

RECOMMENDATION ITU-R SM.1046-1

DEFINITION OF SPECTRUM USE AND EFFICIENCY OF A RADIO SYSTEM

(Question ITU-R 47/1)

(1994-1997)

The ITU Radiocommunication Assembly,

considering

- a) that the spectrum is a limited natural resource of great economic and social value;
- b) that demand for use of the spectrum is increasing rapidly;
- c) that a number of different factors, such as the use of different frequency bands for particular radio services, relevant spectrum management methods for networks in those services, the technical characteristics of transmitters, receivers and antennas used in the services, etc., significantly influence spectrum use and efficiency and through their optimization, particularly in respect of new or improved technologies, significant economies of spectrum can be achieved;
- d) that there is a need for defining the degree and efficiency of spectrum use, as a tool for comparison and analysis for assessing the gains achieved with new or improved technologies, particularly by administrations in the national long-term planning of spectrum utilization and the development of radiocommunications;
- e) that comparison of spectrum efficiency between actual radio systems would be very useful, when developing new or improved technologies and assessing performance of existing systems,

recommends

- 1** that, as a basic concept, the composite bandwidth-space-time domain should be used as a measure of spectrum utilization – the “spectrum utilization factor”, as illustrated in Annex 1 for transmitting and receiving radio equipment;
- 2** that, as a basic concept, spectrum utilization efficiency (SUE), or spectrum efficiency in short, should be measured in terms of a ratio of the amount of information transferred over a distance (or communications achieved) to the spectrum utilization factor, as illustrated in Annex 1. Some examples of how to use this concept may be found in Annex 2;
- 3** that the basic concept of relative spectrum efficiency as outlined in Annex 1 should be used to compare spectrum efficiencies between radio systems;
- 4** that any comparison of spectrum efficiencies should be performed only between similar types of radio systems providing identical radiocommunication services as explained in § 4 of Annex 1;
- 5** that in determining the spectrum efficiency, the interactions of various radio systems and networks within a particular electromagnetic environment should be considered.

ANNEX 1

**General criteria for the evaluation of spectrum utilization factor
and spectrum efficiency****1 Spectrum utilization factor**

Efficient use of spectrum is achieved by (among other things) the isolation obtained from antenna directivity, geographical spacing, frequency sharing, or orthogonal frequency use and time-sharing or time division and these

considerations reflected in definition of spectrum utilization. Therefore, the measure of spectrum utilization – spectrum utilization factor, U , is defined to be the product of the frequency bandwidth, the geometric (geographic) space, and the time denied to other potential users:

$$U = B \cdot S \cdot T \quad (1)$$

where:

B : frequency bandwidth

S : geometric space (usually area) and

T : time.

The geometric space of interest may also be a volume, a line (e.g. the geostationary orbit), or an angular sector around a point. The amount of space denied depends on the spectral power density. For many applications, the dimension of time can be ignored, because the service operates continuously. But in some services, for example, broadcast and single channel mobile, the time factor is important to sharing and all three factors should be considered simultaneously, and optimized.

The measure of spectrum may be computed by multiplication of a bandwidth bounding the emission (e.g. occupied bandwidth) and its interference area, or may take into account the actual shape of the power spectrum density of the emission and the antenna radiation characteristics.

Traditionally, radio transmitters have been considered the users of the spectrum resource. They use the spectrum-space by filling some portion of it with radio power – so much power that receivers of other systems cannot operate in certain locations, times and frequencies because of unacceptable interference. Notice that the transmitter denies the space to receivers only. The mere fact that the space contains power in no way prevents another transmitter from emitting power into the same location; that is, the transmitter does not deny operation of another transmitter.

Receivers use spectrum-space because they deny it to transmitters. The mere physical operation of the receiver interferes with no one (except as it inadvertently acts as a transmitter or power source). Even then the space used physically is relatively small. However, the authorities deny licences to transmitters in an attempt to guarantee interference-free reception. The protection may be in space (separation distance, coordination distance), in frequency (guardbands) or even in time (in the United States of America, some MF broadcasting stations are limited to daylight operation). This denial constitutes “use” of the space by the receiver. The radioastronomy bands are a familiar example of the recognition of receiver use of the spectrum space.

One way to incorporate these facts into a unit of measure of spectrum space is to partition the resource into two spaces – the transmitter space and receiver space – and define dual units to measure the usage of each space. Where simplicity is most important, the two units can be recombined into a single measure for system use.

Further information concerning the general approach to calculate the spectrum utilization factor may be found in Chapter 6 of the National Spectrum Management Handbook (Geneva, 1995).

2 Spectrum utilization efficiency (SUE)

According to the definition of SUE (or spectrum efficiency as a shortened term) of a radiocommunication system, it can be expressed by:

$$SUE = \frac{M}{U} = \frac{M}{B \cdot S \cdot T} \quad (2)$$

where:

M : amount of information transferred over a distance.

3 Relative spectrum efficiency (RSE)

The concept of relative RSE can be used effectively to compare the spectrum efficiencies of two similar types of radio systems providing the same service.

RSE is defined as the ratio of two spectrum efficiencies, one of which may be the efficiency of a system used as a standard of comparison. Hence,

$$RSE = SUE_a / SUE_{std} \quad (3)$$

where:

RSE : relative spectrum efficiency (ratio of SUEs)

SUE_{std} : SUE of a “standard” system

SUE_a : SUE of an actual system.

The likely candidates for a standard system are:

- the most theoretically efficient system,
- a system which can be easily defined and understood,
- a system which is widely used – a *de facto* industry standard.

The RSE will be a positive number with values ranging between zero and infinity. If the standard system is chosen to be the most theoretically efficient system, the RSE will typically range between zero and one.

As an example, the most theoretically efficient system may be characterized according to the principles of information theory. The communication capacity of a communication channel on which a subscriber or a listener receives a wanted communication is determined by the relation:

$$C_0 = F_0 \ln(1 + \rho_0)$$

where:

F_0 : bandwidth of the wanted communication

ρ_0 : signal/noise ratio at the receiver output.

If the signal/noise ratio at the receiver input is equal to the protection ratio ρ_s and the bandwidth of the communication channel over which the signals are transmitted is equal to F_m , then the communication capacity is $C_p = F_m \ln(1 + \rho_s)$. It must exceed or at least be equal to the communication capacity of the channel over which the subscriber receives a wanted communication, i.e. $C_p \geq C_0$. Hence the minimum possible value of the protection ratio ρ_s at which the subscriber will receive a communication with a signal/noise ratio equal to ρ_0 is defined as:

$$\rho_s = (1 + \rho_0)^{F_0/F_m} - 1 \quad (4)$$

The major advantage of directly computing the RSE is that it will often be much easier than computing the SUEs. Since the systems provide the same service, they will usually have many factors (sometimes even physical components) in common. This means that many factors will “cancel out” in the calculation before they need to be actually calculated. Often this will greatly reduce the complexity of the calculation.

Some examples of RSE calculations are presented in Annex 2 below and in Chapter 6 of the National Spectrum Management Handbook (Geneva, 1995).

4 Comparison of spectrum efficiencies

As described in previous sections, values for SUE could be computed for several different systems and could indeed be compared to obtain the relative efficiencies of the systems. Such comparisons, however, will have to be conducted with caution. For example, the SUEs computed for a land mobile radio system and a radar system are very different. The information transfer rate, the receivers and transmitters in these two systems are so different that the two SUEs are not commensurate. It would not be particularly useful to try to compare them. Hence, the comparison of spectrum efficiency should be only done between similar types of systems and which provide identical radiocommunication services. It would be beneficial to conduct the comparison of the spectrum efficiency or utilization of the same system over time to see if there is any improvement in the specific area under study.

It should also be noted that although spectrum efficiency is an important factor, because it allows the maximum amount of service to be derived from the radio spectrum, it is not the only factor to be considered. Other factors to be included in the selection of a technology or a system include the cost, the availability of equipment, the compatibility with existing equipment and techniques, the reliability of the system, and operational factors.

ANNEX 2

Examples**1 Spectrum use by land mobile radio systems****1.1 Spectrum efficiency of an indoor pico-cellular radio system**

In the case of an indoor pico-cellular system in the frequency band between 900 MHz and 60 GHz, the spectrum efficiency can also be derived using equation (2). From this equation, the spectrum efficiency of an indoor pico-cellular radio system may be defined as:

$$\text{Erlangs} / (\text{bandwidth} \times \text{area}) \quad (5)$$

where erlangs is the total voice traffic carried by the pico-cellular system, bandwidth is the total amount of spectrum used by the system and area is the total service area covered by the system. Since the pico-cellular system is to be implemented in a high-rise building, the total floor area is used in the calculation of spectrum efficiency. The number of channels required per cell can then be calculated based on the Erlang B Tables for a given number of users on the floor and traffic per user.

1.1.1 Pico-cellular system covering a building

In order to calculate the total bandwidth required for the whole building, the vertical re-use distance in terms of the number of floors is required. This parameter is dependent on the floor losses and is different for different types of buildings.

The total number of half duplex channels required for the building can then be calculated and is equal to:

$$2 \times \text{No. of channels per cell} \times \text{No. of cells per floor} \times \text{No. of floors of separation}$$

The factor 2 is needed here to reflect the number of channels needed for two-way communications.

The spectrum efficiency, $SUE_{building}$, of the system providing coverage in the building can then be calculated using equation (5):

$$SUE_{building} = \frac{\text{Total traffic carried in the entire building}}{\text{Total No. of channels} \times \text{channel bandwidth} \times \text{total floor area}} \quad (6)$$

Example:

In this indoor system operating at 900 MHz

Bandwidth of a (half duplex) channel	25 kHz
No. of channels per cell	10
No. of cells per floor	4
No. of floors of separation	3
Total No. of channels required	120

At a grade of service of 0.5%, the traffic carried on one floor = $T_f = 16$ E or $2 T_f$ due to both base and mobile stations.

$$SUE_{building} = \frac{16 \times \text{No. of floors}}{120 \times 0.025 \times \text{total floor area}} \quad (7)$$

If the floor is 25 m by 55 m, $SUE_{building} = 3880$ E/MHz/km².

