International Telecommunication Union



Recommendation ITU-R P.676-11 (09/2016)

# **Attenuation by atmospheric gases**

P Series Radiowave propagation



International Telecommunication

#### Foreword

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Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

Electronic Publication Geneva, 2016

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# **RECOMMENDATION ITU-R P.676-11**

# Attenuation by atmospheric gases

(Question ITU-R 201/3)

(1990-1992-1995-1997-1999-2001-2005-2007-2009-2012-2013-2016)

#### Scope

This Recommendation provides methods to estimate the attenuation of atmospheric gases on terrestrial and slant paths using:

- a) an estimate of gaseous attenuation computed by a summation of individual absorption lines that is valid for the frequency range 1-1 000 GHz; and
- b) a simplified approximate method to estimate gaseous attenuation that is applicable in the frequency range 1-350 GHz.

#### **Keywords**

Gaseous absorption, specific attenuation, slant path attenuation, total attenuation, water vapour, oxygen, dry air

The ITU Radiocommunication Assembly,

#### considering

- *a)* the necessity of estimating the attenuation by atmospheric gases on terrestrial and slant paths,
  - recommends

1 that, for general application, the procedure in Annex 1 be used to calculate gaseous attenuation at frequencies up to 1000 GHz;

2 that, for approximate estimates of gaseous attenuation in the frequency range 1-350 GHz, the computationally less intensive procedure given in Annex 2 is used.

#### **Guide to this Recommendation**

This Recommendation provides the following three methods of predicting the specific and path gaseous attenuation due to oxygen and water vapour:

- 1 Calculation of specific and path gaseous attenuation using the line-by-line summation in Annex 1 assuming the atmospheric pressure, temperature, and water vapour density vs. height;
- 2 An approximate estimate of specific and path gaseous attenuation in Annex 2 assuming the water vapour density at the surface of the Earth;
- 3 An approximate estimate of path attenuation in Annex 2 assuming the integrated water vapour content along the path.

These prediction methods can use local meteorological data, or, in the absence of local data, reference atmospheres or meteorological maps corresponding to a desired probability of exceedance that are provided in other ITU-R P-series Recommendations.

#### Specific attenuation

Annex 1 equation (1), which is applicable to frequencies up to 1 000 GHz, or the sum of Annex 2 equations (22) and (23), which is applicable to frequencies up to 350 GHz, may be used to predict

specific attenuation. Both methods require the pressure, temperature, and water vapour density at the applicable location. If local data is not available, a combination of: a) the mean annual global reference atmosphere given in Recommendation ITU-R P.835, b) the map of mean annual surface temperature in Recommendation ITU-R P.1510 and c) the maps of surface water vapour density vs. exceedance probability given in Recommendation ITU-R P.836 may be used in lieu of the standard ground-level surface water vapour density of 7.5 g/m<sup>3</sup>.

#### Slant path (Earth-space) attenuation

Annex 1 equation (20), or Annex 2 equations (28) or (29) may be used.

- Annex 1 equation (20) requires knowledge of the temperature, pressure, and water vapour density profiles along the path. If local profile data is not available, the reference atmospheric profiles given in Recommendation ITU-R P.835 may be used. The surface water vapour density vs. exceedance probability given in Recommendation ITU-R P.836 may be used in lieu of the standard ground-level surface water vapour density of 7.5 g/m<sup>3</sup>.
- Annex 2 equation (28) requires knowledge of the surface pressure, surface temperature, and surface water vapour density. Annex 2 equation (28) is an approximation to equation (20) applicable to frequencies up to 350 GHz assuming a mean annual global reference atmosphere and an arbitrary surface water vapour density with a negative exponential water vapour density profile vs. height. Annex 2 equation (28) can be used to predict: a) the instantaneous gaseous attenuation for a specific value of surface pressure, surface temperature, and surface water vapour density or b) the gaseous attenuation corresponding to the surface water vapour density at a desired probability of exceedance. If local surface water vapour density data is not available, the surface water vapour density maps in Recommendation ITU-R P.836 may be used.
- Annex 2 equation (29) requires knowledge of the surface temperature, surface pressure, and integrated water vapour content along the path. Similar to Annex 2 equation (28), Annex 2 equation (29) can be used to predict: a) the instantaneous gaseous attenuation for a specific value of surface pressure, surface temperature, and integrated water vapour content, or b) the gaseous attenuation corresponding to the integrated water vapour content at a desired probability of exceedance. If local surface integrated water vapour content data is not available, the integrated water vapour maps in Recommendation ITU-R P.836 may be used.

If local surface water vapour density and integrated water vapour content data are both available, Annex 2 equation (29) using local integrated water vapour content is considered to be more accurate than Annex 2 equation (28) using local surface water vapour density data. Similarly, if local data is not available, Annex 2 equation (29) using the maps of integrated water vapour content in Recommendation ITU-R P.836 is considered to be more accurate than Annex 2 equation (28) using the maps of surface water vapour density in Recommendation ITU-R P.836.

