RECOMMENDATION ITU-R P.453-7

THE RADIO REFRACTIVE INDEX: ITS FORMULA AND REFRACTIVITY DATA

(Question ITU-R 201/3)

(1970-1986-1990-1992-1994-1995-1997-1999)

The ITU Radiocommunication Assembly,

considering

a) the necessity of utilizing a single formula for calculation of the index of refraction of the atmosphere;

b) the need for reference data on refractivity and refractivity gradients all over the world;

c) the necessity to have a mathematical method to express the statistical distribution of refractivity gradients,

recommends

1 that the atmospheric radio refractive index, *n*, be computed by means of the formula given in Annex 1;

2 that refractivity data given on world charts in Annex 1 should be used, except if more reliable local data are available;

3 that the statistical distribution of refractivity gradients be computed using the method given in Annex 1.

ANNEX 1

1 The formula for the radio refractive index

The atmospheric radio refractive index, *n*, can be computed by the following formula:

$$n = 1 + N \times 10^{-6} \tag{1}$$

where:

N: radio refractivity expressed by:

$$N = N_{dry} + N_{wet} = \frac{77.6}{T}\left(P + 4810\,\frac{e}{T}\right) \tag{2}$$

with the “dry term” of radio refractivity given by:

$$N_{dry} = 77.6\,\frac{P}{T} \tag{3}$$

and the “wet term” by:

$$N_{wet} = 3.732 \times 10^5\,\frac{e}{T^2} \tag{4}$$

where:

P: atmospheric pressure (hPa)

e: water vapour pressure (hPa)

T: absolute temperature (K).

This expression may be used for all radio frequencies; for frequencies up to 100 GHz, the error is less than 0.5%. For representative profiles of temperature, pressure and water vapour pressure see Recommendation ITU-R P.835.

For ready reference, the relationship between water vapour pressure e and relative humidity is given by:

$$e = \frac{H\, e_s}{100} \tag{5}$$

with:

$$e_s = a \, \exp\left(\frac{b\, t}{t + c}\right) \tag{6}$$

where:

H: relative humidity (%)

t: Celsius temperature (°C)

e_s: saturation vapour pressure (hPa) at the temperature t (°C) and the coefficients a, b, c, are:

for water	*for ice*
a = 6.1121	a = 6.1115
b = 17.502	b = 22.452
c = 240.97	c = 272.55
(valid between –20° to +50°, with an accuracy of ±0.20%)	(valid between –50° to 0°, with an accuracy of ±0.20%)

Vapour pressure e is obtained from the water vapour density ρ using the equation:

$$e = \frac{\rho T}{216.7} \quad \text{hPa} \tag{7}$$

where ρ is given in g/m^3. Representative values of ρ are given in Recommendation ITU-R P.836.

2 Surface refractivity and height dependence

It has been found that the long-term mean dependence of the refractive index n upon the height h is well expressed by an exponential law:

$$n(h) = 1 + N_0 \times 10^{-6} \times \exp(-h/h_0) \tag{8}$$

where:

N_0: average value of atmospheric refractivity extrapolated to sea level

h_0: scale height (km).

N_0 and h_0 can be determined statistically for different climates. For reference purposes a global mean of the height profile of refractivity may be defined by:

$$N_0 = 315$$

$$h_0 = 7.35 \text{ km}$$

These numerical values apply only for terrestrial paths.

This reference profile may be used to compute the value of refractivity N_s at the Earth's surface from N_0 as follows:

