Basic Forecasting Theories

A Brief Introduction

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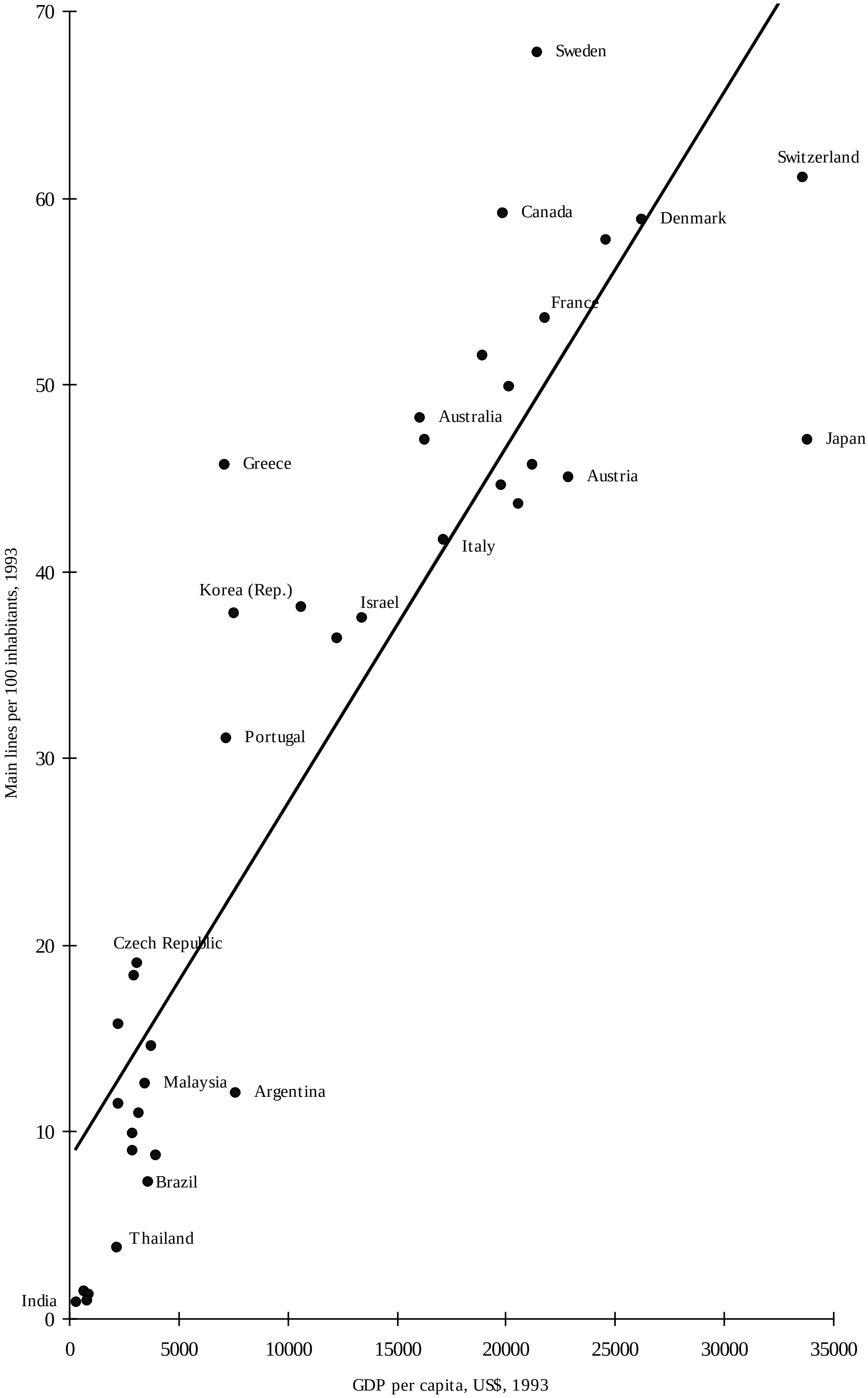
Telephone density and GDP per capita

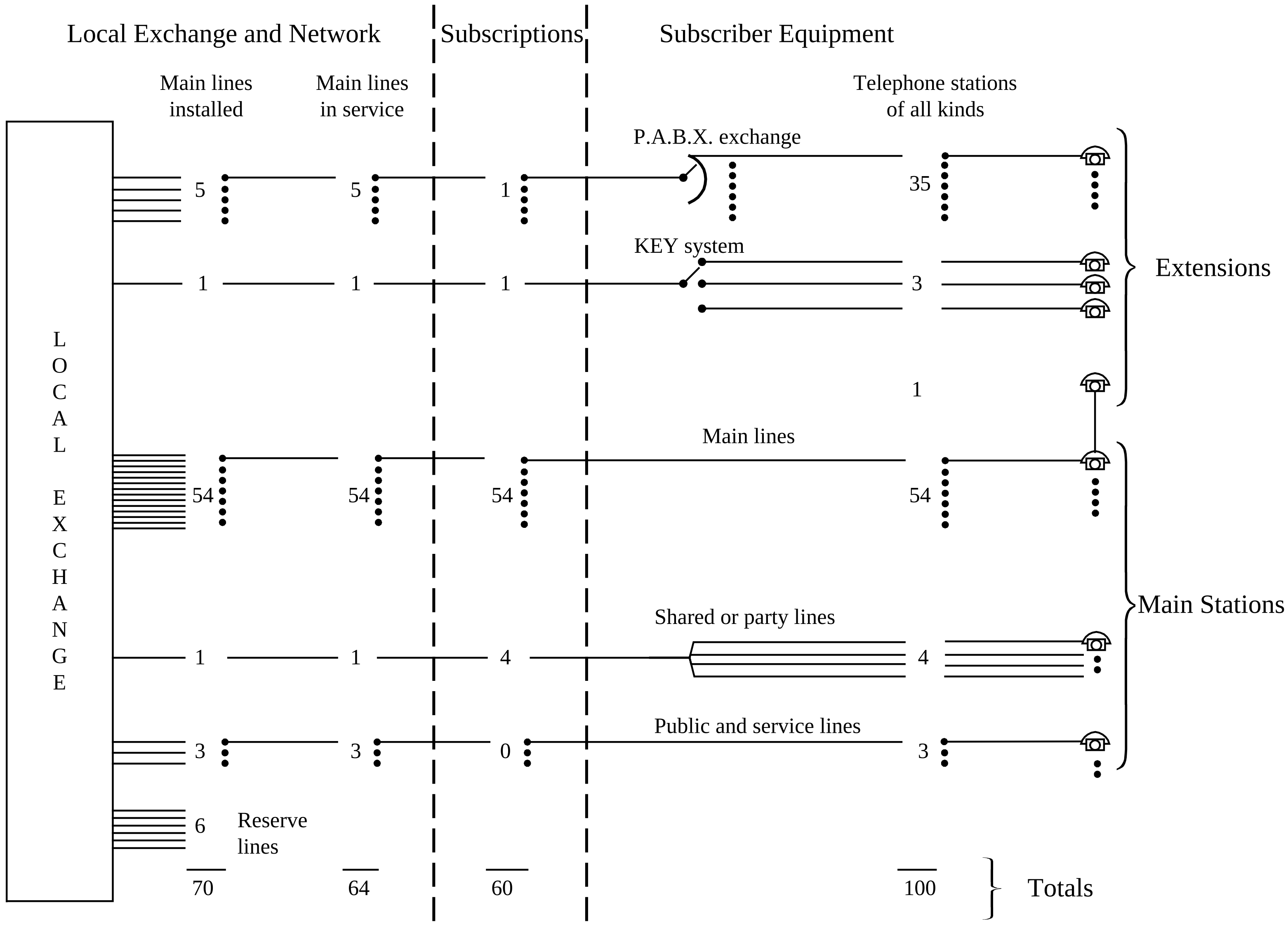


<u>Forecasting subjects</u>

ACCOMMODATION	SITES BUILDINGS
LINES	SUBSCRIBERS' INDIVIDUAL LINES (LINE PLANT FORECAST) JUNCTION NETWORK TRUNK NETWORK
EQUIPMENT	SUBSCRIBERS' TERMINAL EQUIPMENT AUTOMATIC SWITCHING EQUIPMENT SWITCHBOARD EQUIPMENT

Telephone stations and main lines

Relationship between telephone stations of all kinds, the main telephone station and main line.



Types of forecasts

Two kinds of forecasts - overall and detailed forecasts - are needed for exchange and network planning purposes.

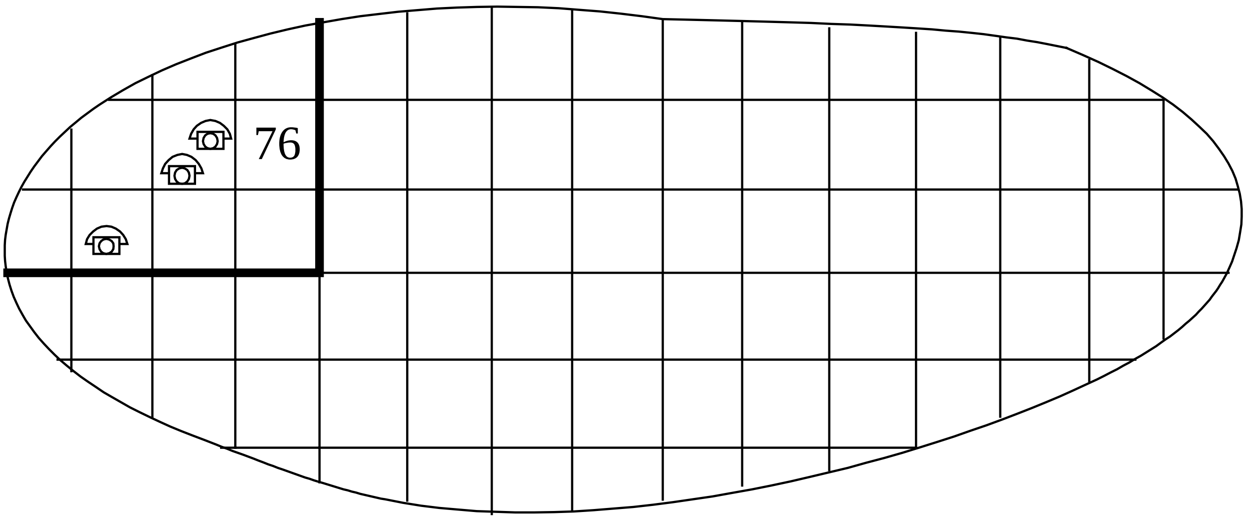
The overall forecast is needed for forecast of future traffic which is the basis for dimensioning the junction network. The overall forecast is purely a numerical description of subscriber development within a country, a city or an exchange area without bothering about the exact (geographical) location of each subscriber.