	Annex 1 Equation (20)	Annex 2 Equation (28)	Annex 2 Equation (29)	
Frequency range	<u>≤</u> 1 000 GHz	<u>&lt;</u> 35	0 GHz	
Accuracy	Best, line-by-line sum	Approximation		
Pressure vs. height				
Temperature vs. height	Arbitrary	Mean Annual Global Reference Atmospheric Profile		
Water vapour density vs. height		Surface value with negative exponential profile vs. height	Integrated water vapour content in lieu of water vapour density vs. height	

# Annex 1

# Line-by-line calculation of gaseous attenuation

# **1** Specific attenuation

The specific attenuation at frequencies up to 1000 GHz, due to dry air and water vapour, can be evaluated most accurately at any value of pressure, temperature and humidity by means of a summation of the individual resonance lines from oxygen and water vapour, together with small additional factors for the non-resonant Debye spectrum of oxygen below 10 GHz, pressure-induced nitrogen attenuation above 100 GHz and a wet continuum to account for the excess water vapour-absorption found experimentally. Figure 1 shows the specific attenuation using the prediction method, calculated from 0 to 1000 GHz at 1 GHz intervals, for a pressure of 1013.25 hPa, temperature of  $15^{\circ}$  C for the cases of a water-vapour density of 7.5 g/m<sup>3</sup> (Standard) and a dry atmosphere (Dry).

Near 60 GHz, many oxygen absorption lines merge together at sea-level pressures to form a single, broad absorption band, which is shown in more detail in Fig. 2. This figure also shows the oxygen attenuation at higher altitudes, with the individual lines becoming resolved as the pressure decreases with increasing altitude. Some additional molecular species (e.g. oxygen isotopic species, oxygen vibrationally excited species, ozone, ozone isotopic species, and ozone vibrationally excited species) are not included in the line-by-line prediction method. These additional lines are insignificant for typical atmospheres, but may be important for a dry atmosphere.

For quick and approximate estimates of specific attenuation at frequencies up to 350 GHz, in cases where high accuracy is not required, simplified algorithms are given in Annex 2 for restricted ranges of meteorological conditions.

The specific gaseous attenuation is given by:

$$\gamma = \gamma_o + \gamma_w = 0.1820 f \left( N''_{Oxygen}(f) + N''_{WaterVapour}(f) \right) \qquad (dB/km)$$
(1)

here  $\gamma_o$  and  $\gamma_w$  are the specific attenuations (dB/km) due to dry air (oxygen, pressure-induced nitrogen and non-resonant Debye attenuation) and water vapour, respectively, *f* is the frequency (GHz), and

 $N''_{oxygen}(f)$  and  $N''_{Water Vapour}(f)$  are the imaginary parts of the frequency-dependent complex refractivities:

$$N_{Oxygen}''(f) = \sum_{i \ (Oxygen)} S_i F_i + N_D''(f)$$
(2a)

$$N_{Water\,Vapour}''(f) = \sum_{i \; (Water\,Vapour)} S_i F_i \tag{2b}$$

 $S_i$  is the strength of the *i*-th oxygen or water vapour line,  $F_i$  is the oxygen or water vapour line shape factor, and the summations extend over all the lines in Tables 1 and 2;

 $N''_D(f)$  is the dry continuum due to pressure-induced nitrogen absorption and the Debye spectrum as given by equation (8).

The line strength is given by:

$$S_{i} = a_{1} \times 10^{-7} \ p \ \theta^{3} \ \exp\left[a_{2} (1 - \theta)\right] \qquad \text{for oxy gen}$$
  
=  $b_{1} \times 10^{-1} \ e \ \theta^{3.5} \ \exp\left[b_{2} (1 - \theta)\right] \qquad \text{for water vapour}$ (3)

where:

- *p*: dry air pressure (hPa)
- *e*: water vapour partial pressure (hPa) (total barometric pressure,  $p_{tot} = p + e$ )

$$\theta = 300/T$$

*T*: temperature (K).



FIGURE 1



Specific attenuation in the range 50-70 GHz at the altitudes indicated, calculated at intervals of 10 MHz, including line centres (0 km, 5 km, 10 km, 15 km and 20 km)

FIGURE 2

If available, local altitude profiles of p, e and T (e.g. using radiosondes) should be used. In the absence of local information, a reference standard atmosphere given in Recommendation ITU-R P.835 should be used. (Note that where total atmospheric attenuation is being calculated, the same-water vapour partial pressure is used for both dry-air and water-vapour attenuations.)

The water-vapour partial pressure, e, at an altitude may be obtained from the water-vapour density,  $\rho$ , and the temperature, T, at that altitude using the expression:

$$e = \frac{\rho T}{216.7} \tag{4}$$

Spectroscopic data for oxygen is given in Table 1, and spectroscopic data for water vapour is given in Table 2. The last entry in Table 2 is a pseudo-line centred at 1 780 GHz whose lower wing represents the combined contribution below 1 000 GHz of water-vapour resonances not included in the line-by-line prediction method (i.e. the wet continuum). The pseudo-line's parameters are adjusted to account for the difference between the measured absorption in the atmospheric windows and the calculated local-line absorption.

