The ITU Radiocommunication Assembly,

*considering*

a) the necessity of utilizing a single formula for calculation of the index of refraction of the atmosphere;
b) the need for reference data on refractivity and refractivity gradients all over the world;
c) the necessity to have a mathematical method to express the statistical distribution of refractivity gradients,

*recommends*

1 that the atmospheric radio refractive index, $n$, be computed by means of the formula given in Annex 1;
2 that refractivity data given on world charts in Annex 1 should be used, except if more reliable local data are available;
3 that the statistical distribution of refractivity gradients be computed using the method given in Annex 1.

ANNEX 1

### 1 The formula for the radio refractive index

The atmospheric radio refractive index, $n$, can be computed by the following formula:

$$ n = 1 + N \times 10^{-6} $$

(1)

where:

$N$: radio refractivity expressed by:

$$ N = N_{\text{dry}} + N_{\text{wet}} = \frac{77.6}{T} \left( P + 4810 \frac{e}{T} \right) $$

(2)

with the “dry term” of radio refractivity given by:

$$ N_{\text{dry}} = 77.6 \frac{P}{T} $$

(3)

and the “wet term” by:

$$ N_{\text{wet}} = 3.732 \times 10^5 \frac{e}{T^2} $$

(4)

where:

$P$: atmospheric pressure (hPa)

$e$: water vapour pressure (hPa)

$T$: absolute temperature (K).
This expression may be used for all radio frequencies; for frequencies up to 100 GHz, the error is less than 0.5%. For representative profiles of temperature, pressure and water vapour pressure see Recommendation ITU-R P.835.

For ready reference, the relationship between water vapour pressure $e$ and relative humidity is given by:

$$ e = \frac{H e_s}{100} $$  \hspace{1cm} (5)

with:

$$ e_s = a \exp \left( \frac{b t}{t + c} \right) $$  \hspace{1cm} (6)

where:

- $H$: relative humidity (%)
- $t$: Celsius temperature (°C)
- $e_s$: saturation vapour pressure (hPa) at the temperature $t$ (°C) and the coefficients $a$, $b$, $c$, are:
  - for water
    - $a = 6.1121$
    - $b = 17.502$
    - $c = 240.97$
  - for ice
    - $a = 6.1115$
    - $b = 22.452$
    - $c = 272.55$

Vapour pressure $e$ is obtained from the water vapour density $\rho$ using the equation:

$$ e = \frac{\rho T}{216.7} \text{ hPa} $$  \hspace{1cm} (7)

where $\rho$ is given in g/m³. Representative values of $\rho$ are given in Recommendation ITU-R P.836.

### 2 Surface refractivity and height dependence

It has been found that the long-term mean dependence of the refractive index $n$ upon the height $h$ is well expressed by an exponential law:

$$ n(h) = 1 + N_0 \times 10^{-6} \times \exp \left( -h / h_0 \right) $$  \hspace{1cm} (8)

where:

- $N_0$: average value of atmospheric refractivity extrapolated to sea level
- $h_0$: scale height (km).

$N_0$ and $h_0$ can be determined statistically for different climates. For reference purposes a global mean of the height profile of refractivity may be defined by:

$$ N_0 = 315 $$

$$ h_0 = 7.35 \text{ km} $$

These numerical values apply only for terrestrial paths.

This reference profile may be used to compute the value of refractivity $N_s$ at the Earth’s surface from $N_0$ as follows:

$$ N_s = N_0 \exp \left( -h_s / h_0 \right) $$  \hspace{1cm} (9)

where:

- $h_s$: height of the Earth’s surface above sea level (km).

It is to be noted, however, that the contours of Figs. 1 and 2 were derived using a value of $h_0$ equal to 9.5 km.
For Earth-satellite paths, the refractive index at any height is obtained using equations (1), (2) and (7) above, together with the appropriate values for the parameters given in Recommendation ITU-R P.835, Annex 1. The refractive indices thus obtained may then be used for numerical modelling of ray paths through the atmosphere.

(Note that the exponential profile in equation (9) may also be used for quick and approximate estimates of refractivity gradient near the Earth’s surface and of the apparent boresight angle, as given in § 4.2 of Recommendation ITU-R P.834.)

3 Vertical refractivity gradients

The vertical gradient of radio refractivity in the lowest layer of the atmosphere is an important parameter for the estimation of path clearance and propagation effects such as ducting, surface reflection and multipath on terrestrial line-of-sight links.

