Rec. ITU-R V.574-4

RECOMMENDATION ITU-R V.574-4

USE OF THE DECIBEL AND THE NEPER IN TELECOMMUNICATIONS*, **, ***

(1978-1982-1986-1990-2000)

Scope

This text recommends the symbols to be used for the logarithmic expression of quantities referring to power, and provides examples of the use as well as the relationship between the decibel and the neper.

The ITU Radiocommunication Assembly,

considering

a) the frequent use by the ITU of the decibel and the neper for expressing quantities;

b) the International Standard IEC 60027-3 of the International Electrotechnical Committee; 1989 on "logarithmic quantities and units";

c) International Standard ISO 31 of the International Standards Organization on quantities and units;

d) the convenience of using only one unit to express in logarithmic form the numerical values of international specifications and the results of measurements in exchanges at international level;

e) the use in radiocommunications of the decibel alone to express the results of measurements in logarithmic form,

recommends

1 that symbols used for the logarithmic expression of quantities that directly or indirectly refer to power should be chosen with the guidance of Annex 1.

ANNEX 1

Use of the decibel and the neper

1 Definition of the decibel

1.1 The *bel* (symbol B) expresses *the ratio of two powers* by the decimal logarithm of this ratio. This unit is not often used, having been replaced by the *decibel* (symbol dB) which is one-tenth of a bel.

1.2 The decibel also may be used to express the ratio of two *field quantities*, such as voltage, current, sound pressure, electric field, charge velocity or density, the square of which in linear systems is proportional to power. To obtain the same numerical value as a power ratio, the logarithm of the field quantity ratio is multiplied by the factor 20, assuming that the impedances are equal.

The relationship between a current or voltage ratio and that of the corresponding power ratio is impedance dependent. Use of the decibel when the impedances are not equal is not appropriate unless adequate information is given concerning the impedances involved.

For example, if P_1 and P_2 are two powers, their ratio expressed in decibels is:

 $10 \lg (P_1/P_2)$

^{*} The Director, BR, is requested to bring this Recommendation to the attention of the ITU-T.

^{**} In this Recommendation, the notation lg is used for the decimal logarithm in accordance with International Standards ISO 31-11 and IEC 60027-3. The notation log₁₀ is also used. The notation log can be used if there is no ambiguity.

^{***} This Recommendation was updated in 2005 for editorial reasons only.

If P_1 and P_2 represent the powers dissipated by currents I_1 and I_2 in resistances R_1 and R_2 :

$$10 \lg \frac{P_1}{P_2} = 10 \lg \frac{I_1^2 R_1}{I_2^2 R_2} = 20 \lg \frac{I_1}{I_2} + 10 \lg \frac{R_1}{R_2}$$

1.3 The decibel may be used to express the ratio of two values of a quantity connected with power by a well-defined relationship. In this case, the logarithm of this ratio must be multiplied by a factor representing the relationship which connects the quantity with a power, and a term representing a multiplying factor may be added to it.

If the ratio of two powers P_1 and P_2 depends on the ratio of values X_1 and X_2 of another quantity X by a relationship of the form $P_1/P_2 = (X_1/X_2)^{\alpha}$, α being any real number, we can express it in decibels as:

$$10 \lg (P_1/P_2) = 10 \alpha \lg (X_1/X_2)$$
 dB

2 Definition of the neper

The *neper* (symbol Np) expresses the ratio of two field quantities such as voltage or current, the square of which is proportional to power by the natural logarithm of this ratio. The value of a power ratio in nepers is one-half of the natural logarithm of the power ratio. The values in nepers of the ratio of two field quantities and of the corresponding powers are equal only if the impedances are equal.

One neper corresponds to the value of e of a field quantity ratio and to the value e^2 of a power quantity ratio.

Sub-multiples such as the decineper (dNp) are also used.

In some disciplines, nepers may be used to express the logarithm of a power ratio without the factor 1/2. An example is optical depth or attenuation in radiometry. Such usage is prohibited in telecommunications in order to prevent ambiguity. Under this definition, the neper would in fact be equal to 4.34 dB, instead of 8.68 dB as is traditionally the case.

3 Use of the decibel and neper

Countries can continue to use either the neper or the decibel for measurement purposes within their own territory and, to avoid conversion of values, countries which prefer to do so may continue to use the neper between themselves by bilateral agreement.

