RECOMMENDATION ITU-R S.1525

Impact of interference from the Sun into a geostationary-satellite orbit fixed-satellite service link

(Question ITU-R 236/4)

(2001)

The ITU Radiocommunication Assembly,

considering

a) that Sun transits are a natural phenomena in geostationary-satellite orbit fixed-satellite service (GSO FSS) networks, which occur over a period of 3-9 days twice a year, depending on the antenna diameter;

b) that GSO FSS earth station operators and customers plan for the Sun transits and implement appropriate means to minimize their impact;

c) that GSO FSS earth station operators should have a methodology available to analyse the magnitude of the interference from the Sun and the timing of the interference events,

recommends

1 that, in designing GSO FSS links, the methodology given in Annex 1 may be used to assess the level of carrier-to-noise ratio (C/N) degradation of a GSO link resulting from Sun transit;

2 that GSO FSS earth station operators may use the method in Annex 2 to predict the date and time of a Sun transit at an earth station.

ANNEX 1

Calculation of the level of interference from the Sun into a GSO FSS link

Sun transits occur twice a year near the spring and autumn equinoxes when the Sun passes close to the main beam of the receiving GSO earth station. During these Sun transits, the microwave radiation from the Sun acts as a source of interference, increasing the effective noise temperature of the satellite link and therefore degrading the link performance. If the amount of degradation exceeds the clear-sky margin of the link, then the link will suffer an outage – usually termed a Sun-outage.

For frequencies below about 30 GHz, emission from the Sun can be considered as having three components: the thermal emission from the "quiet" Sun, a slowly-varying component related to the number and size of sunspots, and occasional intense bursts of emission due to Sun flares. All three components are time-varying, and so it is extremely difficult to use the Sun as a reference source for accurate evaluation of the performance of earth station antennas.

In the satellite communication bands, the thermal emission from the quiet Sun decreases with increasing frequency. The emission is generally unpolarized.

Sunspots are magnetic regions on the Sun, appearing as dark spots on its surface. They typically last for several days, although very large ones may last for several weeks. Although the number of sunspots varies greatly from day to day, there is an underlying pattern with a period of approximately 11 years. Every 11 years, the Sun undergoes a period of activity called the “Sun maximum”, followed by a period of quiet called the “Sun minimum”. During the Sun maximum there are many sunspots, Sun flares, and coronal mass ejections, all of which can affect communications and the weather on Earth. There is a rough correlation between the total solar flux and the number of sunspots. As this increased flux is associated with several small areas of the Sun's surface, it is unsafe to assume a uniform brightness distribution across the face of the Sun. In fact, if the GSO earth station has an antenna beamwidth smaller than the apparent diameter of the Sun, then it could experience varying levels of interference during a single Sun transit event. Emission due to sunspots is somewhat circularly polarized, although this effect is diluted by the random polarization of the thermal emissions from the quiet Sun.

Sun flares or bursts may double or triple the Sun flux, occasionally reaching factors of 100 times the usual level. In the satellite communication bands, most events are fairly short - 5 min to about an hour. Occurrence is unpredictable, but as already noted the events are more frequent around the Sun maximum. Typically there could be a couple of events per day.

The apparent diameter of the Sun at microwave frequencies is slightly larger than the optical diameter. Also, the Earth-Sun distance varies slightly during the year, resulting in a variation in the apparent diameter of the Sun and thus in apparent brightness temperature. However these effects are small compared with other uncertainties (such as the sunspot number), and may thus be neglected. A reasonable estimate of the apparent diameter of the Sun at the equinox is 0.53°.

Although the foregoing material indicates that significant and unpredictable variations exist in the effective level of the Sun flux in the satellite communication bands, a number of simple models have been proposed for the average level. These models are adequate for assessment of the typical levels of interference which can be expected during Sun transits. One expression for the brightness temperature of the quiet Sun at microwave frequencies is:

$$T_{Sun} = 120\,000 \times \gamma \times f^{-0.75}$$

where:

T_{Sun}: equivalent brightness temperature (K)

f: frequency (GHz)

γ: factor to account for the polarization of the emissions from the Sun, which could be taken to be 0.5 for the reasons given above.

This model gives a value of around 21 000 K for the quiet Sun at a frequency of 4 GHz. By comparison, a typical value at sunspot maximum would be 90 000 K.

1 General approach

The Sun transit in the GSO receiver is a phenomenon that can be easily assessed as the geometry is well known. The following method is proposed to fully describe the Sun transit effect on GSO link budgets. The impact of Sun transit is not a fade but an increase of the system noise temperature that can be significant for some low margin, low noise GSO links.

The proposed method is based on the well defined geometry of Sun position relative to a specified location on the Earth: the Sun is approximately a 0.53° diameter disk as seen from an Earth point. The Sun transit effect is significant when the disk intersects the main beam of the reception antenna. The detailed approach (Step 1 a) of this Annex) varies the antenna gain over the optical disc of the Sun in accordance to the assumed antenna gain pattern. The simplified approach (Step 1 b) of this Annex) assumes a constant antenna gain over the optical disc of the Sun corresponding to the gain towards the centre of the Sun's disk. The detailed approach gives greater accuracy but increases the analysis complexity. Whereas the simplified approach is less complex to implement.

Around the period of the spring and autumn equinoxes, the Sun is in line with the earth station receiving FSS antenna and the GSO satellite. This leads to an increase of the antenna noise temperature that affects the FSS receiver figure of merit thereby degrading the *C*/*N*. The following methodology may be used to assess the magnitude of the degradation of link performance.

Step 1: Determine the value of the antenna gain over the Sun disk:

$$\iint_{Sun} G(\theta, \varphi)\, d\Omega$$

where:

θ: off-axis angle

φ: azimuth angle.

a) Detailed approach

As shown in Fig. 1, the Sun is modelled by a disk positioned on a sphere centred on the receiving earth station. The sphere represents the space as seen by the antenna for example, the point M on the Sun is defined by the spherical angles θ and φ.