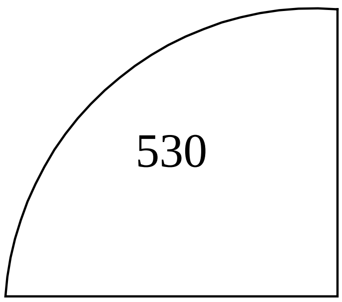
The detailed forecast, on the other hand, has an exact location as its main objective.

The detailed forecast is used for planning local exchanges and their location.

As traffic forecasts are based on the forecast of telephony development, it is important to know exactly the type of measurement that has been used for the telephony development forecast.

The traffic forecast should be based on the number of main lines, if possible divided into different categories with respect to traffic load.





BUS. Subs.		220
RES. - " -		310
Σ	- " -	530

Data needed for planning

The forecaster should be prepared to present the data in different ways, depending on different planning needs, as for example:

***Subscriber distribution in a telecommunication area*:**

a the exact location of the individual subscribers

b the number of subscribers in each grid of a grid map

c the number of subscribers per traffic area in the telecommunication area

d the number of subscribers per subscriber class in each traffic area

***Traffic*:**

e originating and terminating traffic per subscriber

f originating and terminating traffic per subscriber in each traffic area

g originating and terminating traffic per subscriber for each subscriber class

h traffic streams between each two areas

We may define:

A the total traffic quantity related to a group of subscribers

N the number of subscribers in the group

α the calling rate per subscriber

Y the call intensity in the group

S the call holding time

Then, of course, $A = Y \cdot S$

but also: $A = N \cdot \alpha$

Traffic parameters

$$A = Y \cdot S$$

A = Traffic

Y = Call intensity

S = Call holding time

$$A_c^{(t_0)},\quad Y_c^{(t_0)},\quad S_c^{(t_0)} \quad \leftarrow \quad \textit{can be measured}$$
$$A_o^{(t_1)},\quad Y_o^{(t_1)},\quad S_o^{(t_1)} \quad \leftarrow \quad \textit{needed for dimensioning}$$
$$A_d^{(t_2)},\quad Y_d^{(t_2)},\quad S_d^{(t_2)} \quad \leftarrow \quad \textit{needed for planning}$$

c = carried

o = offered

d = demand

t_0 = present situation

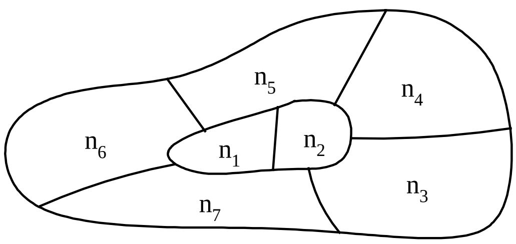
t_1 = future value (short term)

t_2 = future value (long term)

$$A = n \cdot \alpha$$

n = number of subscribers

α =calling rate, erl./subscriber



$$A_i = n_i \cdot \alpha_i$$

$\alpha_a, \alpha_b, \ldots$ } a, b, ... =

$n_a + n_b + \ldots = n$ } sub. categories

$n_{1a} + n_{1b} + \ldots = n_1$

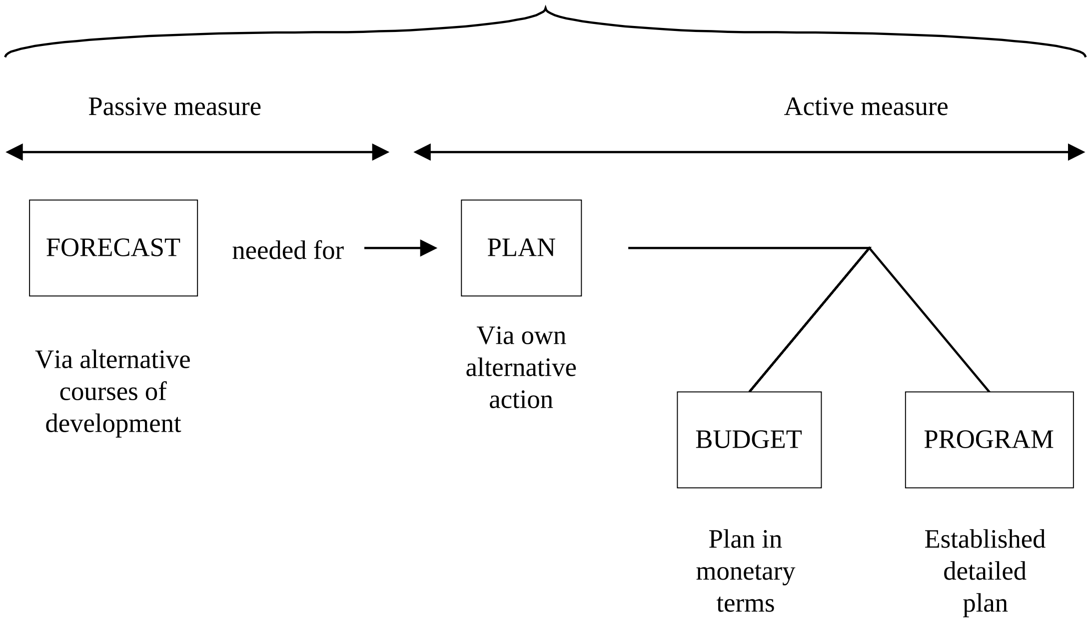
$n_{2a} + n_{2b} + \ldots = n_2$

.. ..

.. ..

$n_a + n_b + \ldots = n$

FACTS PRESENTED TO THE DECISION MAKER



Relation between forecast and plan

Forecast: A forecast is a prediction of the future and is passive from the decision-maker's point of view. Forecasts normally form a basis for planning.

Plan: A plan is a proposal for future action. The plan may contain an evaluation of alternative lines of action and is directed to the activities over which there is control.

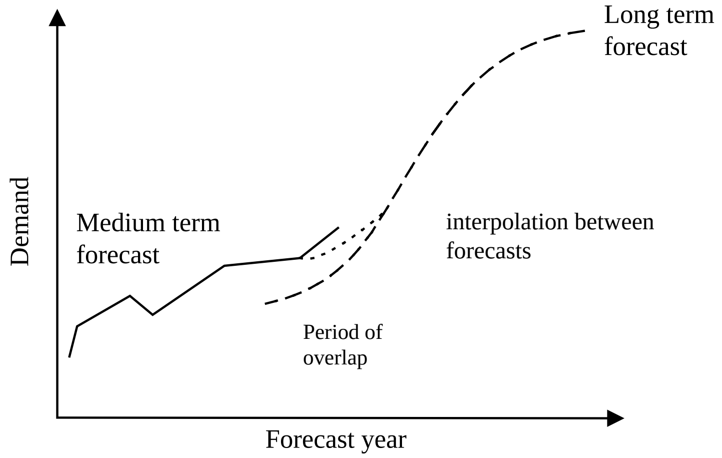
Programme: A programme is a description of the measures, often derived from plans, that have been decided upon.

Planning periods

The period for which forecasts are required depends upon the particular policy question under consideration; demand forecasts are required for decision-making in the following areas:

Customers Apparatus Requirements	1-2 years
Exchange Switching Equipment Provisioning	3-4 years
Local Lines Planning	6-10 years
Ducts	10-15 years
Planning and Construction of Buildings	10-20 years
Site Acquisition and Disposal Policy	up to 50 years

Relationship between long, medium and short term forecasts



How do we start ?

The forecasting process can be divided into the following parts:

- *Definition of the problem*. The purpose of and assumptions for forecasts have to be determined.

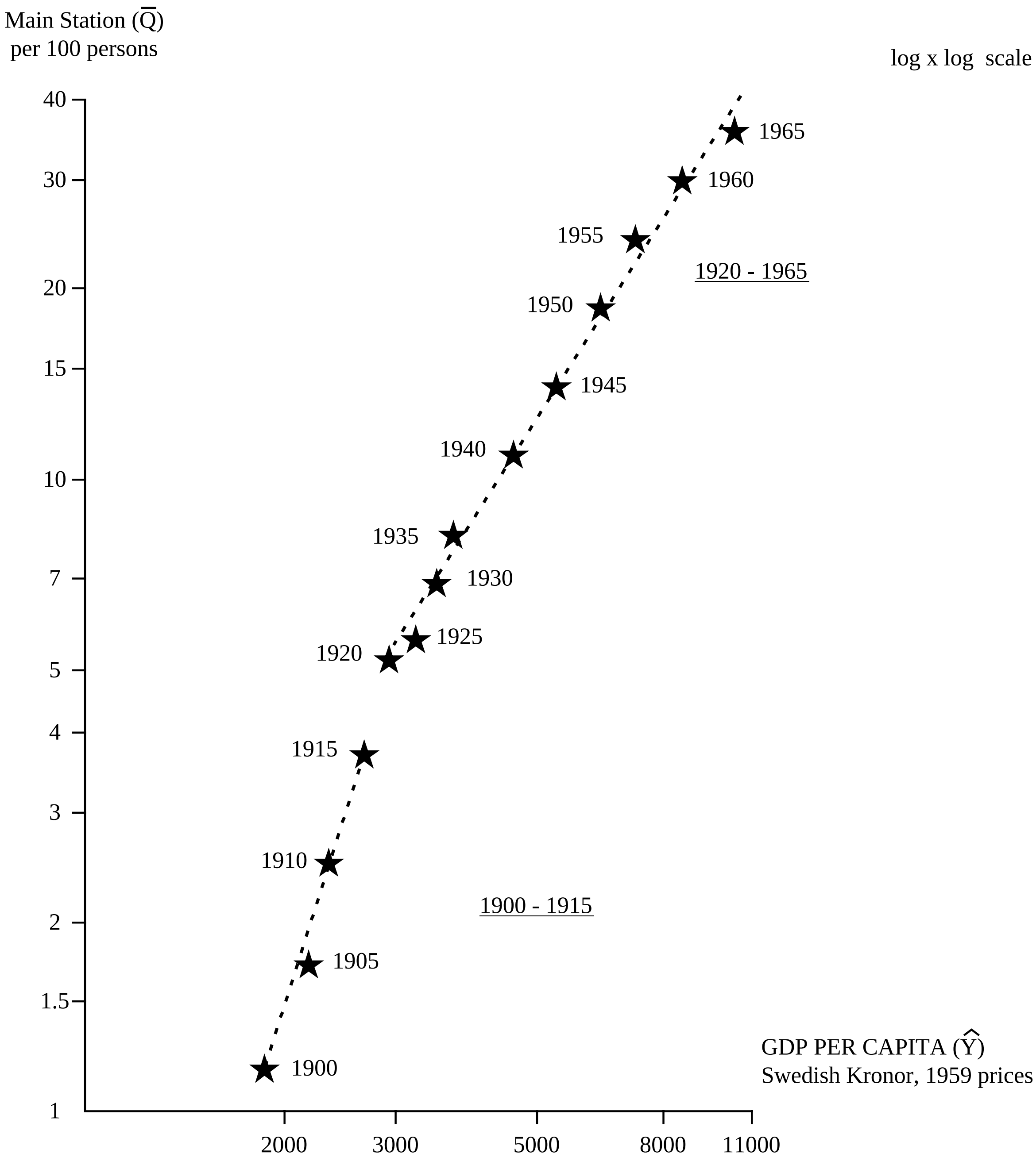
- *Collection of basic data.* Various sources for basic data are investigated. The population and economic growth are studied. The results of recent forecasts are essential facts.