For the international exchange of information concerning transmission measurement and related values and for the international specification of limits for such values, the only logarithmic expression to be used is the decibel. However, it is possible to use the neper for bilateral agreements.

For theoretical or scientific calculations, where ratios are expressed in terms of Napierian logarithms, the neper will always be used, implicitly or explicitly.

As a result of some calculations on complex quantities, a real part in nepers and an imaginary part in radians are obtained. Factors may be applied for converting to decibels or degrees.

The conversion values between the neper and the decibel are as follows:

$1 \operatorname{Np} = (20 \lg e) \operatorname{dB} \approx 8.686$	dB
$1 \text{ dB} = (0.05 \ln 10) \text{ Np} \approx 0.1151$	Np

4 Rules for the use of the symbols where dB is included

Concerning the symbols that include the symbol dB, the following rules should be used as far as possible:

4.1 The symbol dB without additional indication

The symbol dB without additional indication should be used to indicate a ratio of two powers, two power densities, two other quantities clearly connected with power or the difference between two power levels (see § 6).

4.2 The symbol dB followed by additional information within parenthesis

The symbol dB followed by additional information within parenthesis should be used to express an absolute level of power, power flux-density or any other quantity clearly connected with power, in relation to a reference value within the parenthesis. In some cases, however, common use may give rise to simplified symbols, such as dBm instead of dB(mW).

4.3 The symbol dB followed by additional information without parenthesis

The symbol dB followed by additional information without parenthesis should be used to express by convention, special conditions such as measurements through specified filters or at a specified point of a circuit (see § 8).

5 Loss and gain

The *attenuation* or *loss* is a decrease between two points of an electric, electromagnetic or acoustic power. The attenuation is also the quantitative expression of a power decrease expressed by the ratio of the values at two points of a power or of a quantity related to power in a well-defined manner. This ratio is generally expressed in decibels.

The *gain* is the increase between two points of an electric, electromagnetic or acoustic power. The gain is also the quantitative expression of a power increase expressed by the ratio of the values at two points of a power or of a quantity related to power in a well-defined manner. This ratio is generally expressed in decibels.

The exact designation of the loss or gain in question must be given (e.g. image-attenuation coefficient, insertion loss, antenna gain) which in fact refers to the precise definitions of the ratio in question (terminal impedances, reference conditions, etc.).

5.1 Transmission loss

The ratio, expressed in decibels, of the transmitted power (P_t) to the received power (P_r) :

$$L = 10 \lg \left(P_t / P_r \right) \qquad \text{dB}$$

5.2 Antenna gain

The ratio, usually expressed in decibels of the power required at the input of a loss free reference antenna (P_0) to the power supplied to the input of the given antenna (P_a) to produce, in a given direction, the same field strength or the same power flux-density at the same distance in a given direction (when not specified otherwise in the direction of maximum radiation).

$$G = 10 \lg \left(P_0 / P_a \right) \qquad \text{dB}$$

The reference antenna is usually an isotropic antenna, a half-wave dipole, or, in certain cases, a short vertical antenna.

6 Levels

In many cases, the comparison of a quantity, here called x, with a specified reference quantity of the same kind (and dimension), x_{ref} is expressed by the logarithm of the ratio x/x_{ref} . This logarithmic expression is often called "the level of x (with respect to x_{ref})" or "the x-level (with respect to x_{ref})". With the general letter symbol for level L, the level of the quantity x may be written L_x .

Other names and other symbols exist and can be used. x may in itself be a single quantity, e.g. power P, or a ratio, e.g. P/A, where A is area, x_{ref} is here supposed to have a fixed value, e.g. 1 mW, 1 W, 1 μ W/m², 20 μ Pa, 1 μ V/m.

The level representing the quantity x with reference quantity x_{ref} may be indicated by the quantity symbol: L_x (with respect to x_{ref}), and may be expressed in decibels, when the reference quantity is a power, or a quantity linked to power, in a well-defined way.