The line-shape factor is given by:

$$F_{i} = \frac{f}{f_{i}} \left[ \frac{\Delta f - \delta \left(f_{i} - f\right)}{\left(f_{i} - f\right)^{2} + \Delta f^{2}} + \frac{\Delta f - \delta \left(f_{i} + f\right)}{\left(f_{i} + f\right)^{2} + \Delta f^{2}} \right]$$
(5)

where  $f_i$  is the oxygen or water vapour line frequency and  $\Delta f$  is the width of the line:

$$\Delta f = a_3 \times 10^{-4} (p \ \theta^{(0.8 - a_4)} + 1.1 e \theta) \qquad \text{for oxygen}$$
  
=  $b_3 \times 10^{-4} (p \ \theta^{b_4} + b_5 e \theta^{b_6}) \qquad \text{for water vapour}$ (6a)

The line width  $\Delta f$  is modified to account for Zeeman splitting of oxygen lines and Doppler broadening of water vapour lines:

$$\Delta f = \sqrt{\Delta f^2 + 2.25 \times 10^{-6}} \qquad \text{for oxy gen}$$
$$= 0.535\Delta f + \sqrt{0.217\Delta f^2 + \frac{2.1316 \times 10^{-12} f_i^2}{\theta}} \qquad \text{for water vapour} \qquad (6b)$$

 $\delta$  is a correction factor that arises due to interference effects in oxygen lines:

$$\delta = (a_5 + a_6 \theta) \times 10^{-4} (p+e) \theta^{0.8}$$
 for oxy gen  
= 0 for water vapour (7)

# TABLE 1

# Spectroscopic data for oxygen attenuation

	Specific Gata for oxygen attenuation						
$f_0$	<i>a</i> <sub>1</sub>	$a_2$	<i>a</i> <sub>3</sub>	$a_4$	<i>a</i> <sub>5</sub>	<i>a</i> <sub>6</sub>	
50.474214	0.975	9.651	6.690	0.0	2.566	6.850	
50.987745	2.529	8.653	7.170	0.0	2.246	6.800	
51.503360	6.193	7.709	7.640	0.0	1.947	6.729	
52.021429	14.320	6.819	8.110	0.0	1.667	6.640	
52.542418	31.240	5.983	8.580	0.0	1.388	6.526	
53.066934	64.290	5.201	9.060	0.0	1.349	6.206	
53.595775	124.600	4.474	9.550	0.0	2.227	5.085	
54.130025	227.300	3.800	9.960	0.0	3.170	3.750	
54.671180	389.700	3.182	10.370	0.0	3.558	2.654	
55.221384	627.100	2.618	10.890	0.0	2.560	2.952	
55.783815	945.300	2.109	11.340	0.0	-1.172	6.135	
56.264774	543.400	0.014	17.030	0.0	3.525	-0.978	
56.363399	1331.800	1.654	11.890	0.0	-2.378	6.547	
56.968211	1746.600	1.255	12.230	0.0	-3.545	6.451	
57.612486	2120.100	0.910	12.620	0.0	-5.416	6.056	
58.323877	2363.700	0.621	12.950	0.0	-1.932	0.436	
58.446588	1442.100	0.083	14.910	0.0	6.768	-1.273	
59.164204	2379.900	0.387	13.530	0.0	-6.561	2.309	
59.590983	2090.700	0.207	14.080	0.0	6.957	-0.776	
60.306056	2103.400	0.207	14.150	0.0	-6.395	0.699	
60.434778	2438.000	0.386	13.390	0.0	6.342	-2.825	
61.150562	2479.500	0.621	12.920	0.0	1.014	-0.584	
61.800158	2275.900	0.910	12.630	0.0	5.014	-6.619	
62.411220	1915.400	1.255	12.170	0.0	3.029	-6.759	
62.486253	1503.000	0.083	15.130	0.0	-4.499	0.844	
62.997984	1490.200	1.654	11.740	0.0	1.856	-6.675	
63.568526	1078.000	2.108	11.340	0.0	0.658	-6.139	
64.127775	728.700	2.617	10.880	0.0	-3.036	-2.895	
64.678910	461.300	3.181	10.380	0.0	-3.968	-2.590	
65.224078	274.000	3.800	9.960	0.0	-3.528	-3.680	
65.764779	153.000	4.473	9.550	0.0	-2.548	-5.002	
66.302096	80.400	5.200	9.060	0.0	-1.660	-6.091	
66.836834	39.800	5.982	8.580	0.0	-1.680	-6.393	
67.369601	18.560	6.818	8.110	0.0	-1.956	-6.475	
67.900868	8.172	7.708	7.640	0.0	-2.216	-6.545	
68.431006	3.397	8.652	7.170	0.0	-2.492	-6.600	
68.960312	1.334	9.650	6.690	0.0	-2.773	-6.650	
118.750334	940.300	0.010	16.640	0.0	-0.439	0.079	

