RECOMMENDATION ITU-R S.1591

Sharing of inter-satellite link bands around 23, 32.5 and 64.5 GHz between non-geostationary/geostationary inter-satellite links and geostationary/geostationary inter-satellite links

(Question ITU-R 265/4)

(2002)

The ITU Radiocommunication Assembly,

considering

a) that inter-satellite links (ISLs) within geostationary (GSO) satellite systems use, or are planned to use, the inter-satellite service (ISS) frequency allocations 22.55-23.55 GHz, 24.45-24.75 GHz, 32.0-33.0 GHz and 59.3-71.0 GHz;

b) that ISLs between GSO and non-GSO satellites use, or are planned to use, the ISS frequency allocations 22.55-23.55 GHz, 24.45-24.75 GHz, 32.0-33.0 GHz and 59.3-71.0 GHz;

c) that the ITU-R needs criteria and methods of calculation in order to assess the potential for ISLs of the type mentioned in *considering* b) to share frequencies with ISLs of the type mentioned in *considering* a);

d) that the criteria and calculation methods mentioned in *considering* c) could possibly enable the Radiocommunication Bureau (BR) to process notices submitted in accordance with Appendix 4 of the Radio Regulations (RR) for spectrum for ISLs of the type mentioned in *considering* b);

e) that the simulations described in Annex 1 show that instances of significant interference between co-frequency ISLs of the types mentioned in *considerings* a) and b) occur in only a small proportion of cases, and in those cases for only small percentages of the time;

f) that Annex 1 also verifies that the maximum levels of interference occur when the non-GSO/GSO ISL is instantaneously in the Equatorial plane, and that those levels may therefore be calculated manually;

g) that *considerings* e) and f) make it convenient for frequency sharing between the two types of ISL to be facilitated by coordination, and that coordination would not be needed in many cases;

h) that the need to coordinate may be checked by calculating the minimum carrier-to-interference, C/I, ratios and comparing them with a simple threshold C/I,

recommends

1 that frequency sharing between a non-GSO/GSO ISL and a GSO/GSO ISL is practical and in exceptional cases may require coordination;

2 that the need to coordinate should be determined by the method described in Annex 2;

3 that coordination should take place if the minimum C/I ratio in a bandwidth of 1 MHz at any of the four ISL receivers, calculated by the method in Annex 2, is less than either the C/N identified for that receiver in item C.8 *e*) of RR Appendix 4 + 3 dB or, in the absence of a C/N, 30 dB;

4 that, in cases where coordination is found to be necessary from *recommends* 2 and 3, Annex 1 may be used by the coordinating parties for guidance.

ANNEX 1

Investigation of interference between a non-GSO/GSO ISL and a GSO/GSO ISL

1 The feasibility of frequency sharing by ISLs

In general, systems with ISLs in microwave bands have or are expected to have antenna diameters between 0.5 and 2 m. The lowest frequency ISS allocation is 22.55 to 23.55 GHz, and in this frequency range a 0.5 m antenna has a gain of about 39 dBi and a half-power beamwidth of about 2°. For larger diameters and higher frequencies, the beamwidths can be a fraction of a degree.

The angular range over which an ISL antenna may be pointed is large, thus reducing the probability of conjunctions or near conjunctions with interfering ISLs. The number of systems using the ISS allocations in the year 2001 was small. It is anticipated that this number will grow, but will probably be limited by the introduction of optical communications in space, and in any case it could not be expected to be as many as the number of systems with fixed-satellite service (FSS) links, which successfully share frequencies by means of coordination.

Interference, to or from any ISL that has a non-GSO satellite, is dependent on time-varying geometry. Hence, instances of significant interference will occur for only short durations in situations at or very close to the worst-case geometry. At all other times the antenna discrimination at one or both ends of the interference path will be substantial and this will reduce the interference power.

2 Selection of ISL parameters for computation of interference statistics

From a BR database compiled from filings for spectrum assignments, the transmission parameters of ISLs of both types in all three frequency bands associated with a number of MEASAT GSO and MEASAT low Earth orbit (LEO) satellites (Malaysia) were obtained. Also obtained from this source were parameters of 32 GHz ISLs associated with satellites in the LUXSAT series (Luxembourg), and of 67 GHz ISLs associated with satellites in the SMO-GEO series (France). Additionally, relevant information on GSO/GSO ISLs was extracted from Recommendations ITU-R S.1151 (32.5 GHz), ITU-R S.1326 (51 GHz FSS band) and ITU-R S.1327 (62.2 GHz).

At the time of writing the United States of America has several space systems with ISLs in the ISS allocations between 56 GHz and 65 GHz; they are USBL, USFD, USGAE, USGX, USLL, and MILSTAR. Some of these systems are in the planning or design phase, others are under development, and others in operation. The orbits used by these systems include LEOs, high Earth orbits (HEOs) and the GSO. The orbit combinations for their ISLs include, *inter alia*, GSO to GSO, LEO to and from GSO, and HEO to and from GSO.

Finally, reference was made to Recommendation ITU-R S.1328 for orbital and other pertinent data on non-GSO FSS systems employing on-board processing, which might include ISLs in the bands of interest.

To keep the study within practicable bounds it was decided to review all of the available data and then compile sets of parameters for a LEO/GSO ISL, a medium Earth orbit (MEO)/GSO ISL and a GSO/GSO ISL, each of which is hypothetical but has characteristics which are typical of those in the filed systems. Four examples of the GSO/GSO ISL were included, and the parameters produced in this way are shown in Table 1.