1.1.2 Pico-cellular system covering a down-town area

Similarly, the bandwidth required for the whole down-town area may also be calculated if the horizontal re-use distance is known. Again, this parameter is dependent on the building material and the propagation loss of a signal into and out of

a building. This re-use distance directly affects the number of buildings that can be placed in a cluster (or interference group).

In this case, the total number of half duplex channels required in the down-town area is equal to:

$$2 \times \text{No. of channels per building} \times \text{No. of buildings per cluster}$$

Again the factor 2 is needed here to reflect the number of channels needed for two-way communications.

The spectrum efficiency, SUE_{area} , of the system providing coverage to the entire down-town area can then be calculated using equation (5):

$$SUE_{area} = \frac{\text{Total traffic carried in the entire building}}{\text{Total No. of channels} \times \text{channel bandwidth} \times \text{total floor area}} \quad (8)$$

Here, the total service area is the total floor area of the buildings covered by the pico-cellular system.

Example:

In this indoor system operating at 900 MHz

No. of channels per building	120
No. of buildings per cluster	4
Bandwidth of a (half duplex) channel	25 kHz
Total No. of channels required	480

$$SUE_{area} = \frac{16 \times \text{No. of floors} \times \text{No. of floors}}{120 \times 4 \times 0.025 \times \text{total floor area}} = 970 \text{ E/MHz/km}^2 \quad (9)$$

NOTE 1 – Additional information may be found in:

CHAN, G. and HACHEM, H. [September, 1991] Spectrum efficiency of a pico-cell system in an indoor environment. Canadian Conference on Electrical and Computer Engineering, Quebec City, Canada.

HATFIELD, D.N. [August, 1977] Measures of spectral efficiency in land mobile radio. *IEEE Trans. Electromag. Compt.*, Vol. EMC-19, 3, 266-268.

1.2 RSE of land mobile radio systems

RSE values of land mobile radio systems using different types of modulation were compared in the relation to the most theoretically efficient system (see Annex 1, § 3 and equation (4)).

For the sake of simplicity and to obtain finite analytical expressions, calculations were made for the simplest models of a network in the form of an ideal rectangular lattice and propagation conditions typical for the UHF frequency band. However, the general laws will be the same for more complex models of real networks with more sophisticated propagation models.

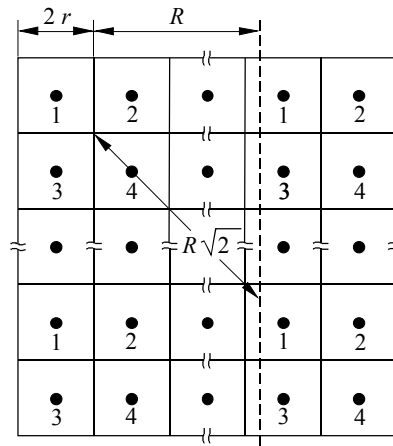
The network model is made up of squares of equal dimensions with the central (base) station being located in the centre of the square (see Fig. 1). The dimension (radius), r , of the service area is considered to be given. In areas bearing the same digit in Fig. 1, the same set of frequency channels can be used if the separation distance, R , between these areas provides sufficient interference attenuation. The antennas of the base stations are not directive ones in the horizontal plane and only use one type of polarization.

In this model, all base station transmitters have the same power and a stable carrier frequency and they do not produce any out-of-band or spurious radiation; base station receivers have ideal selectivity characteristics.

Results of RSE calculations for several specific types of modulation and different signal-to-noise ratios at the receiver output p_0 are presented in Fig. 2. Considered types of modulation are:

- Amplitude modulation – single-side band (AM-SSB),
- Frequency modulation (FM),
- 4 (8) phase phase-shift keying (4(8) PSK),
- 16 state quadrature amplitude modulation (16-QAM).

FIGURE 1
Network model



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As it follows from Fig. 2, the FM land mobile systems have the lowest RSE, since when this type of modulation is used, the bandwidth required for a network development is approximately five times greater than in the case of the most theoretically efficient system (MTES). On the other hand, the type of modulation which is closest to the MTES case for all values of the noise protection ratio is 16-QAM. For a relevant network development it requires only 1.5 times the bandwidth needed for the MTES. If the reception quality requirements are not very high, the closest with respect to the MTES is an AM-SSB. However, the RSE of the AM-SSB drops appreciably as the reception quality requirements are increased, particularly if account is taken of the effect of the frequency instability of real transmitters.

NOTE 1 – Additional information may be found in: Annex IV Report 662-3 (Düsseldorf, 1990).

1.3 SUE of land mobile radio systems

For general dispatched land mobile radio systems the SUE may be obtained using equation (2) in the following way.

$$SUE = \frac{M}{B \cdot S \cdot T} = \frac{Occ}{B \cdot S} \quad (10)$$

where

B : total amount of spectrum considered in the land mobile band of frequencies,

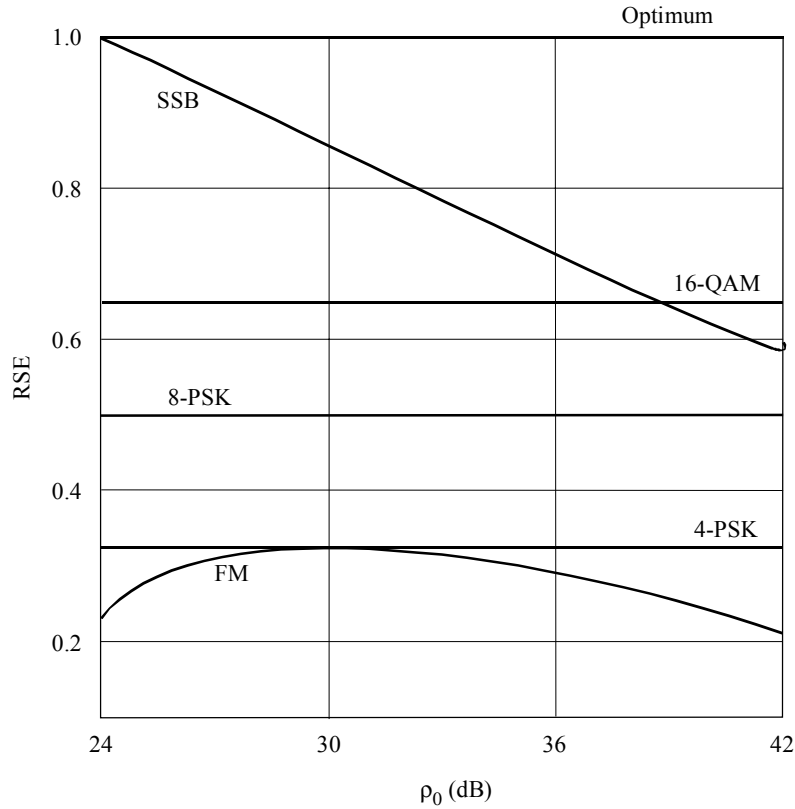
S : the area under study, and

Occ : total occupancy in the area

$$= \text{Occupancy per transmission} \times \text{No. of transmissions in the area} = M/T.$$

The main issue is therefore to calculate the total occupancy in the area. The approach taken is to divide the area under study into a number of cells in which the base stations are located. Depending on the transmitter power and propagation characteristics, the transmitted signal will cover a certain area, in this case, a number of cells. Hence by adding up the cells that are covered by this signal, the occupancy due to this transmission may be calculated. However, if a number of stations share the same frequency, the occupancy will be divided by the number of stations which share the same frequency. All the stations will be accounted for in the total number of transmissions.

FIGURE 2
RSEs in a network with different types of modulation



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In this example, the geographical area of 76 km by 76 km is computationally represented as a matrix of cell values. Each cell is defined as an area of 2 km by 2 km. The cell is considered to be occupied if the coverage circle defined by d (to be further explained in the next section) occupies more than 10% of the area of the cell. The total occupancy of the cell is obtained from each active licence, or station, in the frequency band.

If F_n is the total occupancy of the cell by all n stations in the area, then the Occ is given by:

$$Occ = \sum_0^m F_n \quad (11)$$

where m is the size of the matrix.

The SUE cell index is defined as the total occupancy in the cell by all n stations in that geographical area divided by the total amount of spectrum considered, B , and the area of the cell, a . The SUE average index of a geographical area can hence be obtained from the total occupancy in the city divided by the total amount of spectrum considered and the total area, S .

$$\text{Cell index} = \frac{F_n}{B \cdot a} \quad (12)$$

$$\text{Average index} = \frac{Occ}{B \cdot S} \quad (13)$$

1.3.1 Calculation of the occupied and denied spectrum index

In this analysis, the occupied spectrum index and the composite occupied and denied spectrum index are calculated. The former provides a measure of how a given band of spectrum is utilized, while the latter is an indication as to how the spectrum is used and denied to other users.

As described in the last section, in calculating the index, it is necessary first to estimate the value of the coverage distance, d , based on Recommendation ITU-R P.529:

$$d = \text{anti log} \left[\frac{(P_t + G_r - P_{ibm} - OCR(\Delta f) - 26.16 \log f + 13.82 \log h_t + 1.1 \log f - 0.7)h_r - (1.56 \log f - 0.8)}{44.9 - 6.55 \log h_t} \right] \quad (14)$$

where:

- P_t : e.i.r.p. (dBW)
- G_r : gain of the receiving antenna (dB)
- P_{ibm} : average received power at the mobile (dBW)
- OCR : off-channel-rejection
- f : transmitter frequency (MHz)
- h_t : base station antenna height (m)
- h_r : mobile antenna height (m)

The base station antenna is assumed to be omni-directional. Coordinates of the base station which determine the location of the centre of the coverage circle in the matrix of cells are also used.

To obtain an index for the occupied spectrum, P_{ibm} is -128 dBW and $OCR(\Delta f)$ is equal to zero.

For land mobile radio systems, we are interested not only in the occupied spectrum index, but also the denied spectrum index. The denied spectrum results from the fact that adjacent channels of assigned frequencies cannot be used within a certain distance of separation from the particular base station due to interference. This distance is dependent on the frequency separation, among other parameters. To calculate this distance for various frequency separations, P_{ibm} is assumed to be -145 dBW and various values of $OCR(\Delta f)$ have to be used.