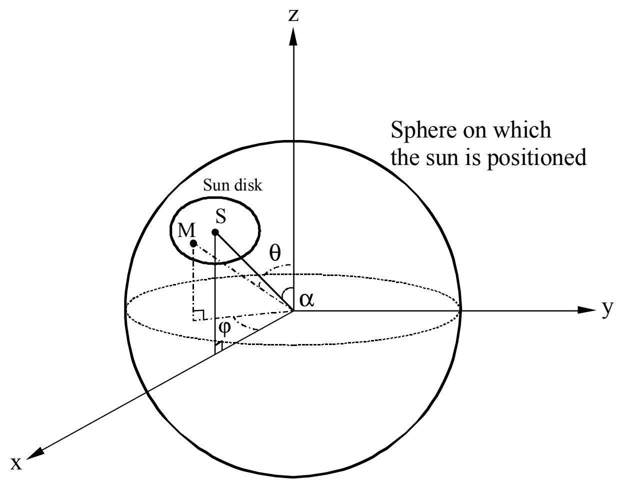
To ease the computation, the centre of the Sun S is in the plane containing the earth station, the x-axis and z-axis. Therefore, the location of S is given by the spherical angle α.

The z-axis is in the direction of the pointing direction of the receive antenna.

The computation can use the axis symmetry of the geometry: the points with the same gain form arcs. These result from the intersection of a plan perpendicular to the axis antenna z with the portion of sphere containing the Sun.

The value of the integral is so determined by the addition of the different lengths of the iso-gain arcs, multiplied by the value of the gain for the arc.

FIGURE 1



φ: azimuth angle
θ: off-axis angle (boresight is defined by z-axis)
α: elevation angle between the centre of the Sun to the z-axis
S: centre point of the sun
M: any point of the sun disk

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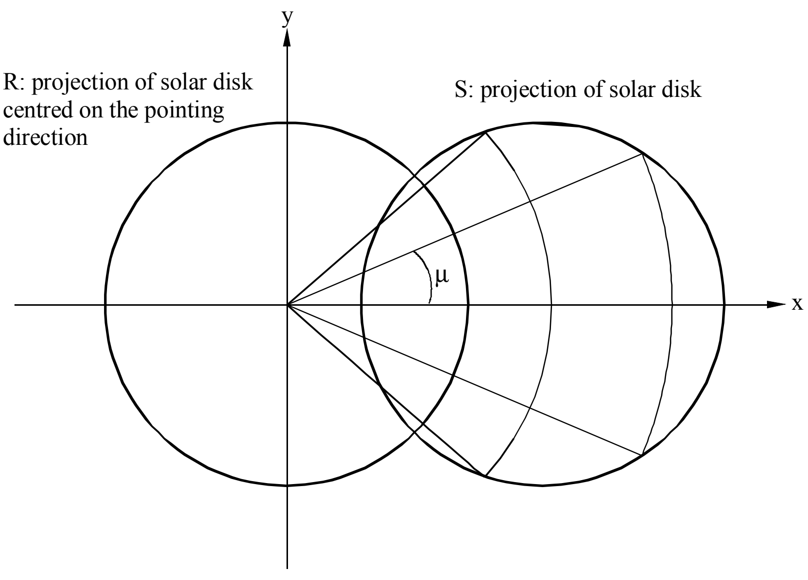
If β is the half angle of view of the Sun, there are two cases:

If α > β:

FIGURE 2

Projection of the Sun on the xy plane*



* For simplification, the picture shows a projection of the solar disk which is circular. In reality it is not circular.

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The disk R is the projection of the Sun when it is centred on the z-axis. When the Sun is not on the z-axis, the calculations of the receive antenna gain in the direction of the Sun are done through the integration over iso-gain arcs, which have an half-aperture μ that can vary from 0 to π. To determine the overall gain in the direction of the Sun disk $\iint_{Sun} G(\theta,\varphi)\,d\Omega$, the following formula applies when $\mu < \pi$.

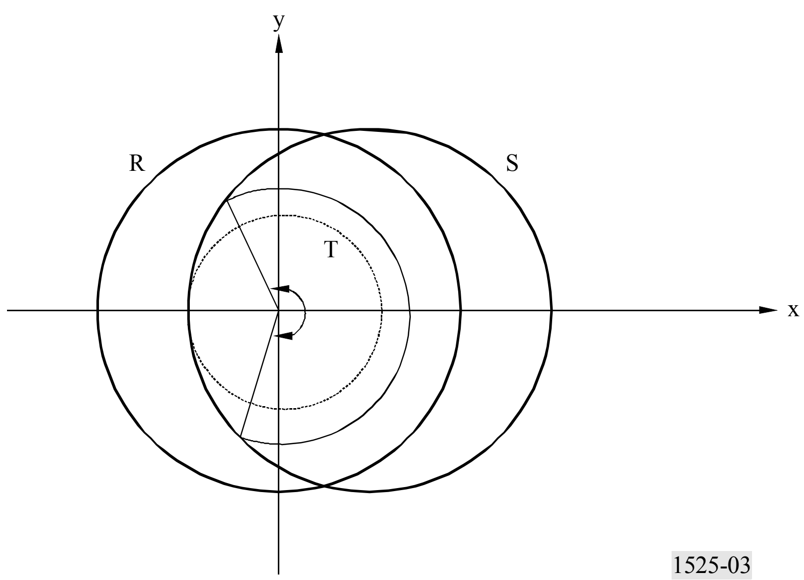
$$\iint_{Sun} G(\theta,\varphi)\,d\Omega = \sum_{\theta=\alpha-\beta}^{\theta=\alpha+\beta} 2\mu \cdot \sin(\theta) \cdot G(\theta) \cdot \Delta\theta$$

where:

$$\mu = A\cos\left\{\frac{[\cos(\beta) - \cos(\theta)\cos(\alpha)]}{\sin(\theta)\sin(\alpha)}\right\}$$

If $\alpha < \beta$:

FIGURE 3



The calculation above is valid for all the arcs which correspond to iso-θ lines of less than ($\beta - \alpha$) (represented by the dotted circle T above). For lower values of θ, the computation of the gain over the portion of the Sun disk, is simplified by the z axial symmetry of the geometry:

$$\iint_{Sun} G(\theta,\varphi)\,d\Omega = \sum_{\theta=0}^{\theta=\beta-\alpha} 2\pi \cdot \sin(\theta) \cdot G(\theta) \cdot \Delta\theta$$

where:

$G(\theta)$: the linear isotropic antenna gain (function of the off-axis angle θ)

$\Delta\theta$: the angular increment.

b) Simplified approach

The Sun only subtends a small angle (θ_{Sun}) as viewed from the Earth and if we assume that over θ_{Sun} the normalized antenna gain (G_n) will average out to be G_n towards the centre of the Sun ($G_{n_{Sun}}$), then $\iint_{Sun} G(\theta, \varphi)\, d\Omega$ can be approximated by:

$$\iint_{Sun} G(\theta, \varphi)\, d\Omega = 2\pi\, G_{n_{Sun}} \left(1 - \cos\left(\frac{\theta_{Sun}}{2}\right)\right)$$

Step 2 : Determine the value of the gain over the entire space:

$$\iint_{Space} G(\theta, \varphi)\, d\Omega$$

Due to the z-axis of the antenna patterns from the ITU-R Recommendations, the calculation is straightforward:

$$\iint_{Space} G(\theta, \varphi)\, d\Omega = \sum_{\theta=0}^{\theta=\pi} 2\pi \cdot \sin(\theta) \cdot G(\theta) \cdot \Delta\theta$$

where:

$G(\theta)$: the linear isotropic antenna gain, only depending on the off-axis angle θ

$\Delta\theta$: the increment of angle.