For the non-GSO end of the non-GSO/GSO link the nearest satellite handover strategy was modelled. The antenna pattern modelled in each case was as defined by Recommendation ITU-R S.672 (single feed, –25 dB first sidelobe). Each ISL was treated as an individual link (i.e. no use was made of transmission gains coupling an ISL with a preceding or following link). The case of multiple satellites in an interfering system transmitting simultaneously toward the same satellite was not modelled; it is expected that this would lead to only slightly smaller minimum values of C/I than those in the present exercise; the minimum C/I instances would occur more often, but in practical cases would still aggregate to small percentages of time.

Since the e.i.r.p. required for a given ISL depends *inter alia* on its length, it was necessary to calculate the e.i.r.p. separately for each link, and this was done assuming an operating C/N of 15 dB at the input to each receiver, as follows:

$$e.i.r.p. - 20 \log (4 \times \pi \times d/\lambda) + G - 10 \log (k\, T\, B) = 15 \text{ dB}$$

where:

d: path length (m) (maximum in the non-GSO/GSO case)

λ: wavelength (m)

G: on-axis gain of the receiving antenna (dBi)

k: Boltzmann's constant (–228.6 dB(W/(Hz · K)))

T: link noise temperature referred to receiver input (K)

B: reference bandwidth (1 MHz).

TABLE 1

Parameters of typical inter-satellite links in ISS bands below 71 GHz

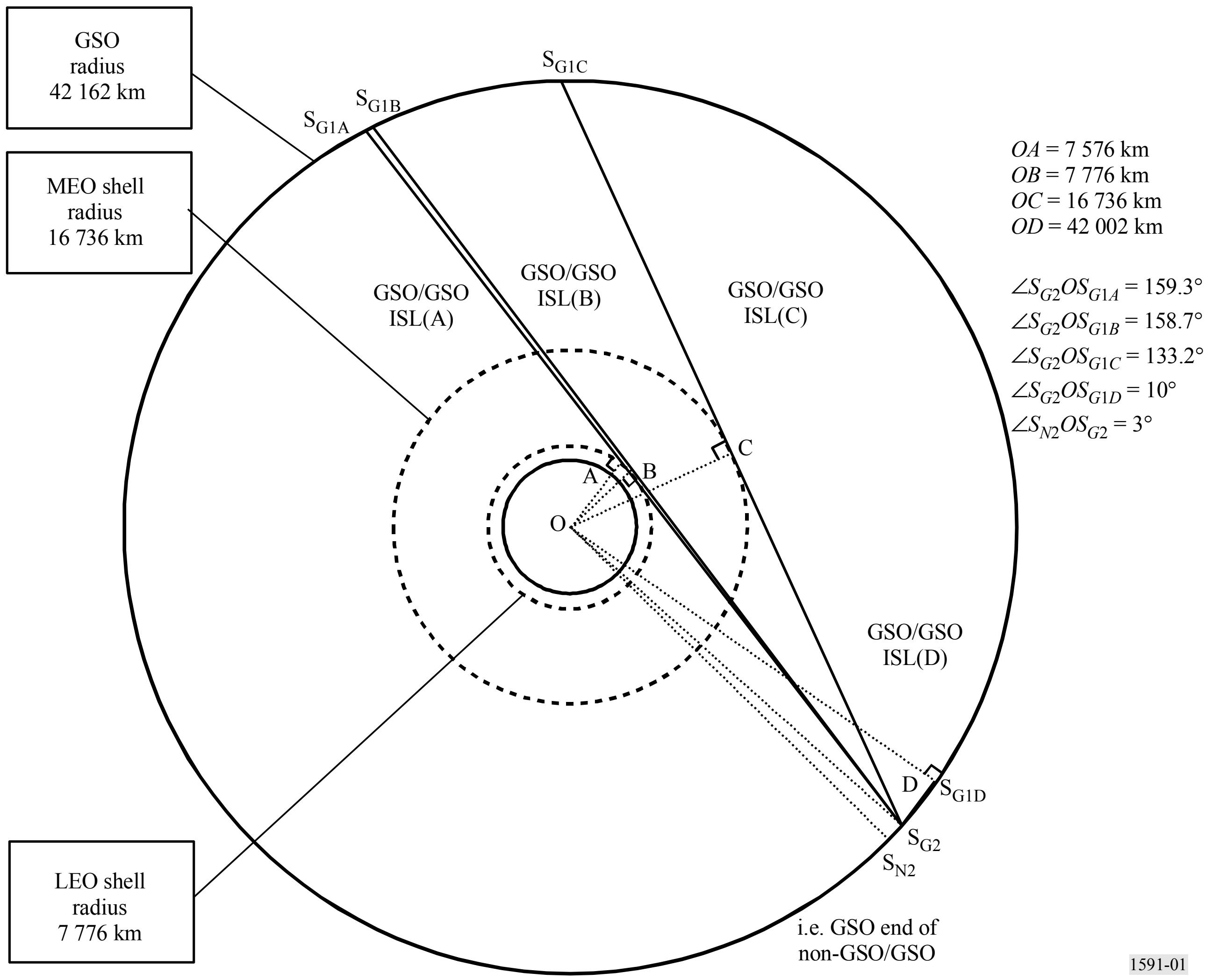
ISL type	LEO/GSO	MEO/GSO	GSO/GSO (A)	GSO/GSO (B)	GSO/GSO (C)	GSO/GSO (D)
Shape of orbits	Circular					
Height of orbits (km)	1 400	10 360	35 786			
Number of orbit planes	7 (+GSO)	4 (+GSO)	1			
Number of satellites per plane	9 (+1 in GSO)	6 (+1 in GSO)	2			
Non-GSO inclination (degrees)	48	82.5	0			
Ascending node displacement (degrees)	25.714	45	0			
True anomaly displacement (degrees)	0		Not available			
Longitude(s) of GSO satellite(s) (°E)	–3		0 and 159.3	0 and 158.74	0 and 133.23	0 and 9.99
Adjacent plane phasing (degrees)	28.57	45	Not available			
Hand-over strategy	Nearest LEO satellite	Nearest MEO satellite	Not available			
Satellite beam pointing method	Tracking		Fixed			
ISL antenna diameter (m)	1					
ISL antenna peak gain (dBi) 23 GHz 32.5 GHz 62.2 GHz	 45.4 48.4 54.0					
ISL antenna pattern	Recommendation ITU-R S.672 $L_N = -25$ dB					
ISL beamwidth (degrees) 23 GHz 32.5 GHz 62.2 GHz	 0.91 0.65 0.34					
ISL length (km)	Variable, maximum 43	Variable, maximum 45	83 128	82 877	77 396	7 339
C/N at receiver input (dB)	15					
e.i.r.p./MHz (dBW)	41.8	42.4	47.6	47.5	46.9	26.5
ISL system noise temperature (K)	700					