- *Choice of forecast method*. The choice of method is made with regard to available information and required accuracy, etc.

- *Analysis and establishing of forecasts*. The analysis consists of methodical preparation of basic data and evaluation of results received.

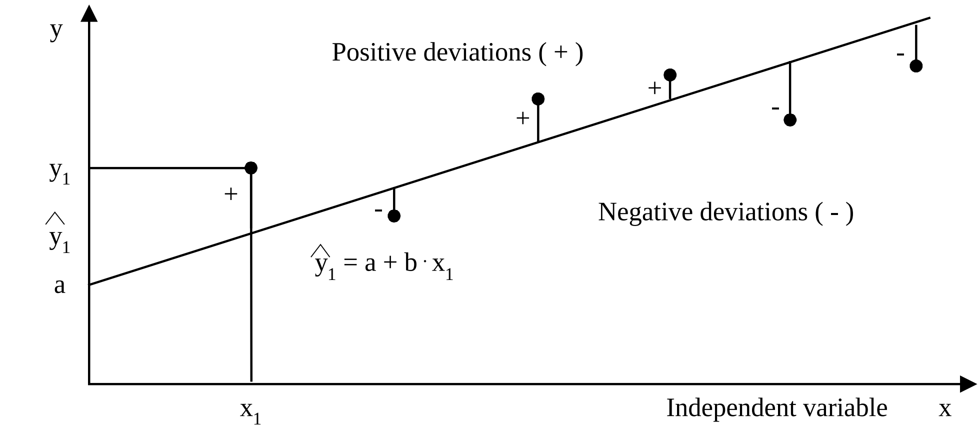
- *Documentation*. The forecast has to be presented in an easily understandable format. The result should contain alternative forecasts. Beside the normal forecast, there should be one optimistic and one pessimistic forecast.

Telephone density and GDP per capita (example)



Statistical demand analysis

Two-variable regression.



Minimize $\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

$y = \alpha + \beta \cdot x + \varepsilon$ $\qquad\qquad \bar{\varepsilon} = 0$

$E\,(y/x) = \alpha + \beta \cdot X$

Let *a, b* be estimates of α, β

$$b = \frac{n \cdot \sum x \cdot y - \sum x \cdot \sum y}{n \cdot \sum x^2 - (\sum x)^2} \qquad\qquad a = \frac{\sum y}{n} - \frac{b \cdot \sum x}{n}$$

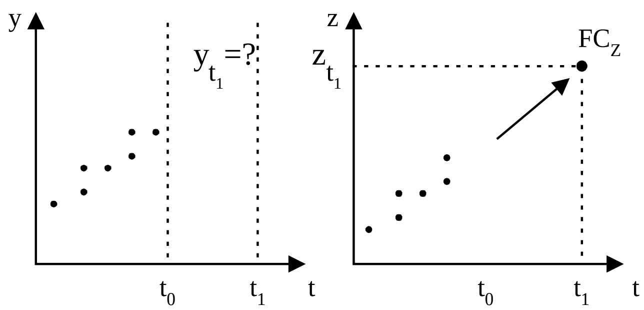
n = number of observations

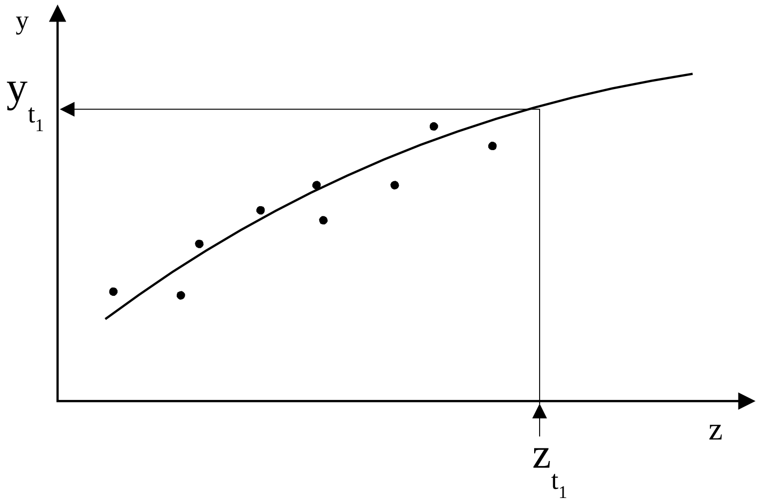
$$R^2 = \frac{[n \cdot \sum (x \cdot y) - \sum x \cdot \sum y]^2}{[n \cdot \sum y^2 - (\sum y)^2] \cdot [n \cdot \sum x^2 - (\sum x)^2]}$$

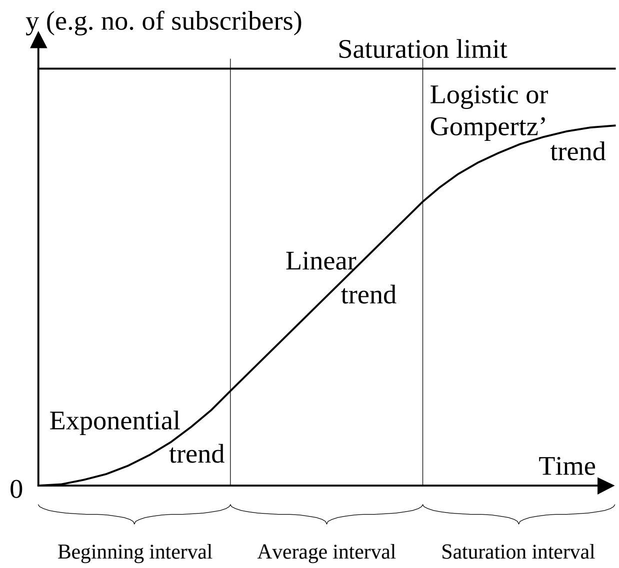
or

$$R^2 = \frac{\Sigma(\hat{y} - \bar{y})^2}{\Sigma(y - \bar{y})^2} = \frac{explained\ variation}{total\ variation}$$

$$\bar{y} = \frac{\sum y}{n}$$
$$-1 \le R \le 1$$




y
y_{t_1}
z
z_{t_1}


y (e.g. no. of subscribers)
Saturation limit
Logistic or Gompertz' trend
Linear trend
Exponential trend
0
Time
Beginning interval
Average interval
Saturation interval

Telecommunication development over time

Development over time in a telecommunication administration.

Curves corresponding to such mathematical expressions are often called growth curves, even if the “growth” is sometimes really a decrease in quantity. Here are some common types of trend curves:

Linear	$y = a + b \cdot t$
Parabolic	$y = a + b \cdot t + c \cdot t^2$
Exponential	$y = a \cdot x \cdot e^{b \cdot t}$
Gompertz	$y = e^{a - b \cdot r^t}$

Denotations

t = point of time (independent variables)

a, b, c, r = parameters to be calculated from historical data

y = item to be forecasted (dependent variable)

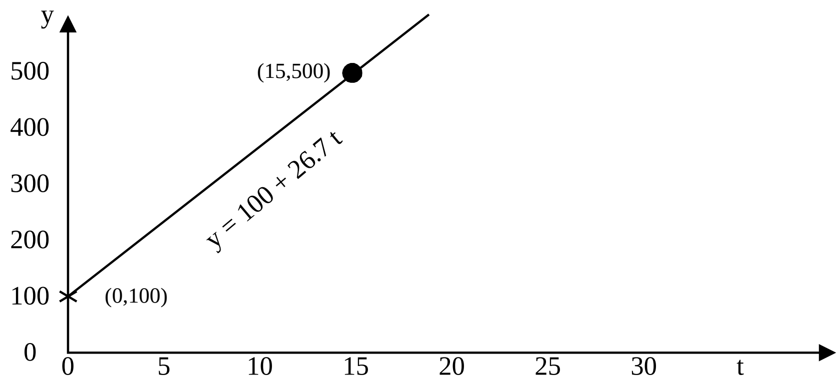
e = the basis for the so called natural logarithms system

Simple trend analysis

There are some simple numerical examples on trends:

1. Linear study y = a + b · t

The formula contains two unknown parameters *a* and *b*, which should be calculated according to basic data. In order to calculate two parameters, you need two equations. These two equations might be obtained by assuming two points in a diagram through which the straight line should pass.