Example:

The statement that the level of a certain power, P, is 15 dB above the level corresponding to 1 W can be written:

$$L_P$$
 (with respect to 1 W) = 15 dB, which means 10 lg (P/P_0) = 15, with P_0 = 1 W,
or 10 lg P (W) = 15

In many cases it is found practical to use a condensed notation based only on the unit, which in this case would be:

$$L_P = 15 \, \text{dB}(1 \, \text{W})$$

The number "1" in the expression of the reference quantity can be omitted, but this is not recommended in cases where confusion may occur. In other words, where no number is shown, the number 1 is to be understood.

There exist condensed notations for special cases, such as dBW for dB(1 W) (see § 8 below).

6.1 Absolute power level

The absolute power level is the ratio, generally expressed in decibels, between the power of a signal at a point in a transmission channel and a specified reference power.

It should be specified in every case whether the power is real or apparent.

It is necessary for the reference power to be indicated by a symbol:

- when the reference power is one watt, the absolute power level is expressed in "decibels relative to one watt" and the symbol "dBW" is used;
- when the reference power is one milliwatt, the absolute power level is expressed in "decibels relative to one milliwatt" and the symbol "dBm" is used.

6.2 Relative power level and related concepts

6.2.1 Definition

The relative power level is the ratio, generally expressed in decibels, between the power of a signal at a point in a transmission channel and the same power at another point in the channel chosen as a reference point, generally at the origin of the channel.

It should be specified in every case whether the power is real or apparent.

Unless otherwise specified, the relative power level is the ratio of the power of a sinusoidal test signal (at 800 or 1 000 Hz) at a point in the channel to the power of that reference signal at the transmission reference point.

6.2.2 Transmission reference point (see ITU-T Recommendation G.101)

In the old transmission plan, the ITU-T had defined "the zero relative-level point" as being the two-wire origin of a long distance circuit (point O of Fig. 1).

In the presently recommended transmission plan the relative level should be -3.5 dBr at the virtual switching point on the sending side of a four-wire international circuit (point V of Fig. 2). The "transmission reference point" or "zero relative level point" (point T of Fig. 2) is a virtual two-wire point which would be connected to V through a hybrid transformer having a loss of 3.5 dB. The conventional load used for the computation of noise on multi-channel carrier systems corresponds to an absolute mean power level of -15 dBm at point T.

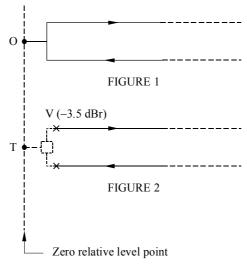
6.2.3 Meaning of "dBm0"

If a measuring signal with an absolute power level L_M (dBm) is applied at point T, the absolute power level of signal appearing at a point X, where the relative level is L_{XR} (dBr), will be $L_M + L_{XR}$ (dBm).

Conversely, if a signal at X has an absolute power level L_{XA} (dBm), it is often convenient to "refer it to a zero relative level point" by computing L_0 (dBm0) by the formula:

$$L_0 = L_{XA} - L_{XR}$$

This formula may be used, not only for signals, but also for noise (weighted or unweighted), which helps in the computation of a signal-to-noise ratio.



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6.3 **Power density**

Definition: Quotient of a power by another quantity, for example, an area, a bandwidth, a temperature.

NOTE 1 – The quotient of a power by an area is called "power flux-density" ("puissance surfacique") and is commonly expressed in "watts per square metre" (symbol: $W \cdot m^{-2}$ or W/m^2).

The quotient of a power by a frequency bandwidth is called "power spectral density" and can be expressed in "watts per hertz" (symbol: $W \cdot Hz^{-1}$ or W/Hz). It can also be expressed with a unit involving a bandwidth characteristic of the technique concerned, for example, 1 kHz or 4 kHz in analogue telephony, 1 MHz in digital transmission or in television; the power spectral density is then expressed in "watts per kilohertz" (W/kHz) or in "watts per 4 kHz" (W/4 kHz) or even in "watts per megahertz" (W/MHz).

The quotient of a power by a temperature, used particularly in the case of noise powers, has no specific name. It is usually expressed as "watts per kelvin" (symbol: $W \cdot K^{-1}$ or W/K).

NOTE 2 – In some cases a combination of several types of power densities can be used, for example a "spectral power flux-density" which is expressed as "watts per square metre and per hertz" (symbol: $W \cdot m^{-2} \cdot Hz^{-1}$ or $W/(m^2 \cdot Hz)$).