3 Simulation model

The geometrical details of the four GSO/GSO ISLs are given in Fig. 1. Although manual calculations of the maximum interference levels in both forward and return directions of transmission were made, in order to check the results of those calculations by simulation it was only necessary to model one direction in the present case. The model was set up to simulate transmissions from satellite S_{G1} to S_{G2} and from S_{N1} to S_{N2}, and hence interference from S_{N1} to S_{G2} and from S_{G1} to S_{N2}. Satellite S_{N1}, not shown explicitly in Fig. 1, is in the non-GSO constellation and changes from time to time as determined by the tracking strategy.

FIGURE 1

Example GSO/GSO ISL



For the main runs the geocentric separation between one end of each GSO/GSO link and the GSO end of each non-GSO/GSO link was 3°, since this is the minimum spacing for most co-frequency, co-coverage GSO/FSS systems. Thus S_{G2} was located at 0° and S_{N2} at 3° W. For GSO/GSO ISL(A) the longitude of the satellite at the other end ($S_{G1(A)}$) was set at 159.3° E, which makes it the longest ISL which could possibly create interference from $S_{G1(A)}$ to S_{N2}; for a longer ISL the interference path would be blocked by the Earth. GSO/GSO ISL(B) was set up to be tangential to the orbit "shell" of the LEO constellation, and GSO/GSO ISL(C) to be tangential to the orbit shell of the MEO constellation. GSO/GSO ISL(D), a short ISL spanning a longitude range of only 10° was included for the sake of completeness.

This model enabled 48 sets of *C*/*I* statistics to be obtained in a single run, i.e.:

a) LEO/GSO ISL interference to GSO/GSO ISL(A) in each of the three ISS bands,

b) LEO/GSO ISL interference to GSO/GSO ISL(B) in each of the three ISS bands,

c) LEO/GSO ISL interference to GSO/GSO ISL(C) in each of the three ISS bands,

d) LEO/GSO ISL interference to GSO/GSO ISL(D) in each of the three ISS bands,

e) MEO/GSO ISL interference to GSO/GSO ISL(A) in each of the three ISS bands,

f) MEO/GSO ISL interference to GSO/GSO ISL(B) in each of the three ISS bands,

g) MEO/GSO ISL interference to GSO/GSO ISL(C) in each of the three ISS bands,

h) MEO/GSO ISL interference to GSO/GSO ISL(D) in each of the three ISS bands,

i) GSO/GSO ISL(A) interference to LEO/GSO ISL in each of the three ISS bands,

j) GSO/GSO ISL(B) interference to LEO/GSO ISL in each of the three ISS bands,

k) GSO/GSO ISL(C) interference to LEO/GSO ISL in each of the three ISS bands,

l) GSO/GSO ISL(D) interference to LEO/GSO ISL in each of the three ISS bands,

m) GSO/GSO ISL(A) interference to MEO/GSO ISL in each of the three ISS bands,

n) GSO/GSO ISL(B) interference to MEO/GSO ISL in each of the three ISS bands,

o) GSO/GSO ISL(C) interference to MEO/GSO ISL in each of the three ISS bands, and

p) GSO/GSO ISL(D) interference to MEO/GSO ISL in each of the three ISS bands.

Each run was continued until about six days of orbit time had been simulated, using time steps of 1 s, thus computing about $6 \times 24 \times 60 \times 60 = 518\,400$ samples of C/I for each of the 48 scenarios. This was sufficient to obtain a smooth cumulative distribution function (CDF) in each case.