Step 3 : Determine the temperature of the Sun:

$$T_{Sun} = 120\,000 \times \gamma \times f^{-0.75}$$

where:

T_{Sun}: equivalent brightness temperature (K)

f: frequency (GHz)

γ: polarization factor, set here to 0.5 as the radiation from the Sun is assumed to have random polarization.

Step 4 : Determine the temperature increase at the receive antenna:

$$\Delta T = \frac{\iint_{Sun} T_{Sun} \times G(\theta, \varphi)\, d\Omega}{\iint_{Space} G(\theta, \varphi)\, d\Omega} = \frac{T_{Sun} \times \iint_{Sun} G(\theta, \varphi)\, d\Omega}{\iint_{Space} G(\theta, \varphi)\, d\Omega}$$

Step 5 : Determine the degradation of the receiver's C/N as follows:

$$\Delta(C/N) = 10 \log \left(\frac{T_0 + \Delta T}{T_0}\right)$$

with T_0 as the total link noise temperature.

2 Application of the methodology to different antenna sizes

The detailed approach described in the previous sections has been applied to different antenna sizes.

Two possible approaches could be used to generate the values for the off-axis angle θ and the azimuth angle φ:

– Full simulation of the motion of the Sun, using for example the algorithm defined in Annex 2.

– A simplified approach based on the fact that the declination angle of the Sun changes at approximately 0.4º per day at the equinoxes and its hour angle changes at approximately 0.25º per minute.

In all the cases the initial noise temperature used is 150 K and the antenna patterns used are according to Recommendation ITU-R S.465 at 11 GHz.

The impact on a link performance depends on the size of the antenna and the initial noise temperature of the link. For large antennas with high gain, the degradation of the *C*/*N* can be up to 15 dB (as shown in Fig. 5) but occurs fewer times than for small antennas with wider beams (Fig. 9).

As was expected the results show that the depth of the degradation of the *C*/*N* is a function of the antenna size and the duration of the Sun transit increases as the antenna diameter decreases.