Assume the following points:	t = 0 y = 100	t = 15 y = 500

This gives the two equations required to give *a* and *b*:

$100 = a + b \cdot 15$ **so *a = 100***

$500 = a + b \cdot 15$

which might be put into the second equation, which gives

$500 = 100 + b \cdot 15$ and

$$b = \frac{500 - 100}{15} = 26.7$$

This gives the trend: ***y = 100 + 26.7 · t***

2. Exponential trend $y = a \cdot e^{b \cdot t}$

Assume the same points be given	t = 0 y = 100	t = 15 y = 500

The two required equations will be

$100 = a \cdot e^{b \cdot 0}$

$500 = a \cdot e^{b \cdot 15}$ **so *a = 100***

Put in this *a*-value in the second equation, which gives

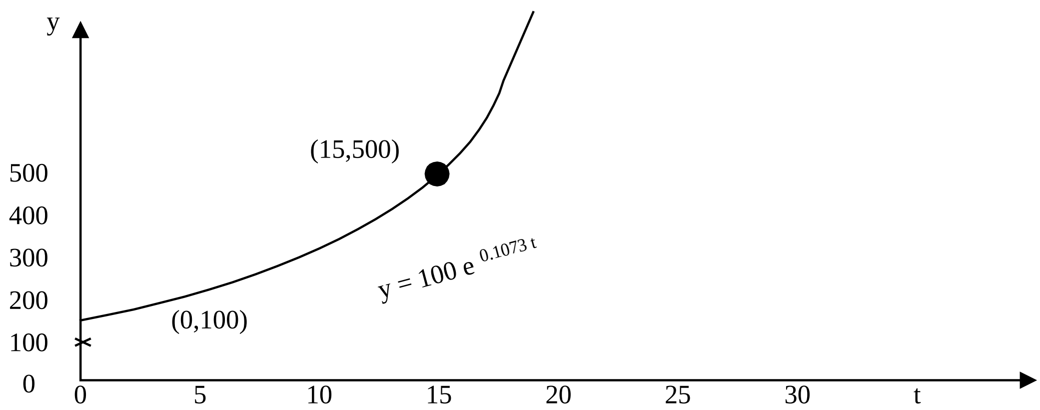
$500 = 100 \cdot e^{b \cdot 15}$ $5 = e^{b \cdot 15}$

Take natural logarithm

$1.609 = 15 \cdot b$ **so *b = 0.1073***

This gives the trend $y = 100 \cdot e^{0.1073 \cdot t}$

The curve is plotted in the following diagram:



3. Gompertz' trend = $e^{a- \ \cdot r^t}$

In this case, we have three parameters *a*, *b* and *r*. The calculation of which requires *three* equations. Two may be obtained by using the two points from the previous example.

$\begin{cases} t = 0 \\ y = 100 \end{cases}$ $\qquad$ $\begin{cases} t = 15 \\ y = 500 \end{cases}$

The third equation might be obtained by assuming a <u>saturation</u> value in infinity that is the point

$\begin{cases} t = \infty \\ y = 3000 \end{cases}$

The three parameters may be calculated in the following way:

$$100 = e^{a- \ \cdot r(0)}$$

$$500 = e^{a- \ \cdot r(1 \ \ )}$$

$$3000 = e^{a-b \cdot r(\infty)}$$

The third equation gives as $r<1$

$3000 = e^{a}:$ $\qquad$ ***<u>a = ln 3000 = 8.006</u>***

The first equation gives, if $a = 8.006$ is inserted

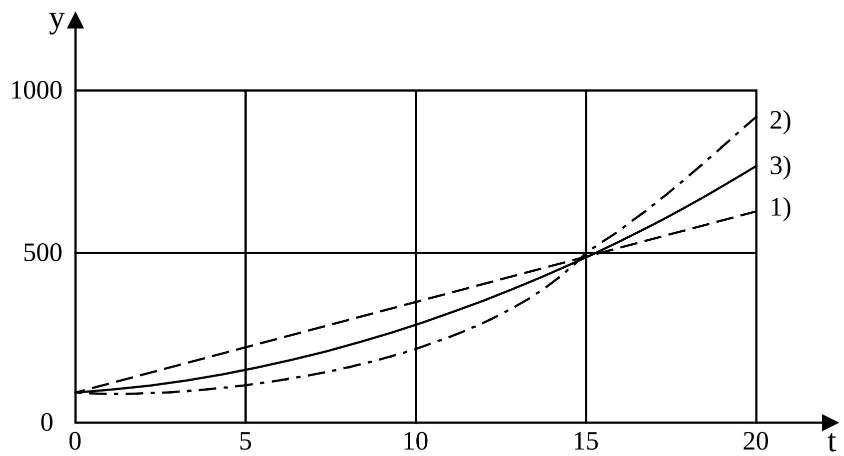
$4.605 = 8.006 - b:$ $\qquad$ ***<u>b = 3.401</u>***

Then, *r* might be calculated from the second equation, putting in

$a = 8.006$ and $b = 3.401$

$$6.215 = 8.006 - 3.401 \cdot r^{15}$$

or $r = \left(\dfrac{8.006 - 6.215}{3.401} \right)^{1/15} = 0.958$



<u>Numerical examples on trends</u>

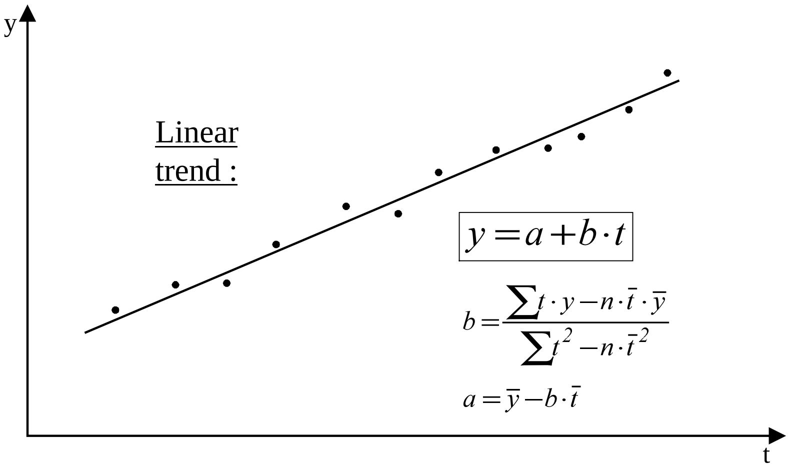
1) Linear trend: $y = 100 + 26.7 \cdot t$

2) Exponential trend: $y = 100 \cdot e^{0.1073 \cdot t}$

3) Gompertz' trend: $e^{8.006 - 3.401 \cdot (0.958)^t}$ (saturation value = 3000)

<u>Time series analysis</u>

Basic methods



<u>Exponential trend</u>:

$$y = a \cdot e^{b \cdot t}$$
$$z = \lambda n\, y$$
$$c = \lambda n\, a$$
$$z = c + b \cdot t$$
$$c = \bar{z} - b \cdot \bar{t}$$
$$b = \frac{\sum t \cdot z - n \cdot \bar{t} \cdot \bar{z}}{\sum t^2 - n \cdot \bar{t}^2}$$
$$y = e^z$$

Statistical checks

<u>Checking the regression model</u>:

1. <u>Significance of the time parameter</u>

$$T = \frac{b \cdot \left(\sum (t - \bar{t})^2 \right)^{1/2}}{s} \qquad s^2 = \frac{\sum y^2 - a \sum y - b \sum t \cdot y}{(n-2)}$$

- 2 > T > + 2

⇒ O.K.