$$N_s = N_0 \exp(-h_s/h_0) \tag{9}$$

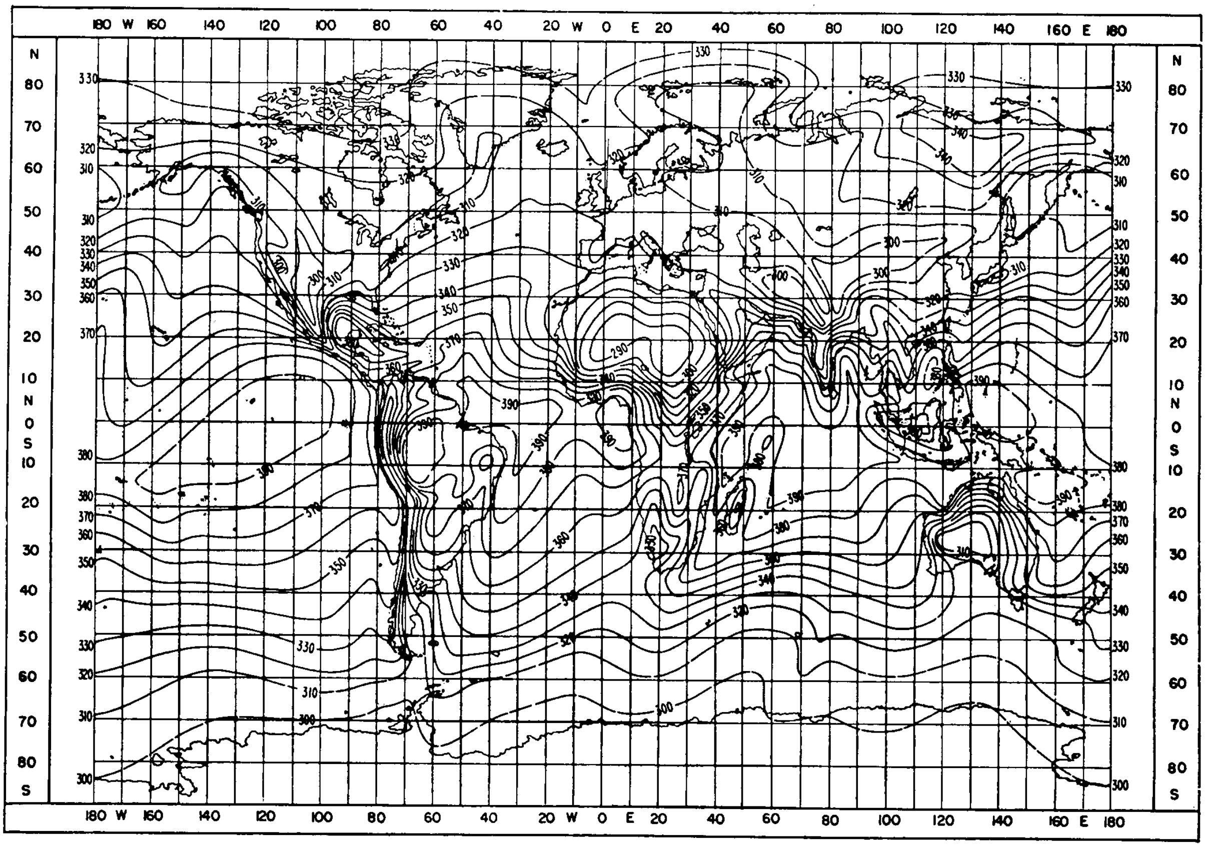
where:

h_s: height of the Earth's surface above sea level (km).

It is to be noted, however, that the contours of Figs. 1 and 2 were derived using a value of h_0 equal to 9.5 km.

FIGURE 1

Monthly mean values of N_0: February



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For Earth-satellite paths, the refractive index at any height is obtained using equations (1), (2) and (7) above, together with the appropriate values for the parameters given in Recommendation ITU-R P.835, Annex 1. The refractive indices thus obtained may then be used for numerical modelling of ray paths through the atmosphere.

(Note that the exponential profile in equation (9) may also be used for quick and approximate estimates of refractivity gradient near the Earth's surface and of the apparent boresight angle, as given in § 4.2 of Recommendation ITU-R P.834.)

3 Vertical refractivity gradients

The vertical gradient of radio refractivity in the lowest layer of the atmosphere is an important parameter for the estimation of path clearance and propagation effects such as ducting, surface reflection and multipath on terrestrial line-of-sight links.

Figures 3 to 6 present isopleths of monthly mean decrease (i.e. lapse) in radio refractivity over a 1 km layer from the surface. The change in radio refractivity, ΔN, was calculated from:

$$\Delta N = N_s - N_1 \tag{10}$$

where N_1 is the radio refractivity at a height of 1 km above the surface of the Earth. The ΔN values were not reduced to a reference surface.

Refractivity gradient statistics for the lowest 100 m from the surface of the Earth are used to estimate the probability of occurrence of ducting and multipath conditions. Where more reliable local data are not available, the charts in Figs. 7 to 10 give such statistics for the world.