FIGURE 4

Daily maximum sky noise temperature increase for a 10 m antenna

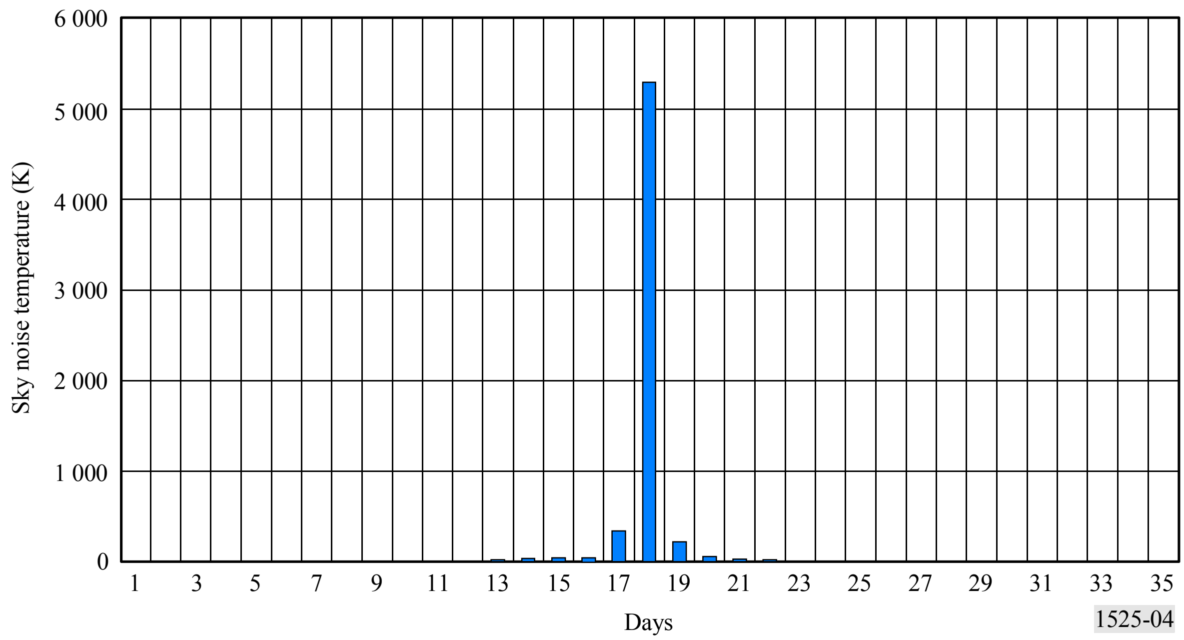


FIGURE 5

Daily maximum degradation of the received *C*/*N* of a 10 m antenna

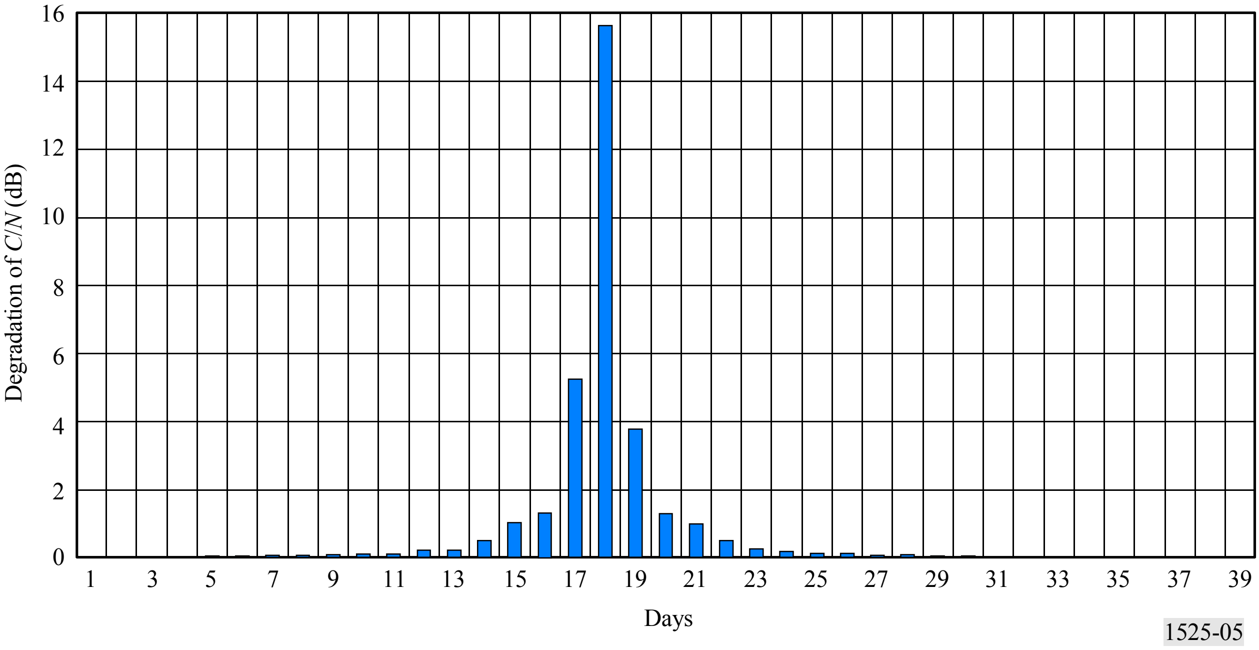


FIGURE 6

Daily maximum sky noise temperature increase for a 3 m antenna

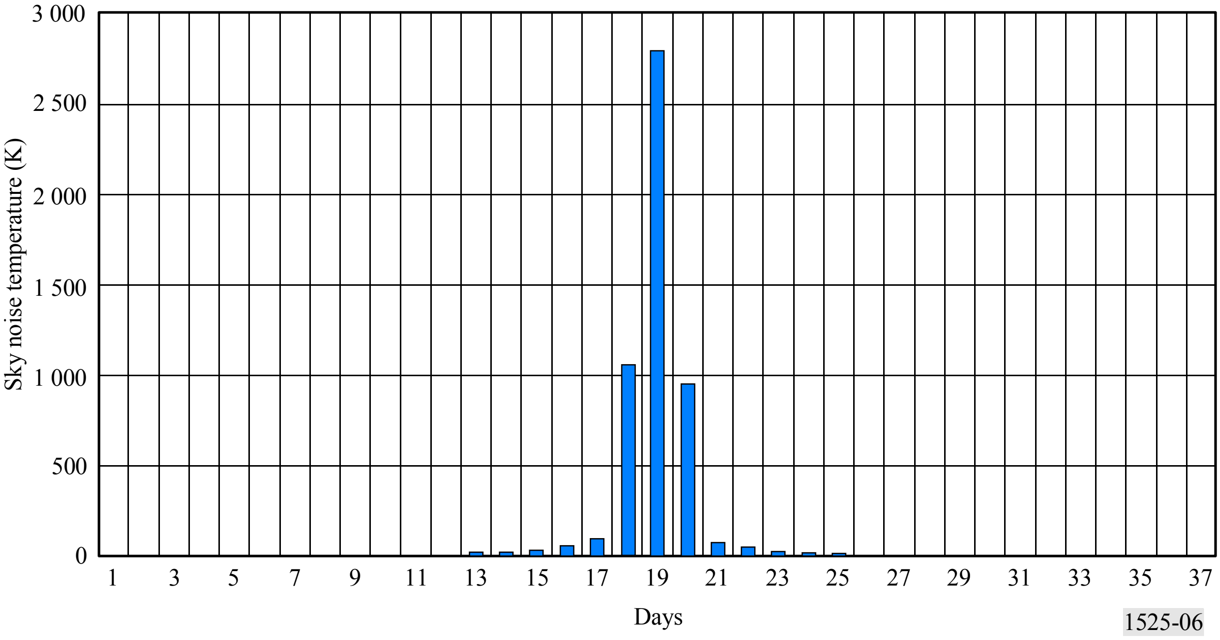


FIGURE 7

Daily maximum degradation of the received *C*/*N* of a 3 m antenna

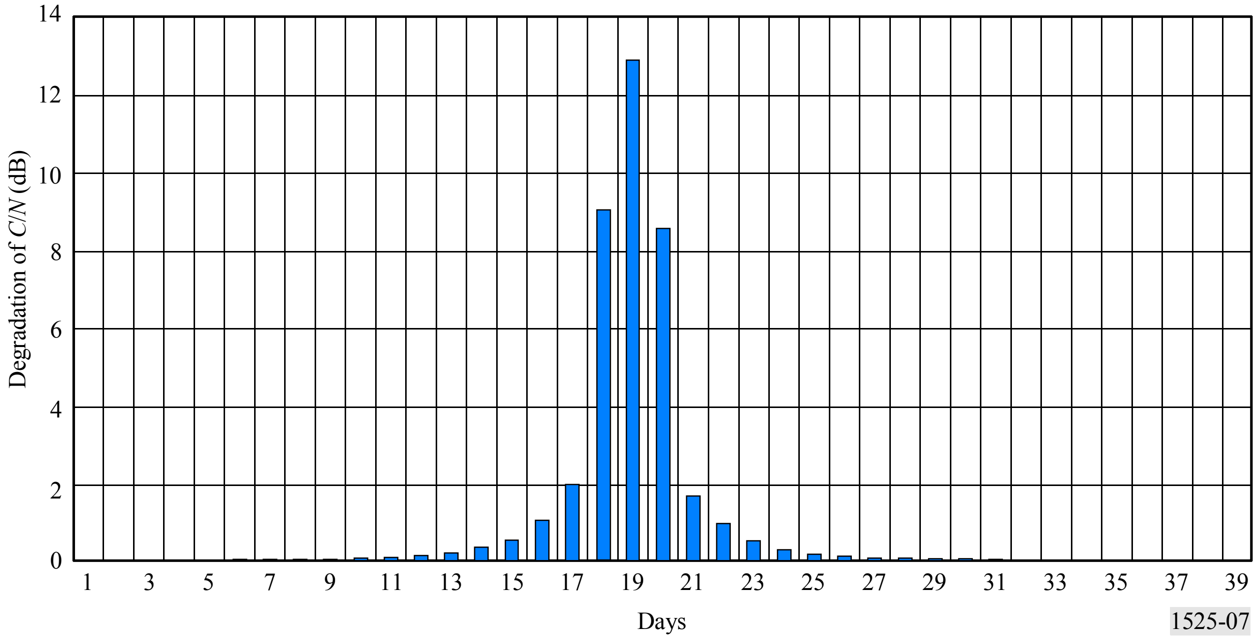


