RECOMMENDATION ITU-R S.1257-1

ANALYTICAL METHOD TO CALCULATE SHORT-TERM VISIBILITY AND INTERFERENCE STATISTICS FOR NON-GEOSTATIONARY-SATELLITE ORBIT SATELLITES AS SEEN FROM A POINT ON THE EARTH'S SURFACE

(Questions ITU-R 206/4 and ITU-R 231/4)

(1997-2000)

The ITU Radiocommunication Assembly,

considering

a) that there may be a need to calculate the probabilities of exceeding a given interference level in interference or sharing studies between non-GSO satellite orbit stations and other stations;

b) that there are computer simulation programmes to derive the required statistics but such programmes may not be commonly available;

c) that computer simulations can be used to provide a large amount of statistics with good accuracy, but they may need expertise in their use and may require considerable time to run;

d) that in some cases analytical methods may be time saving and need less sophisticated means for calculation of limited amount of statistics,

recommends

1 that the analytical methodology given in Annex 1 can be used to obtain short-term visibility statistics for interference and sharing studies between non-GSO networks and other GSO FSS networks (The term "probability" is not used in the strict mathematical sense. It signifies "percentage of time".);

2 that the methodology given in Annex 2 can then be used to obtain short-term interference statistics for interference and sharing studies between non-GSO networks and other GSO FSS networks;

3 that the following Notes should be regarded as part of this Recommendation:

NOTE 1 – The methodology can be used for interference calculations between non-GSO feeder links and GSO stations. They can also be applied to any case where one antenna is pointing to a fixed direction and the other is either on board a non-GSO satellite, or is tracking the non-GSO satellite.

NOTE 2 – The methodology gives an average value for a non-synchronous satellite constellation. It is applicable also for a synchronous orbit constellation (satellite tracks are repeating after a certain period of time) if there are several satellite tracks through the area of interest.

NOTE 3 – The limitations of the methodology described in Annex 1 have to be taken into account.

ANNEX 1

Appendix 1 gives a method to define an area for which the probability that a satellite falls within the area to be calculated.

Appendix 2 gives the method to calculate the probability.

Appendix 3 gives the derivation of the formulae used in Appendix 2.

APPENDIX 1

TO ANNEX 1

Calculation of the needed discrimination angle for a given interference level

The case considered here is non-GSO/GSO sharing.

The area is defined for interference:

a) from non-GSO earth station to GSO satellite;

b) from non-GSO satellite to GSO earth station;

c) from GSO earth station to non-GSO satellite;

d) from GSO satellite to non-GSO earth station.

In the following equations:

ε : elevation angle of a satellite from a station

r : earth radius = 6 378 km

H : altitude of the GEO satellite (km)

h : altitude of the non-GSO satellite (km)

d_I : distance from the interference source to the interfered receiver (km)

d_N : distance to the non-GSO satellite from a station (km)

d_G : distance to the GSO satellite from a station (km)

f : frequency (GHz)

E : e.i.r.p. spectral density (dB(W/Hz))

N_0 : noise spectral density (dB(W/Hz))

$G(\varphi)$: antenna gain to the direction φ (degrees) from the main beam axis (dB)

G : antenna on-axis gain (dB)

Suffix G refers to the GSO network

Suffix N refers to the non-GSO network

Suffix E refers to an earth station

Suffix S refers to a satellite

Suffix T refers to transmitting parameter

Suffix R refers to receiving parameter

PR : protection ratio

Distances to a non-GSO and GSO satellite from a station for the in-line case are:

$$d_N \; = \; \sqrt{r^2 \, \sin^2 \varepsilon \; + \; 2r \, h \; + \; h^2} \; - \; r \sin \varepsilon \tag{1}$$

$$d_G \; = \; \sqrt{r^2 \, \sin^2 \varepsilon \; + \; 2r \, H \; + \; H^2} \; - \; r \sin \varepsilon \tag{2}$$

C_0/I_0 method

$$\left(C_0/I_0\right)_{in-line} = E_{GE} - E_{NE} \qquad \text{for case a)} \qquad (3)$$

$$\left(C_0/I_0\right)_{in-line} = E_{GS} - E_{NS} - 20\log d_G + 20\log d_N \qquad \text{for case b)} \qquad (4)$$

$$\left(C_0/I_0\right)_{in-line} = E_{NE} - E_{GE} \qquad \text{for case c)} \qquad (5)$$

$$\left(C_0/I_0\right)_{in-line} = E_{NS} - E_{GS} - 20\log d_N + 20\log d_G \qquad \text{for case d)} \qquad (6)$$

The GSO and non-GSO earth stations should be either co-located or, in cases a) and d), the GSO satellite antenna discrimination has to be taken into account, and in cases b) and c) the non-GSO satellite antenna discrimination has to be taken into account.

The required antenna discrimination for a non-GSO earth station, in cases a) and d), and for a GSO earth station in cases b) and c), is:

$$G - G(\varphi) = \left(C_0/I_0\right)_{in-line} - PR \qquad (7)$$

To calculate the required avoidance angle φ, the appropriate antenna gain patterns has to be used. For GSO earth stations the side lobe pattern used in equation (8) is given in Recommendation ITU-R S.580. Avoidance angle at side lobe area is:

$$\varphi = 10^{(29 - G(\varphi))/25} \qquad (8)$$

For the main beam area, the discrimination angle may be calculated by:

$$\varphi = \varphi_0 \sqrt{\frac{G - G(\varphi)}{12}} \qquad (9)$$

In case b) the distance from the non-GSO satellite to the GSO earth station changes as the satellite moves on its orbit. For small values of φ and for high elevation angles ε it need not be taken into account but in other cases the distance d_N has to be calculated separately for the highest elevation $(\varepsilon + \varphi)$ and for the lowest elevation $(\varepsilon - \varphi)$ and the new values have to be used in equation (2).

For calculations in Appendices 2 and 3 the angle φ is the radius of the area for which the probability is to be calculated. For the calculation of statistics $\varepsilon_1 = \varepsilon - \varphi/2$ and $\varepsilon_2 = \varepsilon + \varphi/2$.

I_0/N_0 method

To calculate the required avoidance angle φ for the case based on I_0/N_0 requirement, the following equations are used:

$$G - G(\varphi) = \left(I_0/N_0\right)_{in-line} - \left(I_0/N_0\right)_{required} \qquad (10)$$

I_0/N_0 for the in-line case can be calculated by:

$$\left(I_0/N_0\right)_{in-line} = E - N_0 - 20 \log d_I - 20 \log f - 92.5 \tag{11}$$

where:

E : e.i.r.p. of the interfering transmitter

N_0 : noise spectral density of the wanted receiving system.