FIGURE 2

Monthly mean values of N_0: August

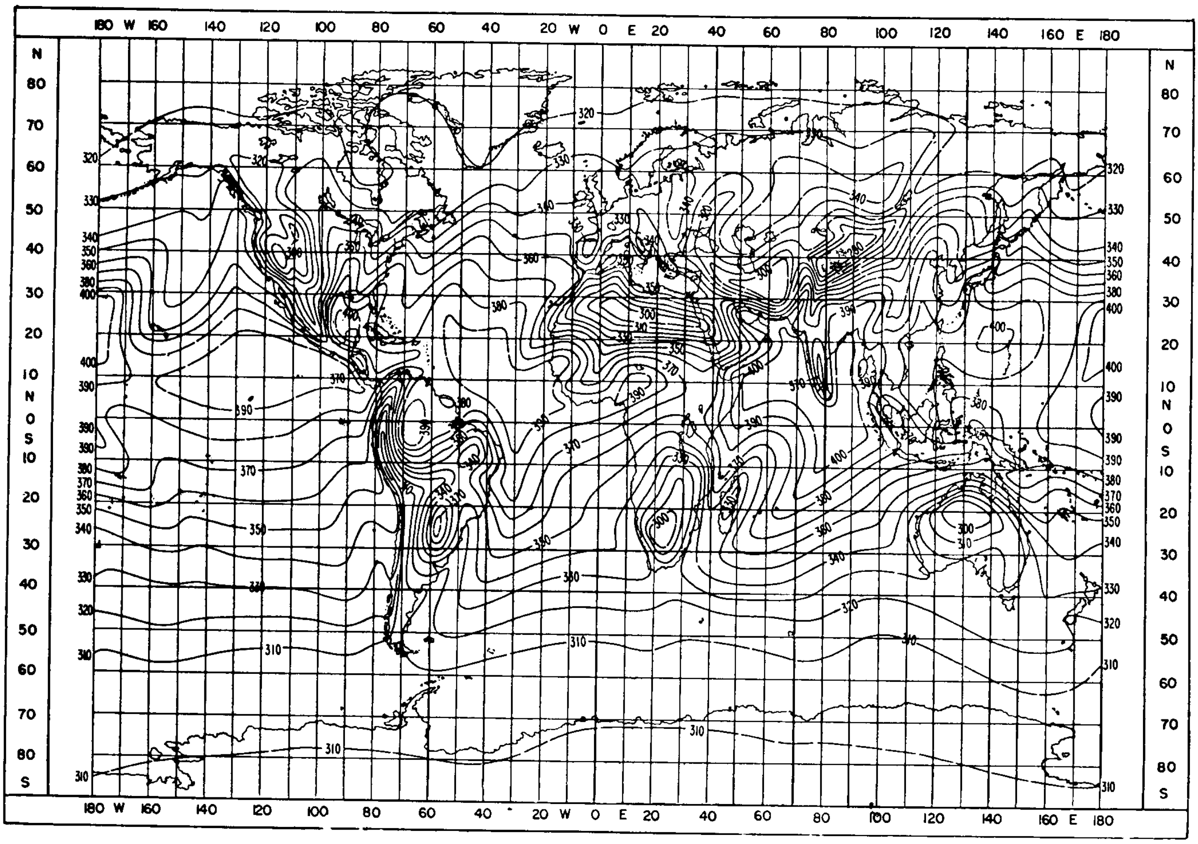

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FIGURE 3

Monthly mean values of Δ*N*: February

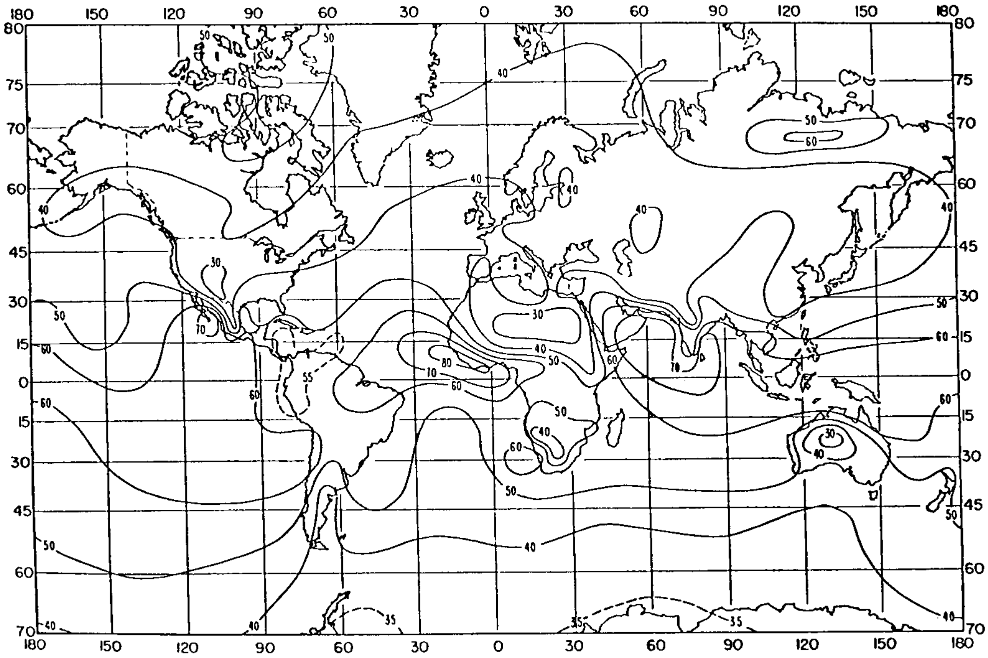