Based on the mask of out-of-band emission, the values used for the OCR factor (dB) at the channel offset of Δf (kHz) are:

Δf	0	± 25	± 50	± 75	± 100
OCR	0	57.1	58.6	58.6	58.6

By using these values it is possible to obtain distances comparable to actual propagation conditions, from one set of sample data and according to the calculation of the coverage distances, the occupied distance is 21.9 km. The corresponding denied distances for $\Delta f = 0, \pm 25$ kHz, ± 50 kHz, and beyond are 69.2 km, 1.5 km, and 1.3 km, respectively.

1.3.2 Results

For illustration of this methodology to calculate the SUE, the result for the 5 776 km² area around the core of the 10 Canadian cities in the 138-174 MHz band is given. Table 1 includes the occupied spectrum index and the denied and occupied spectrum index.

The data used to determine the total occupancy is obtained from the Canadian Assignment and Licensing System database.

The land mobile bands considered in this study include both the VHF band of 138-174 MHz and the UHF bands of 406-430 MHz and 450-470 MHz. The channel spacing for VHF is 30 kHz and that for UHF is 25 kHz.

TABLE 1

Occupied and denied spectrum indices (138-174 MHz)

E/kHz/km ² × 10 ⁻³	Occupied and denied index	Occupied index
Toronto	4.19	1.33
Ottawa	4.54	1.30
Windsor	3.68	0.87
Montreal	3.56	0.88
Saint John	3.24	0.65
Halifax	3.32	0.68
Vancouver	3.20	0.62
Winnipeg	3.31	0.74
Calgary	3.05	0.73
Edmonton	2.99	0.60

Also presented are the graphical results for the city of Vancouver, again in the 138-174 MHz band. A 3-D visualization of a matrix of values, in this case the denied and occupied spectrum, is shown in Fig. 3. The matrix is overlaid on a map of the city to present the utilization information with cartographic detail. This presentation greatly enhances our ability to interpret this information. As shown in Fig. 4, the maximum value of a cell of the occupied spectrum in the centre of the city is 1.7×10^{-3} E/kHz/km². The maximum value of a cell of the denied and occupied spectrum for this band is 4.9×10^{-3} E/kHz/km², which is located just to the north and west of the centroid, as seen in Fig. 5. This area is the highly commercialized core of the city of Vancouver.

FIGURE 3

3-D representation of occupied and denied spectrum index of Vancouver

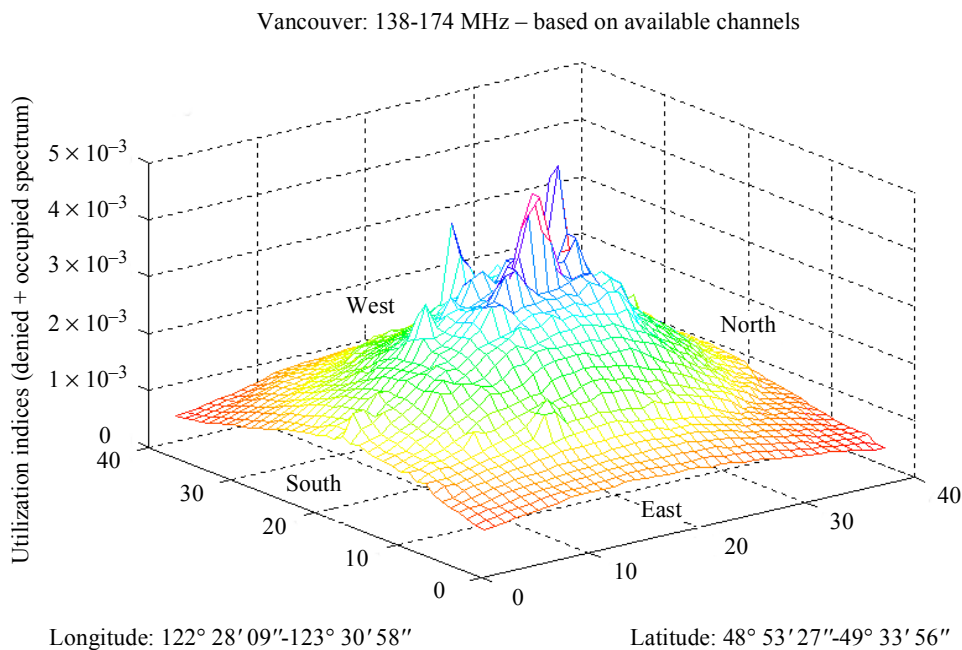
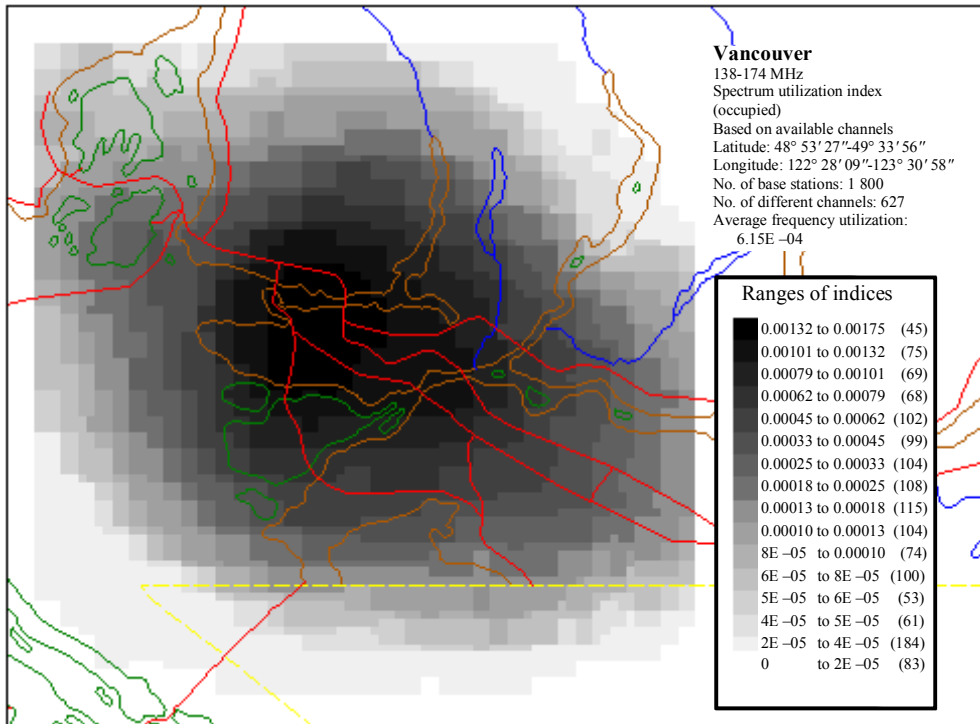
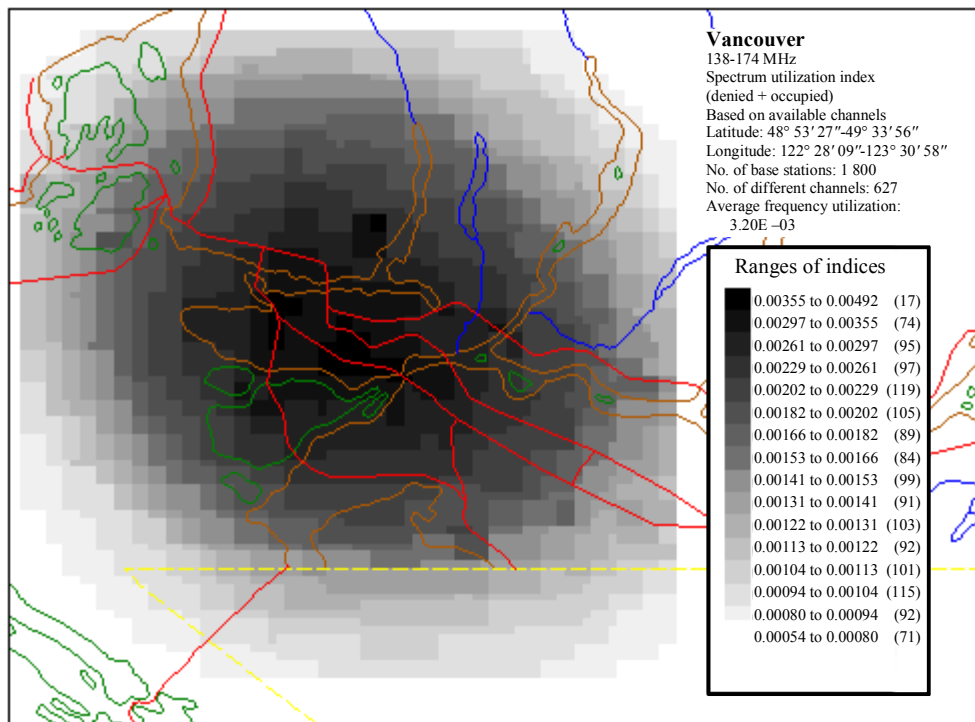


FIGURE 4
2-D plot of occupied spectrum index of Vancouver



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FIGURE 5
2-D plot occupied and denied spectrum index of Vancouver



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2 Spectrum use by radio-relay systems

2.1 Introduction

For radio-relay systems that operate continuously, the dimension of time may be ignored. Referring to equation (2), the SUE can be written as:

$$SUE = \frac{C}{B \cdot S_{\alpha}} \tag{15}$$

where:

- C : measure for communications capacity, for example telephone channels or bit/s
- S_{α} : geometric measure, for example, area, or the angle between branching links at a node.

2.2 SUE for a long artery with branching links at the nodes

Normalized communication capacity which gives the SUE for the terrestrial point-to-point radio-relay system, is defined as:

$$SUE = \frac{N \cdot A}{B_c} \tag{16}$$

where:

- N : allowable number of branching links (that is, two-way radio routes) for one repeater station
- A : transmitting capacity (e.g. number of telephone channels) per radio channel
- B_c : required RF bandwidth per radio channel.