6.4 Absolute power density level

Definition: Expression in logarithmic form, usually in decibels, of the ratio between the power density at a given point and a reference power density.

NOTE 1 – For example, if one watt per square metre is chosen as the reference power flux-density, the absolute power flux-density levels are expressed as "decibels with respect to one watt per square metre" (symbol: $dB(W/m^2)$).

Similarly, if one watt per hertz is chosen as the spectral reference power density, the absolute spectral power density levels are expressed as "decibels with respect to one watt per hertz" (symbol: dB(W/Hz)).

If one watt per kelvin is chosen as the reference for power density per unit temperature, the absolute power density levels per temperature unit are expressed as "decibels with respect to one watt per kelvin" (symbol: dB(W/K)).

This notation can easily be extended to combined densities. For example, the absolute spectral density levels of the flux-density are expressed as "decibels with respect to one watt per square metre and per hertz" for which the symbol is: $dB(W/(m^2 \cdot Hz))$. Some examples are: $dB(W/(m^2 \cdot MHz))$ and $dB(W/(m^2 \cdot 4 \text{ kHz}))$.

6.5 Absolute voltage level

The absolute voltage level is the ratio, generally expressed in decibels, of the voltage of a signal at a point in a transmission channel to a specified reference voltage.

The nature of the voltage in question, e.g. r.m.s. value, should be specified in every case.

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A reference voltage with an r.m.s. of 0.775 volt is generally adopted which corresponds to a 1 milliwatt power dissipated in a resistance of 600 ohms, since 600 ohms represents a rough approximation to the characteristic impedance of certain balanced telephone lines. The absolute voltage level is then expressed in dBu.

If the impedance at the terminals of which the voltage U_1 is measured, is in fact 600 ohms, the absolute voltage level thus defined, corresponds to the absolute power level with respect to 1 milliwatt, and so the number N exactly represents the level in decibels with respect to 1 milliwatt (dBm).

$$L_u = 20 \lg (U_1/U_2) \qquad \text{dBu}$$
$$L_p = 10 \lg (P_1/P_2) \qquad \text{dBm}$$

If the impedance at the terminals of which the voltage U_1 is measured, is *R* ohms, *N* equals the number of dBm increased by the quantity 10 lg (*R*/600).

$$L_u = L_p + 10 \lg (R/600)$$

6.6 Audio frequencies

6.6.1 Absolute audio-frequency noise level

Measurement of audio-frequency noise in broadcasting, sound recording or sound-programme transmission is made, normally through a weighting network and by following the quasi-peak value method of Recommendation ITU-R BS.468 using a reference voltage of 0.775 volt at 1 kHz and a nominal impedance of 600 ohms and expressing the results normally in dBqp (in dBqps if a weighting network is used).

NOTE 1 – The two notations in "dBq" and "dBm" should not be used interchangeably. In sound-programme transmission the notation "dBq" is restricted to level measurements of noise with single or multiple tone bursts whereas the notation "dBm" only applies to sinusoidal signals used for lining up the circuit.

6.6.2 Relative voltage levels

The relative voltage level at a point in a sound-programme transmission chain is the ratio, expressed in dB, of the voltage level of a signal at that point relative to the voltage level of the same signal at the reference point. This ratio is expressed in "dBrs", the "r" indicating "relative level" and "s" indicating that the ratio refers to levels in a "sound-programme" (sound signals) system. At the reference point (the point of zero relative level, 0 dBrs) a test signal at the alignment level has a level of 0 dBu. Note that in some broadcasting chains, there may be no point of zero relative level. However, points of measurements and interconnection may still be given a level (in dBrs) relative to a hypothetical reference point.

6.6.3 Use of the decibel, by extension, for ratios of quantities not connected with power

6.6.3.1 Voltage ratios

In the audio frequency domain, the concept of voltage is sometimes more important than that of power. This is the case, for example, when low output- and high input-impedance two-port networks are associated in tandem. In this way a deliberate departure is made from the impedance matching conditions in order to simplify the formation of these networks. When this is done, only the voltage ratios at different points in the link need to be taken into consideration.