4 Results

The model was first run for the reference scenario, in which satellite S_{N2} was located at 3° W – i.e. 3° from satellite S_{G2}, and the results were plotted as cumulative time distributions of C/I. As expected, the interference varied with time in every case which confirms the wisdom of basing frequency sharing on a short-term criterion. For most of the links even the short-term interference was negligible, and in no case did the C/I fall to a value anywhere near a likely interference threshold. The minimum C/I occurred for interference from the 23 GHz MEO/GSO ISL to GSO/GSO ISL(B) and was about 31.5 dB. As examples the results for this case and for the "best" case – GSO/GSO ISL(D) – are shown in Figs. 2 and 3; the other plots (14 Figures each with three graphs) were prepared.

FIGURE 2

MEO/GSO ISL interference to GSO/GSO ISL(B)

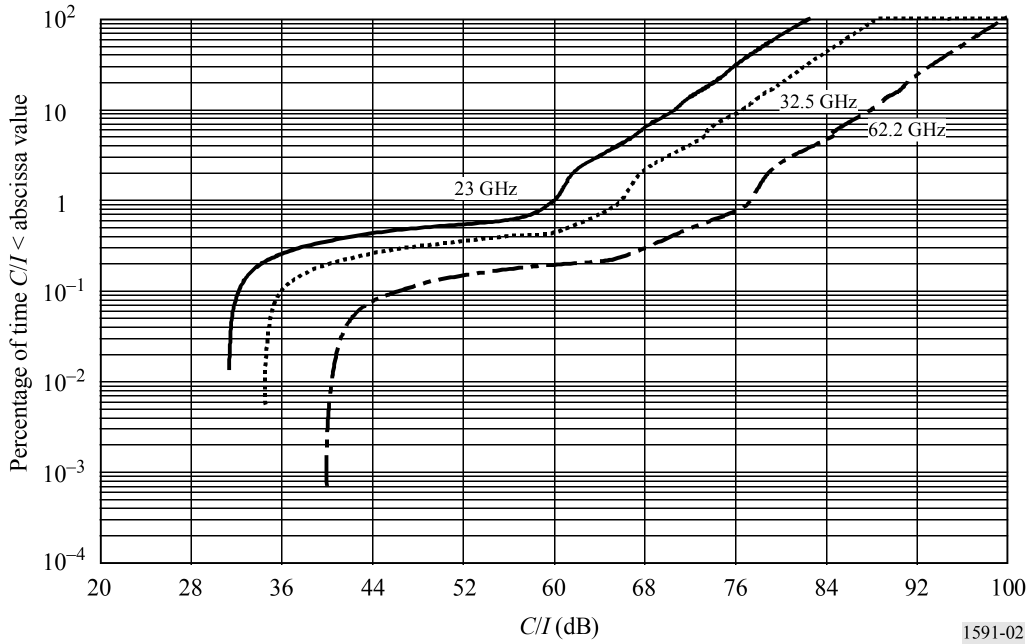
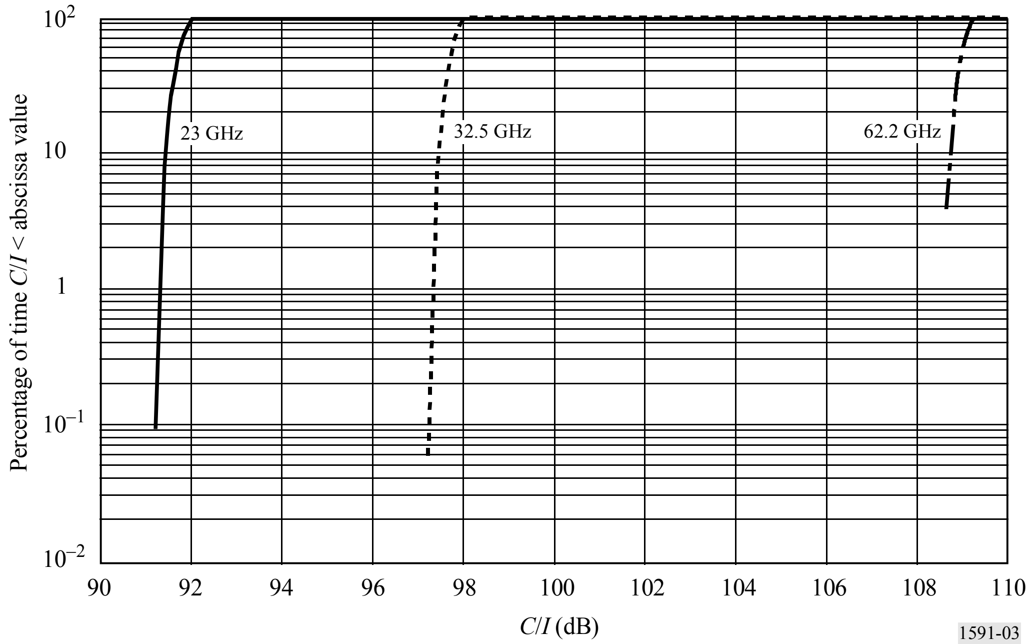


FIGURE 3

GSO/GSO ISL(D) interference to LEO/GSO ISL



NOTE 1 – Figure 2 shows the lowest of the 48 sets of *C*/*I* statistics and Fig. 3 shows the highest.

By carrying out further runs with satellite S_{N2} in various longitudes relative to S_{G2} it was confirmed that, as anticipated, the lowest *C*/*I* ratios occur when S_{N2} is close to S_{G2}. It was therefore decided to run the simulation for successively smaller S_{N2}-S_{G2} separations, in order to determine the circumstances under which frequency sharing might be difficult. The lowest *C*/*I* ratios thus obtained are listed in Table 2. These are all for 23 GHz; the ratios increase with increasing frequency in every case because of increasing antenna gain.