FIGURE 4

Monthly mean values of Δ*N*: May

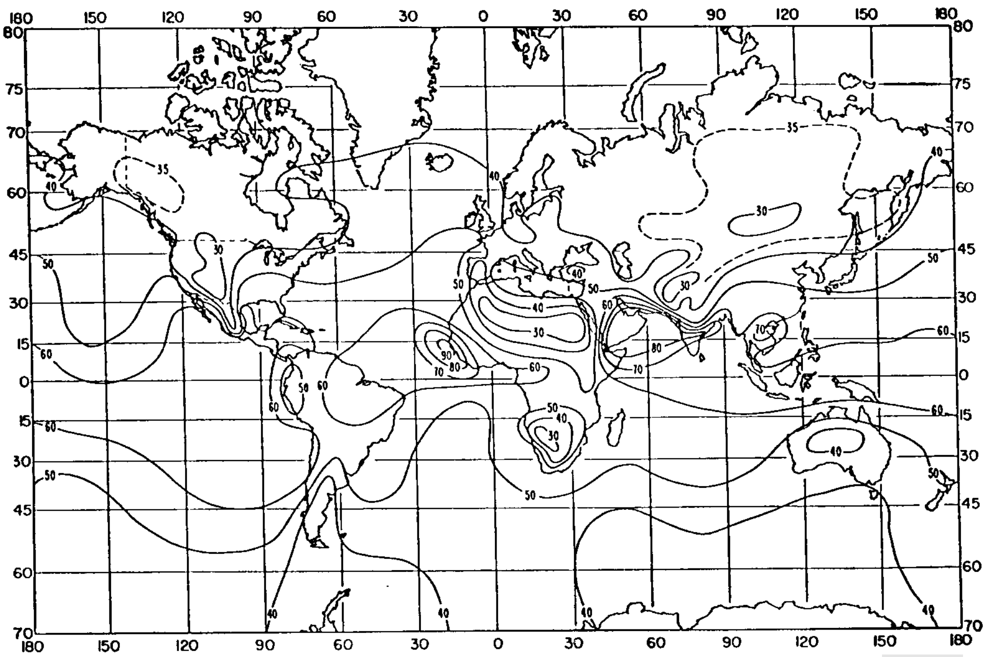

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FIGURE 5

Monthly mean values of Δ*N*: August

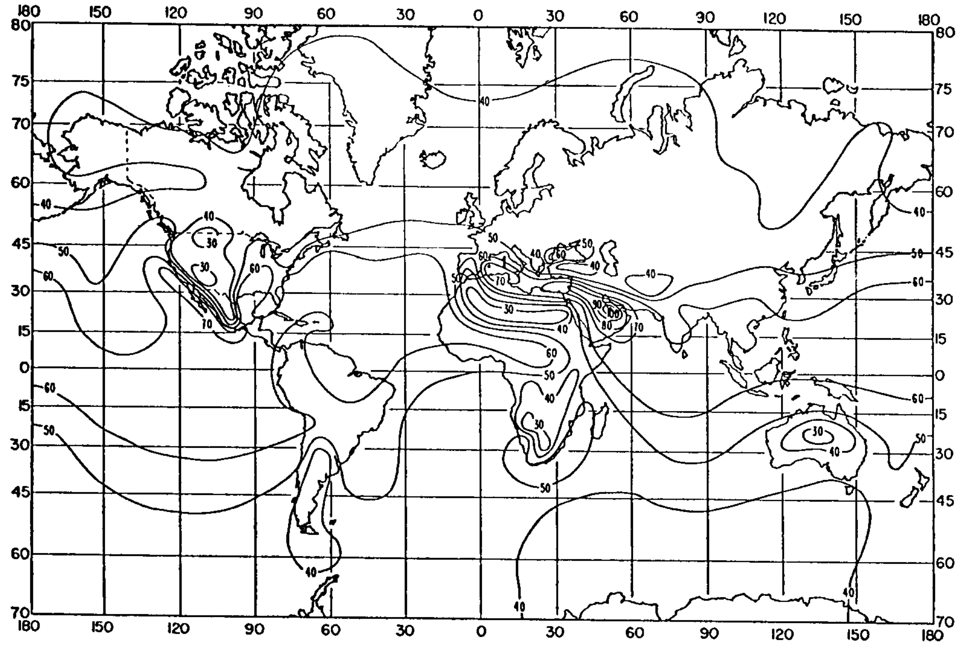