$f_0$	<i>a</i> <sub>1</sub>	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>	$a_4$	<i>a</i> <sub>5</sub>	<i>a</i> <sub>6</sub>
368.498246	67.400	0.048	16.400	0.0	0.000	0.000
424.763020	637.700	0.044	16.400	0.0	0.000	0.000
487.249273	237.400	0.049	16.000	0.0	0.000	0.000
715.392902	98.100	0.145	16.000	0.0	0.000	0.000
773.839490	572.300	0.141	16.200	0.0	0.000	0.000
834.145546	183.100	0.145	14.700	0.0	0.000	0.000

TABLE 1 (end)

# TABLE 2

# Spectroscopic data for water-vapour attenuation

$f_0$	<b>b</b> 1	<b>b</b> 2	<b>b</b> 3	<b>b</b> 4	<b>b</b> 5	<b>b</b> 6
*22.235080	.1079	2.144	26.38	.76	5.087	1.00
67.803960	.0011	8.732	28.58	.69	4.930	.82
119.995940	.0007	8.353	29.48	.70	4.780	.79
*183.310087	2.273	.668	29.06	.77	5.022	.85
*321.225630	.0470	6.179	24.04	.67	4.398	.54
*325.152888	1.514	1.541	28.23	.64	4.893	.74
336.227764	.0010	9.825	26.93	.69	4.740	.61
*380.197353	11.67	1.048	28.11	.54	5.063	.89
390.134508	.0045	7.347	21.52	.63	4.810	.55
437.346667	.0632	5.048	18.45	.60	4.230	.48
439.150807	.9098	3.595	20.07	.63	4.483	.52
443.018343	.1920	5.048	15.55	.60	5.083	.50
*448.001085	10.41	1.405	25.64	.66	5.028	.67
470.8889999	.3254	3.597	21.34	.66	4.506	.65
474.689092	1.260	2.379	23.20	.65	4.804	.64
488.490108	.2529	2.852	25.86	.69	5.201	.72
503.568532	.0372	6.731	16.12	.61	3.980	.43
504.482692	.0124	6.731	16.12	.61	4.010	.45
547.676440	.9785	.158	26.00	.70	4.500	1.00
552.020960	.1840	.158	26.00	.70	4.500	1.00
*556.935985	497.0	.159	30.86	.69	4.552	1.00
620.700807	5.015	2.391	24.38	.71	4.856	.68
645.766085	.0067	8.633	18.00	.60	4.000	.50
658.005280	.2732	7.816	32.10	.69	4.140	1.00
*752.033113	243.4	.396	30.86	.68	4.352	.84
841.051732	.0134	8.177	15.90	.33	5.760	.45
859.965698	.1325	8.055	30.60	.68	4.090	.84
899.303175	.0547	7.914	29.85	.68	4.530	.90
902.611085	.0386	8.429	28.65	.70	5.100	.95
906.205957	.1836	5.110	24.08	.70	4.700	.53

f0	b1	b2	b3	b4	b5	b6
916.171582	8.400	1.441	26.73	.70	5.150	.78
923.112692	.0079	10.293	29.00	.70	5.000	.80
970.315022	9.009	1.919	25.50	.64	4.940	.67
987.926764	134.6	.257	29.85	.68	4.550	.90
*1 780.000000	17506.	.952	196.3	2.00	24.15	5.00

TABLE 2 (end)

The dry air continuum arises from the non-resonant Debye spectrum of oxygen below 10 GHz and a pressure-induced nitrogen attenuation above 100 GHz.

$$N_D''(f) = f \ p \ \theta^2 \left[ \frac{6.14 \times 10^{-5}}{d \left[ 1 + \left(\frac{f}{d}\right)^2 \right]} + \frac{1.4 \times 10^{-12} \ p \ \theta^{1.5}}{1 + 1.9 \times 10^{-5} \ f^{1.5}} \right]$$
(8)

where *d* is the width parameter for the Debye spectrum:

$$d = 5.6 \times 10^{-4} (p+e) \theta^{0.8} \tag{9}$$

#### 2 Path attenuation

#### 2.1 Terrestrial paths

For a terrestrial path, or for slightly inclined paths close to the ground, the path attenuation, *A*, may be calculated as:

$$A = \gamma r_0 = (\gamma_o + \gamma_w) r_0 \qquad \text{dB}$$
(10)

where  $r_0$  is the path length (km).

#### 2.2 Slant paths

This section provides a method to integrate the specific attenuation calculated using the line-by-line model given above, at different pressures, temperatures and humidities through the atmosphere. By this means, the path attenuation for communications systems with any geometrical configuration within and external to the Earth's atmosphere may be accurately determined simply by dividing the atmosphere into horizontal layers, specifying the profile of the meteorological parameters pressure, temperature and humidity along the path. In the absence of local profiles, from radiosonde data, for example, the reference standard atmospheres given in Recommendation ITU-R P.835 may be used, either for global application or for low (annual), mid (summer and winter) and high latitude (summer and winter) sites.

Figure 3 shows the zenith attenuation calculated at 1 GHz intervals with this model for the global reference standard atmosphere given in Recommendation ITU-R P.835, with horizontal layers 1 km

thick and summing the attenuations for each layer, for the cases of a moist atmosphere (Standard) and a dry atmosphere (Dry).