TABLE 2

Lowest *C*/*I* ratios from simulations with S_{N2} close to S_{G2}

Angle between S_{N2} and S_{G2} (degrees)	Interfering ISL	Victim ISL	Lowest *C*/*I* (dB)
3	MEO/GSO	GSO/GSO(B)	31.3
2	GSO/GSO(A)	MEO/GSO	19.3
1.5	GSO/GSO(A)	MEO/GSO	13.0
1.0	GSO/GSO(A)	MEO/GSO	14.4
0.5	MEO/GSO	GSO/GSO(A)	4.6

From Table 2 it is evident that, for co-frequency operation of typical ISLs, satellite S_{N2} could be safely located at longitudes with respect to S_{G2} down to 3° without creating unacceptable interference peaks, and hence coordination would not be necessary if the geocentric angle between S_{N2} and S_{G2} was greater than 3°.

Finally, checks were made to ensure that the minimum values of *C*/*I* occur at some of the instants when the non-GSO satellite in a non GSO/GSO ISL is passing through the Equatorial plane. This was done by carrying out a run to identify when the minimum *C*/*I* on a given link occurred, and then repeating that part of the run whilst plotting *C*/*I* against time. An example of this process is illustrated in Fig. 4, where both the upper and lower diagrams were printed from the simulation for the 23 GHz non-GSO/GSO ISLs interfering with GSO/GSO ISL(A) when S_{N2} was separated from S_{G2} by only 0.5°. The lower diagram is a view from a point in space in the Equatorial plane at 90° E longitude, looking down at the Earth, at the time-step corresponding to *T* in the upper diagram. Satellite S_{N1} in the MEO/GSO ISL can be seen to be passing through the Equatorial plane at this instant. Satellite S_{N1} in the LEO/GSO ISL is south of the Equatorial plane at this instant, but by a similar exercise it was confirmed that the minimum *C*/*I* occurs during certain crossings of the Equatorial plane in that case also.

5 Conclusions

The foregoing results suggest that in the great majority of cases non GSO/GSO ISLs will be able to operate co-frequency with GSO/GSO ISLs without the need for coordination, provided that the geostationary satellite in the non-GSO/GSO ISL is not located within 3° of either satellite in the GSO/GSO ISL. It has also been shown that the minimum *C*/*I* ratios occur at some instants when the non-GSO satellite in the non-GSO/GSO ISL is passing through the Equatorial plane. Given a decision that frequency sharing should be facilitated by a coordination procedure, these two factors lead to a simple method which may be used for determining whether or not coordination is necessary, and which is described in Annex 2. The method requires a short-term *C*/*I* threshold as a basis for the coordination trigger.

In the light of this study the following answers to the *decides* in Question ITU-R 265/4 seem appropriate, insofar as the sharing of non-GSO/GSO ISLs with GSO/GSO ISLs in the ISS bands near 23 GHz, 32.5 GHz and 62.2 GHz is concerned.

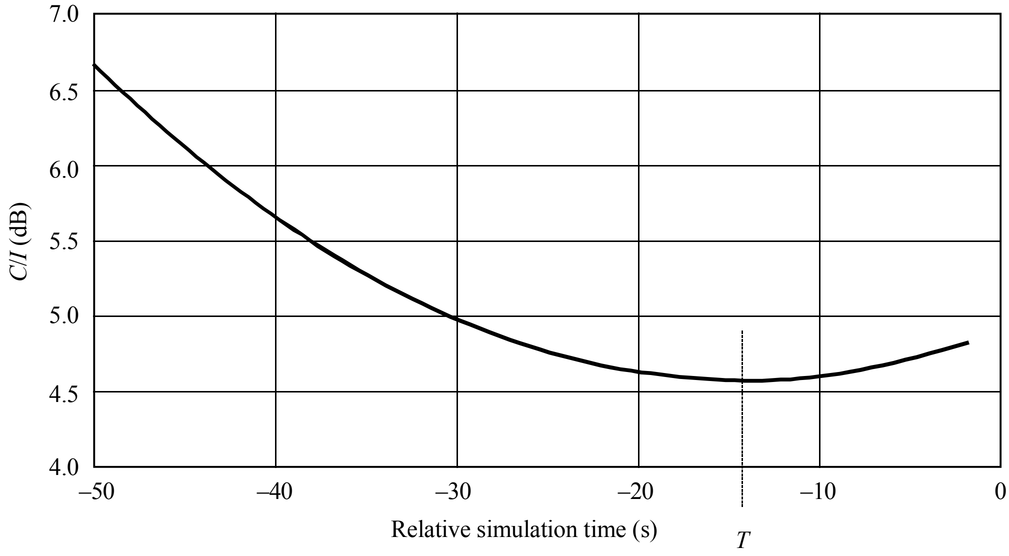
Since the method for determining the need to coordinate described in Annex 2 is based on calculating the minimum *C*/*I* ratio, and this occurs for only a small proportion of the time (e.g. < 0.1% as illustrated in Fig. 2), a threshold 3 dB above the *C*/*N* ratio corresponding to the performance objective would be a sufficiently conservative trigger level. Therefore, for use in the method of Annex 2: coordination should take place if the minimum *C*/*I* ratio in a bandwidth of 1 MHz at any of the four ISL receivers, is less than either the *C*/*N* identified for that receiver in item C.8 *e*) of RR Appendix 4 + 3 dB or, in the absence of a *C*/*N*, 30 dB.