It is then convenient to express these voltage ratios in a logarithmic scale, e.g. to the base 10, by defining the number N of corresponding units by means of the equation:

$$N = K \lg \left(U_1 / U_2 \right)$$

In this equation the coefficient K is a priori arbitrary. However, by analogy with the operation:

$$N = 20 \lg (U_1/U_2)$$

which expresses in decibels the ratio of the power loss as in two equal resistances at the terminals of which the voltages U_1 and U_2 respectively, are applied, one is led to adopt the value 20 for the coefficient K. The number N then expresses in decibels the power ratios which would correspond to the voltage ratios, if the latter were applied to equal resistances, although in practice this is not generally the case.

6.6.3.2 Absolute voltage level

If the impedance at the terminals of which the voltage is measured is not specified, the corresponding power level cannot be calculated. However, a number N can be defined conventionally in accordance with § 6.6.3.1 with respect to a reference voltage and can be expressed in decibels. To avoid any confusion, it is essential to specify that an absolute voltage level is concerned and the symbol dBu must be used. The symbol dBu appears to create no confusion with the use defined in § 6.7 as the absolute level of the electromagnetic field referred to 1 microvolt per metre. If, however, there is any risk of confusion, the expression dB(775 mV) must be written, at least the first time.

6.7 Absolute level of an electromagnetic field

The strength of an electromagnetic field can be expressed by a power flux-density (P/A), by an electric field-strength E or by a magnetic field-strength H. The absolute field-strength level L_E is the logarithm of the ratio of E and a reference field-strength, usually 1 μ V/m. It is usually expressed in decibels.

$$L_E = 20 \lg (E/E_0) \qquad dB(\mu V/m)$$

6.8 Sound pressure level

This is the logarithm, generally expressed in decibels, of the ratio of sound pressure and a reference pressure, often $20 \,\mu$ Pa.

$$L_p = 20 \log (p/p_0)$$
 dB(20 µPa)

7 Ratios expressing transmission quality

7.1 Signal-to-noise ratio

This is either the ratio of the signal power (P_s) to the noise power (P_n) , or the ratio of the signal voltage (U_s) to the noise voltage (U_n) measured at a given point with specified conditions. It is expressed in decibels:

$$R = 10 \lg (P_s/P_n)$$
 dB or $R = 20 \lg (U_s/U_n)$ dB

The ratio of the wanted signal to the unwanted signal is expressed in the same way.

7.2 Protection ratio

This is either the ratio of the wanted signal power (P_w) to the maximum permissible interfering signal power (P_i) , or the ratio of the wanted signal field-strength (E_w) to the maximum permissible interfering signal field-strength (E_i) . It is expressed in decibels:

$$A = 10 \lg (P_w/P_i)$$
 dB or $A = 20 \lg (E_w/E_i)$ dB

7.3 Carrier to spectral noise density ratio (C/N_0)

This is the ratio $P_c/(P_n/\Delta f)$ where P_c is the carrier power, P_n the noise power, Δf the corresponding frequency bandwidth. This ratio has a dimension of frequency, it cannot be expressed without caution in terms of decibels, for power is not linked with frequency on a well-defined basis.

This ratio could be expressed in relation with a reference quantity such as 1 W/(W/Hz) which clearly indicates the origin of the result.

For example, with $P_c = 2$ W, $P_n = 20$ mW and $\Delta f = 1$ MHz, for the logarithmic expression corresponding to C/N_0 we have:

$$10 \lg \frac{P_c}{P_n / \Delta f} = 50 \qquad \text{dB}(W/(W/\text{kHz}))$$

This expression can be abbreviated to read 50 dB(kHz) if it is not liable to give rise to any misunderstanding.

7.4 Energy to spectral noise density ratio

In the case of the ratio "energy per bit to spectral noise density", E/N_0 , which is used in digital transmission, the ratio is made between two quantities homogeneous with spectral power density, and this ratio may normally be expressed in decibels, like power ratios (see § 7.1). However, it is necessary to ensure that the units used for the expression of both terms in the ratio are equivalent; for example, joule (J) for energy and watts per hertz (W/Hz) for spectral noise density.

7.5 Figure of merit (*M*)

The figure of merit (M) characterizing a receiving radio station is a logarithmic expression which is related to the antenna gain G (in decibels) and the overall noise temperature T (in kelvins) in the following way:

$$M = \begin{bmatrix} G - 10 \lg (T/1 \text{ K}) \end{bmatrix} \qquad \text{dB}(W/(W \cdot K))$$

The decibel notation may be abbreviated to read $dB(K^{-1})$ which should however be avoided if it is liable to give rise to misunderstanding.