2. Systematic errors

For example, wrong curve shape, discontinuity in data

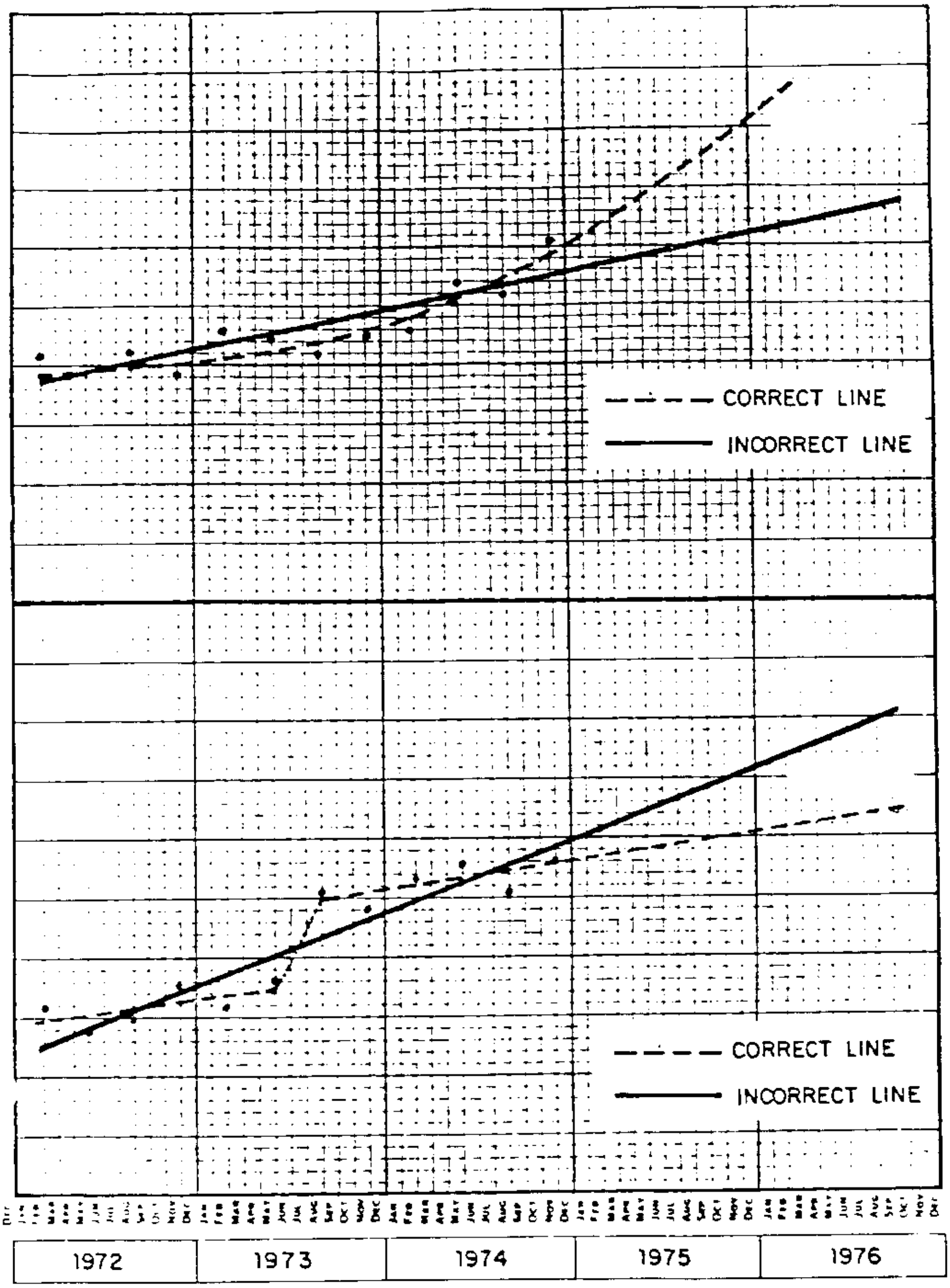
$$DW = 2 - 2\frac{w}{v}$$

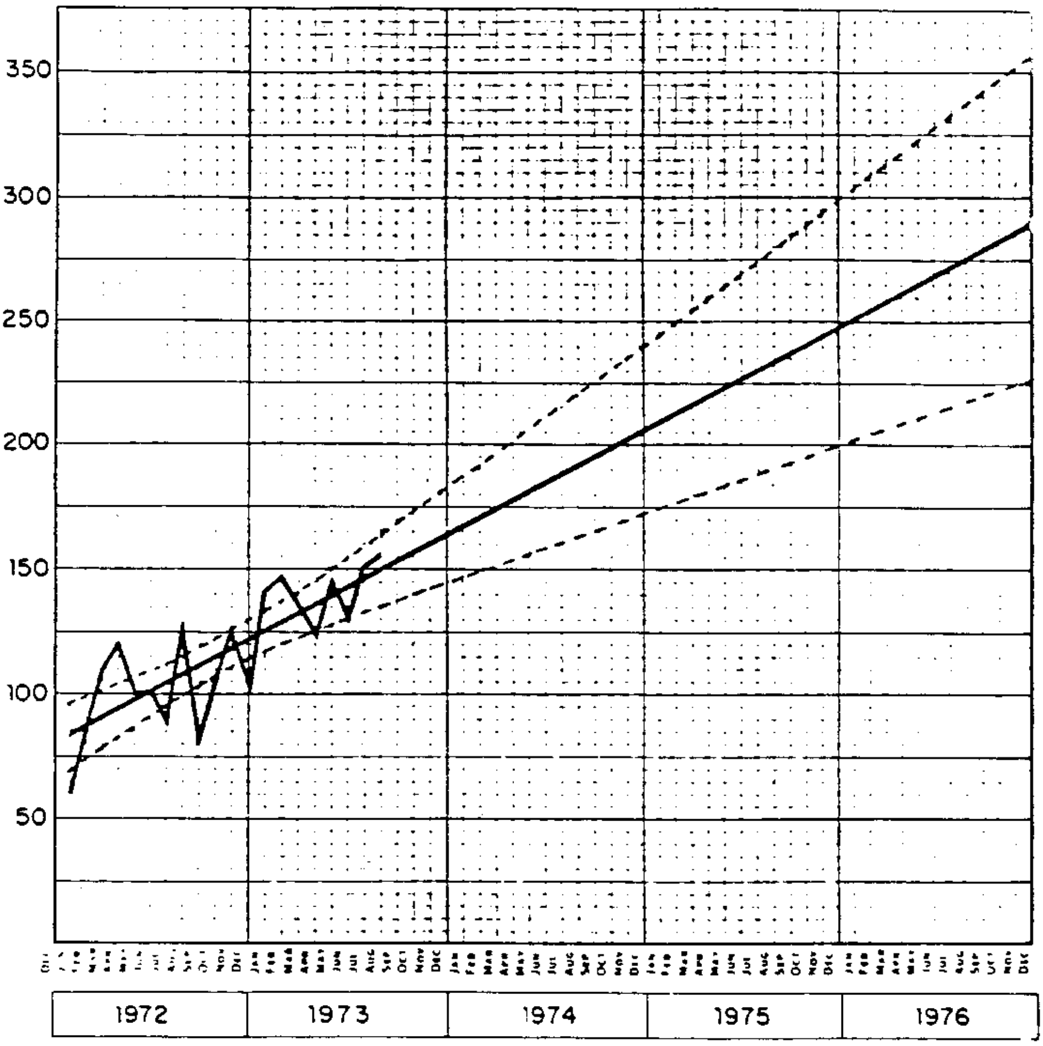
$$W = \sum_{i=1}^{n-1} (y_i - \hat{y}_i)(y_{i+1} - \hat{y}_{i+1})$$

$$V = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

(1.7) 1.5 < DW < 2.3 (2.5)

$\Rightarrow$ O.K.





Forecasting future levels

FC for time $t_F = y_F$

Confidence interval:

$$y_F \pm 2 \cdot \sqrt{u}$$

$$u = s^2\left[1 + \frac{1}{n} + \frac{(t_F - \bar{t})^2}{\sum(t-\bar{t})^2}\right]$$

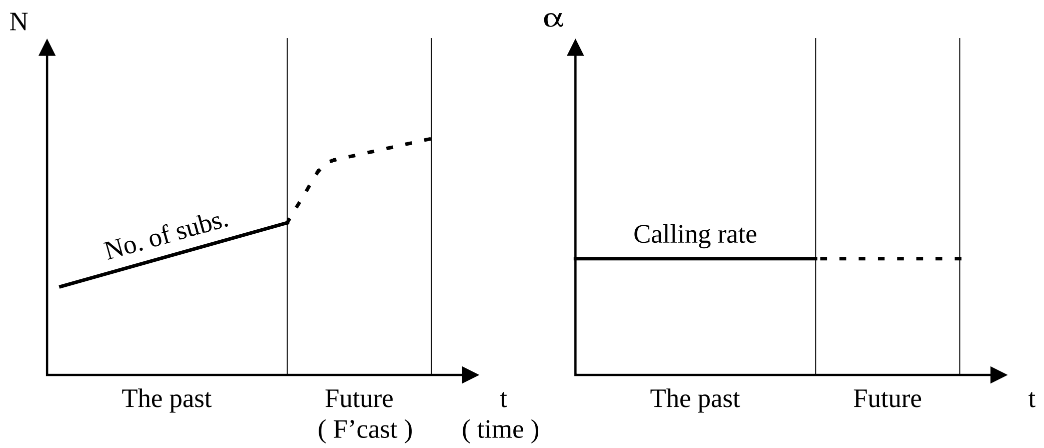
Check of the forecast

We must always ask ourselves how good and applicable the specific forecast is:

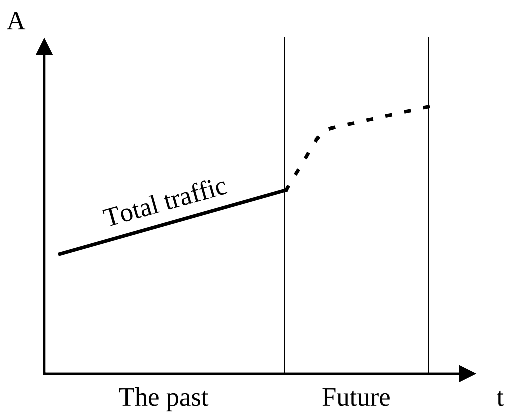
i. *Is the forecast VALID (= relevant) ?*

<u>Example</u>

The total traffic forecast for an exchange has always been based on the forecasts of the total number of subscribers and the calling rate per subscriber. The situation is now the following:



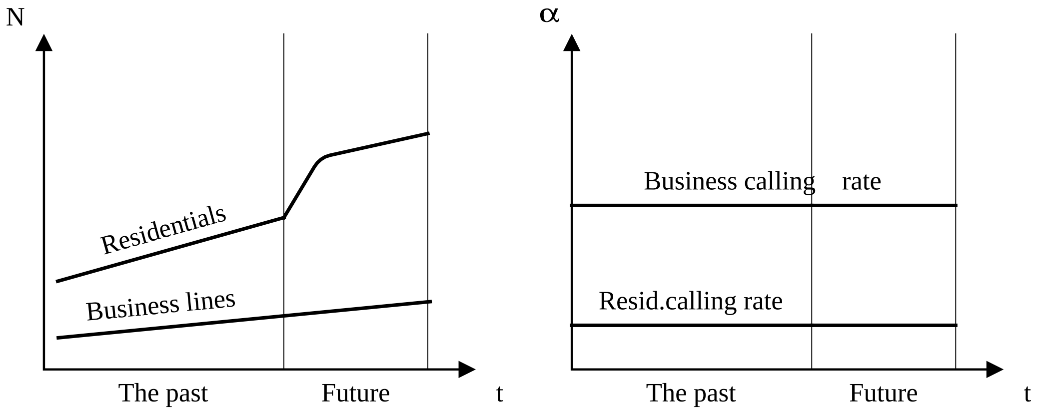
A valid, accurate, reliable and credible forecast of the future number of subscribers (N) has been made, showing that the trend will change due to a community policy decision to modernize the building blocks in the whole area. Since the calling rate has remained constant throughout the years, the forecaster assumes that it will remain unchanged, so that he produces the following traffic forecast by calculating $A = N \cdot \alpha$



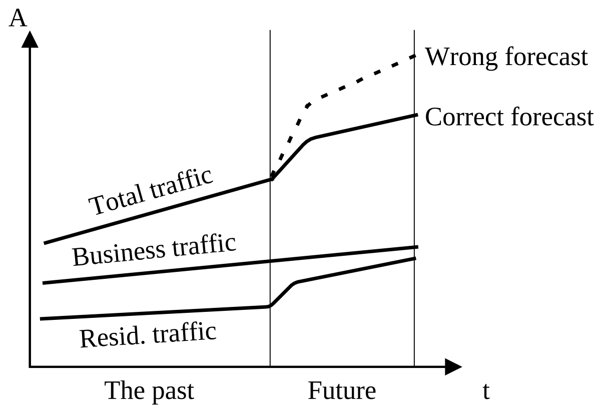
This forecast happens to be invalid.

The intention of the policy decision was to create a lot of new flats for people working in a town nearby. Therefore, the expected sharp increase relates to residential lines and not to business lines.