APPENDIX 2

TO ANNEX 1

Calculation of in-area statistics for non-GSO satellites

1   Introduction

The method given in this Appendix can be used in calculating the probability to find a satellite of a constellation in a circular or rectangular area (azimuth/elevation or latitude/longitude). A circular area may be for example a radio-relay link or satellite earth station antenna main beam or side lobe area. If the area is based on an offset angle where certain interference level is achieved, the result is the probability that a given interference level is exceeded. The method can be used to calculate the interference between non-GSO and GSO networks. The method can also be used in calculating the probability of interference between a non-GSO earth station and a receiving fixed satellite station or FSS earth station. The calculation uses the discrimination angle to define an area where the permissible interference level is exceeded.

The method can be used for any observation point (e.g. earth station) latitude and for any satellite altitude, inclination, azimuth and elevation, but only in the case that the satellite may be visible in the defined area. (See § 5.)

2   Symbols used (in Appendix 2 and Appendix 3)

T_s :   Satellite orbital period (min)

T_e :   Earth rotation time (min)

L_0 :   Latitude of the observation point (rad)

L :   Latitude of the area (rad)

l_m :   Medium length of the tracks through a circle

b :   Length of the area in longitudinal direction (rad)

i :   Inclination of satellite orbit (rad)

r_c :   Radius of the circular area (rad)

α :   Angle between ground track and latitude line (rad)

A :   Area on a spherical surface (sterad)

A_c :   Area of a circle on a spherical surface (sterad)

P_h :   Probability to hit the area (one satellite calculation)

P_i :   Probability to be inside the area if hit during the revolution (one satellite calculation)

P: Probability for a satellite to be inside the defined area (one satellite calculation)

P_c: Probability that any one of the satellites in a constellation is inside the area

N: Number of satellites in the constellation

$k =$ $r / (r + h)$

Λ: Azimuth of the centre of the area (rad)

ε: Elevation of the centre of the area (rad)

r: Radius of the Earth

h: Altitude of the satellite

γ: Nadir angle from subsatellite point (see Fig. 3) (rad)

θ_ε: Geocentric angle in elevation direction corresponding to ε (rad)

$\theta_{\varepsilon 2}$:   Geocentric angle for the highest point of the area in elevation direction (rad)

$\theta_{\varepsilon 1}$:   Geocentric angle for the lowest point of the area in elevation direction (rad)

$\Delta\theta_\beta$:   Geocentric angle difference in direction perpendicular to θ_ε (rad)

β: Width of the area in azimuthal direction (e.g. antenna beamwidth in azimuthal direction)

$\varepsilon_1, -\varepsilon_2$:   The highest and lowest elevation of the area (rad) ($\varepsilon_1, -\varepsilon_2$ is e.g. the antenna beamwidth in the elevation direction)

3   Calculation of probability

The following formulae are a collection of those in Appendix 3 and given here are only those which are needed for the probability calculation. The numbering of formulae is the same as in Appendix 3, explanation is given for the calculated parameters.

$$k \;=\; \frac{r}{r \;+\; h} \tag{21}$$

$$\theta_{\varepsilon_1} \;=\; \pi \;-\; \gamma \;-\; (\pi/2 \;+\; \varepsilon_1) \;=\; \arccos{(k \cos \varepsilon_1)} \;-\; \varepsilon_1 \tag{22}$$

$$\theta_{\varepsilon_2} \;=\; \pi \;-\; \gamma \;-\; (\pi/2 \;+\; \varepsilon_2) \;=\; \arccos{(k \cos \varepsilon_2)} \;-\; \varepsilon_2 \tag{22}$$

$$\Delta\theta_\varepsilon \;=\; \theta_{\varepsilon 2} \;-\; \theta_{\varepsilon 1} \tag{23}$$

$$\theta_\varepsilon \;=\; \frac{\theta_{\varepsilon 1} \;+\; \theta_{\varepsilon 2}}{2} \tag{24}$$

$$\Delta\theta_\beta \;=\; 2 \arctan \frac{\tan{(\beta / 2)} \; \sin \theta_\varepsilon}{\cos \varepsilon} \tag{25}$$

$$L = \arcsin{(\cos \theta_\varepsilon \cdot \sin L_0 \;+\; \sin \theta_\varepsilon \cdot \cos L_0 \cdot \cos \Lambda)} \tag{27}$$

$$\alpha \;=\; \arccos \frac{\cos i}{\cos L} \tag{15}$$

$$P \;=\; \frac{A}{2\pi^2} \; \frac{1}{\sin \alpha} \; \frac{1}{\cos L} \tag{19a}$$

$$P_c \;=\; N P$$

For a circular area:

$$A = A_c$$

$$A_c = \frac{\pi}{4} \Delta\theta_\varepsilon \ \Delta\theta_\beta$$

For a rectangular area:

$$A = \beta (\varepsilon_2 - \varepsilon_1) \tag{26}$$

APPENDIX 3*

TO ANNEX 1

Derivation of the formulae given in Appendix 2

1 Probability that a satellite is in a given area

Figure 1 shows a non-GSO satellite orbit around the Earth. A simple case is the orbit over the poles. If the satellite is projected on the Earth surface, ground tracks are created. If the Earth would be stationary, there would be only one track over the poles. However, as the Earth rotates during the time it takes the satellite to make one revolution, the next ground track will be shifted by a longitude difference equal to:

$$T_s / T_e \ 2\pi \tag{12}$$

Figure 1 shows several tracks over a longer period of time. The rectangular area is an area for which the probability is to be calculated. The probability that the satellite hits the area is the portion of length $2 b$ from the whole length of the shadowed band around the Earth (see Fig. 1). The multiple 2 takes into account the fact that the satellite crosses the band twice during one revolution around the Earth. The length of the band is $2\pi \cos L$ and the probability to hit a rectangular area as in Fig. 2 is:

$$P_h = \frac{b}{\pi} \ \frac{1}{\cos L} \tag{13}$$

In this equation, the value of b is in radians and it is the actual length expressed as a geocentric angle. If the longitude difference is used, then:

$$b = (\text{Longitude difference}) \times \cos L \tag{13a}$$

It should be noted that the area used in calculations is the area at the satellite orbit shell. For a circular antenna beam that area is an ellipsoid whose major axis is in elevation direction. The latitude L used in calculations is not the station latitude but the latitude of a point from the orbit shell projected to the Earth surface. In this case, the projected point is the centre of the area.

For a common case, when the satellite orbit inclination is different than 90°, the probability of a satellite to be inside the defined area is the probability to hit the area multiplied by the medium length of the tracks inside the area and divided by the length of one revolution. The probability to hit the area is dependent on the length of its projection on a latitude line crossing the middle of the area. The projection is made parallel to the ground tracks. Calculations are here presented for a circular area case, because it is more illustrative.

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* The symbols used in this Appendix are given in § 2 of Appendix 2.

According to Fig. 2, the length of b is:

$$b = \frac{2r_c}{\sin \alpha} \tag{14}$$

$$\alpha = \text{arc cos} \frac{\cos i}{\cos L} \tag{15}$$

Angle α is the angle between the satellite track and latitude line.