The total slant path attenuation,  $A(h, \varphi)$ , from a station with altitude, h, and elevation angle,  $\varphi$ , can be calculated as follows when  $\varphi \ge 0$ :

$$A(h,\phi) = \int_{h}^{\infty} \frac{\gamma(H)}{\sin \phi} dH$$
(11)

where the value of  $\Phi$  can be determined as follows based on Snell's law in polar coordinates:

$$\Phi = \arccos\left(\frac{c}{(r+H) \times n(H)}\right)$$
(12)

where:

$$c = (r+h) \times n(h) \times \cos \varphi \tag{13}$$

where n(h) is the atmospheric radio refractive index, calculated from pressure, temperature and watervapour pressure along the path (see Recommendation ITU-R P.835) using equations (1) and (2) of Recommendation ITU-R P.453.

When  $\phi < 0$ , there is a minimum height,  $h_{min}$ , at which the radio beam becomes parallel with the Earth's surface. The value of  $h_{min}$  can be determined by solving the following transcendental equation:

$$(r+h_{min}) \times n(h_{min}) = c \tag{14}$$

This can be easily solved by repeating the following calculation, using  $h_{min} = h$  as an initial value:

$$h'_{min} = \frac{c}{n(h_{min})} - r \tag{15}$$

Therefore,  $A(h, \phi)$  can be calculated as follows:

$$A(h, \varphi) = \int_{h_{min}}^{\infty} \frac{\gamma(H)}{\sin \Phi} \, \mathrm{d}H + \int_{h_{min}}^{h} \frac{\gamma(H)}{\sin \Phi} \, \mathrm{d}H$$
(16)

In carrying out the integration of equations (11) and (16), care should be exercised since the integrand becomes infinite at  $\Phi = 0$ . However, this singularity can be eliminated by an appropriate variable conversion, for example, by using  $u^4 = H - h$  in equation (11) and  $u^4 = H - h_{min}$  in equation (16).

A numerical solution for the attenuation due to atmospheric gases can be implemented with the following algorithm.

To calculate the total attenuation for a satellite link, it is necessary to know not only the specific attenuation at each point of the link but also the length of path that has that specific attenuation. To

derive the path length it is also necessary to consider the ray bending that occurs with a spherical Earth.

Using Fig. 4 as a reference,  $a_n$  is the path length through layer *n* with thickness  $\delta_n$  that has refractive index  $n_n$ .  $\alpha_n$  and  $\beta_n$  are the entry and exit incidence angles.  $r_n$  is the radius from the centre of the Earth to the beginning of layer *n*.  $a_n$  can then be expressed as:

$$a_n = -r_n \cos \beta_n + \frac{1}{2} \sqrt{4 r_n^2 \cos^2 \beta_n + 8 r_n \delta_n + 4 \delta_n^2}$$
(17)

The angle  $\alpha_n$  can be calculated from:

$$\alpha_n = \pi - \arccos\left(\frac{-a_n^2 - 2r_n\,\delta_n - \delta_n^2}{2a_n\,r_n + 2a_n\,\delta_n}\right) \tag{18}$$

 $\beta_1$  is the incidence angle at the ground station (the complement of the elevation angle  $\varphi$ ).  $\beta_{n+1}$  can be calculated from  $\alpha_n$  using Snell's law as follows:

$$\beta_{n+1} = \arcsin\left(\frac{n_n}{n_{n+1}}\sin\alpha_n\right) \tag{19}$$

where  $n_n$  and  $n_{n+1}$  are the refractive indexes of layers n and n + 1.

Equation (19) may become invalid at very low elevation angles ( $\varphi < 1^{\circ}$ ) when radiosonde data from certain regions of the world susceptible to ducting conditions are used as input. In such cases, air layers with radio refractivity gradients less than -157 N/km are present and the ray-tracing algorithm (equations (17) to (19)), which is based on geometrical optics, is no longer applicable. The arcsine function in equation (19) becomes complex under these anomalous conditions since its argument is then slightly larger than 1. It should be noted that equation (19) is valid for all elevation angles when the reference standard atmospheres given in Recommendation ITU-R P.835 are used as input, since these idealized atmospheres – clearly without strong negative refractivity gradients – do not support such anomalous propagation conditions.



The total attenuation,  $A_{qas}$ , can be derived using:

$$A_{gas} = \sum_{n=1}^{k} a_n \gamma_n \qquad \text{dB}$$
(20)

where  $\gamma_n$  is the specific attenuation of the n<sup>th</sup> layer per equation (1).

To ensure an accurate estimate of the path attenuation, the thickness of the layers should increase exponentially, from 10 cm at the lowest layer (ground level) to 1 km at an altitude of 100 km, according to the following equation:

$$\delta_i = 0.0001 \exp\left\{\frac{i-1}{100}\right\} \qquad \text{km} \tag{21}$$

for i = 1 to 922, noting that  $\delta_{922} \cong 1.0$  km and  $\sum_{i=1}^{922} \delta_i \cong 100$  km.