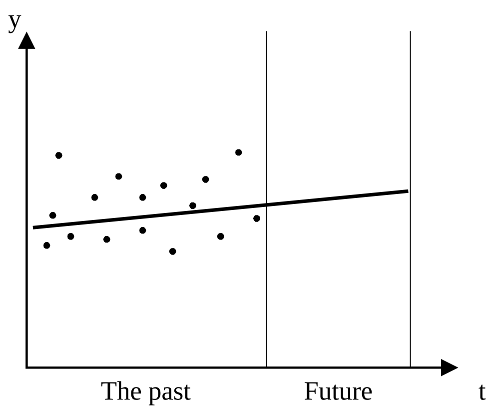
The appropriate procedure should have been to separate the subscriber forecasts for residentials and business lines and then add them together.



Forecast:



Another example:

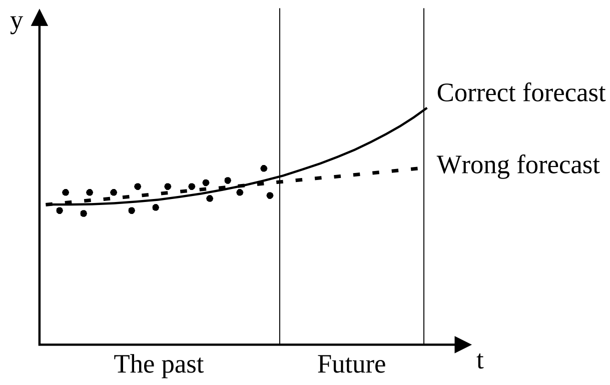


The forecast is invalid if a significance test shows that time is not a significant variable.

ii. *Accuracy*

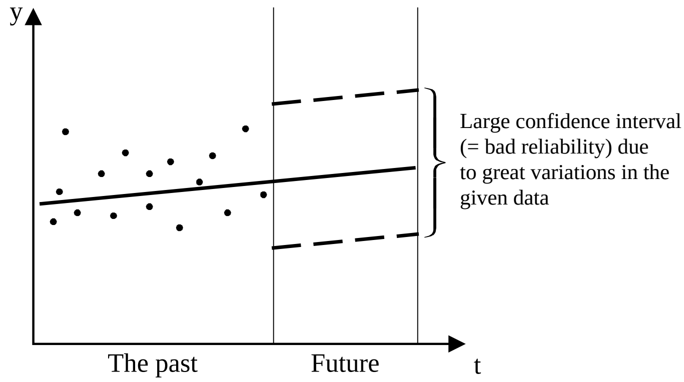
Example:

Historical material is given. The forecaster has to choose a curve-form to represent the trend. An inaccurate curve may then seem to agree rather well with plots, but may give a completely wrong forecast.



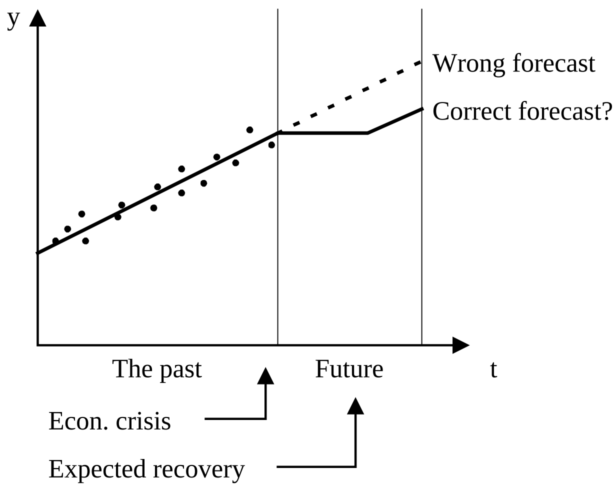
iii. *Reliability (= precision)*

Example:



iv. *Credibility*

Example:



v. *Internal evidence in data*

We must also look for and analyse:

- inconsistencies (e.g. different data sets are contradictory)
- irregular patterns
- unlikely or impossible values

Individual judgment

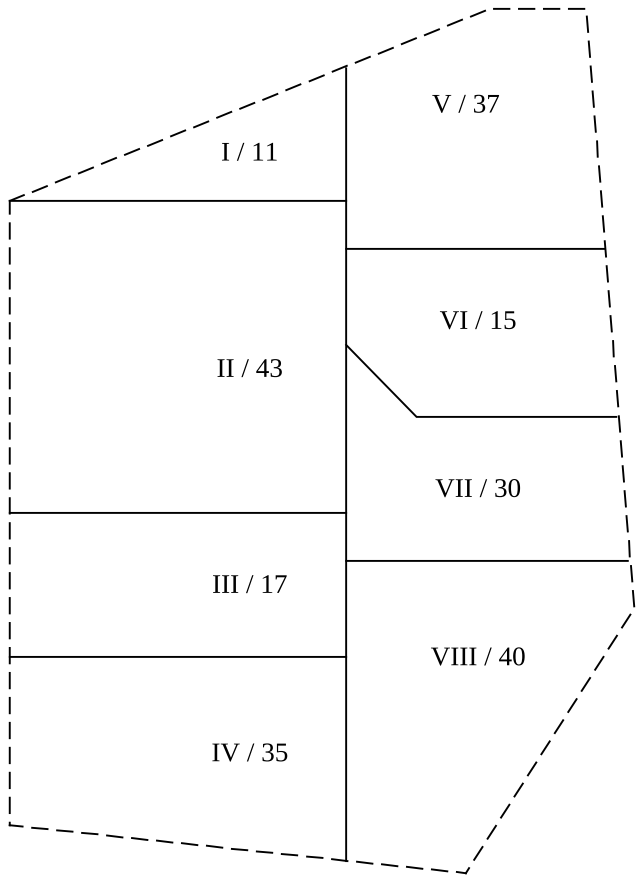
Assessment of number of subscribers per building of different kinds

This example is valid for a developing country with 4-8 subscribers per 100 inhabitants.

Type of building	Number of subscribers
Official buildings and offices, banks, insurance companies, big hotels, clubs, big restaurants, hospitals, large commercial houses	Investigate
Small hotels, restaurants, grocery stores, boarding houses	1 - 2
Chemists, doctors, lawyers, etc.	1 - 1.5
Shops	0.5 - 1
Large factories	Investigate
Small factories, workshops	0.5 - 1.5
Cinemas, petrol stations	1 - 2
Private houses of highest class	1
Private houses of lower class	0.3 - 0.5
Terrace houses	0.3 / apartment
Blocks of flats, highest class	0.5 - 1 / apartment
Blocks of flats, lower class	0.2 / apartment

Assessment of number of subscribers per hectare in built-up areas of different kinds. This example is valid for a developing country with 4-8 subscribers per 100 inhabitants.

	Type of built-up area	Subscribers per hectare
A.	Slum	0.25
B.	Parks, gardens, etc.	0.5
C.	Old private houses with large gardens	1
D.	Poor workers’ residential districts	1.5
E.	Better class workers’ residential districts	2
F.	Modern private houses with large gardens	3
G.	Modern workers’ residential districts	4
H.	Industrial areas	5
I.	Modern private houses with small gardens	7
J.	Non-detached houses of older type	8
K.	Area consisting of working class dwellings and small workshops	10
L.	Modern non-detached houses	13
M.	Non-detached 1-2 storey residential buildings and small shops	18
N.	Blocks of flats up to 4 storeys	25
O.	Blocks of flats and shops of up to 4 storeys	28
P.	Business centre in residential area	30
Q.	Blocks of flats of more than 4 storeys	40
R.	Office buildings up to 3 storeys	80
S.	Office buildings of 4-6 storeys	150
T.	Office buildings of more than 6 storeys	250

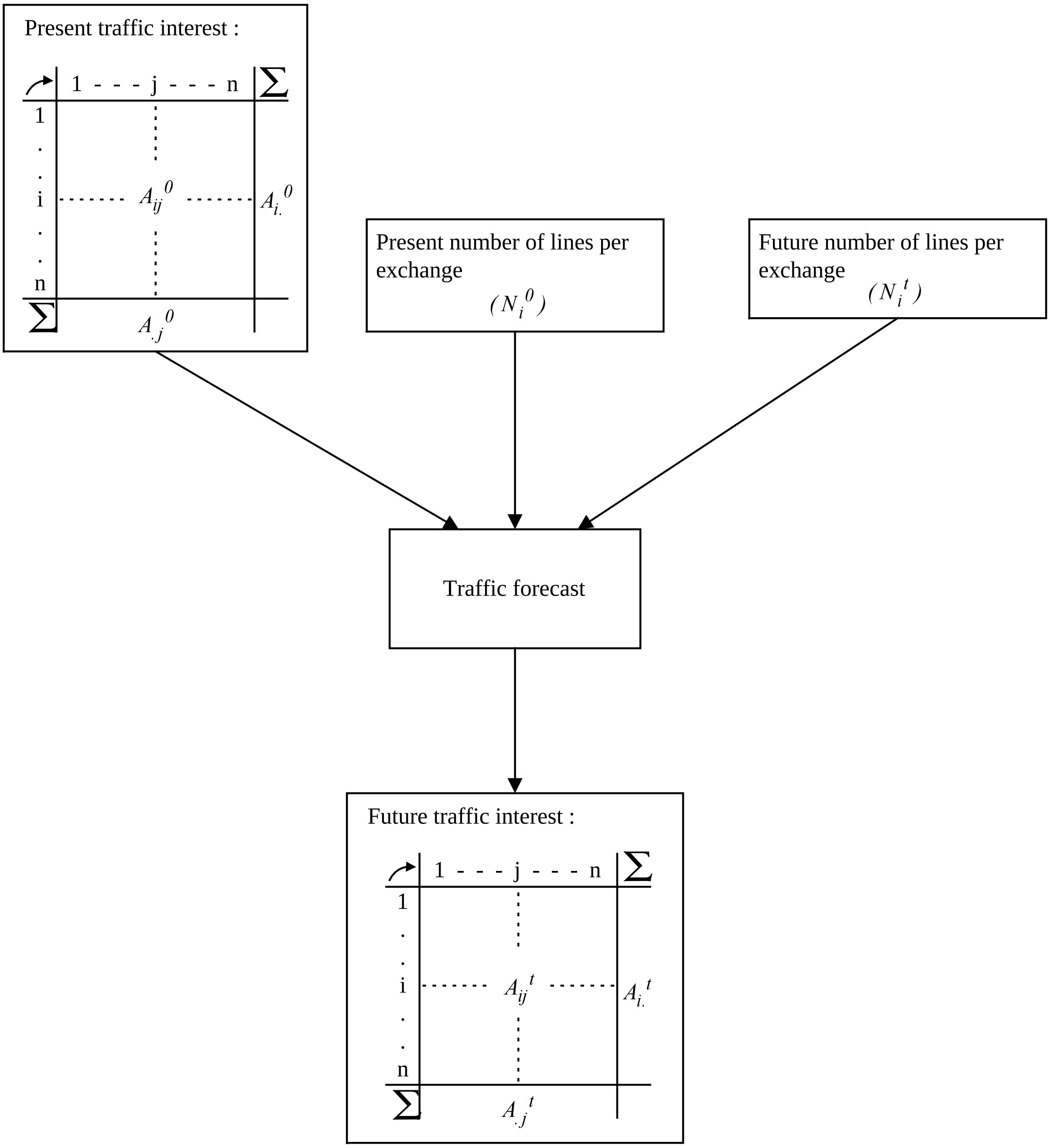


Exchange area divided into sections for building forecast

-------------	Exchange boundary
__________	Section boundary
III/17	Section number/area in hectares