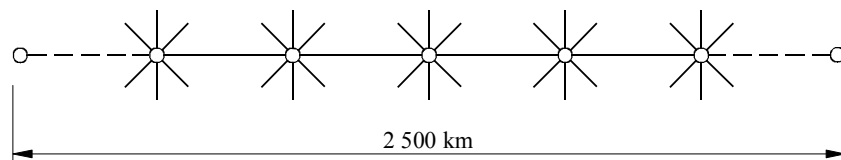
This formula includes the geometric measure, N (N depends on the allowable angle between branching links).

Spectrum use efficiency in the terrestrial point-to-point radio-relay system was calculated for telephone transmission using the above formula.

The assumptions used are:

- telephone signal is transmitted;
- probability of fading is the same as that given in Recommendation ITU-R P.530;
- circuit length is 2 500 km; and circuit model is as shown in Fig. 6;

FIGURE 6
Circuit model



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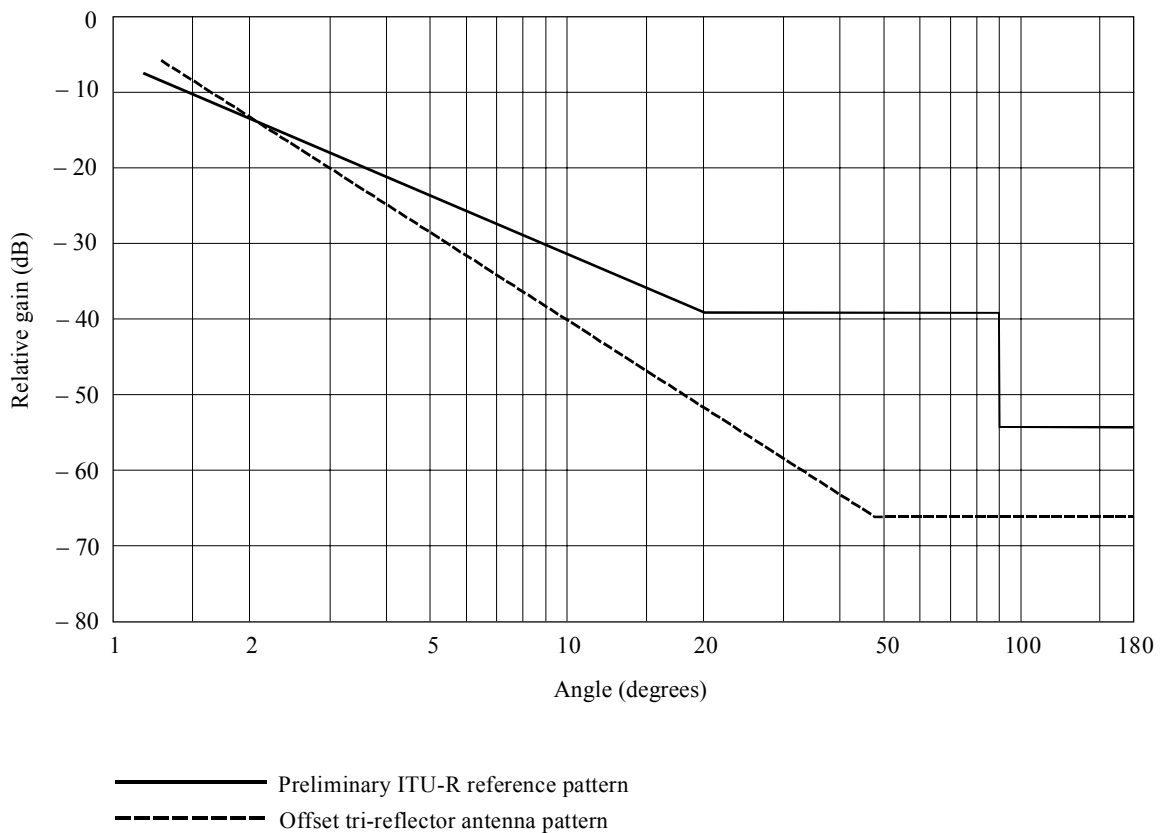
- required carrier-to-noise ratio, C/N , is expressed as:

$$C/N = 10 \log [(2^n - 1) / 3] + 11.8 \quad \text{dB} \tag{17}$$

where n is n -state QAM;

- one tenth of the overall radio-relay circuit noise for the 2 500 km circuit is assigned as the interference noise from other routes;
- interference from other routes has the same frequency as the wanted signal;
- a reference antenna diagram for a circular antenna in Recommendation ITU-R F.699 and a dual offset tri-reflector antenna used in Japan for a digital microwave radio, as shown in Fig. 7 are used;
- links with random branching angles.

FIGURE 7
Antenna pattern



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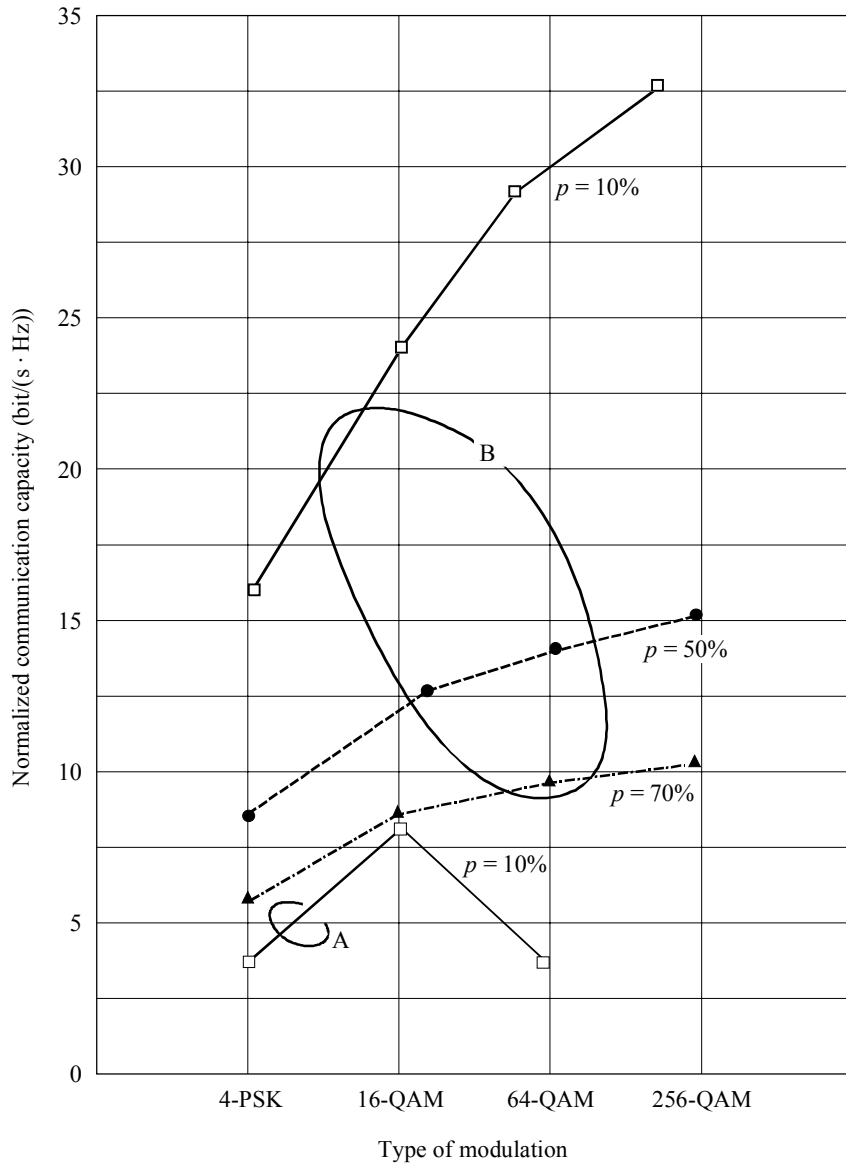
The normalized communication capacities for these two types of antenna were calculated and are shown in Fig. 8. The performance of the circular antenna in Recommendation ITU-R F.699 is insufficient to estimate the spectrum use efficiency of high-level modulation systems. As the results depend on antenna performance, if a high performance antenna can be used, higher level modulation such as 256-QAM is effective.

2.3 SUE in randomly arranged radio-relay links

2.3.1 Formulation

Figure 9 shows a radio-relay link X-Y with another radio station Z operating on the same frequency. Station Z is randomly located on a circle around station Y.

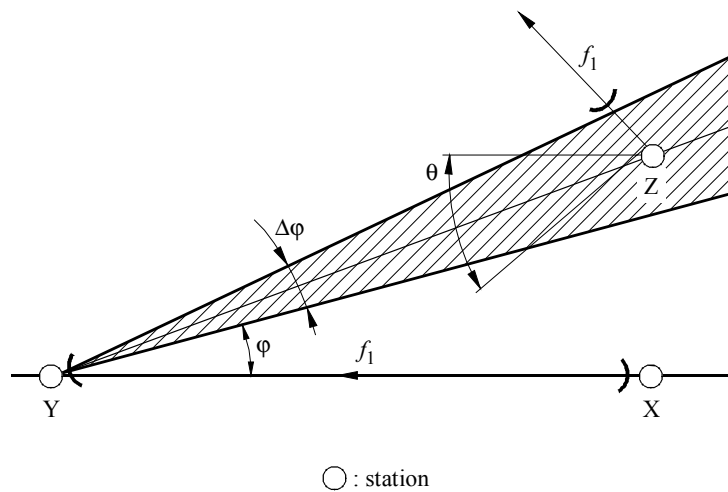
FIGURE 8
Normalized communication capacity



Fading margin: 20 dB
 Repeater spacing: 50 km
 Channel capacity: 64 kbit/s
 Allotment to branch noise: 10%

p : probability of interference
 A: preliminary ITU-R reference pattern
 B: offset tri-reflector antenna pattern

FIGURE 9
Random layout of stations



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Station Y receives a desired signal of frequency f_1 from station X. Station Z transmits a signal of the same frequency f_1 in an arbitrary direction.

The normalized communication capacity which gives the spectrum utilization efficiency, is defined as:

$$SUE = \frac{N \cdot A}{B_c} = \frac{\bar{p}}{p} \frac{A}{B_c} \quad (18)$$

where:

N : number of radio links possible using the same frequency: $N \approx \bar{p}/p$

A : transmitting capacity per radio channel.