It is to be noted that the first part of this threshold adapts to the need of each ISL receiver, while the second part, applied only when RR Appendix 4 item C.8 *e*) is not available for a particular receiver, gives a threshold that satisfies the requirements that have been stated by administrations.

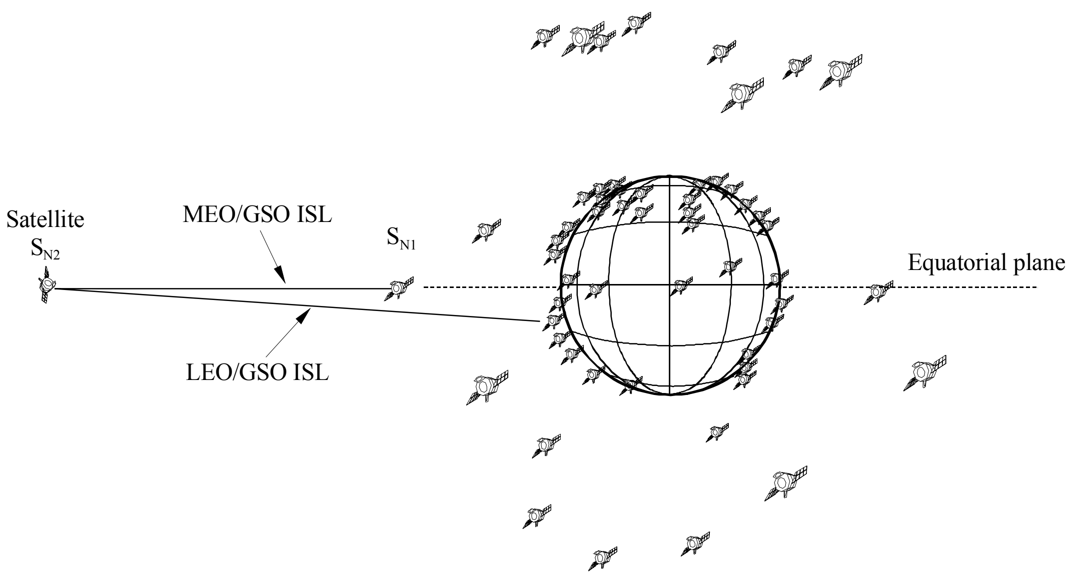
FIGURE 4

Example of passage through Equatorial plane corresponding to minimum *C*/*I*

MEO/GSO ISL interference to GSO/GSO ISL(A) at 23 GHz
satellite S_{N2} 0.5° from satellite S_{G2} in the GSO



Direction of interfering ISL at time *T*



Note – GSO/GSO ISLs not shown.

1591-04

ANNEX 2

Method for determining whether coordination is necessary for a non-GSO/GSO ISL sharing frequency with a GSO/GSO ISL

The method applies to the ISS bands between 22 GHz and 71 GHz, if there is frequency overlap between a carrier in a non-GSO/GSO ISL and a carrier in a GSO/GSO ISL. If there is no frequency overlap coordination is of course unnecessary.

A diagram similar to Fig. 5 should be drawn, but using the actual longitudes proposed for the geostationary satellites at each end of the GSO/GSO ISL and at the GSO end of the non-GSO/GSO ISL. Note that Fig. 5 illustrates an instant when S_{N1} is in line with S_{G1} and S_{N2}.

If angle $\theta \geq \pm 3°$ and the line $S_{G1}S_{N2}$ does not intersect the orbit shell of the non-GSO system concerned, then it should be concluded that coordination is not necessary.

Regardless of the value of angle θ, if the line $S_{G1}S_{N2}$ intersects the orbit shell of the non-GSO system concerned or if angle $\theta < \pm 3°$ and the line $S_{G1}S_{N2}$ does not intersect the orbit shell of the non-GSO system concerned, then path lengths $S_{G1}S_{G2}$, $S_{N1}S_{N2}$, $S_{G1}S_{N2}$ and $S_{N1}S_{G2}$ should be calculated (km), and off-axis angles φ_{G1}, φ_{G2} and φ_{N1} should also be calculated (degrees), in the manner explained in Fig. 5 and the subsequent equations. Note that, if the line $S_{G1}S_{N2}$ intersects the orbit shell of the non-GSO system, then angle $\varphi_{N2} = 0°$ at each in-line instant. The e.i.r.p. per MHz (*E*) of each ISL, in each direction of transmission should then be calculated by subtracting 10 log (carrier bandwidth (MHz)) from the operational carrier e.i.r.p.. Finally, the *C*/*I* should be calculated as follows:

$$(C/I)_{G1} = E_{G2} - E_{N2} + D_T(\varphi_{N2}) + D_R(\varphi_{G1}) + 20 \times \log[(S_{N2}S_{G1})/(S_{G2}S_{G1})] \quad \text{dB}$$

$$(C/I)_{G2} = E_{G1} - E_{N1} + D_T(\varphi_{N1}) + D_R(\varphi_{G2}) + 20 \times \log[(S_{N1}S_{G2})/(S_{G1}S_{G2})] \quad \text{dB}$$

$$(C/I)_{N1} = E_{N2} - E_{G2} + D_T(\varphi_{G2}) + D_R(\varphi_{N1}) + 20 \times \log[(S_{G2}S_{N1})/(S_{N2}S_{N1})] \quad \text{dB}$$