Forecast model for network planning

Components in a traffic forecasting model



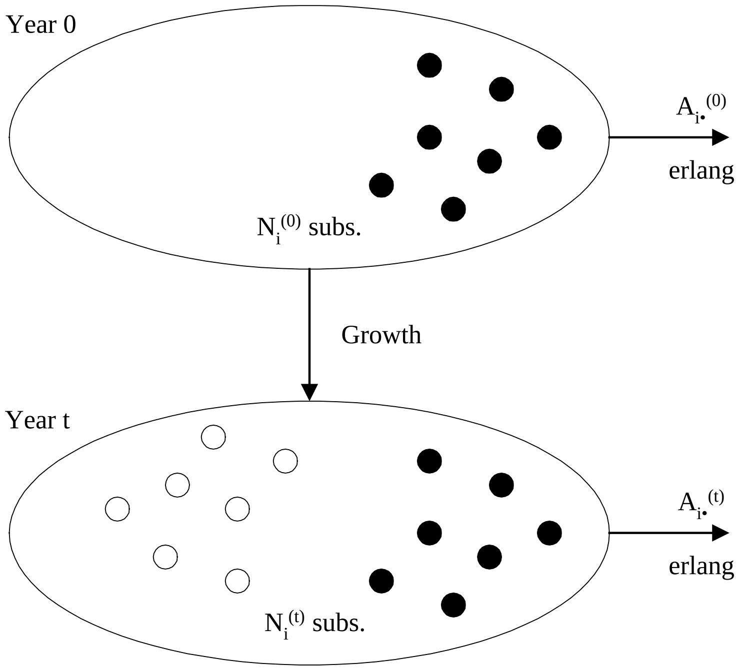
Traffic interest matrix

		j		Σ
i		A_{ij}		$A_{i.}$
Σ		$A_{.j}$		$A_{..}$

$A_{i.}$ = Total orig. traffic in area i

$A_{.j}$ = Total term. traffic in area j

A_{ij} = Traffic interest from area i to area j (point-to-point traffic)



<u>Models for total traffic</u>

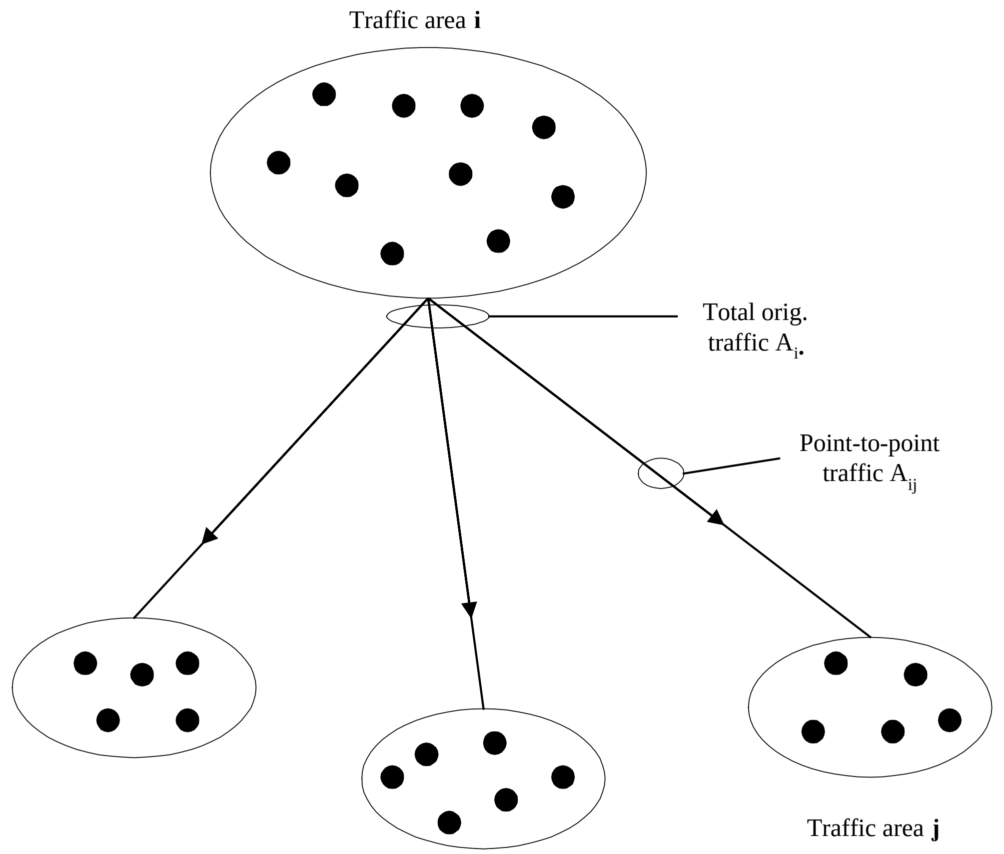
$$A_{i\cdot}^{(t)} = A_{i\cdot}^{(0)} \frac{N_i^{(t)}}{N_i^{(0)}} \cdot \alpha \qquad \alpha \geq 1$$

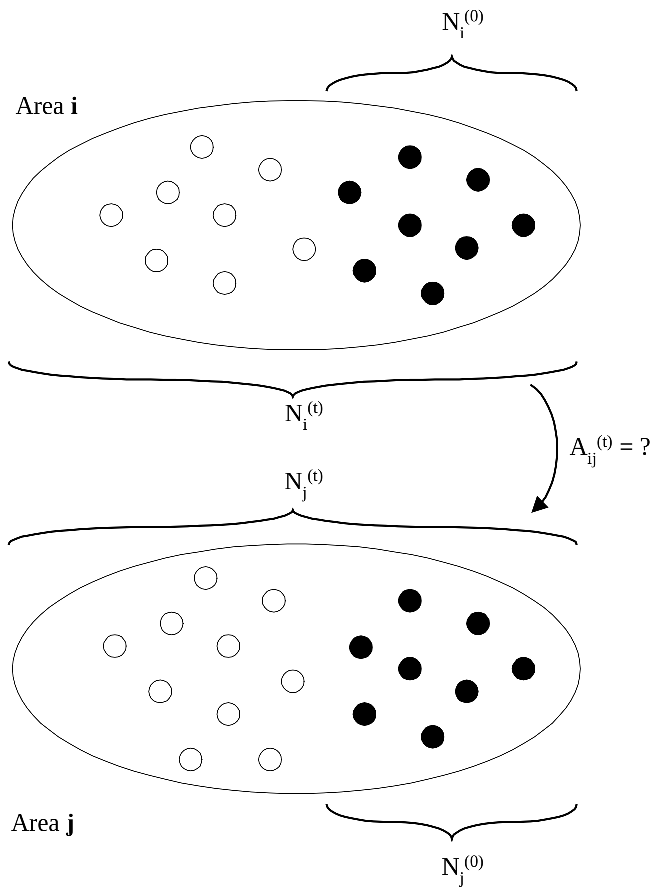
If $\alpha = 1$, traffic per main line is assumed to be constant.

$$A_{i\cdot}^{(t)} = A_{i\cdot}^{(0)} \left(\frac{N_i^{(t)}}{N_i^{(0)}} \right)^{\alpha} \qquad \alpha \geq 1$$

The percentage growth of traffic is assumed to be equal to the percentage growth of the number of main lines, times α. (same formulae for $A_{\cdot j}$)

Models for point-to-point traffic: Weight Growth

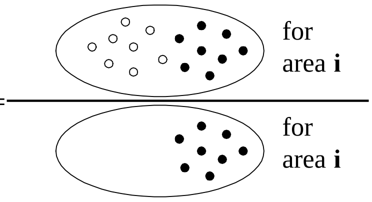




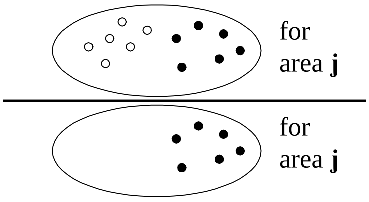
<u>*Basic expression for*</u> $A_{ij}^{(t)}$ *:*

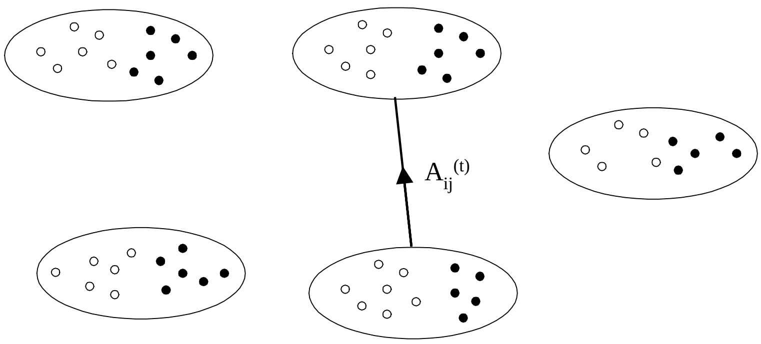
$$A_{ij}^{(t)} = A_{ij}^{(0)} \cdot \frac{W_i \cdot G_j + W_j \cdot G_i}{W_i + W_j}$$