The probability p that station Y receives interference exceeding the acceptable limit is calculated by considering the combination of the antenna pattern of stations Y and Z and \bar{p} is the maximum permissible probability of interference.

As the accumulation of interference from two or more stations has been neglected, some margin should be provided in any actual application.

2.3.2 Application: spectrum efficiency in 2 GHz band radio-relay systems

The SUE for a small-capacity terrestrial point-to-point radio-relay system operating in the 2 GHz band was calculated for telephone transmission using the above formula.

The relative spectrum utilization efficiency for 1.8 m diameter antennas was calculated using the permissible interference ratio and corresponding efficiency for each type of modulation in Table 2. The results are shown in Fig. 10.

The digital system is superior to the analogue system for smaller fading margins. In this study, the attenuation due to fading is the same as the degradation of W/U (wanted signal level to unwanted signal level ratio) caused by interference. If space-diversity techniques are used, the necessary fading margin is lower. In general, digital systems tend to deliver superior spectrum utilization efficiency.

For digital modulation, a change from 2-phase to multi-phase or multi-state requires less bandwidth, but it may have lower spectrum utilization efficiency when interference is high. The exact value depends on the antenna characteristics, etc., but the 4-PSK system may be optimum from the macroscopic viewpoint in cases where other radio links operating around the repeater station are randomly located in an area.

TABLE 2

Parameters of various modulation types in the 2 GHz band

Modulation type		Permissible S/N or error ratio	Interference reduction factor (IRF)		Permissible wanted signal/unwanted signal ratio W/U	Parameters related to B		Spacing to adjacent channels B	Number of channels A	$A/B^{(1)}$ (channels/kHz)
Analogue transmission	MF	58 dB	20 dB		38 dB	Frequency deviation for test tone: 100 kHz r.m.s.		520 kHz	24	0.046
	SSB	58 dB	9.5 dB		48.5 dB	Highest baseband frequency: 108 kHz Filter coefficient: $\times 2$ Frequency tolerance: 20 kHz		236 kHz	24	0.1
Digital transmission	2-PSK	10^{-6}	(C/N)	(Degradation)	16.2 dB	Clock frequency	Filter coefficient	2 MHz	24	0.012
			10.7 dB	5.5 dB		1 544 kHz	$\times 1.3$			
	4-PSK	10^{-6}	13.7 dB	5.5 dB	19.2 dB	772 kHz	$\times 1.4$	1.1 MHz	24	0.022
	8-PSK	10^{-6}	19.1 dB	5.5 dB	24.6 dB	515 kHz	$\times 1.5$	0.77 MHz	24	0.031
	QPRS	10^{-6}	16.8 dB	5.5 dB	22.3 dB	722 kHz	$\times 1.1$	0.85 MHz	24	0.028
16-QAM	10^{-6}	21.4 dB	5.5 dB	26.9 dB	386 kHz	$\times 1.6$	0.62 MHz	24	0.039	

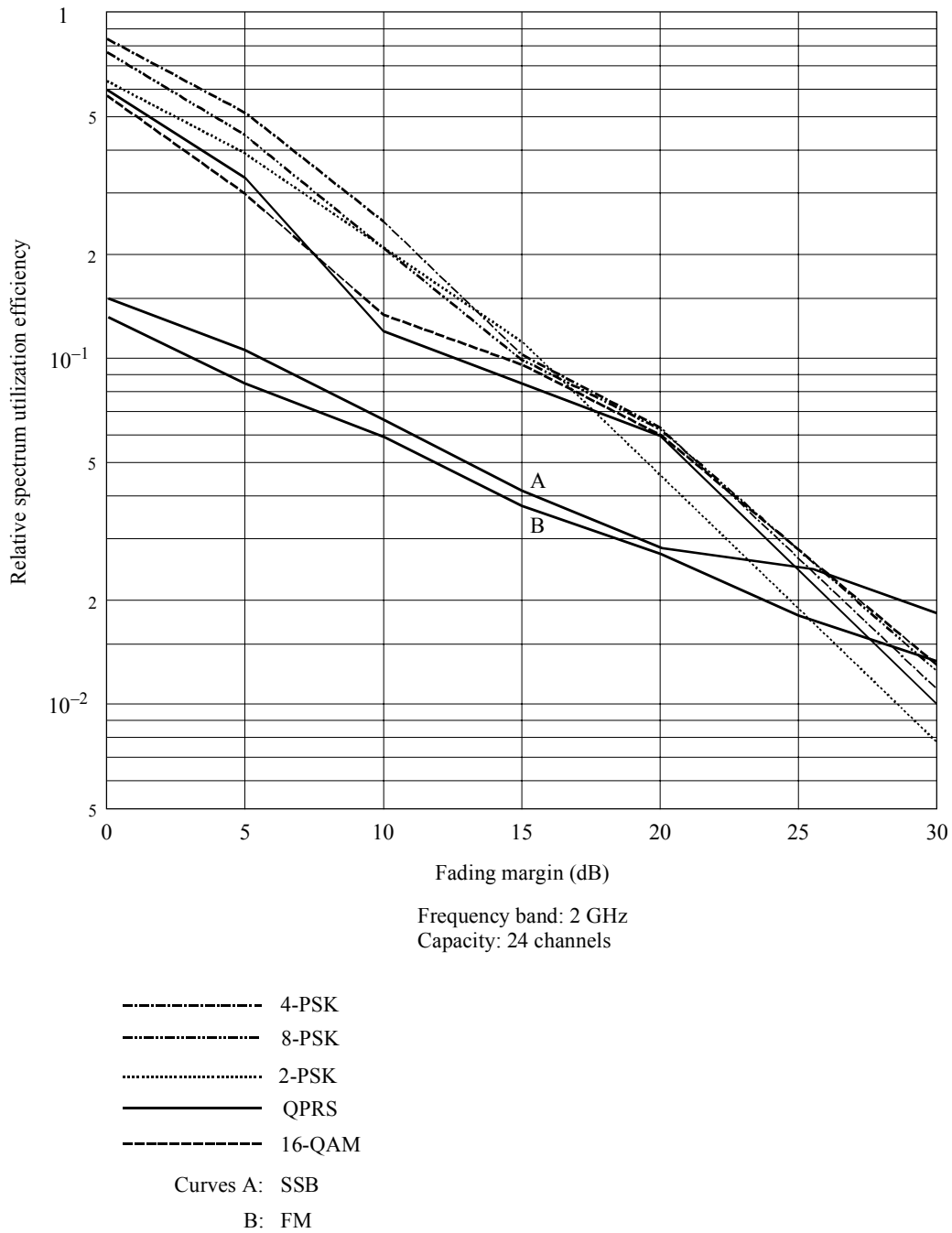
⁽¹⁾ The proper efficiency for each type of modulation.

QPRS: quadrature partial-response system.

The assumptions used are:

- acceptable interference and spectrum efficiency for each modulation type are as shown in Table 1. 80% of the total circuit noise is allotted to interference;
- distances between a station subject to interference (station Y) and the interfering stations are assumed to be the same; this assumption is considered to cause little error in efficiency calculation since the free-space losses of two links differ by only 6 dB even if they differ in length by a factor of two;
- fading in the wanted signal and in the interfering signals is assumed to have no correlation;
- the antenna radiation pattern is the reference diagram in Recommendation ITU-R F.699;
- all stations have the same transmitting output power;
- the limit on the probability of interference, $\bar{p} = 0.1$.

FIGURE 10
SUE of random layout



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2.3.3 SUE in a random mesh network

In order to perform a fair comparison of modulation techniques, one can assume an interleaved frequency plan with a channel spacing corresponding to a given performance degradation caused by adjacent channel interferences. Table 3 gives tentative values of the normalized channel spacing, X defined in ex-CCIR Report 608 (Kyoto, 1978) and the corresponding spectrum efficiency (bit/(s · Hz)). Even if different results could be derived, based on other assumptions, it should be noted that the calculated results of Table 3 are quite near the values which could be derived from specific

channel arrangements, as suggested by ITU-R Recommendations (for example 140 Mbit/s, with 16-QAM modulation and 40 MHz channel spacing between cross-polarized channels). Measured values might be different from these calculated values.

TABLE 3

Modulation method	Normalized channel spacing, X	Spectrum efficiency (bit/(s · Hz))
4-PSK	1.88	2.13
8-PSK	2.16	2.77
16-QAM	2.23	3.59

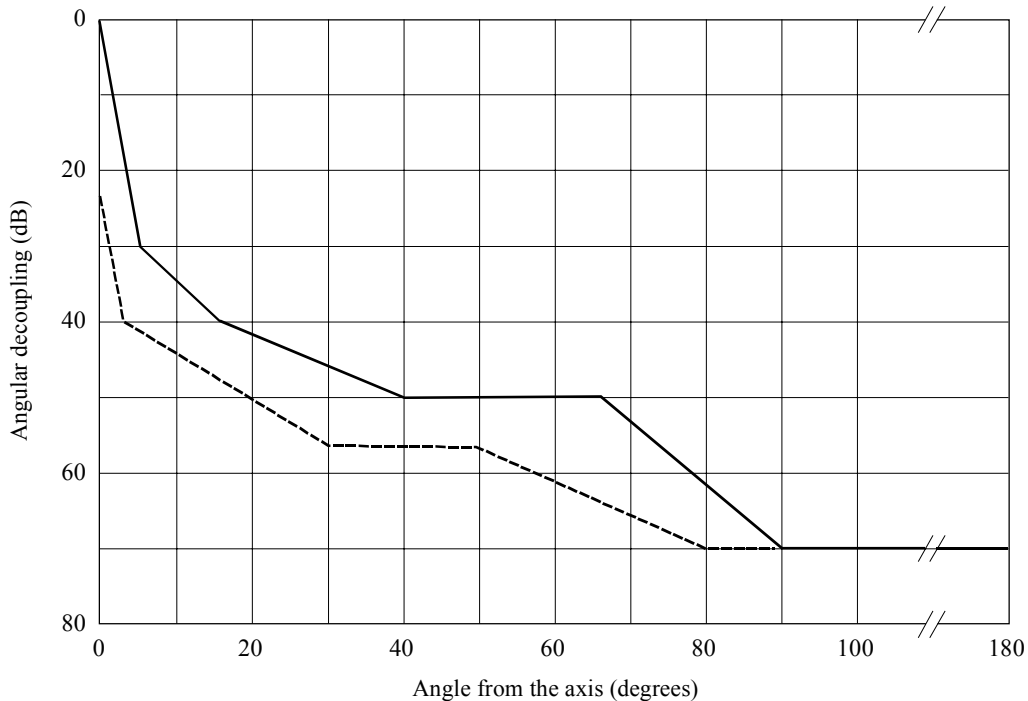
NOTE 1 – Degradation due to adjacent channel interference: 0.5 dB.