FIGURE 6

Monthly mean values of Δ*N*: November

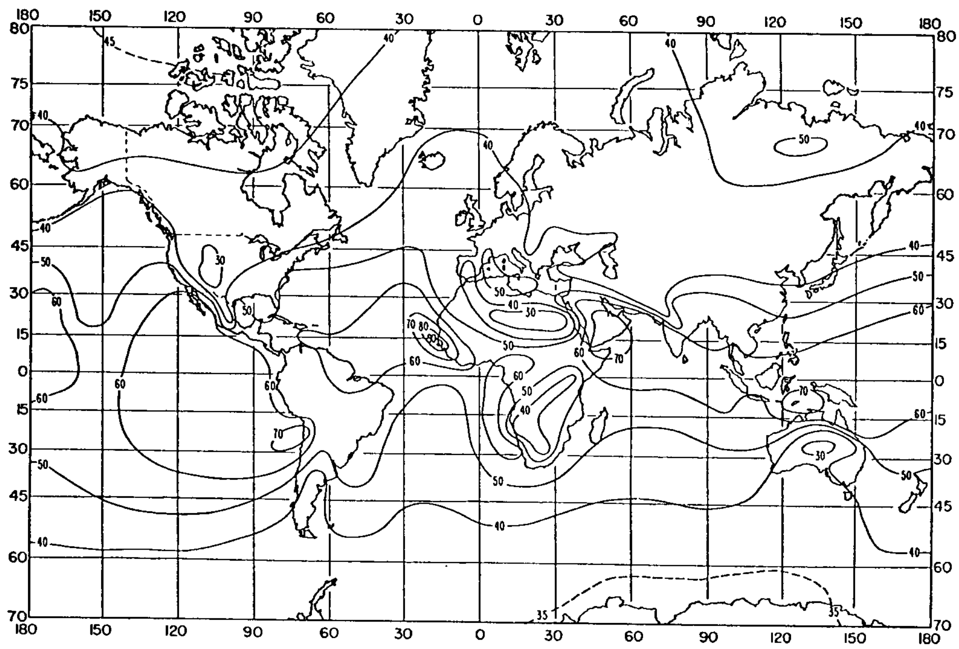

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FIGURE 7
Percentage of time gradient ≤ – 100 (N-units/km): February