FIGURE 8

Daily maximum sky noise temperature increase for a 0.6 m antenna

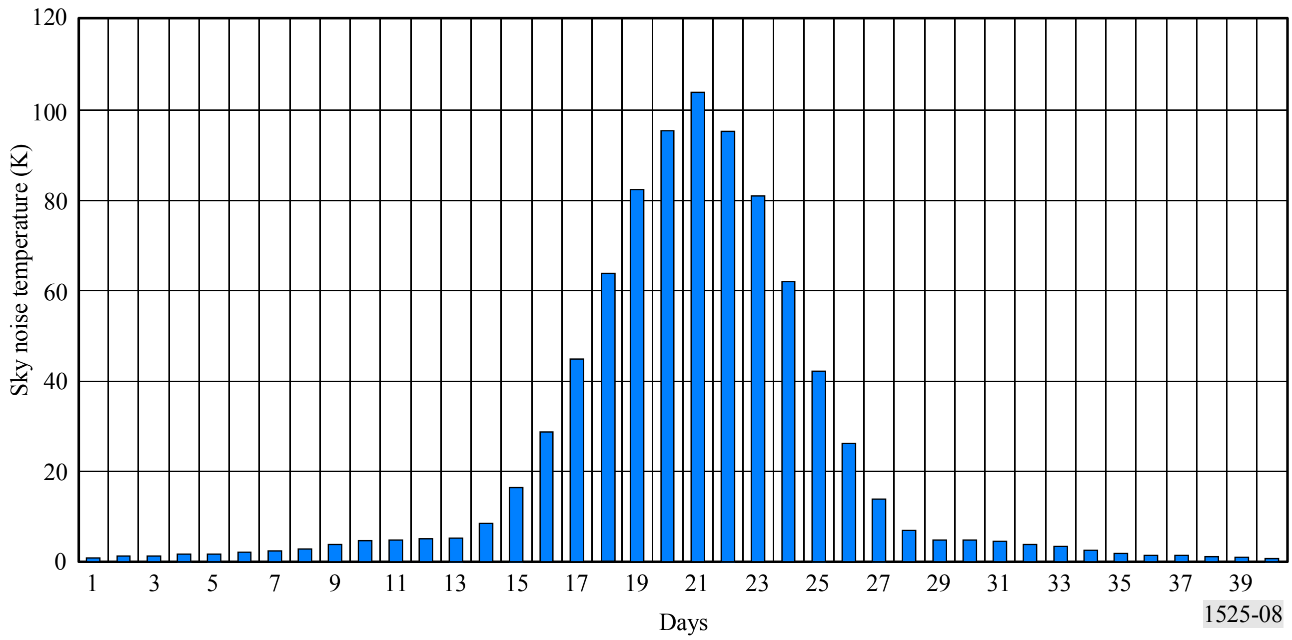
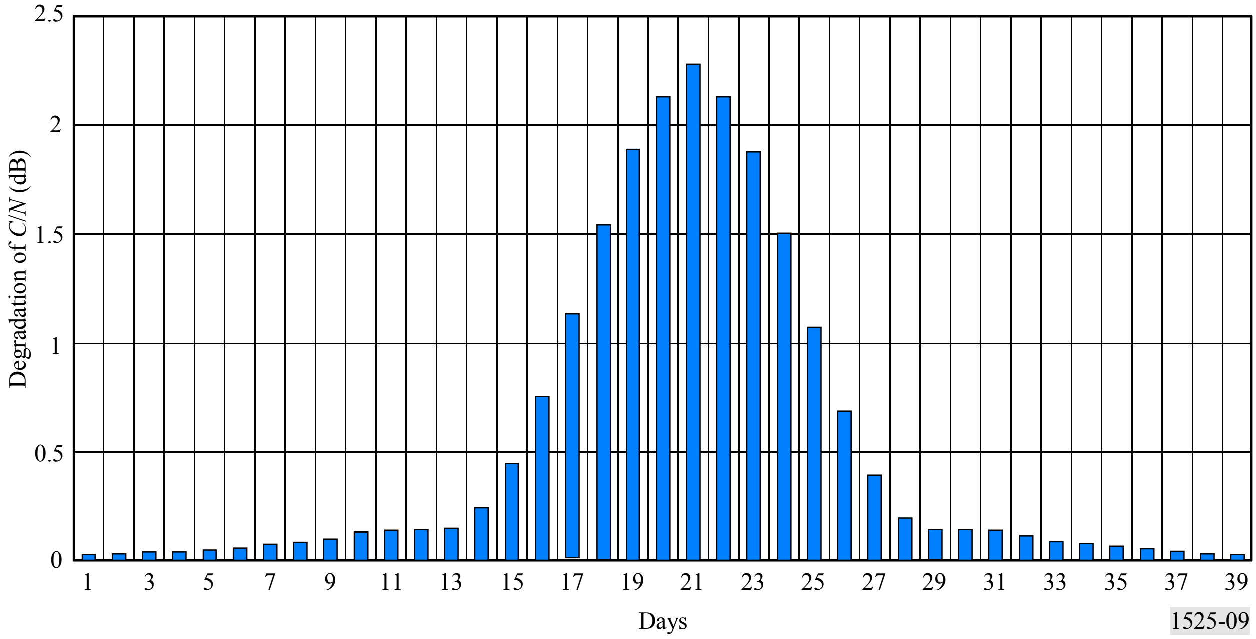


FIGURE 9

Daily maximum degradation of the received *C*/*N* of a 0.6 m antenna



3 Variation during a day

Computations have been done to show a time profile of the degradation of *C*/*N* as a function of the time of the day close to the equinox period. The time step is set to 1 s.