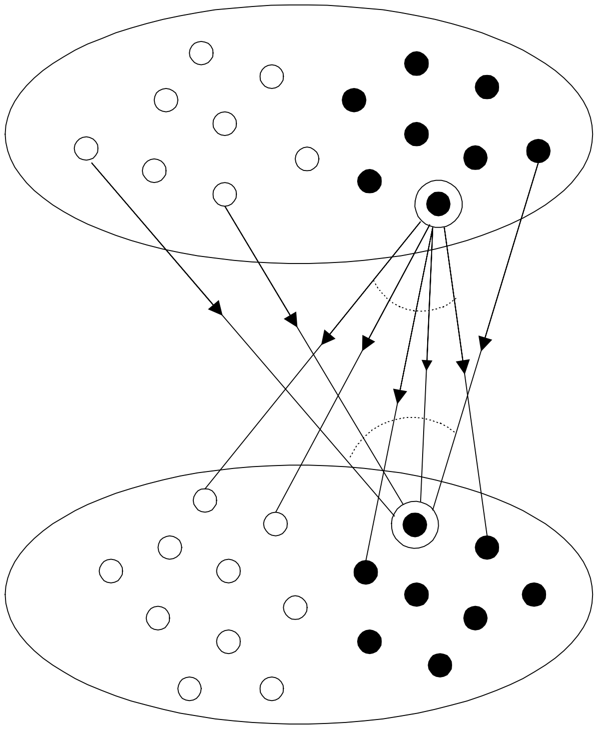
$G_i = \dfrac{N_i^{(t)}}{N_i^{(0)}} =$



$G_j = \dfrac{N_j^{(t)}}{N_j^{(0)}} =$







Rapp's formula 1:

$$W_i = N_i^{(t)}, \quad W_j = N_j^{(t)}$$

Rapp's formula 2:

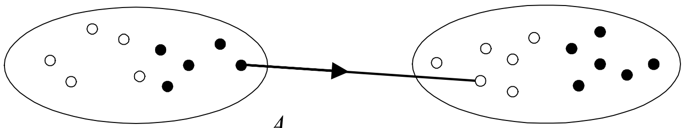
$$W_i = \left(N_i^{(t)}\right)^2, \quad W_j = \left(N_j^{(t)}\right)^2$$

$$\left(\frac{A_{i\cdot}^{(0)}}{N_i^{(0)}} - \frac{A_{i\cdot}^{(t)}}{N_i^{(t)}}\right)^2 + \left(\frac{A_{\cdot j}^{(0)}}{N_j^{(0)}} - \frac{A_{\cdot j}^{(t)}}{N_j^{(t)}}\right)^2 = min.$$

APO

$$W_i = \frac{N_i^{(0)} + N_i^{(t)}}{2}$$

$$W_j = \frac{N_j^{(0)} + N_j^{(t)}}{2}$$



$$\frac{A_{ij}}{N_i \cdot N_j} = constant :$$

$$A_{ij}^{(t)} = A_{ij}^{(0)} \frac{N_i^{(t)}}{N_i^{(0)}} \cdot \frac{N_j^{(t)}}{N_j^{(0)}}$$

$$A_{ij}^{(t)} = A_{ij}^{(0)} \cdot N_i^{\alpha} \cdot N_j^{\beta}$$

Traffic distribution methods: Kruithof's double factor method

A critical survey of a number of methods for prediction of the distribution on traffic in a telephone network has been made by D. Bear.

The most common method is *Kruithof's Double Factor Method*. The mathematical performance of Kruithof's Double Factor Method is illustrated by the following example:

Example: Consider a telephone network with two exchanges.

Given:

1. The present traffic interests $\left[A_{ij}^{(0)}\right]$

j / i	1	2	sum
1	10	20	30
2	30	40	70
sum	40	60	100

2. Forecasted values on the future total originating and terminating traffics per exchange

$\left[A_{i.}^{(t)} \quad and \quad A_{.j}^{(t)}\right]$:

j / i	1	2	sum
1			40
2		?	105
sum	50	100	150

Problem:

Estimate the traffic values $A_{ij}^{(t)}$ by using Kruithof's method.

Solution:

Iteration 1 Row multiplication.
$A_{i.}^{(t)}$ is distributed according to the present traffic interest.

Result: $A_{ij}^{(1)}$

j / i	1	2	sum
1	15	30	45
2	45	60	105
sum	60	90	150

$$A_{ij}^{(1)} = \frac{A_{ij}^{(0)}}{A_{i\cdot}^{(0)}} \cdot A_{i\cdot}^{(t)}$$

After row multiplication, the sums of the columns differ too much from the forecast.

Iteration 2 Colum multiplication.

$A_{.j}^{(\;)}$ is distributed according to the traffic interest matrix given by iteration 1.

Result: $A_{ij}^{(2)}$

i \ j	1	2	sum
1	12.5	33.33	45.83
2	37.5	66.67	104.17
sum	50	100	150

$$A_{ij}^{(2)} = \frac{A_{ij}^{(1)}}{A_{.j}^{(1)}} \cdot A_{.j}^{(t)}$$

After column multiplication, sums of the rows differ from the forecasted values.

Iteration 3 Row multiplication.

$A_{i.}^{(t)}$ is distributed according to the traffic interest matrix given by iteration 2.

Result: $A_{ij}^{(3)}$

i \ j	1	2	sum
1	12.27	32.73	45
2	37.80	67.20	105
sum	50.07	99.93	150

$$A_{ij}^{(3)} = \frac{A_{ij}^{(2)}}{A_{i.}^{(2)}} \cdot A_{i.}^{(t)}$$

Iteration 4 Column multiplication.

$A_{.j}^{(\;)}$ is distributed according to the traffic interest matrix given by iteration 3.

Result: $A_{ij}^{(4)}$

i \ j	1	2	sum
1	12.25	32.75	45
2	37.75	67.25	105
sum	50	100	150

$$A_{ij}^{(4)} = \frac{A_{ij}^{(3)}}{A_{.j}^{(3)}} \cdot A_{.j}^{(t)}$$

After 4 iterations, the sums of rows and columns are equal to the forecasted values. We can put:

$A_{ij}^{(\;)} = A_{ij}^{(4)}$

Note: $A_{i.} = \sum_j A_{ij}; \quad A_{.j} = \sum_i A_{ij}$

<u>Example 2</u>

The present traffic matrix has been estimated:

	To	Exch. No. j			
From:		1	2	3	sum
Exch.	1	25	30	45	100
No. i	2	35	55	110	200
	3	60	85	155	300
sum		120	170	310	600

$= (A_{ij}^{(0)})$

The number of main lines per exchange in the year *t* has been forecasted:

Exch. No.	$N_i^{(0)}$	$N_i^{(t)}$
1	2000	3000
3	3500	3500
3	6800	7500

The main lines have not been classified into different categories since the proportion of high-traffic subscribers is expected to be the same in the future.

The total originating and terminating traffic per exchange is therefore forecasted by the models:

$$A_{i.}^{(t)} = N_i^{(t)} \cdot \frac{A_{i.}^{(0)}}{N_i^{(0)}}$$

$$A_{.j}^{(t)} = N_j^{(t)} \cdot \frac{A_{.j}^{(0)}}{N_j^{(0)}}$$

Exchange No.	$A_{i.}^{(t)}$	$A_{.j}^{(t)}$
1	150.0	180.0
2	200.0	170.0
3	331.9	341.9
Sum	681.9	691.9

Since the sum of $A_{i.}^{(t)}$ and the sum of $A_{.j}^{(\ )}$ differ, we can use the mean value of these sums as an estimate of $A^{(\ )}$ and adjust the $A_{i.}^{(t)}$ and $A_{.j}^{(\ )}$. This will give:

Exchange No.	$A_{i.}^{(t)}$	$A_{.j}^{(\ )}$
1	151.1	178.7
2	201.5	168.8
3	334.3	339.4
Sum	686.9	686.9

The weighted growth factor models can now be used to forecast the point-to-point traffic.

The growth factors are equal to:

Exchange 1 $$G_1 = \frac{N_1^{(t)}}{N_1^{(0)}} = 1.5$$

Exchange 2 $G_2 = 1.0$

Exchange 3 $G_3 = 1.1$

A. *Forecast according to Rapp's formula 1:*

To From	Exchange 1	 2	 3	 Sum
Exch. 1	37.5	38.1	62.4	138.0
Exch. 2	44.4	55.0	113.5	212.9
Exch. 3	83.1	87.7	170.5	341.3
Sum	165.0	180.8	346.4	692.2

B. *Forecast according to Rapp's formula 2:*

To From	Exchange 1	 2	 3	 Sum
Exch. 1	37.5	38.6	65.0	141.1
Exch. 2	45.1	55.0	112.0	212.1
Exch. 3	86.7	92.6	170.5	349.8
Sum	169.3	186.2	347.5	703.0

C. *Forecast according to APO's formula:*

To From	Exchange 1	 2	 3	 Sum
Exch. 1	37.5	38.8	62.8	139.1
Exch. 2	45.2	55.0	113.6	213.8
Exch. 3	83.8	87.8	170.5	342.1
Sum	166.5	181.9	346.9	695.0

For exchange 1, one can see that the total originating traffic per main line has decreased from the present value of 0.050 to between 0.046 and 0.047.

For exchange 2, on the other hand, the total originating traffic per main line has increased from 0.057 to about 0.060.

As the traffic per main line in this case was considered as constant during the forecast period, the obtained matrices have to be reconciled with the estimate of the total originating and the total terminating traffic for each exchange. Kruithof's Double Factor Method is used for this purpose.

According to Rapp's formula 1:

To / From	Exchange 1	2	3	Sum
Exch. 1	44.5	39.1	67.5	151.1
Exch. 2	45.8	49.0	106.7	201.5
Exch. 3	88.4	80.7	165.3	334.3
Sum	178.7	168.8	339.4	686.9

According to Rapp's formula 2:

To / From	Exchange 1	2	3	Sum
Exch. 1	43.6	38.3	69.6	151.1
Exch. 2	46.2	48.5	106.7	201.5
Exch. 3	89.2	82.0	163.1	334.3
Sum	178.7	168.8	339.4	686.9

According to APO's formula:

To / From	Exchange 1	2	3	Sum
Exch. 1	44.0	39.5	67.6	151.1
Exch. 2	46.2	48.8	106.5	201.5
Exch. 3	88.5	80.5	165.3	334.3
Sum	178.7	168.8	339.4	686.9