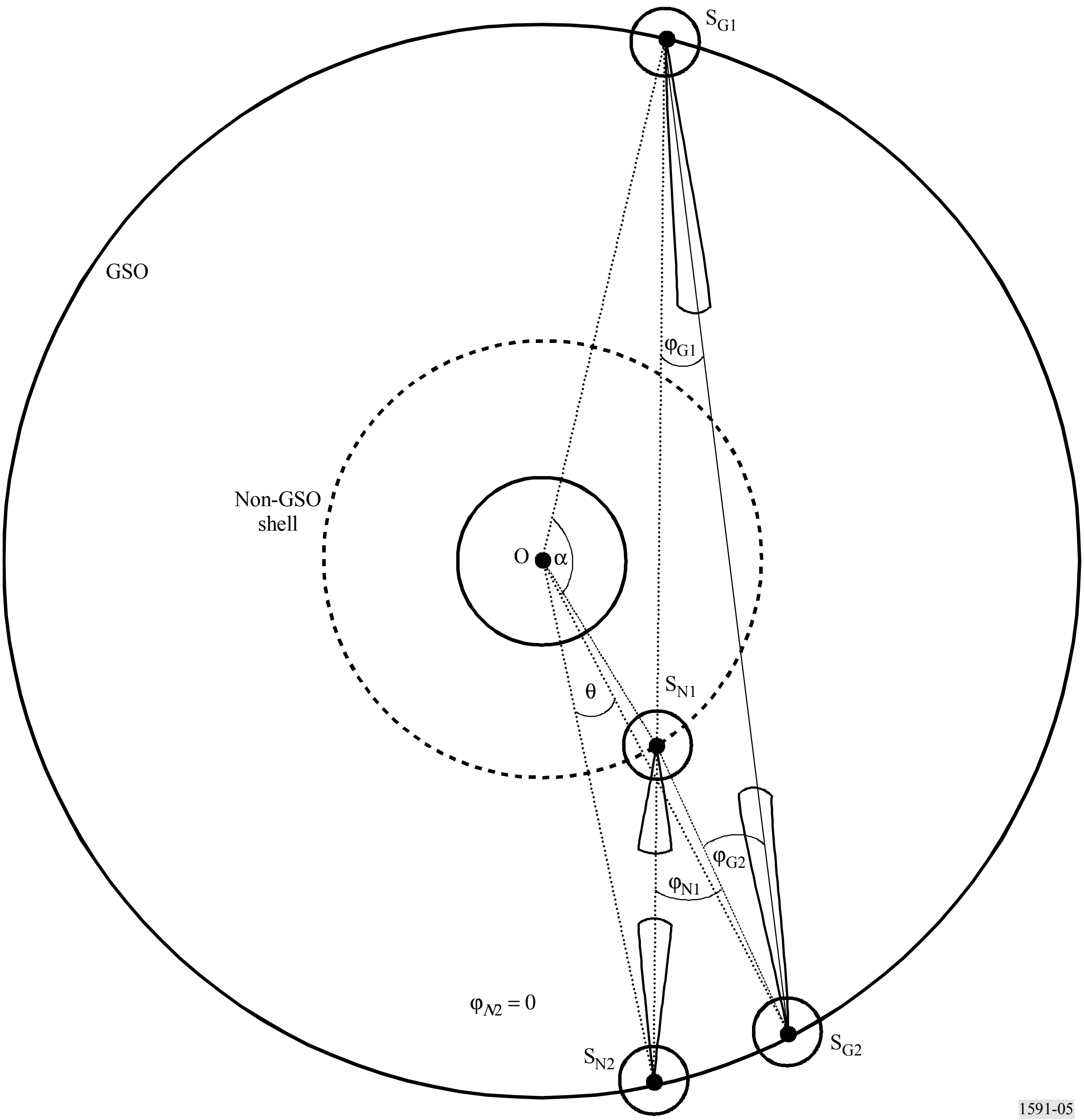
$$(C/I)_{N2} = E_{N1} - E_{G1} + D_T(\varphi_{G1}) + D_R(\varphi_{N2}) + 20 \times \log[(S_{G1}S_{N2})/(S_{N1}S_{N2})] \quad \text{dB}$$

where D_T is the discrimination (dB) given by the on-axis gain minus the gain at the off-axis angle concerned of the antenna transmitting the interference, and similarly D_R (dB) is the discrimination of the antenna receiving the interference. Note that, if the line $S_{G1}S_{N2}$ intersects the orbit shell of the non-GSO system, then $D_T(\varphi_{N2}) = D_R(\varphi_{N2}) = 0$ dB at each in-line instant. (It is assumed that the ISL antenna patterns would be included in the information supplied in RR Appendix 4. Otherwise the single-feed pattern in Recommendation ITU-R S.672 could be used.)

If these four *C*/*I* ratios are all greater than or equal to the threshold, it should be concluded that coordination is not necessary; if one or more is less than the threshold it should be concluded that coordination is required. The threshold for each of the four ISL receivers is the *C*/*N* identified for that receiver in item C.8 *e*) of RR Appendix 4 + 3 dB or, in the absence of a *C*/*N*, 30 dB.

FIGURE 5

Geometry for coordination trigger



The bold variables below are those needed for the *C*/*I* equations in the text above Fig. 5.

