Cost Comparisons of

Voice Frequency Cable vs. PCM System

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Cost Comparison of Voice

Frequency Cable vs. PCM System

1. The problem

Two analogue exchanges in a metropolitan area are linked by a voice frequency cable which is completely used up. There are two alternatives to provide facilities: either to use PCM systems equipped with the appropriate signalling interface unit on existing cable or to lay another voice frequency cable to expand the existing facilities.

The problem is to determine the “break-even distance” between the two alternatives.

To set up a PCM link, two voice-frequency pairs are needed. These pairs will be selected among those of the existing cable serving the traffic of the exchanges under consideration. The actual capacity K' of PCM will be:

$$K' = K - 2 \qquad (1)$$

Here, K is the capacity of PCM system.

2. Evaluation of present worth of expenditures

2.1 *PCM Equipment*

Let C_P be the cost of an installed pair of PCM terminals with the necessary signalling interface units and line terminating equipment. Charges associated to service life and maintenance costs have also been included.

Assume that C_{LE} represents the cost per kilometer of the line equipment installed. Resultant costs due to service life and maintenance have been taken into account.

The cost of a complete PCM link is given by

$$C_{PCM} = C_P + C_{LE} \cdot \lambda \qquad (2)$$

Every time a system is used up, a new one should be installed to cater for the future demand. The period between successive installations is

$$t_p = k' / \lambda \qquad (3)$$

λ is the demand growth. This is considered constant over time.

For an infinite number of expansions, the cash-flow is illustrated in Figure 1.

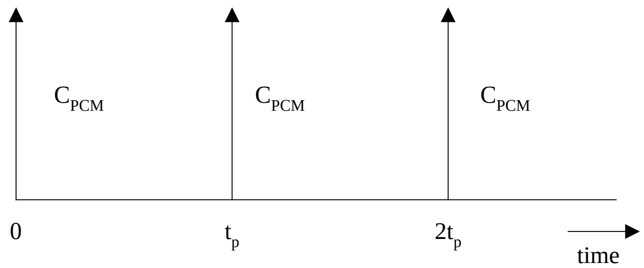

Figure 1

The present worth of expenditures is written down

$$PW_P = C_{PCM} + C_{PCM}(1+i)^{-t_p} + \cdots = C_{PCM}[1+(1+i)^{-t_p} + (1+i)^{-2t_p} + \cdots]$$

The sum in brackets is the infinite geometric progression with ratio $(1+i)^{-t_p}$. We get:

$$PW_p = \frac{C_{PCM}}{1-(1+i)^{-t_p}} \quad (4)$$

The above relationship can also be written down

$$PW_P = \frac{C_P + C_{LE}\cdot\lambda}{1-e^{-rt_p}} = \frac{C_P + C_{LE}\cdot\lambda}{1-e^{-rK'_p/\lambda}} \quad (5)$$

where $r = ln(1+i)$.

2.2 *Relay sets*

Every year, the relay sets needed are 2λ.

Let C_R be the total cost of an installed relay set. We point out that charges associated to service life and maintenance cost have been included. The cost of relay sets for one year is then:

$$2\lambda C_R$$

Assuming that the installation of relay sets takes place every year, the cash-flow is then as in Figure 2.

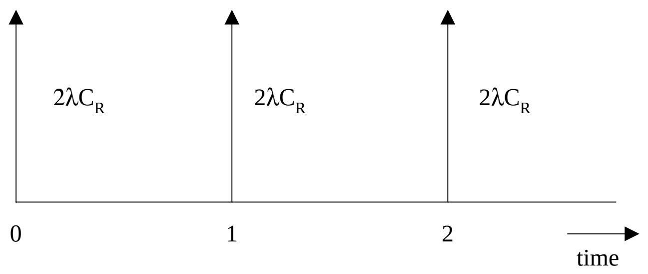

Figure 2

The present worth of expenditures is:

$$PW_R = 2\lambda C_R[1+(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + \cdots]$$

$$PW_R = \frac{2\lambda C_R}{1-(1+i)^{-1}} = \frac{2\lambda C_R}{1-e^{-r}} \quad (6)$$

2.3 *Cable*

The cost of installed cable of length λ is:

$$C_c = (a+bs)\lambda \quad (7)$$

a and *b* are the basic and incremental capital cost in which all kinds of charges have been taken into account. The optimal size is given by

$$S = \frac{\lambda}{r} ln(1 + p + \sqrt{2p}) \qquad (8)$$

where $p = ar / b\lambda$

The provisioning time is

$$t_c = S / \lambda \qquad (9)$$

This means that the cable must be expanded every t_c years.