For Earth-to-space applications, the integration should be performed up to at least 30 km, and up to 100 km at the oxygen line-centre frequencies.

#### FIGURE 4 Path through the atmosphere



#### **3** Dispersive effects

In addition to the attenuation described in the previous paragraph, which is based on the imaginary part of the frequency-dependent complex refractivity, oxygen and water vapour also introduce dispersion, which is based on the real part of the frequency-dependent complex refractivity. This effect is described in terms of phase vs. frequency (degree/km) or group delay vs. frequency (ps/km); and, similar to attenuation, can be calculated for slant paths. The effects of dispersion are discussed in Chapter 6 of the ITU-R Handbook on Radiometeorology, which contains a model for calculating dispersion based on the line-by-line prediction method. For practical purposes, dispersive effects should not impose serious limitations on millimetric terrestrial communication systems operating with bandwidths of up to a few hundred MHz over short ranges (for example, less than about 20 km), especially in the window regions of the spectrum, at frequencies removed from the centres of major absorption lines. For satellite communication systems, the longer path lengths through the atmosphere will constrain operating frequencies to be within the window regions, where both the atmospheric attenuation and the corresponding dispersion are low.

#### Annex 2

# Approximate estimation of gaseous attenuation in the frequency range 1-350 GHz

This Annex contains simplified algorithms for approximate estimates of gaseous attenuation for a limited range of meteorological conditions and a limited variety of geometrical configurations.

#### **1** Specific attenuation

The specific attenuation due to dry air and water vapour, from sea level to an altitude of 10 km, can be estimated using the following simplified algorithms, which are based on the oxygen and water vapour specific attenuation from the line-by-line calculation and the effective oxygen and water vapour heights. These approximations have good agreement with the line-by-line calculation. However, for altitudes higher than 10 km, and in cases where higher accuracy is required, the line-by-line calculation should be used.

The specific attenuation for dry air,  $\gamma_o$  (dB/km), and the specific attenuation for moist air,  $\gamma_w$  (dB/km), are given by the following equations:

$$\gamma_o = 0.1820 f\left(\sum_{i(oxygen)} S_i F_i + N_D^{\prime\prime}(f)\right)$$
(22)

$$\gamma_w = 0.1820 f \sum_{i(Water \, Vapour)} S_i F_i \tag{23}$$

where  $S_i$ ,  $F_i$ , and  $N_D''(f)$  are defined in equations (3), (5), (6a), (7), (8), and (9) for oxygen, and  $S_i$ , and  $F_i$  are defined in equations (3), (4), (5), (6a), and (7) for water vapour. Equation (6b) is not included since Zeeman splitting of oxygen lines and Doppler broadening of water vapour lines does not need to be considered at altitudes below 10 km. The summation for oxygen is over all oxygen lines in Table 1, and the summation for water vapour is over the subset of 9 water vapour lines in Table 2 denoted with an asterisk.

The dry pressure, p, and the temperature, T, are the dry pressure and temperature at the surface of the Earth. If local data is not available, the mean annual global reference atmosphere given in Recommendation ITU-R P.835 can be used to determine the dry pressure and temperature at the altitude of the surface of the Earth.

Figure 5 shows the dry air (Dry), water vapour only with a density of 7.5 g/m<sup>3</sup> (Water Vapour), and total (Total) specific attenuation from 1 to 350 GHz at sea-level for the mean annual global reference atmosphere given in Recommendation ITU-R P.835.

# 2 Path attenuation

# 2.1 Terrestrial paths

For a horizontal path, or for slightly inclined paths close to the ground, the path attenuation, *A*, may be calculated as:

$$A = \gamma r_0 = (\gamma_o + \gamma_w) r_0 \quad dB \tag{24}$$

where  $r_0$  is the path length (km).



Specific attenuation due to atmospheric gases



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### 2.2 Slant paths

This section contains reduced complexity algorithms for estimating the gaseous attenuation along slant paths through the Earth's atmosphere, by defining an equivalent height by which the specific

attenuation calculated in paragraph 1 may be multiplied to obtain the zenith attenuation. The equivalent heights are dependent on pressure, and can hence be employed for determining the zenith attenuation from sea level up to an altitude of about 10 km. The resulting zenith attenuations are accurate to within  $\pm 10\%$  for dry air and  $\pm 5\%$  for water vapour from sea level up to altitudes of about 10 km, using the pressure, temperature and water-vapour density appropriate to the altitude of interest. For altitudes higher than 10 km, and particularly for frequencies within 0.5 GHz of the centres of resonance lines at any altitude, the procedure in Annex 1 should be used. Note that the Gaussian function in equation (25b) describing the oxygen equivalent height in the 60 GHz band can yield errors higher than 10% at certain frequencies, since this procedure is not intended to accurately reproduce the structure shown in Fig. 7. The expressions below were derived from zenith attenuations calculated with the procedure in Annex 1, integrating the attenuations numerically over a bandwidth of 500 MHz; the resultant attenuations effectively represent approximate minimum values in the 50-70 GHz band. The path attenuation at elevation angles other than the zenith may then be determined using the procedures described later in this section.

