Security assessment and key management in a quantum key distribution network **Xiongfeng Ma** xma@tsinghua.edu.cn **Center for Quantum Information** In collaboration with Hongyi Zhou, Kefan Lyu, Longbo Huang



Outline

- Background
 - Quantum key distribution (QKD)
 - Current implementations
 - Field tests of QKD networks
- Security assessment
- Key management

Background

Quantum Cryptography

- Quantum communication
 - Quantum key distribution, teleportation, secret sharing, ...
- Quantum cryptography
 - Secure against quantum computing, informationtheoretical
- Quantum key distribution cannot replace current communication means
 - Many other tasks, such as authentication and signature, done well with classical means
- In near future, quantum communication cannot replace classical ones



Quantum key distribution (QKD)

- BB84 (Bennett & Brassard 1984)
 Point-to-point protocol
- What if introducing multiple users?

QKD network



Current implementations

- USA QKD network: DARPA, Battelle, Smartgrid, Since 1004
- Japan: Tokyo network, 2015
- Europe: SECOQC, 2008
- Switzerland: Geneva network, 2009
- South Africa: Durban, 2009
- Australia: Melbourne network, 2010



China's Quantum Secure

Backbone project

- Total length 2000 km
 - 2013.6-2016.12
 - 32 trustable relay nodes and 31 fiber links
 - GDP 35.6% and population 25.8%
- Metropolitan networks
 - Existing: Hefei, Jinan
 - New: Beijing, Shanghai
- Customers
 - China Industrial & Commercial Bank; Xinhua News Agency; China Banking Regulatory Commission ...



Practical Tasks in QKD Network

QKD Network Structure

- Host layer
- Key Management Layer
- QKD Network Layer
- QKD Link Layer

Tysowski et al. Quantum Science and Technology, 2018, 3(2): 024001.



Network topological settings

Different scenarios employ different settings



Alleaume, Roueff, Diamanti, Lutkenhaus, New J. Phys. 11 075002 (2009)

Practical Tasks

- Network issues
 - Latency, reliability, scalability, cost
 - Security
 - Key consumption
- Two simple examples
 - In trusted nodes scenario, some of the nodes may be compromised, where the keys are insecure
 - Balance key distribution and consumption
 - Data routing and scheduling



Task I: compromised nodes

- Designing the classical communication scheme in highest security level with sufficient keys
- Strongest attack
 - Eve may eavesdrop arbitrary nodes and obtain their keys
 - The strongest attack is a cut between Alice and Bob in the graph
 - Eve's strategy: minimum cut problem in graph theory



Communication in highest security

- Communication scheme in highest security level can only be hacked by the strongest attack
 - Each node (except Alice and Bob) broadcast the exclusive-or result of all keys from connected channels.
 - Alice broadcasts the exclusive-or result of all keys from connected channels and the message to be sent.



Task II: Key management and data scheduling

- QKD network: graph description $G = \{N, L\}$
 - User: node
 - Distributed key: edge
- Share key => transfer message along a single link
- QKD network state at time t
 - For nodes $n \in N$, how much data to transfer to $c \in N$, $Q_n^c(t)$?
 - For channels $[n, m] \in L$, how much key stored, $E_{[n,m]}(t)$?



Utility optimization

- Data transmission from n to c: $R_n^c(t)$
- Utility: the value of the transmitted data: $U_n^c(R_n^c(t))$
 - Concave with $R_n^c(t)$
 - Defined according to practical cases: e.g. $\log_2 R_n^c(t)$
- Utility optimization: Optimize data scheduling $R_n^c(t)$ and key management $P_{[n,m]}(t)$ to maximize the total utility $\sum_{n,c} U_n^c(R_n^c(t))$,

$$\max_{\substack{R_n^c(t), P_{[n,m]}(t) \\ r_n^c}} \sum_{\substack{n,c \\ \tau}} U_n^c(r_n^c)$$
$$r_n^c = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{\tau} R_n^c(\tau),$$

given the following dynamics and constraints.

Dynamics

• Dynamic queue: how the data (key) size in t+1 evolves from t $Q_n^c(t+1) = Q_n^c(t) + \sum_b \mu_{[n,b]}^c(t) - \sum_a \mu_{[a,n]}^c(t) + R_n^c(t)$ $E_{[n,m]}(t+1) = E_{[n,m]}(t) + K_{[n,m]}(t) - P_{n,m}(t)$



Constraints

 Stability constraint: a well-defined problem in field of network -- residual data is convergent with time

$$\bar{Q} = \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n,c} Q_n^c(\tau) < \infty$$

Solution: Lyapunov Function and drift,

$$L(t) = \frac{1}{2} \sum_{n,c} [Q_n^c(t)]^2 + \frac{1}{2} \sum_{[n,m]} [E_{[n,m]}(t) - \theta_{[n,m]}]^2$$
$$\Delta(t) = L(t+1) - L(t)$$

• Key availability constraint: consumption is less than storage

$$P_{[n,m]}(t) \le E_{[n,m]}(t)$$

Tanaka K, Hori T, Wang H O. A multiple Lyapunov function approach to stabilization of fuzzy control systems[J]. IEEE Transactions on Fuzzy Systems, 2003, 11(4):582-589.

Optimization algorithm

• Target function

$$\Delta(t) - \sum_{n,c} U_n^c \left(R_n^c(t) \right) \le B + C + D + E$$

- Main idea:
 - Minimize target function
 - Obtain optimal data scheduling $R_n^c(t)$ and key management $P_{[n,m]}(t)$
 - Decouple $R_n^c(t)$ and $P_{[n,m]}(t)$



B=*constant*

$$C = \sum_{[n,m]} (E_{[n,m]}(t) - \theta_{[n,m]}) K_{[n,m]}(t)$$

$$D = -\sum_{n,c} \left[V U_n^c \left(R_n^c(t) \right) - Q_n^c(t) R_n^c(t) \right]$$

$$E = -\sum_{[n,m]} \{\mu_{[n,m]}(t) [Q_n^c(t) - Q_b^c(t)] + (E_{[n,m]}(t) - \theta_{[n,m]})P_{[n,m]}(t)\}$$

Huang L, Neely M J. Utility optimal scheduling in energy-harvesting networks[J]. IEEE/ACM Transactions on Networking (TON), 2013, 21(4): 1117-1130.

Optimization algorithm

- Input of the algorithm. Initialize $\theta_{[n,m]}$. At every time slot t, observe $Q_n^c(t)$, $E_{[n,m]}(t)$.
- Key generation. Minimize C. Obtain an optimized key generation strategy $K_{[n,m]}(t)$.
- **Data transmission.** Minimize D. Obtain an optimized data scheduling strategy $R_n^c(t)$.
- Key management. Minimize E. Obtain an optimized key management strategy $P_{[n,m]}(t)$.
- Queue update. Use the dynamics of data queue and key storage queue. Obtain $Q_n^c(t+1)$, $E_{[n,m]}(t+1)$.

Conclusion and Outlook

- More network issues should be taken into account
- Current graph theory techniques can be employed in QKD network
- Current network techniques can be employed in QKD network
- Generalized to quantum network
 - Entanglement layer
 - Quantum computing

Thank you!

- International graduate student fellowships are available!
- Welcome to join and visit!