Figures 3 to 6 present isopleths of monthly mean decrease (i.e. lapse) in radio refractivity over a 1 km layer from the surface. The change in radio refractivity, $\Delta N$, was calculated from:

$$\Delta N = N_3 - N_1$$

where $N_1$ is the radio refractivity at a height of 1 km above the surface of the Earth. The $\Delta N$ values were not reduced to a reference surface.

Refractivity gradient statistics for the lowest 100 m from the surface of the Earth are used to estimate the probability of occurrence of ducting and multipath conditions. Where more reliable local data are not available, the charts in Figs. 7 to 10 give such statistics for the world.
FIGURE 2
Monthly mean values of $N_0$: August
FIGURE 3
Monthly mean values of $\Delta N$: February

FIGURE 4
Monthly mean values of $\Delta N$: May
FIGURE 5
Monthly mean values of $\Delta N$: August

FIGURE 6
Monthly mean values of $\Delta N$: November
FIGURE 7
Percentage of time gradient $\leq -100$ (N-units/km): February

FIGURE 8
Percentage of time gradient $\leq -100$ (N-units/km): May
FIGURE 9
Percentage of time gradient \( \leq -100 \) N-units/km: August

FIGURE 10
Percentage of time gradient \( \leq -100 \) N-units/km: November
4 Statistical distribution of refractivity gradients

It is possible to estimate the complete statistical distribution of refractivity gradients near the surface of the Earth over the lowest 100 m of the atmosphere from the median value \( \text{Med} \) of the refractivity gradient and the ground level refractivity value \( N_s \) for the location being considered.

The median value \( \text{Med} \) of the refractivity gradient distribution may be computed from the probability \( P_0 \) that the refractivity gradient is lower than or equal to \( D_n \) using the following expression:

\[
\text{Med} = \frac{D_n + k_1}{(1/P_0 - 1)^{1/E_0}} - k_1
\]

(11)

where:

\[
E_0 = \log_{10} \left( |D_n| \right)
\]

\[
k_1 = 30
\]

Equation (11) is valid for the interval \(-300 \) N-units/km \( \leq D_n \leq -40 \) N-units/km.

If this probability \( P_0 \) corresponding to any given \( D_n \) value of refractivity gradient is not known for the location under study, it is possible to derive \( P_0 \) from the world maps in Figs. 7 to 10 which give the percentage of time during which the refractivity gradient over the lowest 100 m of the atmosphere is less than or equal to \(-100 \) N-units/km.

Where more reliable local data are not available, \( N_s \) may be derived from the global sea level refractivity \( N_0 \) maps of Figs. 1 and 2 and equation (9).

For \( D_n \leq \text{Med} \), the cumulative probability \( P_1 \) of \( D_n \) may be obtained from:

\[
P_1 = \frac{1}{1 + \left[ \left( \frac{|D_n - \text{Med}|}{B} + k_2 \right) k_3 \right]^{E_1}}
\]

(12)

where:

\[
B = \left| \frac{0.3 \text{Med} - 210}{2} \right|
\]

\[
E_1 = \log_{10}(F + 1)
\]

\[
F = \frac{2 \times |D_n - \text{Med}|}{\left( \frac{B}{67} \right)^{6.5} + 1}
\]

\[
k_2 = \frac{1.6B}{120}
\]

\[
k_3 = \frac{120}{B}
\]

Equation (12) is valid for values of \( \text{Med} \geq -120 \) N-units/km and for the interval \(-300 \) N-units/km \( \leq D_n \leq 50 \) N-units/km.
For $D_n > Med$, the cumulative probability $P_2$ of $D_n$ is computed from:

$$P_2 = 1 - \frac{1}{1 + \left[\left(\frac{|D_n - Med|}{B} + k_4\right)k_4\right]E_1}$$  \hspace{1cm} (13)$$

where:

$$B = \frac{|0.3 Med - N_s + 210|}{2}$$

$$E_1 = \log_{10}(F + 1)$$

$$F = \frac{2 \times |D_n - Med|}{\left(\frac{B}{67}\right)^{6.5}} + 1$$

$$k_4 = \left[\frac{100}{B}\right]^{2.4}$$

Equation (13) is valid for values of $Med > –120$ N-units/km and for the interval $–300$ N-units/km $< D_n < 50$ N-units/km.