– Channel filters: raised cosine roll-off 0.5.

– Decoupling between cross-polarized channels (residual cross-polar discrimination (XPD)): 12 dB.

The antenna radiation pattern used in the analysis is shown in Fig. 11; it is for a typical parabolic antenna. It has been assumed that performance degradation (and a bit error ratio (BER) of 1×10^{-3}) due to co-channel interference from other links is not greater than 1 dB. It is assumed that the interfered-with link is at the threshold, with 40 dB fade margin, while the interfering link is receiving its nominal value.

FIGURE 11
Antenna radiation masks



Parabolic antenna, $D/\lambda = 75$

— co-polarization
 - - - cross-polarization

A normalized network density γ has been defined as:

$$\gamma = \frac{2N\rho^2}{\text{overall area covered by the network}} \tag{19}$$

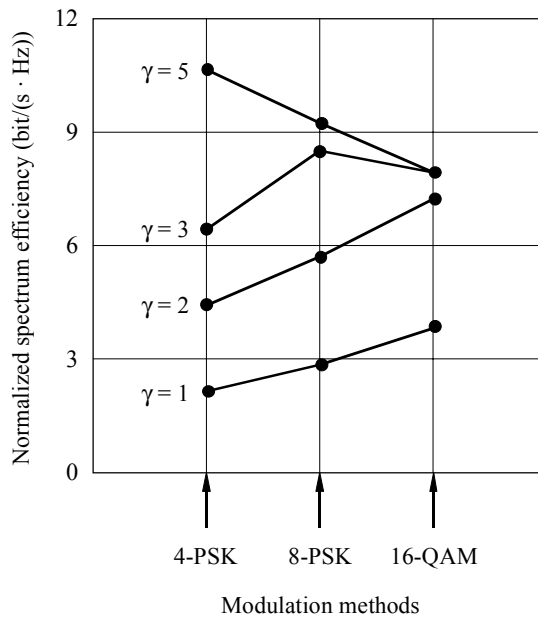
where:

N : number of radio nodes in the network

ρ : mean square hop length.

The results of Fig. 12 show that in high density networks the highest efficiency is achieved with 4-PSK modulation. However, the modulation method moves in favour of 8-PSK or even 16-QAM when the network density is lower. This shows that the SUE of modulation methods depends on the interference environment.

FIGURE 12
Spectrum efficiency in a mesh network



Antenna radiation mask of Fig. 11.
Performance degradation due to frequency re-use: 1 dB

1046-12

NOTE 1 – Additional information may be found in:

DODO, J., KUREMATSU, H. and NAKAZAWA, I. [8-12 June, 1980] Spectrum use efficiency and small capacity digital radio-relay system in the 2 GHz band. IEEE International Conference on Communications (ICC '80), Seattle, WA, United States of America.

TILLOTSON, L. C. *et al.* [1973] Efficient use of the radio spectrum and bandwidth expansion. *Proc. IEEE*, 61, 4.

2.4 Assessing spectrum conserving properties of new technology for digital radio-relay systems

2.4.1 Introduction

To assess the spectrum conserving properties of various design factors or technology options a computer model was designed. The relative SUE that can be achieved must be quantitatively evaluated. The concept of SUE can be extended and defined as:

$$SUE = VC/(T \cdot A \cdot B) \quad (20)$$

where:

VC : number of voice channels

T : fraction of time a system is used (defined to be equal to 1 for this analysis)

A : denial area (km²)

B : occupied bandwidth (MHz).

Equation (20) was chosen because it takes into account both spectrum and spatial (area) denial in assessing the spectrum-conservation properties of a system. The denial area is the area in which another system cannot operate without degradation in system performance below a specified performance criteria. The denial area is a function of the system antenna-pattern characteristics, transmitter output power and the receiver interference threshold level.

The algorithm used to calculate the denial area involves the segmentation (quantization) of the transmitter antenna gain pattern into a number of segments, angular sectors, which accurately represent the antenna pattern. The transmitter antenna gain pattern is an input to the model which calculates the denial area by summing the area in each segment. Geometrically, each segment is an angular sector, the area of which may be calculated using the formula:

$$\text{Area of angular sector} = \pi R^2 \theta / 360 \quad (21)$$

where:

R : radius of sector (R_1, R_2, \dots, R_n)

θ : vortex angle of sector ($\theta_1, \theta_2, \dots, \theta_n$)

n : number of angular segments.

The radii of R_n for each segment were calculated using the relationship:

$$L(R) = P_t + G_t(n) + G_r - I_{max} \quad (22)$$

where:

$L(R)$: required propagation loss (dB)

P_t : transmitter output power (dBm)

$G_t(n)$: transmitter antenna gain for sector n (dBi)

G_r : receiver antenna gain -10 dBi

I_{max} : maximum permissible interference level (dBm).

Then using a smooth earth inverse propagation model, the distance R corresponding to the required loss is determined. This facilitates the evaluation of the denial area for each angular sector (see equation (21)).

To apply equation (20) to point-to-point radio-relay systems, it is necessary to establish characteristics of a reference system between two microwave sites. These characteristics include path length, path attenuation, antenna gain, insertion losses, fade margin and system gain. It is also necessary to establish certain modulation characteristics for the

modulation types addressed. The digital modulations considered in this investigation are 16-QAM, 64-QAM, and 256-QAM. The characteristics assumed for the digital radio-relay systems for this analysis are based on the North American Standard and are as follows:

Digital radio-relay system parameters (see Note 1)

- Voice channels: 1 344 for 16-QAM
 2 016 for 64-QAM
 2 688 for 256-QAM
- Bit rate: 90 Mbit/s for 16-QAM
 135 Mbit/s for 64-QAM
 180 Mbit/s for 256-QAM
- BER: 1×10^{-6}
- Receiver noise figure, F : 4 dB
- System gain, G_s : 103 dB.

The analysis utilized theoretical transmission efficiency and input carrier-to-noise $(C/N)_i$ levels for the different modulation types to ensure a just comparison.

The following is a discussion of the application of equation (20) to the major design areas of antennas, modulation types and signal processing.

NOTE 1 – The system parameters used in this analysis have been selected to provide an indication of a single path analysis of spectrum efficiency for the various cases considered. As such, the parameters may not be representative of realizable systems, particularly those using higher order modulation schemes. The results therefore are illustrative of an application of the concept of spectrum efficiency to radio-relay systems, and administrations should employ representative parameters in any analysis of spectrum efficiency.

2.4.2 Antennas

Spatial denial is a key factor in addressing spectrum conservation. One of the major radiocommunication system components contributing to spatial denial is its antenna. In recent years, significant advances in the antenna-design areas of polarization discrimination and sidelobe reduction have provided the capability for enhanced spectrum efficiency in point-to-point microwave radiocommunications.

Frequency re-use can be achieved by implementing antenna-design spectrum-conservation techniques. Spatial denial can be minimized if sidelobe levels are minimized. The antenna radiation patterns, and therefore sidelobe distributions, vary with antenna type. Three antenna types commonly used in point-to-point microwave transmission are:

- Standard-dish (STD)
- Shrouded-dish (SHD)
- Conical horn reflector (CHR).

Typical radiation patterns for these antennas, with a 43 dBi gain, are shown in Fig. 13. The antenna pattern characteristics shown in Fig. 13 were used in the model.

A plot of the transmitter output power versus denial area for a receiver interference threshold of -102.5 dBm is shown in Fig. 14 for the three types of antennas. Although the mainbeam gain for all the antennas is the same, the results shown in Fig. 14 indicate that the CHR antenna has less denial area than the other two antennas. Also, the difference in denial area for the three antennas is small until the transmitter power is greater than 30 dBm. This is understandable because the contribution to the denial area caused by sidelobe/backlobe antenna characteristics is small until the transmitter power is increased beyond 30 dBm. At transmitter powers greater than 30 dBm, the difference in denial area for the three antennas is significant. The denial area is also a function of the receiver interference threshold.