The rotation speed of the Earth does not need to be taken into account in the probability calculation, but it should be taken into account if the real angle, in relation to the rotating Earth, is needed.

The average length of a large number of equally spaced paths traversing a circle is:

$$l_m = \frac{\pi \, r_c}{2} \tag{16}$$

If the satellite hits the area during one revolution, the probability that it is inside the area is the length of the track inside the area divided by the length of one revolution.

$$P_i = \frac{l_m}{2\pi} = \frac{r_c}{4} \tag{17}$$

The total probability is then:

$$P = P_h \, P_i = \frac{r_c}{\pi} \, \frac{1}{\sin \alpha} \, \frac{1}{\cos L} \, \frac{r_c}{2} \tag{18}$$

From that follows:

$$P = \frac{A_c}{2\pi^2} \, \frac{1}{\sin \alpha} \, \frac{1}{\cos L} \tag{19a}$$

$$P = \frac{A_c}{2\pi^2} \, \frac{1}{\sqrt{\sin^2 i - \sin^2 L}} \tag{19b}$$

Part of the area may be under the horizon as in the case of the radio-relay link antenna. In that case, only the area above the horizon is used.

The last part of equation (19b) may be taken to be a conversion factor:

$$c = \frac{1}{\sqrt{\sin^2 i - \sin^2 L}} \tag{19c}$$

Values of this factor are given in Fig. 5.

Other calculation or simulation results may be converted to other latitudes and inclinations for the same satellite altitude by using conversion factor:

$$c_{21} = \frac{\sqrt{\sin^2 i_1 - \sin^2 L_1}}{\sqrt{\sin^2 i_2 - \sin^2 L_2}} = \frac{c_2}{c_1} \tag{19d}$$

The rotation speed of the Earth does not need to be taken into account for the probability calculation but if the time the satellite is inside an area is to be calculated, the vector sum of the satellite speed and the local Earth speed should be used.

It can be shown that the result is independent of the shape of the area A, which in the case of a circle is $\pi \, r^2$. For a rectangular area (azimuth, elevation) the area is:

$$A = \beta \, (\varepsilon_2 - \varepsilon_1)$$

2   Calculation of the area

In the following, the area which has been defined by elevation and azimuth values is projected onto the spherical orbit shell of the satellite. From the triangle in Fig. 3:

$$\frac{\sin\left(\varepsilon + \pi/2\right)}{r + h} = \frac{\sin\gamma}{r} \tag{20}$$

$$k = \frac{r}{r + h} \tag{21}$$

$$\theta_\varepsilon = \pi - \gamma\left(\pi/2 + \varepsilon\right) = \arccos\left(k\cos\varepsilon\right) - \varepsilon \tag{22}$$

The angle is calculated separately for the highest point of the area, $\theta_{\varepsilon2}$, and for the lowest point of the area, $\theta_{\varepsilon1}$. The geocentric angle between these two points, corresponding to the "height" of the area, is thus:

$$\Delta\theta_\varepsilon = \theta_{\varepsilon2} - \theta_{\varepsilon1} \tag{23}$$

In Fig. 4 the distance from the observation point, O, to the orbit shell changes only slowly with the azimuth angle and may be taken to be constant. The geocentric angle corresponding to the "width" of the area can then be calculated by:

$$\theta_\varepsilon = \frac{\theta_{\varepsilon1} + \theta_{\varepsilon2}}{2} \tag{24}$$

$$\Delta\theta_\beta = 2\arctan\frac{\tan\left(\beta/2\right)\sin\theta_\varepsilon}{\cos\varepsilon} \tag{25}$$

The values $\Delta\theta_\varepsilon$ and $\Delta\theta_\beta$ are used to calculate the area A. For an antenna beam, which is circular, the area on the orbit shell is an ellipsoid and:

$$A_c = \frac{\pi}{4}\Delta\theta_\varepsilon\ \Delta\theta_\beta \tag{26}$$

The latitude of the centre of the area is given by:

$$L = \arcsin\left(\cos\theta_\varepsilon\ \sin L_0 + \sin\theta_\varepsilon\ \cos L_0\ \cos\Lambda\right) \tag{27}$$

It should be checked that no parts of the area have higher latitudes than the inclination of the orbit, since no satellite would pass through those parts.

3   Simulation programme

A simulation programme was used to verify the results. The programme can make a simulation of one satellite on a true circular or elliptical orbit, taking into account also the ascending node drift. The programme uses small time steps to calculate the satellite's longitude on its orbit. For every point, the subsatellite point longitude and latitude are calculated and they are converted to azimuth and elevation angles. Step size is given as a geocentric angle. For a circular orbit any step size can be given, but for an elliptical orbit the minimum value is 0.1°. The step used in verification of circular orbits was 0.01°, which corresponds 0.2 s to some low orbits. In case of smaller time percentages the step size was reduced to 0.02 s.

The size and location of an area, which can be circular or rectangular, can be defined in an elevation-azimuth scale. The simulation programme creates a log of entering and leaving the area and calculates the total time inside the area. Proper step size and number of tracks inside the area can be checked from the graphical display.

FIGURE 1

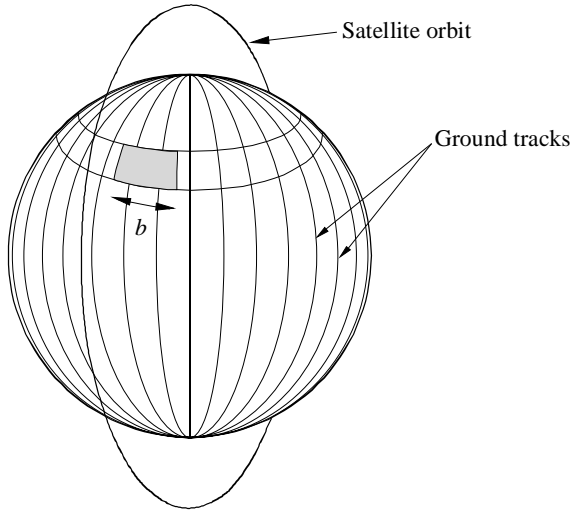**Tracks of polar orbit satellite
(Rectangular area for calculation)**