$OS_{G1} = OS_{G2} = OS_{N2} = 42\,162$ km. $OS_{N1} = (6\,376 + h)$ km, where h is height of non-GSO (km).

$\therefore\ \boldsymbol{S_{G1}S_{G2}} = [2 \times (42\,162)^2 \times \{1 - \cos(\alpha)\}]^{0.5}$ and $\angle OS_{G1}S_{G2} = \sin^{-1}[(42\,162) \times \sin(\alpha)/(S_{G1}S_{G2})]$.

In the equations below, except of the last one the plus (+) sign is to be used if S_{N2} is outside the arc between S_{G1} and S_{G2} (i.e. the arc associated with angle θ) and the minus (–) sign is to be used if S_{N2} is inside that arc.

$\boldsymbol{S_{G1}S_{N2}} = [2 \times (42\,162)^2 \times \{1 - \cos(\alpha \pm \theta)\}]^{0.5}$ and $\angle OS_{G1}S_{N2} = \sin^{-1}[(42\,162) \times \sin(\alpha \pm \theta)/(S_{G1}S_{N2})]$.

Then $\boldsymbol{\varphi_{G1}} = \pm\,(\angle OS_{G1}S_{G2} - \angle OS_{G1}S_{N2}) = \theta/2$

If the line $S_{G1}S_{N2}$ intersects the orbit shell of the non-GSO system concerned, then continue with the equations labelled "equations for intersection". If angle $\theta < 3°$ and the line $S_{G1}S_{N2}$ does not intersect the orbit shell of the non-GSO system concerned, then continue with the equations labelled "equations for no intersection".

Equations for intersection

$\angle OS_{N1}S_{G1} = \sin^{-1}[(42\,162) \times \sin(\angle OS_{G1}S_{N2})/(6\,376 + h)]$, and

$\angle S_{G1}OS_{N1} = 180° - \angle OS_{G1}S_{N2} - \angle OS_{N1}S_{G1}$

$\therefore\ S_{G1}S_{N1} = [(42\,162)^2 + (6\,376 + h)^2 - 2 \times (42\,162) \times (6\,376 + h) \times \cos(\angle S_{G1}OS_{N1}))]^{0.5}$,
and $\boldsymbol{S_{N1}S_{N2}} = S_{G1}S_{N2} - S_{G1}S_{N1}$

Also $\boldsymbol{S_{N1}S_{G2}} = [(S_{G1}S_{N1})^2 + (S_{G1}S_{G2})^2 - 2 \times (S_{G1}S_{N1}) \times (S_{G1}S_{G2}) \times \cos(\varphi_{G1})]^{0.5}$

$\therefore\ \boldsymbol{\varphi_{G2}} = \sin^{-1}\ [(S_{G1}S_{N1}) \times \sin(\varphi_{G1})/(S_{N1}S_{G2})]$, and $\boldsymbol{\varphi_{N1}} = \varphi_{G1} + \varphi_{G2}$

$\varphi_{N2} = \angle S_{G1}S_{N2}S_{N1} = 0°$

Equations for no intersection

For this case the position of S_{N1} is indeterminate since it is not on the line between S_{G1} and S_{N2}. To fix a location for S_{N1}, place it as close as possible to the line $S_{G1}S_{G2}$, so that OS_{N1} is perpendicular to $S_{G1}S_{G2}$.

$\therefore\ \angle S_{G1}OS_{N1} = \angle S_{G2}OS_{N1} = \alpha/2$

$\therefore\ S_{G1}S_{N1} = [(42\,162)^2 + (6\,376 + h)^2 - 2 \times (42\,162) \times (6\,376 + h) \times \cos(\angle S_{G1}OS_{N1})]^{0.5}$

$\angle S_{N2}OS_{N1} = \alpha/2 \pm \theta$

$$\begin{aligned}\therefore\ \boldsymbol{S_{N1}S_{N2}} &= [(42\,162)^2 + (6\,376 + h)^2 - 2 \times (42\,162) \times (6\,376 + h) \times \cos(\angle S_{N2}OS_{N1})]^{0.5}\\ &= [(42\,162)^2 + (6\,376 + h)^2 - 2 \times (42\,162) \times (6\,376 + h) \times \cos(\alpha/2 \pm \theta)]^{0.5}\end{aligned}$$

$\boldsymbol{S_{N1}S_{G2}} = S_{N1}S_{G1} = \{[(42\,162 \times \cos(\alpha/2) - (6\,376 + h))]^2 + [S_{G1}S_{G2}/2]^2\}^{0.5}$

$\boldsymbol{\varphi_{G2}} = \cos^{-1}[(S_{G1}S_{G2}/(2 \times S_{N1}S_{G2})]$

$\boldsymbol{\varphi_{N1}} = 180 - (90 - \varphi_{G2}) - \angle OS_{N1}S_{N2} = 90 + \varphi_{G2} - \sin^{-1}\{[42\,162 \times \sin(\alpha/2 \pm \theta)]/S_{N1}S_{N2}\}$

In the equation below, the plus (+) sign is to be used if S_{N2} is inside the arc between S_{G1} and S_{G2} (i.e. the arc associated with angle θ) and the minus (–) sign is to be used if S_{N2} is outside that arc.

$\varphi_{N2} = \angle S_{G1}S_{N2}S_{N1} = \sin^{-1}\ \{[S_{G1}S_{N1} \times \sin(\varphi_{G1} \pm \varphi_{G2})]\ /S_{N1}S_{N2}\}$