FIGURE 13
 Typical radiation patterns for STD, SHD and CHR antennas

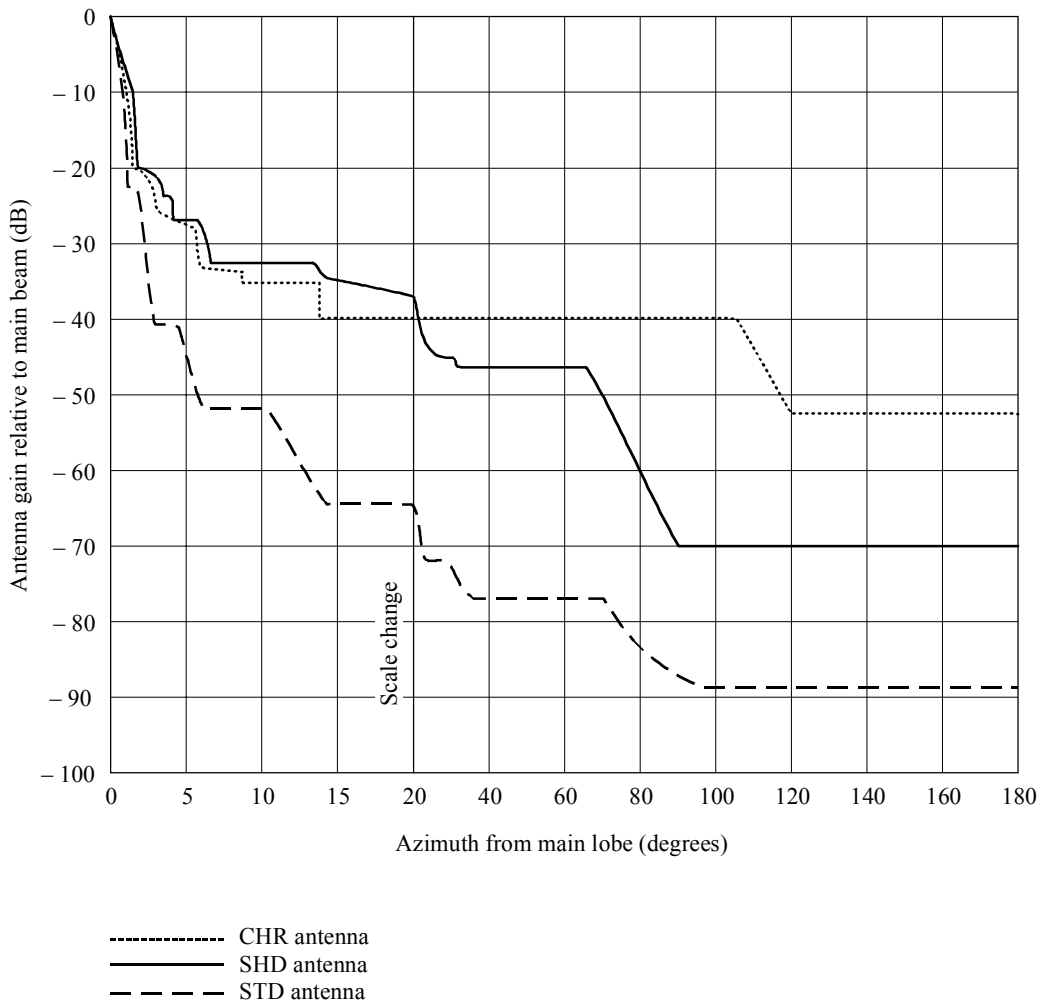
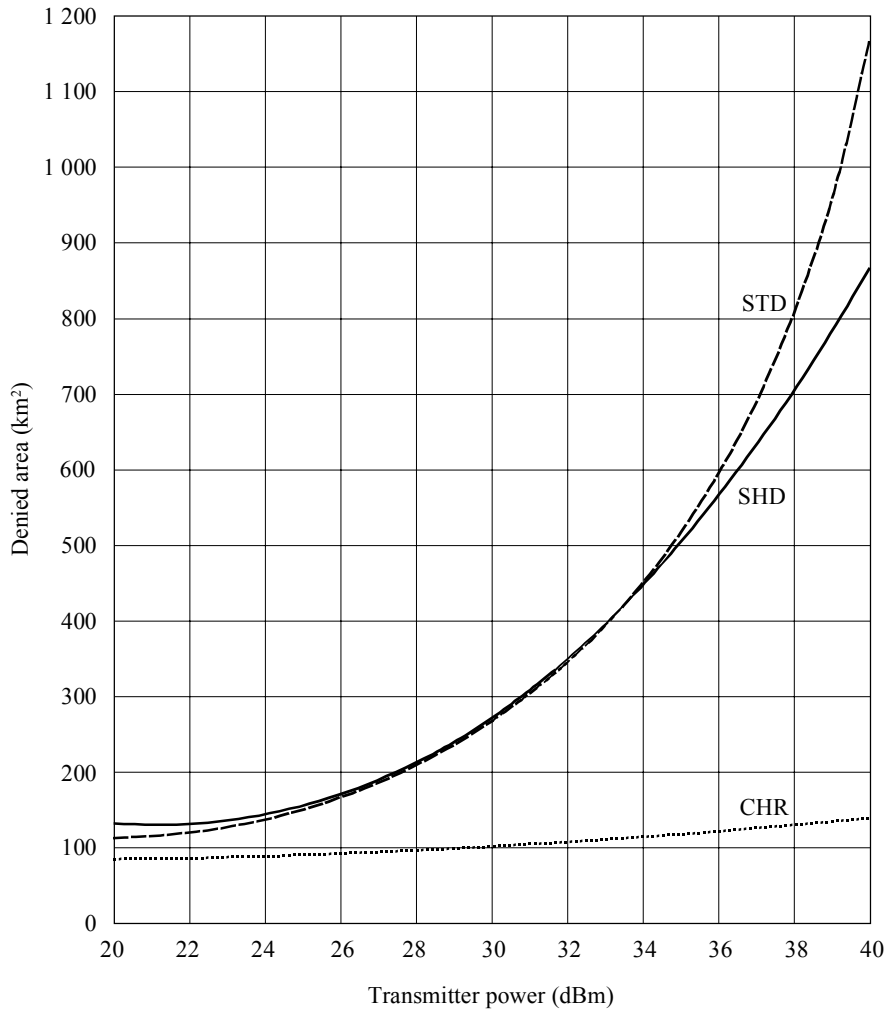


FIGURE 14

Denied area as a function of antenna type and transmitter output power



Calculations based on transmitter and receiver antenna heights of 50 m over smooth earth terrain

$$I_{max} = -102.5 \text{ dBm}$$

$$G_t (\text{mainbeam}) = 43 \text{ dBi}$$

$$G_r = -10 \text{ dBi}$$

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Since the denied area for the three antennas is a function of P_t and I_{max} , the spectrum conserving properties for the three antennas must also be related to the system modulation type. Thus the spectrum efficiency enhancement properties of the STD, SHD, CHR antennas will be discussed in the modulation section.

2.4.3 Modulation

The evaluation of spectrum conservation properties for different modulation schemes is very complex in that both spectrum and spatial denial are affected by the choice of modulation type used in a system. In general, system parameters, such as occupied bandwidth, required receiver input carrier-to-noise $(C/N)_i$, and I_{max} are all functions of the modulation type and have a direct bearing on spectrum utilization.

This analysis is based on theoretical transmission efficiency and receiver $(C/N)_i$ for the different modulation types to ensure a just comparison. To evaluate the spectrum-conservation properties of the different modulation types, the occupied bandwidth, B , and required P_t for each were determined. These parameters are given in Table 4.

TABLE 4

Digital system parameters

Modulation type	Transmission efficiency (bit/(s · Hz))	Occupied bandwidth, B (MHz)	Required input carrier-to-noise, $(C/N)_i$ (dB)	Noise level (dBm)	Minimum carrier level (dBm)	Transmitter output power level, P_t (dBm)
16-QAM	4	22.5	21.0	-96.5	-75.5	27.5
64-QAM	6	22.5	27.0	-96.5	-69.5	33.5
256-QAM	8	22.5	33.0	-96.5	-63.4	39.5

The occupied bandwidth B for the digital modulations was determined using the relationship:

$$B \text{ (MHz) } = \text{Bit rate (Mbit/s)/transmission efficiency (bit/(s · Hz))} \quad (23)$$

where the bit rate (see digital system parameters) and the transmission efficiency (see Table 4) are functions of the modulation type.

To establish the required P_t for each modulation type, the required receiver $(C/N)_i$ for specified performance criteria was determined. A BER of 1×10^{-6} was used as the performance criteria and the theoretical required $(C/N)_i$ was obtained from literature.

The receiver input noise level, N_i , given in Table 4 (-96.5 dBm) was determined using a receiver bandwidth of 22.5 MHz and a receiver noise figure of 4 dB. The required minimum carrier level (C_{min}) at the receiver input was then determined from the relationship:

$$C_{min} \text{ (dBm) } = (C/N)_i + N_i \quad (24)$$

The required transmitter power level, P_t , given in Table 4 was determined using the expression:

$$P_t \text{ (dBm) } = C_{min} + G_s \quad (25)$$

where G_s represents system gain, which is set to equal 103 dB.

The denied area is also a function of the victim receiver I_{max} . The receiver I_{max} associated with each modulation was determined assuming that the victim receiver has the same modulation type as the interfering transmitter. In this analysis, the I_{max} was determined using the criteria established in the Telecommunications Industry Association (TIA) Telecommunications System Bulletin No. 10-E. For the digital systems, the performance criteria was an increase in BER from 1×10^{-6} to 1×10^{-5} , which corresponds to approximately a 1 dB increase in receiver noise level. This is equivalent to a receiver input interference-to-noise ratio $(I/N)_i = -6$ dB (i.e., $I_{max} = -96.5 \text{ dBm} - 6\text{dB} = -102.5 \text{ dBm}$ for 16-QAM, 64-QAM and 256-QAM).

Table 5 contains the calculated SUE values using the bandwidth and transmitter output power given in Table 4 and $I_{max} = -102.5 \text{ dBm}$. The entries for SUE in Table 5 are for the three different modulation types and the three antennas. Systems with higher SUE values are more efficient from the spectrum utilization point-of-view. It should be emphasized that the calculated results clearly point out that the SUE varies considerably from one antenna type to another. For example, the SUE for 64-QAM is 0.201 for the STD antenna as compared to 0.212 and 0.811 for the SHD and the CHR antennas respectively. Therefore, the results shown in Table 5 clearly indicate the SUE can be optimized only when the effects of the antenna and modulation are both considered.

TABLE 5

Spectrum utilization efficiency

Ranking order	SUE for different antenna types		
	STD	SHD	CHR
1	16-QAM (0.307)	16-QAM (0.282)	256-QAM (0.841)
2	64-QAM (0.201)	64-QAM (0.212)	64-QAM (0.811)
3	256-QAM (0.112)	256-QAM (0.144)	16-QAM (0.709)

Also, the analysis results show the SUE for 64-QAM to be higher than for 256-QAM for the STD and SHD antennas, but not for the CHR. Table 6 helps provide an explanation of why 64-QAM is more spectrum efficient than 256-QAM for an ultra high performance SHD antenna. The input parameters to the model are provided in the table. The number of VCs is 2016 for 64-QAM and 2688 for 256-QAM. The required system bandwidth, B , is the same for both 64- and 256-QAM ($B = 22.5$ MHz). However, the required P_t for 256-QAM is significantly higher than 64-QAM (39.6 dBm as compared to 33.5 dBm). Since the transmitter power has a major effect on denied area to another user (see Fig. 14), the denied area for 256-QAM is significantly more than for the 64-QAM modulation thus causing the 64-QAM modulation to be more spectrum efficient than 256-QAM.