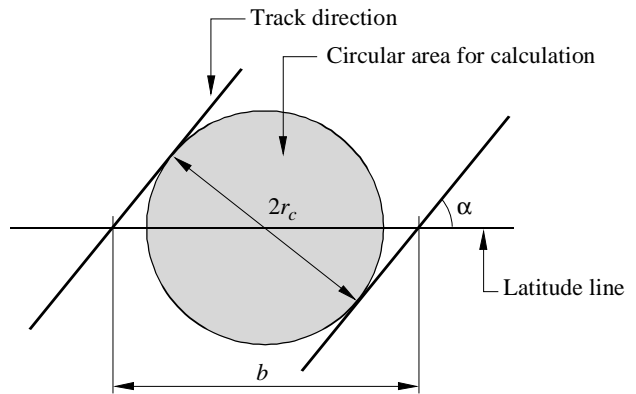FIGURE 2

**Length *b* of a circular
area projection on latitude line**
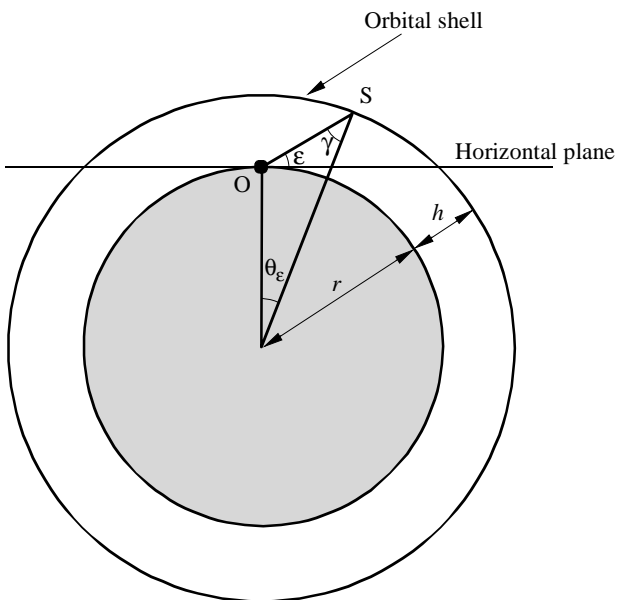
FIGURE 3

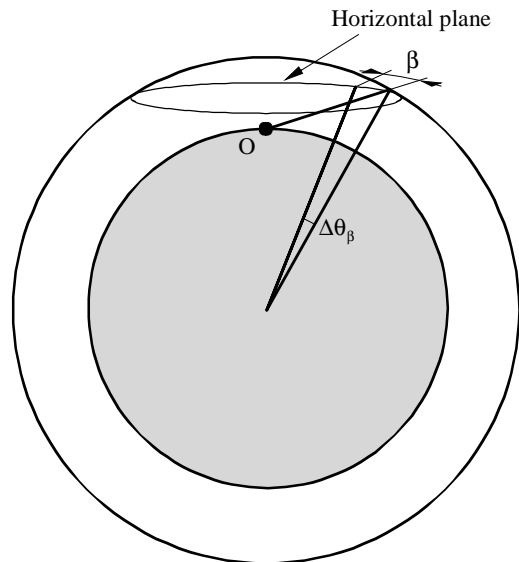**Angles in elevation direction**

FIGURE 4

**Angles for azimuthal movement
of satellite**

O: observation point
S: satellite

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4   Verification of the formulae

For verification purposes, in the simulation the Earth rotation was stopped in the programme and the drift of the ascending node was set to an artificial value of 0.06°/rotation or smaller, so that there was at least 20 tracks through the area. The step size was 0.01°. For Table 1 these values correspond to a simulation of about 450 days and 216 000 000 satellite positions.

Table 1 makes comparison of calculated and simulated values for two FSS earth station latitudes and several antenna elevations and azimuths, which are all towards the GSO. The earth station beamwidth is 2°. The satellite constellation has 48 satellites, orbit altitude is 1 406.8 km and inclination 52°.

TABLE 1

FSS earth station latitude (degrees)	Earth station antenna elevation angle (degrees)	Earth station antenna azimuth (degrees)	Probability for the whole constellation (%)	
			Simulation	Calculation
50	2.0	103.0	0.219	0.219
50	19.9	129.4	0.087	0.089
50	32.6	176.7	0.049	0.049
50	21.4	227.6	0.082	0.082
50	2.0	257.0	0.219	0.219
10	3.6	92.2	0.143	0.143
10	24.5	96.4	0.0479	0.0480
10	67.0	119.6	0.0115	0.0118
10	63.9	244.5	0.0123	0.0125
10	26.6	263.1	0.0433	0.0434
10	2.0	268.1	0.155	0.155

In Table 2, values are presented for cases where the calculation method would be more inaccurate because of the low altitude of the satellite and because of the latitude of the station and more difficult azimuth angles. Satellite orbit is circular, altitude 780 km, inclination 86° and number of satellites 66. The area is circular and its diameter is 2°.

TABLE 2

Station latitude (degrees)	Antenna elevation angle (degrees)	Antenna azimuth (degrees)	Probability for the whole constellation (%)	
			Simulation	Calculation
50	1	65.5	0.277	0.277
60	1	45	0.381	0.385
60	1	10	1.683	1.674
60	41	10	0.0267	0.0267

In Table 3 the same satellite as in Table 2 is used to show the calculation accuracy as the diameter of the circular area is increased. The station latitude and its elevation and azimuth angles are selected to that the projected latitude of the centre of the area at the orbit shell is about 60°, and the satellite motion is approximately parallel with the major axis of the ellipse formed by the intersection of the circular area with the orbit shell.

TABLE 3

Earth station latitude (degrees)	Antenna elevation angle (degrees)	Antenna azimuth (degrees)	Diameter of area (degrees)	Probability for the whole constellation (%)	
				Simulation	Calculation
40	5	10	2	0.217	0.219
40	5	10	10	5.660	5.658
40	10	10	20	15.236	15.555

The following calculations and simulations were made to test how the time percentage changes if the earth station is at 65° latitude and the azimuth is near the highest latitude of the satellite orbit. Satellite was at 1 406.85 km altitude, inclination 52°. The area used was circular, diameter 2° and elevation angle 1°.

TABLE 4

Station latitude (degrees)	Azimuth (degrees)	Area latitude (degrees)	Probability for the whole constellation (1 000%)	
			Simulation	Calculation
0	90	0	3.36	3.36
65	180	31	4.43	4.43
65	83	51.27	29.58	23.7
65	86	50.15	15.18	14.90
65	90	48.7	11.2	11.1

It can be concluded that the formulae give a good accuracy if the latitudes of the area are at least a few degrees less than the orbit inclination. In practice it means that the slope of the curves in Fig. 5 should not change considerably inside the area.

Another conclusion is that with increasing subsatellite point latitude the probability increases. For one specific case in Table 4 the probability was 8.8 times higher than at the Equator.

Examples of statistics for LEO-F for a particular case are given in Figs. 7a and 7b. They are results of simulations and show the probability dependence on antenna azimuth and elevation.

5 Visibility and worst-case azimuth

Figures 6a) and 6b) are results from computer simulations. They show satellite positions in simulation time steps. The figures show the areas where satellites are not visible. In such areas the equation (19a) is not applicable and the probability is zero. The same areas can be found in Figs. 7a and 7b which represent results from a computer simulation of LEO-F for a particular case.

From equations it can be concluded that at a given elevation angle if:

a) $i < |L_0| - \theta_\varepsilon$ a satellite will not be visible at any azimuth;

b) $i \geq |L_0| + \theta_\varepsilon$ a satellite will be visible at all azimuths;

c) $|L_0| - \theta_\varepsilon \leq i < |L_0| + \theta_\varepsilon$ a satellite will be visible at some but not all azimuths.

