdot dt$$

where

$$r = ln(1+i)$$

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