For dry air, the equivalent height is given by:

$$h_o = \frac{6.1}{1 + 0.17 r_p^{-1.1}} (1 + t_1 + t_2 + t_3)$$
(25a)

where:

$$t_1 = \frac{4.64}{1 + 0.066 r_p^{-2.3}} \exp\left[-\left(\frac{f - 59.7}{2.87 + 12.4 \exp((-7.9 r_p))}\right)^2\right]$$
(25b)

$$t_2 = \frac{0.14 \exp(2.12 r_p)}{(f - 118.75)^2 + 0.031 \exp(2.2 r_p)}$$
(25c)

$$t_{3} = \frac{0.0114}{1 + 0.14 r_{p}^{-2.6}} f \frac{-0.0247 + 0.0001f + 1.61 \times 10^{-6} f^{2}}{1 - 0.0169f + 4.1 \times 10^{-5} f^{2} + 3.2 \times 10^{-7} f^{3}}$$
(25d)

with the constraint that:

$$h_o \le 10.7 r_p^{0.3}$$
 when  $f < 70 \,\text{GHz}$  (25e)

and, for water vapour, the equivalent height is:

$$h_{w} = 1.66 \left( 1 + \frac{1.39\sigma_{w}}{(f - 22.235)^{2} + 2.56\sigma_{w}} + \frac{3.37\sigma_{w}}{(f - 183.31)^{2} + 4.69\sigma_{w}} + \frac{1.58\sigma_{w}}{(f - 325.1)^{2} + 2.89\sigma_{w}} \right) (26a)$$
  
for  $f \le 350 \text{ GHz}$   
 $\sigma_{w} = \frac{1.013}{1 + \exp\left[-8.6\left(r_{p} - 0.57\right)\right]}$  (26b)

where:

 $r_p = (p + e)/1013.25$ 

The zenith attenuation between 50 to 70 GHz is a complicated function of frequency, as shown in Fig. 7, and the above algorithms for equivalent height can provide only an approximate estimate, in general, of the minimum levels of attenuation likely to be encountered in this frequency range. For greater accuracy, the procedure in Annex 1 should be used.

The concept of equivalent height is based on the assumption of an exponential atmosphere specified by a scale height to describe the decay in density with altitude. Note that scale heights for both dry air and water vapour may vary with latitude, season and/or climate, and that water vapour distributions in the real atmosphere may deviate considerably from the exponential profile, with corresponding changes in equivalent heights. The values given above are applicable up to altitudes of about 10 km.

The total zenith attenuation is then:

$$A = \gamma_o h_o + \gamma_w h_w \qquad \text{dB} \tag{27}$$

Figure 6 shows the total zenith attenuation at sea level (Total), as well as the attenuation due to dry air (Dry) and water vapour (Water Vapour), using the mean annual global reference atmosphere given in Recommendation ITU-R P.835. Between 50 and 70 GHz, greater accuracy can be obtained from the 0 km curve in Fig. 7 which was derived using the line-by-line calculation described in Annex 1.

#### 2.2.1 Elevation angles between 5° and 90°

#### 2.2.1.1 Earth-space paths

For an elevation angle,  $\phi$ , between 5° and 90°, the path attenuation is obtained using the cosecant law, as follows:

For path attenuation based on surface meteorological data:

$$A = \frac{A_o + A_w}{\sin \phi} \qquad \text{dB} \tag{28}$$

where  $A_o = h_o \gamma_o$  and  $A_w = h_w \gamma_w$ 

and for path attenuation based on integrated water vapour content:

$$A = \frac{A_o + A_w}{\sin \phi} \qquad \text{dB} \tag{29}$$

where  $A_w$  is given in § 2.3.

#### 2.2.1.2 Inclined paths

To determine the attenuation values on an inclined path between a station situated at altitude  $h_1$  and another at a higher altitude  $h_2$ , where both altitudes are less than 10 km above mean sea level, the values  $h_o$  and  $h_w$  in equation (28) must be replaced by the following  $h'_o$  and  $h'_w$  values:

$$h'_{o} = h_{o} \left[ e^{-h_{1}/h_{o}} - e^{-h_{2}/h_{o}} \right]$$
 km (30)

$$h'_{w} = h_{w} \left[ e^{-h_{1}/h_{w}} - e^{-h_{2}/h_{w}} \right]$$
 km (31)

it being understood that the value  $\rho$  of the water-vapour density used in equation (23) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \times \exp\left(h_1/2\right) \tag{32}$$

where  $\rho_1$  is the value corresponding to altitude  $h_1$  of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).

Equations (30), (31) and (32) use different normalizations for the dry air and water-vapour equivalent heights. While the mean air pressure referred to sea level can be considered constant around the world

(equal to 1 013.25 hPa), the water-vapour density not only has a wide range of climatic variability but is measured at the surface (i.e. at the height of the ground station). For values of surface water-vapour density, see Recommendation ITU-R P.836.

#### 2.2.2 Elevation angles between 0° and 5°

#### 2.2.2.1 Earth-space paths

In this case, Annex 1 of this Recommendation should be used. Annex 1 should also be used for elevations less than zero.