In case a) the probability P is zero for all azimuths and equation (19a) does not apply. In case b) the probability P is highest at either 0° or 180° azimuth, respectively at the northern or southern hemisphere. In case c) the highest probability occurs at the point where the subsatellite latitude $L = \pm i$ and the probability does not depend on the latitude of the station.

The azimuths where the probability is highest are called worst-case azimuths.

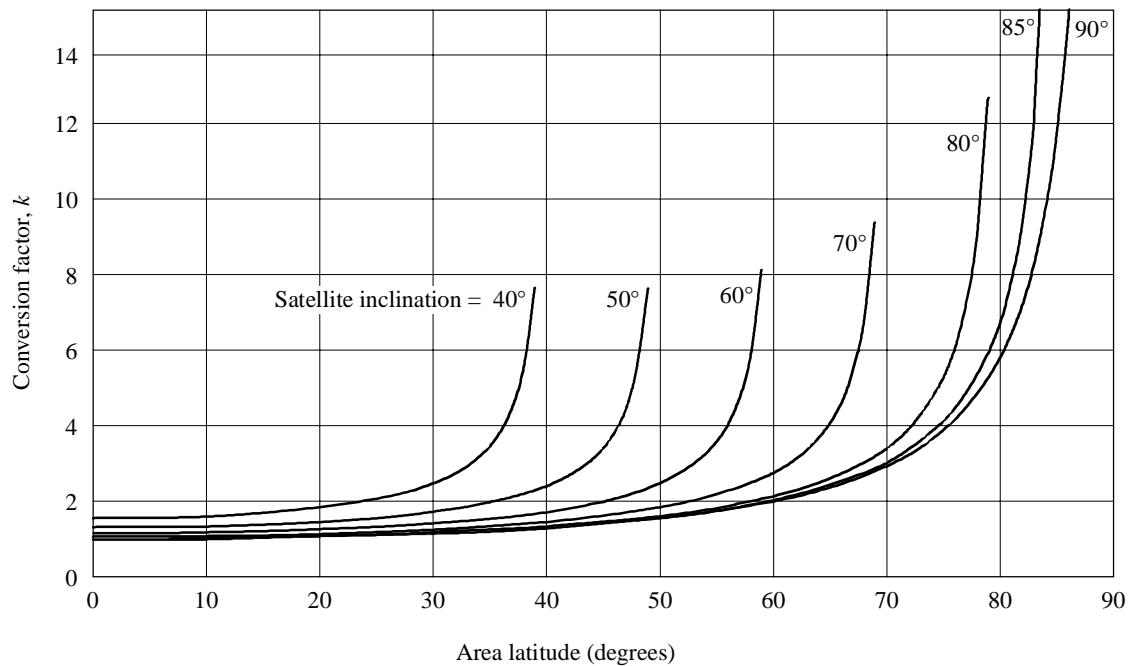In the case of a zero area size, the corresponding worst-case azimuths are:

$$\Lambda_1 = \text{arc cos} \frac{\sin i - \cos \theta_\varepsilon \sin L_0}{\sin \theta_\varepsilon \cos L_0} \tag{28a}$$

$$\Lambda_2 = 2\pi - \Lambda_1 \tag{28b}$$

$$\Lambda_3 = \text{arc cos} \frac{-\sin i - \cos \theta_\varepsilon \sin L_0}{\sin \theta_\varepsilon \cos L_0} \tag{29a}$$