TABLE 6

SUE comparison of 64- and 256-QAM for SHD antennas

Parameter	64-QAM	256-QAM
VC	2016	2688
B (MHz) (see Table 4)	22.5	22.5
P_t (dBm) (see Table 4)	33.5	39.5
I_{max} (dBm)	-102.5	-102.5
A (km ²) (see Fig. 11)	421	830
SUE (see Table 5)	0.212	0.144

However, the spectrum conserving potential of a system is a function of several design factors all of which must be taken into consideration when evaluating the spectrum efficiency of a system. That is, one cannot say that a system with a particular modulation is more spectrum conserving than a system with another modulation without considering all other design factors (e.g., antennas, signal processing, RF filters, etc.).

Table 5 can also be used to determine the relative improvement in spectrum conservation of using a SHD antenna or CHR antenna over a STD antenna. As stated earlier, the improvement in spectrum conservation for the SHD and CHR antennas is dependent on the modulation type. This is due to the fact that the denial area produced by a particular antenna type is a function of P_t which is modulation dependent (see Fig. 14). Table 7 shows the percentage improvement in the SUE for the various modulations addressed using the SUE data in Table 5. For the digital modulations, the greatest improvement occurs for 256-QAM modulation with a 28% and 533% increase for the SHD and CHR antennas respectively.

TABLE 7

SUE improvement for SHD and CHR antennas as a function of modulation

Modulation type	Improvement in SUE	
	SHD antenna	CHR antenna
16-QAM	-8%	130%
64-QAM	6%	338%
256-QAM	28%	533%

2.4.4 Signal processing

In fixed radio-relay systems, signal processing is done at the transmitter and receiver terminal. Signal processing consists of electrical operations on a signal in order to produce certain desired characteristics. Signal processing can affect such parameters as amplitude, frequency, phase, signal level and reliability. The use of signal processing techniques can improve the processing gain of a system, permitting lower P_t for specified receiver output performance criteria. Thus, through the use of signal processing techniques, the P_t can be lowered reducing the spatial (area) denied to other systems. However, it should be noted that signal processing techniques are used by the microwave link designers to improve link reliability and are not generally considered for the purpose of spectrum conservation.

2.4.5 Error correction/coding

Forward error correction (FEC) coding is a method of improving BER performance of digital microwave systems, particularly when the system is power limited. The utilization of FEC coding techniques permits a limited number of errors to be corrected at the receiving end by means of a special coding and software (or hardware) implemented at both ends of a circuit. This improvement in BER can be traded off for a reduction in required receiver $(C/N)_i$ to meet a specified BER performance, thus reducing the denial area to other systems. The reduction in $(C/N)_i$ is referred to as coding gain. The performance of a coding technique is described by the coding gain and coding rate. However, the coding rate has an impact on the system occupied bandwidth, thus increasing the denied spectrum to other users of the spectrum.

To show the effect of coding on spectrum conservation, 64-QAM was selected as the modulation for study. Four types of FEC codes were selected. Table 8 shows the coding rate, the bandwidth expansion factor (1/coding rate), the occupied bandwidth after coding, the obtainable reduction in $(C/N)_i$ for a BER of 1×10^{-6} and the required P_t after taking into consideration the obtainable reduction in $(C/N)_i$. The values for bandwidth and power, shown in Table 8, were input to the SUE model to evaluate coding as a spectrum conservation technique. Table 9 shows the SUE for the STD, SHD, CHR antennas. The SUE for 64-QAM without coding is also shown in the Table 9 for a baseline comparison of with and without coding.

TABLE 8

Error correction/coding (64-QAM modulation)

Signal processing	Coding rate	Bandwidth expansion factor	Occupied bandwidth, B (MHz)	Reduction in C/N (dB)	Transmitter output power, P_t (dBm)
Error correction coding	1/2	2	45.00	6.0	27.5
	3/4	1.333	30.00	3.5	30.0
	7/8	1.142	25.70	2.0	31.5
	18/19	1.055	23.74	3.0	30.5

In summary, the SUE values given in Table 9 indicate that signal processing techniques such as error correction/coding which utilize RF bandwidth versus C/N trade-offs only provide significant improvement in spectrum conservation, higher SUE values, when high-efficiency coding techniques (i.e., coding techniques with high coding rates and coding gain) are used. Also, the relative improvement in spectrum conservation is greater when the system has a STD antenna than a SHD or CHR antenna. This is due to the fact that the reduction in denied area is greater for STD antennas because of the higher sidelobe/backlobe characteristics.

TABLE 9

SUE for error correction/coding (64-QAM modulation)

Signal processing type		SUE for different antenna types		
		STD	SHD	CHR
Without signal processing		0.201	0.212	0.811
Error correction/coding	Coding rate			
	1/2	0.230	0.211	0.532
	3/4	0.249	0.240	0.673
	7/8	0.235	0.237	0.754
	18/19	0.294	0.285	0.838

2.4.6 Adaptive/transversal equalizers

Adaptive/transversal equalizers improve the digital system performance in the presence of multipath fading, linear distortion, or both. The equalizers can only mitigate the dispersive aspects of multipath fading. These adaptive equalizers reshape the pulse so as to minimize the intersymbol interference. An approximate 4 to 6 dB improvement in the composite fade margin can be achieved with these equalizers in 64-QAM receivers. The major drawback of adaptive equalizers is their expense. The model was run for a system bandwidth of 22.5 MHz and P_t of 29.5 dBm (A 4 dB reduction in P_t for 64-QAM). Table 10 shows the SUE for the three types of antennas. The SUE values without adaptive equalizers are also shown in the table for comparison with adaptive equalizers.

TABLE 10

SUE improvement for adaptive equalizers (64-QAM modulation)

Signal processing type		SUE for different antenna types		
		STD	SHD	CHR
Without signal processing		0.201	0.212	0.811
With adaptive equalizers		0.355	0.337	0.930

For 64-QAM, the use of adaptive equalizers can improve the spectrum conservation properties of a system from approximately 15% to 75% with the greatest improvement in systems that use STD antennas.

2.4.7 Error correction/coding and adaptive equalizers

Some digital systems utilize both error correction/coding and adaptive equalizers to improve system performance. For 64-QAM, the utilization of error correction/coding (18/19 coding rate) and adaptive equalizers can reduce the required $(C/N)_i$ ratio by 7 dB for a BER = 1×10^{-6} . The application of the SUE model for an occupied bandwidth of 23.74 MHz and a P_t of 26.5 dBm (A 7 dB reduction in P_t for 64-QAM) is shown in Table 11. Table 11 shows that the use of error correction/coding and adaptive equalizers can improve the spectrum conserving properties of a system from 30% to 150% with the greatest improvement in systems that use dish type antennas.

TABLE 11

SUE error correction/coding and adaptive equalizers (64-QAM modulation)

Signal processing type	SUE for different antenna types		
	STD	SHD	CHR
Without signal processing	0.201	0.212	0.811
Error correction/coding and equalizers	0.503	0.441	1.066

2.4.8 Summary

2.4.8.1 The spectrum conserving potential of a system is a function of several design factors all of which must be taken into consideration when evaluating the SUE of a system. That is, one cannot say that a system with a particular modulation is more spectrum conserving than a system with another modulation without considering all other design factors such as antennas, signal processing, RF filters, etc.

2.4.8.2 The higher order digital modulations (modulations with higher transmission efficiency, bit/(s · Hz)) require higher P_t levels. Therefore, when the definition of spectrum use and efficiency is used which takes into consideration denied area, modulations which have a higher transmission efficiency may not necessarily be more spectrum conserving. Thus, the transmission efficiency of the digital modulation may not suffice as an indicator of spectrum efficiency.

NOTE 1 – Additional information may be found in:

HINKLE, R.L. and FARRAR, A.A., [May 1989] "Spectrum-conservation techniques for fixed microwave systems". NTIA Report TR-89-243. National Telecommunication and Information Administration. US Dept. of Commerce, United States of America.

2.5 RSE of single-hop rural radio-relay links

RSE values of single-hop rural point-to-multipoint radio-relay systems using different types of modulation were compared to the MTES (see Annex 1, § 3 and equation (4)).

In the model adopted for such a network, each central station has a specific service area within which it communicates by single-section links with four local stations situated in different rural sites.

The RSE in such a network in the relation to the MTES was considered as a function of a large number of parameters: the frequency band used, the type of frequency and polarization-space planning of the network, the height at which the antenna is suspended and the antenna diameter under different types and parameters of the modulation (FM and pulse-code modulation (PCM) with Multiple-PSK).

Table 12 presents some results of those calculations in the case of a network operation in the 8 GHz frequency band with antenna suspension heights of 45 m, antenna diameters, D , of 1.5 m and 3.0 m, 2- and 4-frequency operations ($K = 2$ and $K = 4$ respectively) and two types of polarization plan - with the use in the network of one (1P) and two (2P) types of polarization. For PCM the data is given for 2-PSK ($M = 2$), 4 PSK ($M = 4$) and for a value (M_{max}) which ensures maximum value of the RSE. The dashes in Table 12 indicate that with the given combinations of parameters the performance characteristic standards cannot be met. As Table 12 shows, the use of PCM with PSK produces a gain in the RSE only with the optimum modulation conditions ($M_{max} \geq 8$) and antennas with a diameter of 3 m.

NOTE 1 – Additional information may be found in: Annex IV Report 662-3 (Düsseldorf, 1990).

TABLE 12
RSE of single-hop rural radio-relay links

Modulation	Frequency plan	Parameters of modulation	RSE			
			$D = 1.5$ m		$D = 3.0$ m	
			1P	2P	1P	2P
FM	$K = 2$		0.27	0.3	0.285	0.285
	$K = 4$		0.2	0.25	0.25	0.25
PCM	$K = 2$	$M = 2$	–	–	0.055	0.049
		$M = 4$	–	–	–	–
		M_{max}	–	–	0.055	0.055
	$K = 4$	$M = 2$	0.125	0.0625	0.0625	0.0625
		$M = 4$	0.125	0.125	0.125	0.125
		M_{max}	0.25	0.25	0.25	0.5