The present worth of expenditures was found in previous paragraphs to be:

$$PW_c = \frac{(a + bs)\lambda}{1 - e^{-rs/\lambda}} \qquad (10)$$

3. <u>Break-even distance</u>

The break-even distance $\bullet_o$ is the distance that the present worths of both alternatives become equal. Thus, we get

$$PW_p = PW_r + PW_c \qquad (11)$$

This equation eventually becomes:

$$\frac{C_P + C_{LE} \cdot \lambda_o}{1 - e^{-rk'/\lambda}} = \frac{2\lambda C_r}{1 - e^{-r}} + \frac{(a + bS)\lambda_o}{1 - e^{-rs/\lambda}} \qquad (12)$$

By solving this equation with respect to $\bullet_o$, we get the break-even distance $\bullet_o$ as a function of demand growth

$$\lambda_o = \frac{C_P - Y(\lambda)X(\lambda)}{Z(\lambda)X(\lambda) - C_{LE}} \qquad (13)$$

where: $X(\lambda) = 1 - e^{-rk'/\lambda} \qquad (14)$

$$Y(\lambda) = \frac{2\lambda C_R}{1 - e^{-r}} \qquad (15)$$

$$Z(\lambda) = \frac{a + bS}{1 - e^{-rs/\lambda}} \qquad (16)$$

4. Numerical example

Data regarding costs and service lines are as follows:

PCM Systems:

- Cost of provisioning of two terminals equipped with the appropriate signalling interface units and line terminating equipment: 1800 *MU*
- Installation of two terminals and alignment of equipment: 500 *MU*
- Taxes: 20 % on the imported equipment
- Cost of one both-way regenerator: 100 *MU*
- Cost of housing for one regenerator: 25 *MU*
- Cost of installation and alignment of regenerator: 70 *MU*
- Average regenerator spacing: 1.81 km
- Operating plus maintenance: 5 %
- Service life: 20 years
- System capacity: 30 Ch

Voice frequency cable:

- Purchasing cost, basic cost: 100 *MU*/km
- Incremental cost: 4.8 *MU*/km/pair
- Digging cost: 500 *MU*/km
- Jointing cost: 40 *MU*/km
- Placement cost: 20 *MU*/km
- No taxes because cables are locally produced: --
- Service life: 40 years
- Maintenance and operating cost: 2 %

Relay sets:

- Purchasing cost of relay sets: 20 *MU*/relay set
- Installation cost: 4 *MU*/relay set
- Taxes: 20 % of purchasing cost
- Service life: 20 years
- Maintenance: 7 %
- Interest rate: 10 %

The scrap value for all dismantled equipment is assumed to be negligible.

Calculation of costs

PCM Systems

Total capital cost of terminal

$$C_p = 1800 \cdot \mu_p + (\text{taxes + installation + alignment}) \cdot \left(1 + \frac{1}{(1+i)^{Tp} - 1}\right)$$

The μ_p for PCM is

$$\mu_p = 1 + \frac{1}{(1+i)^{Tp-1}} + \frac{u}{i} = 1.675$$

Thus, we get

$$C_p = 1800 \cdot 1.675 + [1800 \cdot 0.2 + 500]\left[1 + \frac{1}{(1.1)^{20} - 1}\right] = 4025\, MU$$

Total cost of line equipment per km

C = [Regenerators + housing] μ_p + [(Regenerators + housing) . taxes

+ Installation and alignment] $\cdot \left(1 + \frac{1}{(1+i)^{Tp} - 1} \right) = (100+25)\cdot 1.675$

$+ \ [(100+25)\cdot 0.2 + 70]\cdot \left(1 + \frac{1}{1\cdot 1^{20} - 1} \right) =$

$321\,MU$ per regenerator spacing.

The cost of line equipment per km is:

$C_{LE} = 321 / 1.81 = 177\,MU / km$

1.81 is the average repeater spacing.