$$\Lambda_4 = 2\pi - \Lambda_3 \tag{29b}$$

FIGURE 5

Conversion factors for different area latitudes and inclinations

$$c = \frac{1}{\sqrt{\sin^2 i - \sin^2 L}} \tag{Equation (19c)}$$

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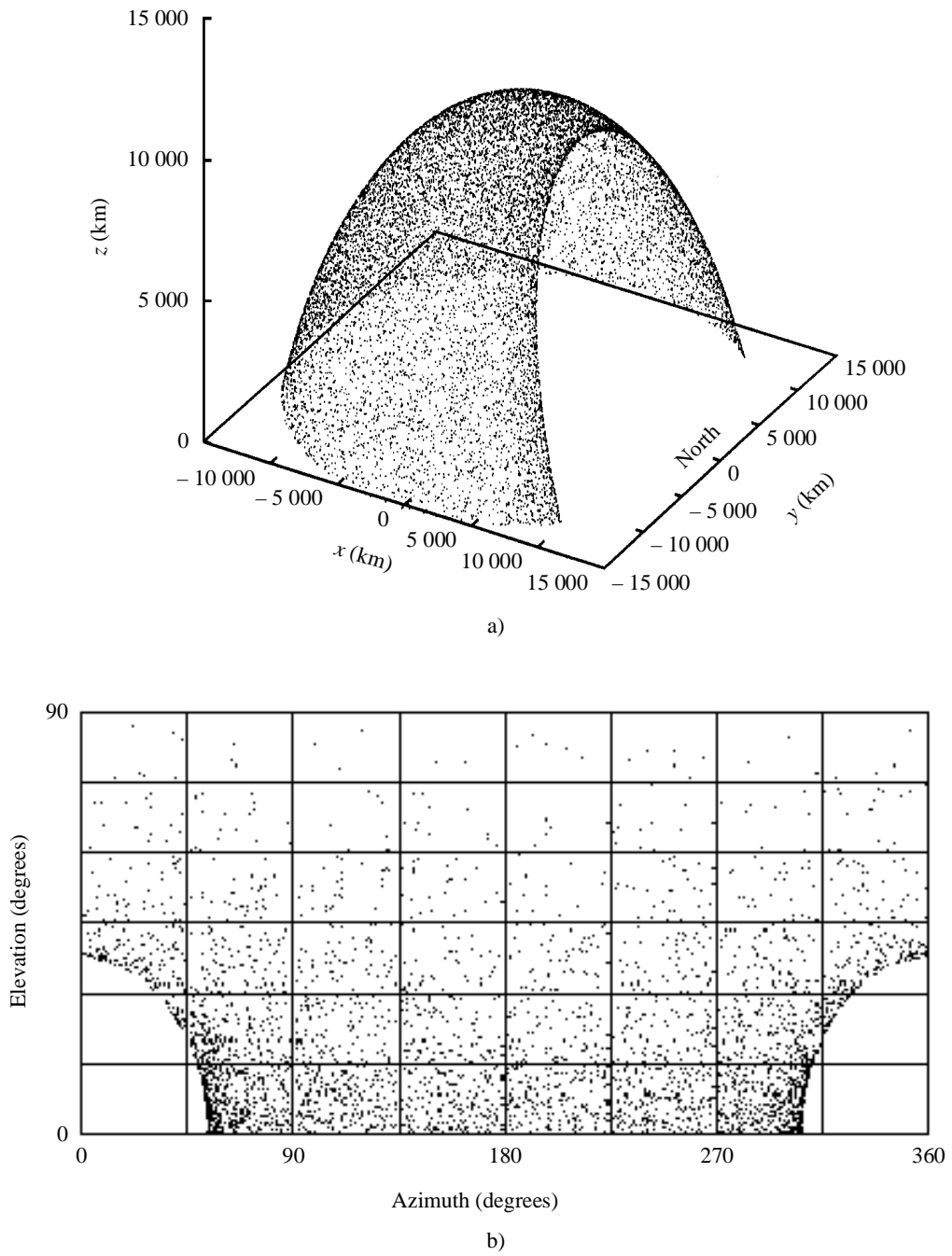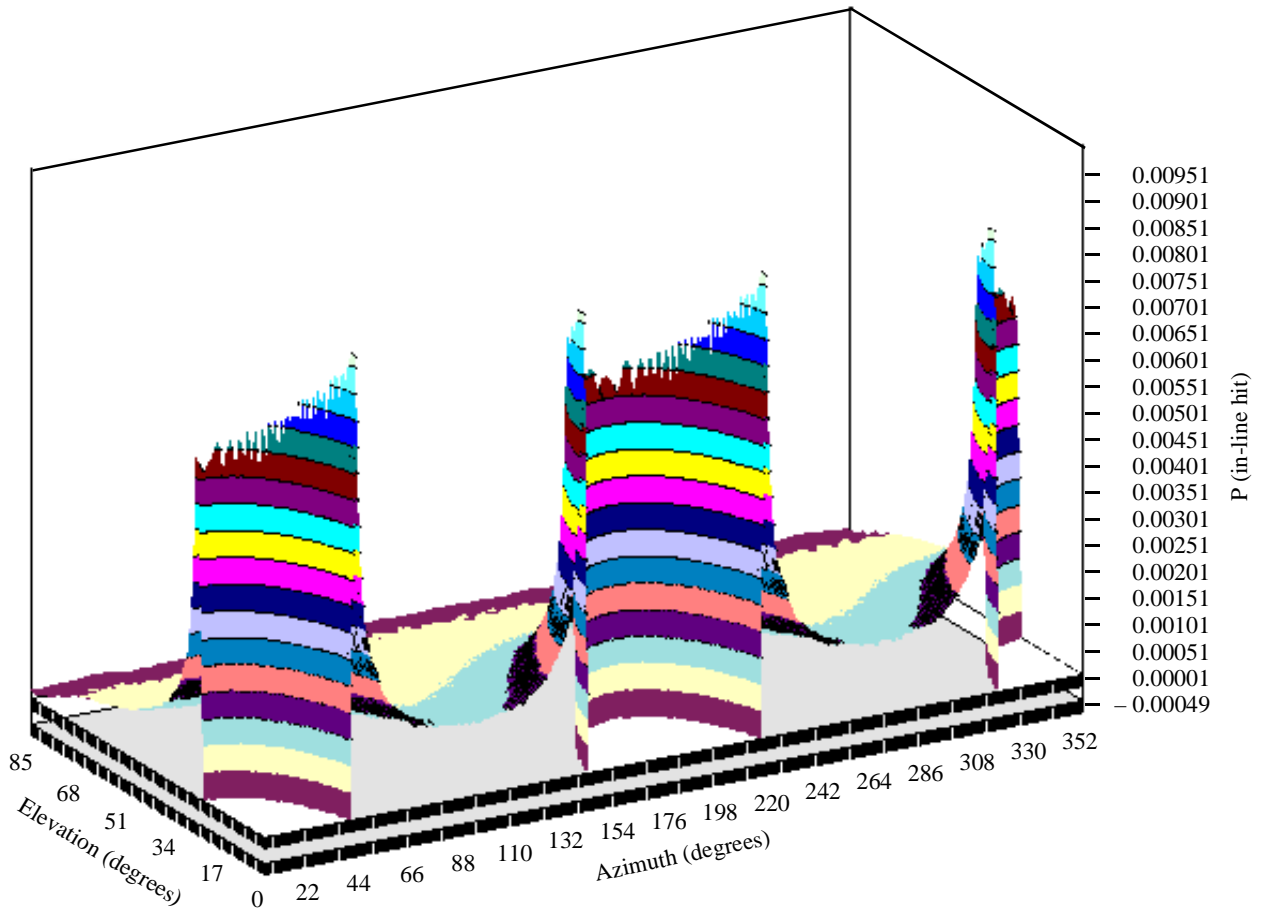
FIGURE 6

Two typical sky coverages for non-GSO satellite

a)

b)

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FIGURE 7a

In-line statistics for different pointing directions:
case LEO-F, $L_0 = 0°$, beam side = 3°

☐ 0.00901-0.00951	☐ 0.00401-0.00451
☐ 0.00851-0.00901	☐ 0.00351-0.00401
☐ 0.00801-0.00851	☐ 0.00301-0.00351
☐ 0.00751-0.00801	☐ 0.00251-0.00301
☐ 0.00701-0.00751	☐ 0.00201-0.00251
☐ 0.00651-0.00701	☐ 0.00151-0.00201
☐ 0.00601-0.00651	☐ 0.00101-0.00151
☐ 0.00551-0.00601	☐ 0.00051-0.00101
☐ 0.00501-0.00551	☐ 0.00001-0.00051
☐ 0.00451-0.00501	– 0.00049-0.00001