FIGURE 10

Degradation of the received *C*/*N* for a 10 m antenna during a day

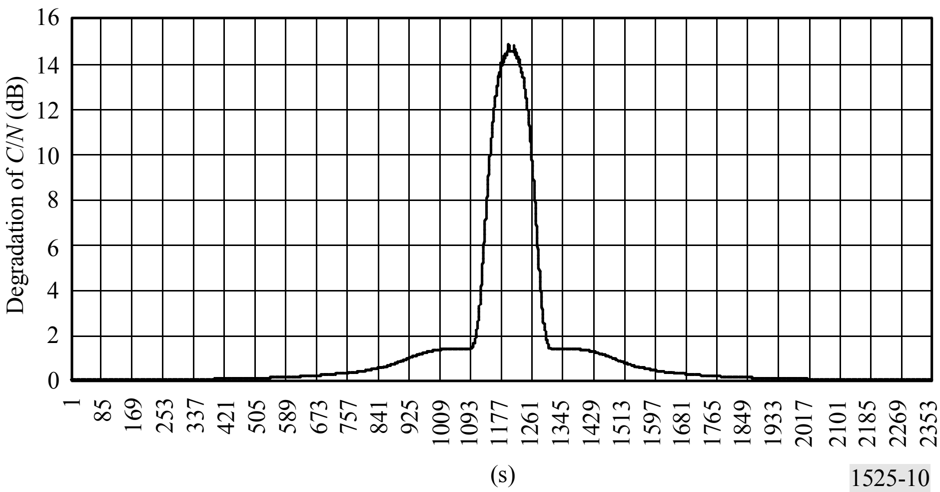
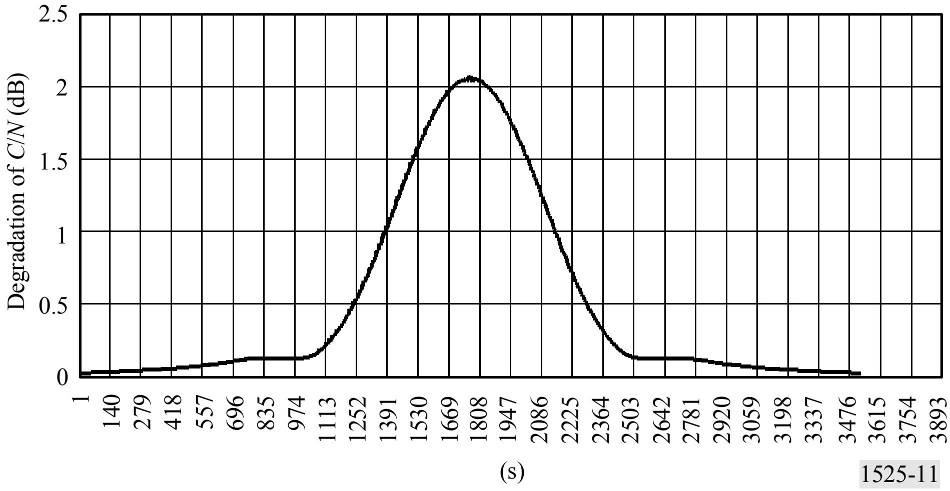


FIGURE 11

Degradation of the received *C*/*N* for a 0.6 m antenna during a day



ANNEX 2

Simplified method to calculate the Sun transit period for a GSO earth station

1 Introduction

GSO FSS earth station operators have accepted that Sun interference is a natural phenomenon that occurs for a short period 0 to 21 days before or after the equinoxes, depending on whether the station is located in the Northern or Southern Hemisphere.

Most operators use simplified algorithms such as the one given below, which does not require a specific carrier link budget, to estimate the day and times when a Sun transit will occur. With this information, they can take proactive action to mitigate the effects of Sun interference.

2 Satellite ephemeris data

Satellite operators use a number of different mathematical models to represent the movement of a satellite. A simplified approach was developed by one GSO satellite operator whereby, instead of computing all physical effects acting on a satellite, these effects are defined in terms of three equations. This approximation contains 11 parameters obtained with a least-squares curve fit. It has been demonstrated that this simplified model approximates the full prediction results to better than 0.01° for a period of up to seven days.

In this approach, the three equations that predict the satellites position at any relative time, t, from the start of the epoch are:

Satellite east longitude:

$$L = L_0 + L_1 t + L_2 t^2 + (L_c + L_{c1} t)\cos(Wt) + (L_s + L_{s1} t)\sin(Wt) + (K/2)(l_{c2} - l_{s2})\sin(2Wt) - K\, l_c l_s \cos(2Wt)$$

Satellite geocentric latitude:

$$l = (l_c + l_{c1} t)\cos(Wt) + (l_s + l_{s1} t)\sin(Wt)$$

Satellite radius:

$$r_{sat} = r_g\,(1 - 2L_1/3(W - L_1))\,(1 + KL_c \sin(Wt) - KL_s \cos(Wt))$$

where:

$W =$ L_1 + 360.98564 degrees/day

$r_g =$ 42 164.57 km

$K =$ $\pi/360$

t: time in days

and the eleven parameters are:

L_0: mean longitude (East of Greenwich) (degrees)

L_1: drift rate (degrees/day)

L_2: drift acceleration (degrees/day/day)

L_c: longitude oscillation-amplitude for the cosine term (degrees)

L_{c1}: rate of change of longitude, for the cosine term (degrees/day)

L_s: longitude oscillation-amplitude for the sine term (degrees)

L_{s1}: rate of change of longitude, for the sine term (degrees/day)

l_c: latitude oscillation-amplitude for the cosine term (degrees)

l_{c1}: rate of change of latitude, for the cosine term (degrees/day)

l_s: latitude oscillation-amplitude for the sine term (degrees)

l_{s1}: rate of change of latitude, for the sine term (degrees/day).

With the satellite position defined as a function of time in terms of a geocentric system aligned with the Greenwich Meridian, the satellite position with respect to the earth station and the appropriate pointing angles are calculated as follows:

$$\Delta r = r_{satellite} - r_{station}$$

$$\Delta r_x = r_{sat}\cos(\varphi_{sat})\cos(\theta_{sat} - \theta_{sta}) - R_a$$

$$\Delta r_y = r_{sat}\cos(\varphi_{sat})\sin(\theta_{sat} - \theta_{sta})$$

$$\Delta r_z = r_{sat}\sin(\varphi_{sat}) - R_z$$

where:

φ_{sat}: satellite latitude (geocentric)

θ_{sat}: satellite longitude

θ_{sta}: station longitude East of Greenwich

R_a: station radial distance from Earth rotational axis

R_z: station axial distance above Earth equatorial plane.