Relay sets

Total capital cost $\underline{\underline{C_R}}$ =(purchasing cost) μ_r +(purchasing cost $\cdot$ taxes+installation) $\cdot \left(1 + \frac{1}{(1+i)^{T_r} - 1} \right)$

The pvf is:

$$\mu_r = 1 + \frac{1}{1.1^{20} - 1} + \frac{0.07}{0.1} = 1.875$$

$$C_r = 20 \cdot 1.875 + (20 \cdot 0.2 + 4)\left(1 + \frac{1}{1.1^{20} - 1} \right) = 46.9\,MU / relay \quad set$$

Cable

Total capital basic cost

$a =$ (purchasing cost) μ_c + (digging + jointing + placement) $\cdot \left(1 + \frac{1}{(1+i)^{T_c} - 1} \right)$

$$= 100\mu_c + (500 + 40 + 20)\left(1 + \frac{1}{1.1^{40} - 1} \right)$$

$$\mu_c = 1 + \frac{1}{1.1^{40} - 1} + \frac{0.02}{0.1} = 1.222$$

$a = 694\,MU / km$

Total incremental capital cost

$b =$ (purchasing) $\mu_c = 4.8 \cdot 1.222 = 5.87\,MU / pair / km$

With the above calculated costs and by using Eg (10), we elaborated Table 1, providing break-even distance as a function of λ. Figure 3 illustrates the curve between λ and $\bullet_o$.

No.	λ	$\mathbf{t_c}$	**S**	**X(λ)**	**Y(λ)**	**Z(λ)**	$\mathbf{1_o}$
1	5	17.7	90	0.410	4690	1492	4.80
2	10	13.5	130	0.230	9380	2016	6.50
3	15	11.5	180	0.162	14070	2573	7.30
4	20	10.1	200	0.124	18760	3046	8.46
5	25	9.2	230	0.101	23450	3508	9.47
6	30	8.5	250	0.085	28140	3952	10.30
7	35	7.9	280	0.073	32830	4390	11.35
8	40	7.45	200	0.064	37520	4818	12.36
9	45	7.1	320	0.057	42210	5238	13.32
10	50	6.7	340	0.052	46900	5652	13.60

Table 1

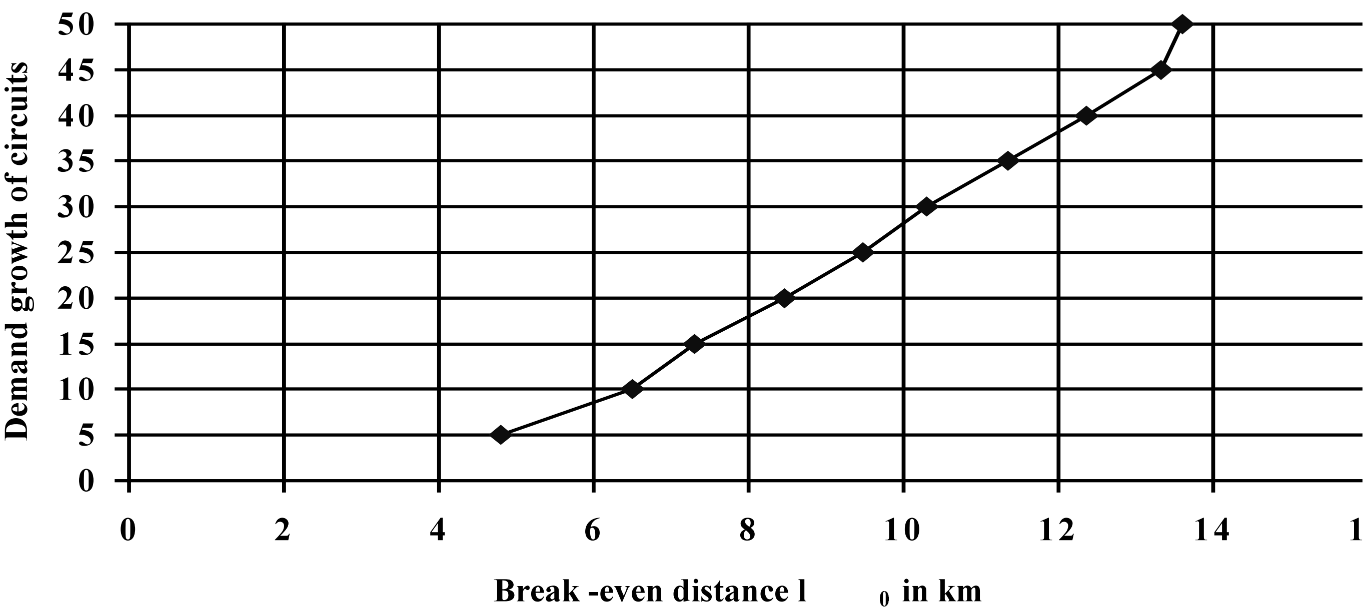

Figure 3