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FIGURE 7b

Topographical view of in-line statistics for different pointing directions:
case LEO-F, $L_0 = 0°$, beam side = $3°$
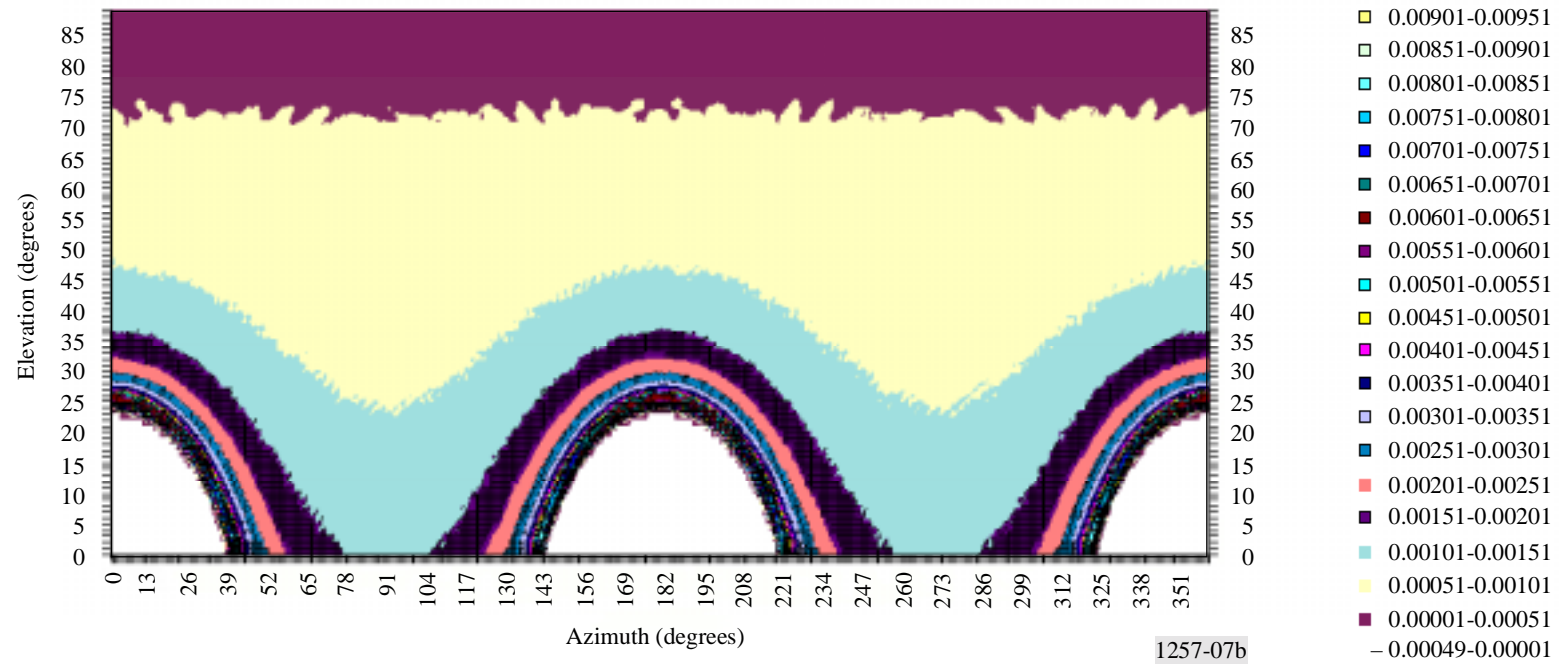
The azimuths from equations (28) and (29) are symmetrical with respect to the north ($\Lambda = 0°$) and the south ($\Lambda = 180°$) and depend only on parameters L_0, ε, h and i.

These worst-case azimuths exist on the condition that the arc cos x function exists. It exists if the argument $|x| \leq 1$ and thus the azimuths exist if:

for equation (28a):

$$\sin i \ \leq \ \sin (L_0 \ + \ \theta_\varepsilon) \qquad \text{and}$$
$$\sin i \ \geq \ \sin (L_0 \ - \ \theta_\varepsilon) \tag{30a}$$

for equation (29a):

$$- \sin i \ \leq \ \sin (L_0 \ + \ \theta_\varepsilon) \qquad \text{and}$$
$$- \sin i \ \geq \ \sin (L_0 \ - \ \theta_\varepsilon) \tag{30b}$$

If for a given elevation angle ε the condition of equation (30a) is not met, then equations (28) do not apply and give no worst-case azimuth. The same applies to equation (30b) and equations (29). Either equation, (28) or (29), may apply independently. If in equation (30a) or (30b) the inequality applies, then the equation (28) or correspondingly equation (29) gives two maxima. If equality applies, the corresponding equation gives one maximum. Even if one of the equations (28) or (29) does not apply there may be a lower maximum at 0° or 180° azimuth.

It should be noted that equations (28) give accurate worst-case azimuths only for an interested area size of zero. For a practical case, as for a satellite earth station antenna beam, the exact value of worst-case azimuth depends e.g. on the beamwidth of the terrestrial station antenna, but the equations give an estimate which normally does not deviate more than about half of the beamwidth from the real worst-case azimuth. No exact relationship between the beamwidth and worst-case azimuth has been presented. Further, in practical case the maximums may be of different amplitude depending on the beamwidth.

6 Conclusion

From the above Tables it can be concluded that the calculation method in this Recommendation gives results which are accurate enough for most of the practical cases. Even if it is not tested here, the method may be expected to be inaccurate near the highest and the lowest latitudes of a satellite orbit. This is basically related to the fact that some parts of the area may be outside of the visibility of the satellite.

ANNEX 2

1 Summary

It has often been observed that software simulation produces short-term downlink interference statistics which have a shape closely resembling that of the radiation pattern of the GSO FSS antenna which suffers the interference. Also, it has been observed that a family resemblance exists between the downlink interference curves for different antenna diameters. This Annex shows that there is a simple mathematical explanation for these effects. Further, it is shown that the short-term visibility statistics derived in Annex 1 of this Recommendation can be used to generate continuous curves of short-term downlink interference.

2 Derivation

For the receive antenna of the GSO FSS earth station, represent the main lobe of the antenna radiation pattern by a cone with half angle φ. This cone subtends a topocentric solid angle $2\pi(1 - \cos \varphi)$. For small values of φ the solid angle is approximately $\pi \varphi^2$ (with φ in radians).

If, in the area of sky close to the main lobe of the antenna, the probability of seeing a non-GSO satellite is P_0 per unit topocentric solid angle, then the probability of finding a non-GSO satellite in the main lobe of the antenna is approximately:

$$P = P_0 \, \pi \, \varphi^2$$

In the RR and in ITU-R Recommendations, the main lobe of the GSO FSS antenna radiation pattern is usually represented by:

$$G(\varphi) = G_{max} - 0.0025 \left(\frac{D \, \varphi}{\lambda} \right)^2 \qquad \text{dB} \qquad \left(\text{with } \varphi \text{ in degrees}\right)$$

or

$$G(\varphi) = G_{max} - \left(\frac{D \, \varphi}{20 \, \lambda} \right)^2 \qquad \text{dB} \qquad \left(\text{with } \varphi \text{ in degrees}\right)$$

or

$$\varphi^2 = \Delta G \left(\frac{20 \, \lambda}{D} \right)^2 \qquad \text{degrees}^2 \qquad \left(\text{with } \Delta G \text{ in dB}\right)$$

$$= \Delta G \left(\frac{\pi \, \lambda}{9 \, D} \right)^2 \qquad \text{rad}^2$$

so:

$$P = P_0 \, \pi \cdot \Delta G \cdot \left(\frac{\pi \, \lambda}{9D} \right)^2 \tag{31}$$

where:

$$\Delta G = G_{max} - G(\varphi)$$

Note that P is the cumulative distribution function (CDF) of the value of ΔG, and the term ΔG represents the variation of antenna gain within the main lobe of the antenna, which is the same as the variation in downlink interference level within the main lobe region. Therefore, given the values of P_0 and D/λ which apply for a GSO FSS receive antenna, equation (31) may be used to generate a continuous curve of P versus ΔG which represents the short-term downlink interference experienced by the GSO FSS receive antenna.
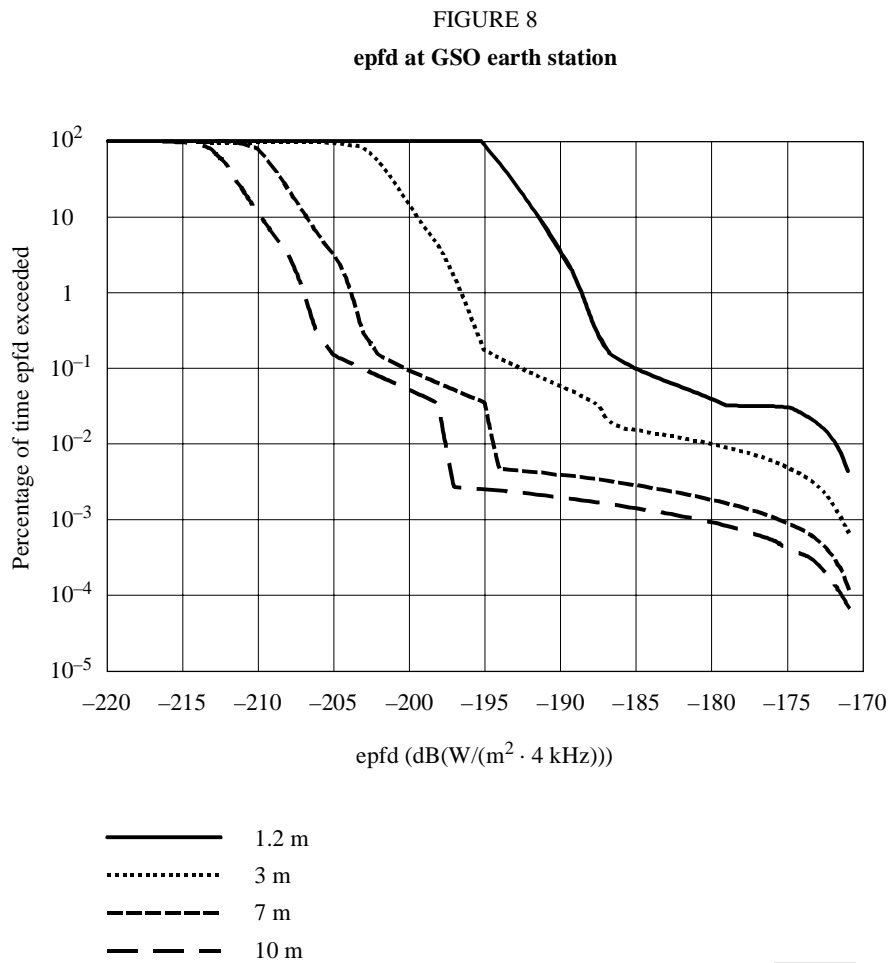
Annex 1 of this Recommendation provides a procedure for calculating P_c, the probability of finding one spacecraft of a non-GSO constellation in the main lobe of a GSO earth station antenna with beamwidth β rad. The relationship between the term P_c of Annex 1 and the term P_0 used in this Annex is therefore:

$$P_0 = P_c \, / \, 2\pi(1 - \cos [\beta / 2]) \tag{32}$$

3   Verification

A computer simulation was performed for a constellation of 64 non-GSO satellites in 16 planes, with planes inclined at 55° to the Equator. The earth station was in Japan (35° N: 135° E), operating to a GSO FSS satellite at 110° E. The RF parameters used in the simulation were such that the downlink interference for an in-line event resulted in a downlink equivalent power flux-density (epfd) of –171 dB(W/(m^2 · 4 kHz)). The simulation was run for 33.4 M time steps, with 1 s intervals between steps.

Figure 8 shows the set of epfd curves produced by the simulation program for a range of antenna diameters.

FIGURE 8

epfd at GSO earth station

The value of P_c for the non-GSO constellation was calculated using the methodology described in Annex 1 of this Recommendation, and equation (32) of this Annex was then used to obtain P_0. A value of 0.384 was obtained for P_0: that is, a probability of 38.4%.

The CDFs for all antennas were then calculated using equation (31). Table 5 shows the results of this procedure. As expected, there is excellent agreement between the two sets of values of P.

TABLE 5

epfd	1,2 m		3 m		7 m		10 m	
	Simulation	Model	Simulation	Model	Simulation	Model	Simulation	Model
−193							0.00219	0.00215
−192							0.00209	0.00215
−191							0.00199	0.00196
−190					0.00377	0.00381	0.00187	0.00187
−189					0.00359	0.00361	0.00179	0.00177
−188					0.00343	0.00342	0.00170	0.00168
−187					0.00332	0.00323	0.00161	0.00158
−186					0.00306	0.00303	0.00149	0.00149
−185					0.00285	0.00284	0.00142	0.00139
−184					0.00266	0.00265	0.00133	0.00130
−183			0.01337	0.01335	0.00248	0.00245	0.00124	0.00120
−182			0.01232	0.01230	0.00231	0.00226	0.00106	0.00111
−181			0.01120	0.01124	0.00208	0.00206	0.00100	0.00101
−180			0.01023	0.01019	0.00189	0.00187	0.00089	0.00092
−179			0.00917	0.00913	0.00169	0.00168	0.00082	0.00082
−178			0.00810	0.00808	0.00149	0.00148	0.00072	0.00073
−177			0.00705	0.00703	0.00131	0.00129	0.00060	0.00063
−176			0.00604	0.00597	0.00104	0.00110	0.00056	0.00054
−175	0.03067	0.03075	0.00492	0.00492	0.00088	0.00090	0.00041	0.00044
−174	0.02471	0.02417	0.00382	0.00387	0.00070	0.00071	0.00033	0.00035
−173	0.01798	0.01758	0.00279	0.00281	0.00052	0.00052	0.00026	0.00025
−172	0.01134	0.01100	0.00177	0.00176	0.00031	0.00032	0.00015	0.00016
−171	0.00450	0.00441	0.00068	0.00071	0.00012	0.00013	0.00007	0.00006

4   Range of applicability

The derivation of § 2 was based on the fact that short-term downlink interference statistics are determined interference from non-GSO satellites close to the main lobe of the GSO FSS antenna. This will be true if the non-GSO satellites outside of the exclusion zone cannot generate high levels of interference.

The short-term interference region may be defined in the following way. Suppose that the non-GSO network implements a 10° exclusion zone, and that the non-GSO satellite has side-lobe levels at least 25 dB below the main lobe. Also, assume that the non-GSO satellites outside of the exclusion zone should affect the short-term interference levels by no more than 1 dB. This condition can be met within ΔG_{max} of the boresight of the GSO earth station antenna, where:

$$\Delta G_{max} = G_{max} - (G(10°) + 25 + 7) \qquad \text{dB}$$

Provided this criterion is met, the short-term epfd level is unaffected by non-GSO satellites which are outside of the 10° exclusion zone.

The current reference radiation pattern is based on side-lobe levels of 29 − 25 log(φ) dBi, so the gain at a 10° offset angle is 4 dBi.

Hence:

$$\Delta G_{max} = G_{max} - (4 + 25 + 7) = G_{max} - 36 \qquad \text{dB}$$