From the above, azimuth and elevation pointing is determined as follows:

Earth station azimuth angle:

$$AZ = \arctan\left(\Delta r_y / \Delta r_{North}\right)$$

Earth station elevation angle:

$$EL_{geometric} = \arctan\left(\Delta r_{zenith} / (\Delta r_{North}^2 + \Delta r_y^2)^{1/2}\right)$$

where:

$$\Delta r_{North} = \Delta r_x \sin(\varphi_{sta}) + \Delta r_z \cos(\varphi_{sta})$$

$$\Delta r_{zenith} = \Delta r_x \cos(\varphi_{sta}) + \Delta r_z \sin(\varphi_{sta})$$

3 Predicted Sun transit periods

In order to calculate the timing of the interference from the Sun, earth station pointing angles in terms of the equatorial coordinate system (ECS) are required, which will be described in more detail in the following paragraph.

This is the same coordinate system used for polar antenna mounts. The hour angle and declination in the ECS are calculated from AZ and EL above as:

$$\text{Hour angle} = \arctan\left(\frac{-\cos(EL)\cdot\sin(AZ)}{\sin(EL)\cdot\cos(\varphi_{sta}) - \cos(EL)\cdot\sin(\varphi_{sta})\cdot\cos(AZ)}\right)$$

$$\text{Declination} = \arcsin(\sin(EL)\cdot\sin(\varphi_{sta}) + \cos(EL)\cdot\cos(\varphi_{sta})\cdot\cos(AZ))$$

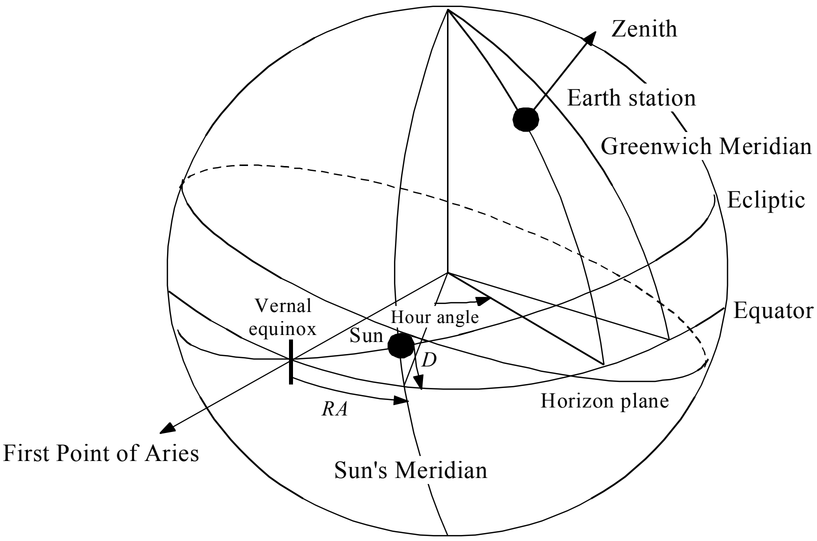
3.1 Coordinate system for calculating Sun transit

The calculations for Sun interference are based on the equatorial coordinate system, where the Earth's equator is the reference plane and the vernal equinox is the reference direction. The vernal equinox or The First Point of Aries, is the intersection of the ecliptic (mean plane of the Earth's orbit) with the celestial equator, at which the Sun crosses the equator from south to north. The Earth's centre is the origin of this system, which is illustrated in Fig. 12.

Declination and right ascension give the coordinates of the Sun. Declination is the angle between the equatorial plane and the Sun. Right ascension is the angle, measured counterclockwise relative to celestial north in the equatorial plane from the vernal equinox to the current position of the Sun. The hour angle is the angular difference between the observer's longitude and the longitude of the Sun.

FIGURE 12

Celestial sphere



D: Declination with respect to the Equator
RA: Right ascension of the Sun

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3.2 Calculating the coordinates of the Sun

Time is measured with respect to the period of the Earth's rotation in terms of the Sun day, which is the period between successive Sun passages through the observer's meridian. Since the Earth also circles the Sun in a one-year period, the Sun day is not the true period of the Earth's rotation. The Earth travels 1/365 of the way around its orbit in one day, but the Earth has to rotate slightly more than one complete turn relative to a fixed star during this interval.

Sidereal time is based on the Earth's rotation with respect to a fixed star. Variations in the Earth's rotation, due to precession and nutation, affect the length of the sidereal day. Precession is the circular movement of the Earth's North Pole around the ecliptic pole. The radius of the circle is approximately 23.5° and a period of 26 000 years is required to complete one revolution. Nutation is a complicated oscillation of the Earth's North Pole about the mean North Pole. The difference between the mean sidereal time and apparent sidereal time is called the equation of the equinox.

The right ascension and declination of the Sun, as well as the Earth's rotation relative to the vernal equinox are related to Universal Time (UT). Universal Time, which is the basis for civil timekeeping, is closely related to the mean diurnal motion of the Sun and is directly related to sidereal time. Local apparent sidereal time (LAST) is the hour angle of the true equinox, the intersection of the true equator and the ecliptic of the date, which is affected by the nutation of the Earth's axis. Local mean sidereal time (LMST) is the hour angle of the mean equinox or the intersection of the mean equator with the ecliptic of the date, which is affected by the precession of the Earth's axis.

The right ascension and declination of the Sun can be derived from the obliquity of the ecliptic, Ω, and the ecliptic longitude, L_{Sun}, as shown in Fig. 13 as:

$$L_{Sun} = L_{mean} + 1.915° \sin(M_e) + 0.020° \sin(2 \times M_e)$$

where the mean longitude of the Sun is:

$$L_{mean} = 280.460° + 0.9856474° \times d$$

and the mean anomaly of Earth's orbit is:

$$M_e = 357.528° + 0.9856003° \times d$$

The time d is expressed in terms of Julian days (JD) referenced to 1200 UT, 1 January 2000, which is Julian day 2 451 545 and $d = JD - 2\ 451\ 545$.