#### 2.2.2.2 Inclined paths

The attenuation on an inclined path between a station situated at altitude  $h_1$  and a higher altitude  $h_2$  (where both altitudes are less than 10 km above mean sea level), can be determined from the following:

$$A = \gamma_{o} \sqrt{h_{o}} \left[ \frac{\sqrt{R_{e} + h_{1}} \cdot F(x_{1}) e^{-h_{1}/h_{o}}}{\cos \varphi_{1}} - \frac{\sqrt{R_{e} + h_{2}} \cdot F(x_{2}) e^{-h_{2}/h_{o}}}{\cos \varphi_{2}} \right]$$
$$+ \gamma_{w} \sqrt{h_{w}} \left[ \frac{\sqrt{R_{e} + h_{1}} \cdot F(x_{1}') e^{-h_{1}/h_{w}}}{\cos \varphi_{1}} - \frac{\sqrt{R_{e} + h_{2}} \cdot F(x_{2}') e^{-h_{2}/h_{w}}}{\cos \varphi_{2}} \right]$$
dB (33)

where:

- $R_e$ : effective Earth radius including refraction, given in Recommendation ITU-R P.834, expressed in km (a value of 8 500 km is generally acceptable for the immediate vicinity of the Earth's surface)
- $\varphi_1$ : elevation angle at altitude  $h_1$
- F: function defined by:

$$F(x) = \frac{1}{0.661 x + 0.339 \sqrt{x^2 + 5.51}}$$
(34)

$$\varphi_2 = \arccos\left(\frac{R_e + h_1}{R_e + h_2}\cos\varphi_1\right)$$
(35a)

$$x_i = \tan \varphi_i \sqrt{\frac{R_e + h_i}{h_o}} \qquad \text{for } i = 1, 2 \tag{35b}$$

$$x'_{i} = \tan \varphi_{i} \sqrt{\frac{R_{e} + h_{i}}{h_{w}}} \qquad \text{for } i = 1, 2 \qquad (35c)$$

it being understood that the value  $\rho$  of the water vapour density used in equation (23) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \cdot \exp\left(\frac{h_1}{2}\right) \tag{36}$$

where  $\rho_1$  is the value corresponding to altitude  $h_1$  of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).



Frequency (GHz)

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Zenith oxygen attenuation from the altitudes indicated, calculated at intervals of 10 MHz, including line centres (0 km, 5 km, 10 km, 15 km and 20 km)

FIGURE 7

Values for  $\rho_1$  at the surface can be found in Recommendation ITU-R P.836.

The different formulation for dry air and water vapour is explained in § 2.2.2.2.

## 2.3 Zenith path water-vapour attenuation

The above method for calculating slant path attenuation relies on knowledge of the surface watervapour density. If integrated water vapour content,  $V_t$ , is known, the total water-vapour attenuation can be estimated as follows:

$$A_{w} = \begin{cases} \frac{0.0176 V_{t} \gamma_{w}(f, p_{ref}, \rho_{v, ref}, t_{ref})}{\gamma_{w}(f_{ref}, \rho_{ref}, \rho_{v, ref}, t_{ref})}, & 1 \text{ GHz} \le f \le 20 \text{ GHz} \\ \frac{0.0176 V_{t} \gamma_{w}(f, p_{ref}, \rho_{v, ref}, t_{ref})}{\gamma_{w}(f_{ref}, \rho_{ref}, \rho_{v, ref}, t_{ref})} (ah^{b} + 1), & 20 \text{ GHz} < f \le 350 \text{ GHz} \end{cases}$$
(37)

where:

$$a = 0.2048 \exp\left[-\left(\frac{f - 22.43}{3.097}\right)^2\right] + 0.2326 \exp\left[-\left(\frac{f - 183.5}{4.096}\right)^2\right] + 0.2073 \exp\left[-\left(\frac{f - 325}{3.651}\right)^2\right] - 0.113$$
  
$$b = 8.741 \times 10^4 \exp(-0.587f) + 312.2f^{-2.38} + 0.723$$
  
$$h = \begin{cases} h_s & h_s \le 4 \ km \\ 4 & h_s > 4 \ km \end{cases}$$

and

$$f$$
: frequency (GHz)  
 $f_{ref}$ : 20.6 (GHz)  
 $p_{ref} = 815$  (hPa)

$$\rho_{v,ref} = \frac{V_t}{3.67} \text{ (g/m^3)}$$
$$t_{ref} = 14 \ln \left(\frac{0.22 V_t}{3.67}\right) + 3 \text{ (°C)}$$

- $V_t$ : integrated water vapour content from: a) local radiosonde or radiometric data or b) at the required percentage of time (kg/m<sup>2</sup> or mm) obtained from the digital maps in Recommendation ITU-R P.836 (kg/m<sup>2</sup> or mm)
- $\gamma_{W}(f, p, \rho, t)$ : specific attenuation as a function of frequency, pressure, water-vapour density, and temperature calculated from equation (23) (dB/km)
  - $h_s$ : station height above mean sea level (a.m.s.l) (km).