8 Special notations

Examples of special notations, the use of which may be continued, are given below. These notations are often made in addition to other notations.

For absolute power level (see § 6.1, 6.2 and 6.6)

dBW:	absolute power level with respect to 1 watt, expressed in decibels;
dBm:	absolute power level with respect to 1 milliwatt, expressed in decibels;
dBm0:	absolute power level with respect to 1 milliwatt, expressed in decibels, referred to a point of zero relative level;
dBm0p:	absolute psophometric power level (weighted for telephony) with respect to 1 milliwatt, expressed in decibels, referred to a point of zero relative level;
dBm0s:	absolute power level with respect to 1 milliwatt, expressed in decibels, referred to a point of zero relative level in sound programme transmission;
dBm0ps:	absolute psophometric power level (weighted for sound-programme transmission) with respect to 1 milliwatt, expressed in decibels, referred to a point of zero relative level in sound programme transmission.

For absolute level of an electromagnetic field (see § 6.7)

dB μ or dBu: absolute level of the electromagnetic field with respect to 1 μ V/m, expressed in decibels.

For absolute voltage level including the audio-frequency noise level (see § 6.6)

dBu:	absolute voltage level with respect to 0.775 V;
dBu0:	absolute voltage level with respect to 0.775 V, referred to a point of zero relative level;
dBu0s:	absolute voltage level with respect to 0.775 V, referred to a point of zero relative level in sound-programme transmission;
dBq:	absolute noise voltage level with respect to 0.775 V;
dBqps:	absolute weighted voltage level in sound-programme transmission;
dBq0ps:	absolute weighted voltage level with respect to 0.775 V referred to a point of zero relative level in sound-programme transmission;
dBq0s:	absolute unweighted voltage level in sound-programme transmission with respect to 0.775 V referred to a point of zero relative level.

For relative power level (see § 6.2)

dBr.

For relative audio frequency voltage level (see § 6.6)

dBrs: relative voltage level expressed in decibels, referred to another point in sound-programme transmission.

For absolute acoustic pressure level (see § 6.8)

dBA, dBB or dBC: weighted acoustic pressure level with respect to 20 µPa, mentioning the weighting curve used (curves A, B or C, see IEC Publication 651).

For antenna gain (see § 5.2)

- dBi: in relation to an isotropic antenna;
- dBd: in relation to a half-wave dipole.

In Appendix 1 is given the principle of the notation recommended by the IEC for expressing the level of a quantity with respect to a specified reference. The notations used in the present Recommendation are applications of this principle.

APPENDIX 1

Notation for expressing the reference of a level

(Part 5 of IEC Publication 27-3)

A level representing the quantity x with the reference quantity x_{ref} may be indicated by:

 L_x (with respect to x_{ref}) or by L_x/x_{ref} .

Examples:

The statement that a certain sound pressure level is 15 dB above the level corresponding to a reference pressure of $20 \,\mu$ Pa can be written as:

$$L_p (\text{re } 20 \,\mu\text{Pa}) = 15 \,\text{dB}$$
 or as $L_{p/20 \,\mu\text{Pa}} = 15 \,\text{dB}$

The statement that the level of a current is 10 Np below 1 ampere can be written as:

 L_I (with respect to 1 A) = -10 Np.

The statement that a certain power level is 7 dB above 1 milliwatt can be written as:

 L_p (with respect to 1 mW) = 7 dB.

The statement that a certain electric field-strength is 50 dB above 1 microvolt per metre can be written as:

 L_E (with respect to 1 μ V/m) = 50 dB.

In presenting data, particularly in tabular form or in graphical symbols, a condensed notation is often needed for identifying the reference value. Then, the following condensed form, illustrated by application to the above examples, may be used:

15 dB(20 μPa) -10 Np(1 A) 7 dB(1 mW) 50 dB(1 μV/m)

The number 1 in the expression of a reference quantity is sometimes omitted. This is not recommended in cases when confusion may occur.

When a constant level reference is used repeatedly in a given context and explained in the context, it may be omitted.*

^{*} The omission of the reference level, permitted by the IEC, is not permitted in ITU-R and ITU-T texts.