The obliquity of the ecliptic (the angle between the ecliptic and equator) is nearly constant and is expressed as:

$$\Omega = 23.439°$$

From the law of sines in spherical trigonometry the declination, D, is:

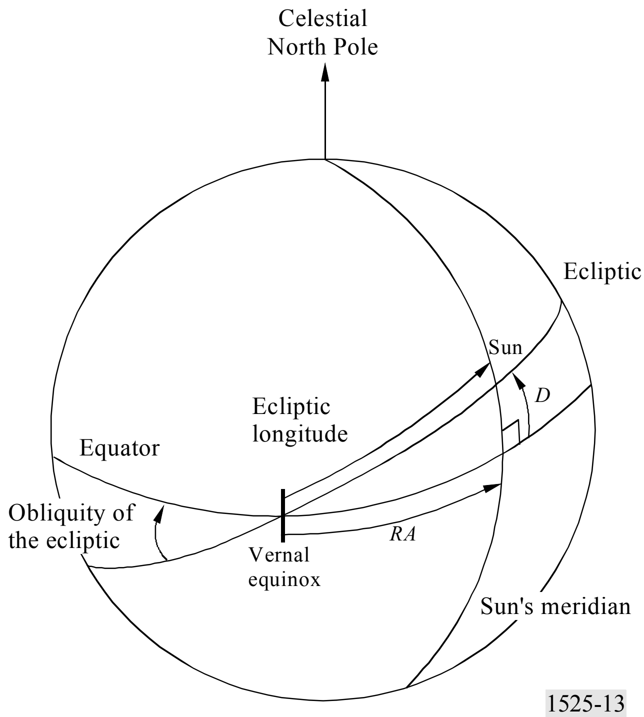
$$D = \arcsin\left(\sin(\Omega) \cdot \sin(L_{Sun})\right)$$

and the right ascension, RA, is:

$$RA = \arctan\left(\cos(\Omega) \cdot \tan(L_{Sun})\right)$$

FIGURE 13

The equatorial coordinate system



For this purpose, the declination of the Sun can be approximated by:

$$D_{Sun} = 23.5° \sin(360\,(p - x)/365)$$

where 23.5° is the approximate obliquity of the ecliptic, p is the day of the year and the value x is 80 for the vernal equinox and 83.5 for days near the autumnal equinox. Because the number of days from the vernal to autumnal equinox is not equal, the value x takes on these two different values.

One condition for Sun transit is when the declination of the Sun is very close to the declination of the satellite as derived in above in terms of the equatorial coordinate system. The approximate date for Sun transit, P_0, can be obtained to the nearest integer of p, by substituting the declination angle for D_{Sun} in the equation above.

3.3 Calculating the right ascension of the Sun

Using the value of P_0 as the starting point of an iterative calculation that increments the Julian day, the calculation proceeds until:

$$|D_{Sun} - \text{Declination}\,| \leq 0.17°$$

where:

$$D_{Sun} = \arcsin(\sin(\Omega_i) \cdot \sin(L_{Sun_i}))$$

At the time that the declination of the Sun and the earth station declination angle are very close, the value of right ascension of the Sun is obtained from:

$$RA_i = \arctan(\cos(\Omega_i) \cdot \tan(L_{Sun_i}))$$

3.4 Calculating the time of Sun transit

Another condition for Sun transit is coincidence of the right ascension of the Sun and the satellite. The declination of the satellite look angle is a fixed value; however, the equivalent right ascension of the satellite varies with time as follows:

$$RA_{sat} = LAST - \text{Hour angle}$$

or

$$LAST = RA_{sat} + \text{Hour angle}$$

where the hour angle was calculated previously above. In terms of Greenwich apparent sidereal time (GAST):

$$GAST = LAST - \theta_{sta}$$

therefore, equating $RA_{Sat} = RA_{Sun}$ and substituting the value RA from above:

$$GAST = RA + \text{Hour angle} - \theta_{sta} = -351.774°$$

The time in Coordinated Universal Time (UTC) can then be computed from:

$$GAST = 100.4602346 + 0.985647348 \times (JDO_{sat} - 2451545) + 15.041068 \times UT$$

or

$$UT = \left[\frac{GAST - 100.4602346 - 0.985647348 \times (JDO_{sat} - 2451545)}{15.0410}\right]$$

where JDO_{Sat} is the Julian date at midnight, that is, it must be a half integer.

It was found through measurements conducted at an operating earth station that the above method can estimate the peak Sun transit to ±15 s.

3.5 Approximate values

While the above calculations provide an accurate estimation of the time of a Sun transit for an earth station, the actual number days and the length of the time that significant levels of Sun interference will be experienced at a particular earth station will depend on the signal characteristics, performance objectives, and link margins for each carrier. The following sections derive approximate values for the number of affected days, maximum duration per day, and the total duration of the Sun transit effects at each equinox.

3.5.1 The number of affected days

Because the declination angle of the Sun changes at approximately 0.4º per day at the equinoxes, the number of affected days at each equinox can be approximated by:

$$\text{Affected days} = \frac{\theta_{3\text{dB}} + 0.48°}{0.4°}$$

where 0.48° is the approximate optical diameter of the Sun and the earth station antenna half power beamwidth, θ_{3dB} (degrees) is estimated as:

$$\theta_{3\text{dB}} = 70 \times \lambda / d_{ant}$$

where:

d_{ant}: antenna diameter

λ: wavelength in the same units.

For an 11 m antenna at a frequency of 11 GHz, θ_{3dB} = 0.17°, and so Sun transits through the 3 dB beamwidth of the antenna would typically occur on 1 or 2 successive days at each equinox.

3.5.2 The maximum duration

Because the hour angle of the Sun changes at approximately 0.25º per minute, the maximum duration of a Sun transit can be approximated by:

$$\text{Affected minutes} = \frac{\theta_{3\text{dB}} + 0.48°}{0.25°}$$

For the 11 m antenna, θ_{3dB} = 0.17°, and so the maximum duration of a Sun transit through the antenna 3 dB beamwidth would be approximately 2.5 min.

3.5.3 Total duration at each equinox

When a large number of parallel lines intersect a circle, the average length of the intersection is $\pi D/4$, where D is the diameter of the circle. The total length of the intersections is $\pi D^2/4p$, where p is the separation between the parallel lines.

From this last expression, and since the declination angle of the Sun changes at approximately 0.4º per day at the equinoxes and its hour angle changes at approximately 0.25º per min, the total duration of the Sun transits at each equinox can be approximated by:

$$\pi \times (\theta_{3\text{dB}} + 0.48°)^2 \,/\, (4 \times 0.4° \times 0.25°)$$

For the 11 m antenna, θ_{3dB} = 0.17°, and so the total duration of Sun transits through the antenna 3 dB beamwidth would be approximately 3.5 min at each equinox.