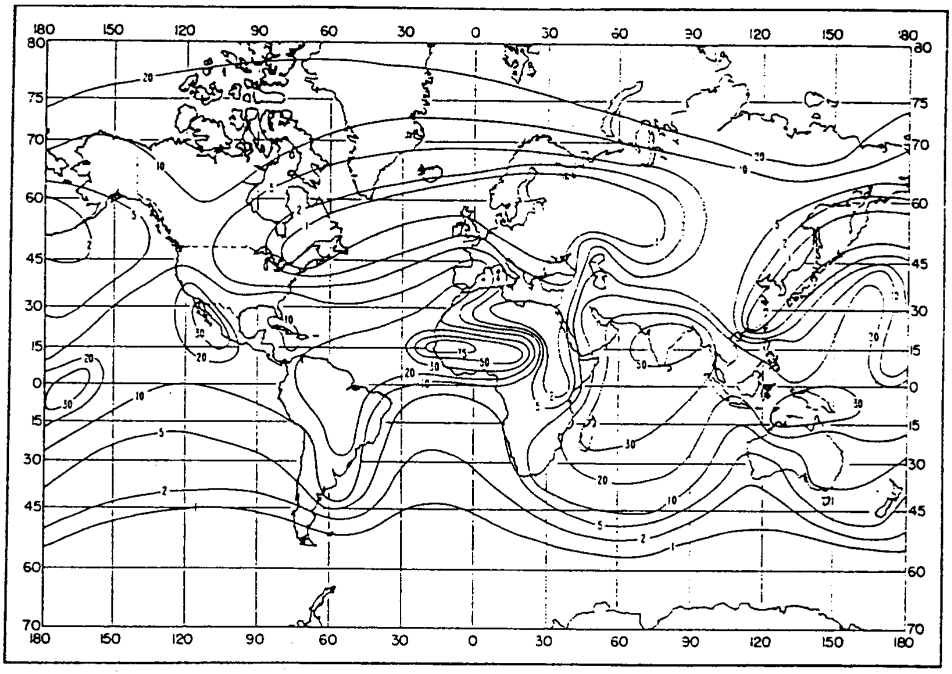

FIGURE 8
Percentage of time gradient ≤ – 100 (N-units/km): May

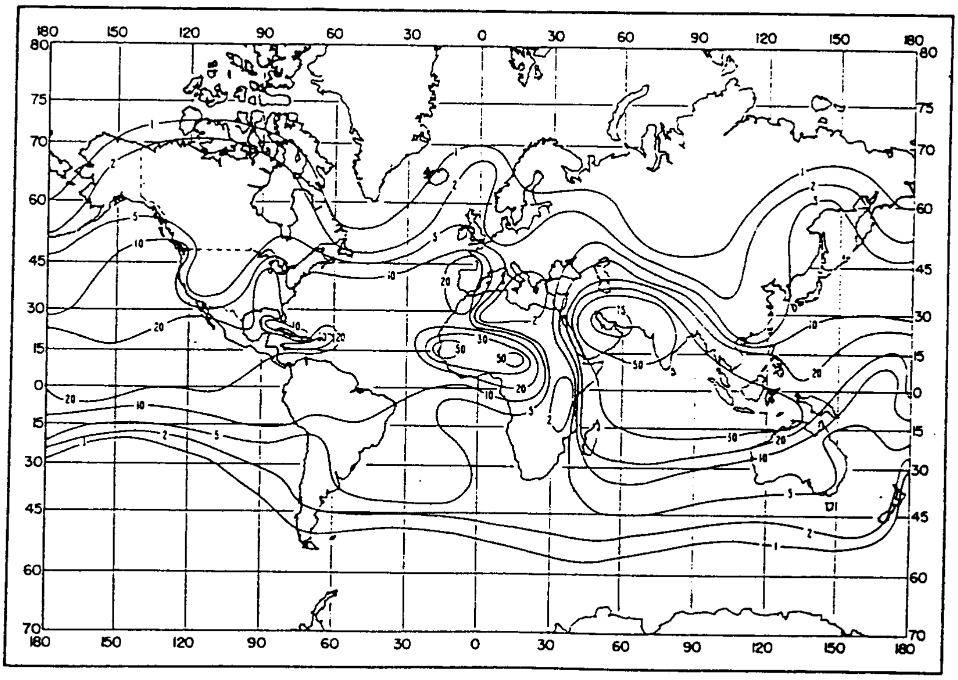

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FIGURE 9

Percentage of time gradient ≤ – 100 N-units/km: August

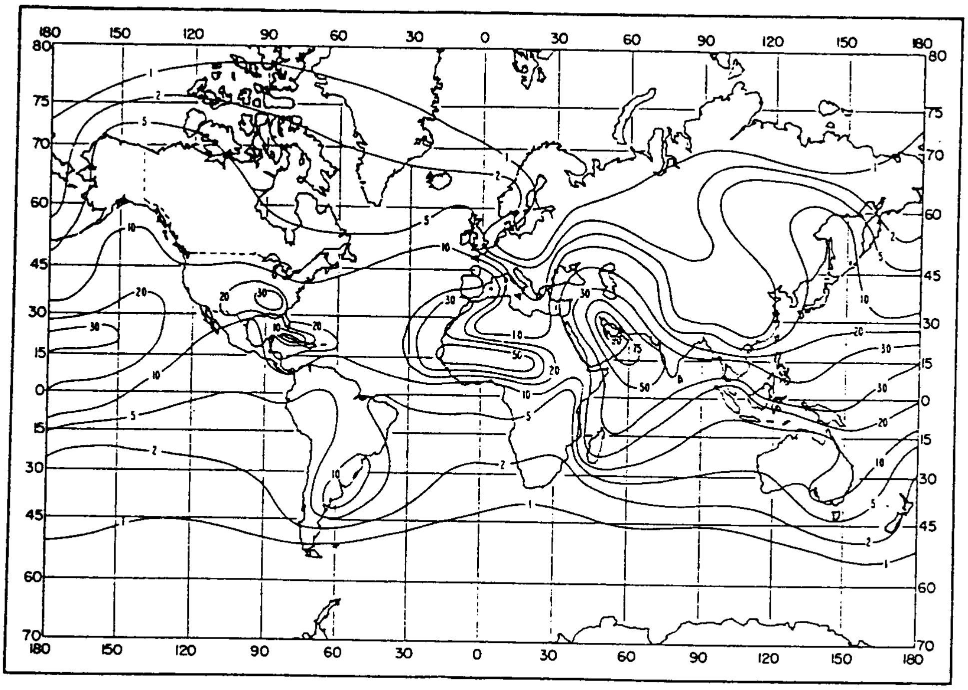
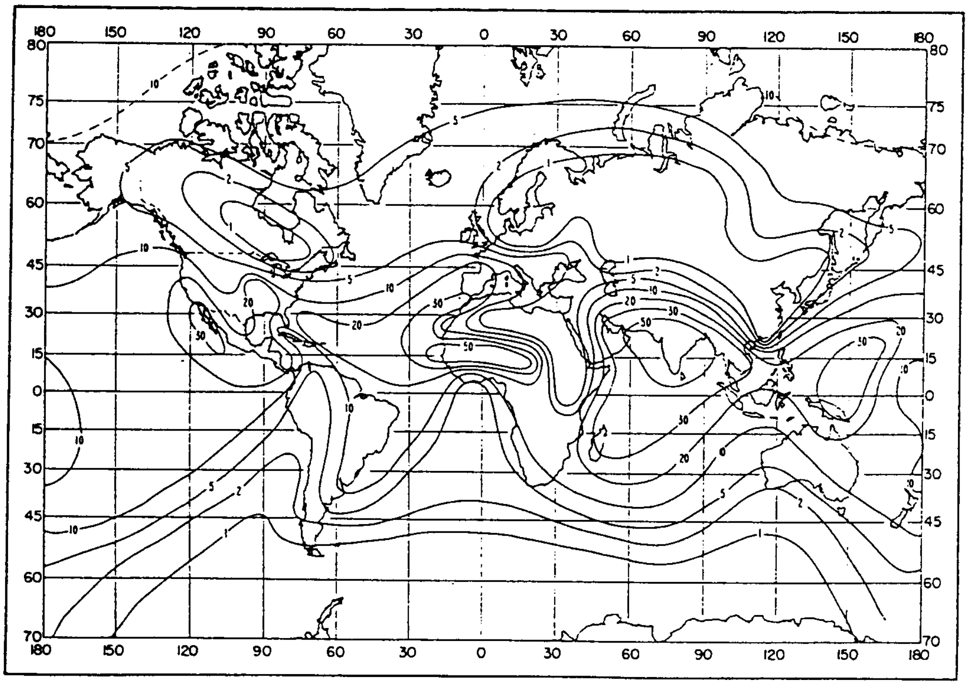


FIGURE 10

Percentage of time gradient ≤ – 100 N-units/km: November



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4 Statistical distribution of refractivity gradients

It is possible to estimate the complete statistical distribution of refractivity gradients near the surface of the Earth over the lowest 100 m of the atmosphere from the median value *Med* of the refractivity gradient and the ground level refractivity value N_s for the location being considered.

The median value *Med* of the refractivity gradient distribution may be computed from the probability P_0 that the refractivity gradient is lower than or equal to D_n using the following expression:

$$Med = \frac{D_n + k_1}{(1/P_0 - 1)^{1/E_0}} - k_1 \tag{11}$$

where:

$$E_0 = \log_{10}(|D_n|)$$

$$k_1 = 30$$

Equation (11) is valid for the interval –300 N-units/km ≤ D_n ≤ –40 N-units/km.

If this probability P_0 corresponding to any given D_n value of refractivity gradient is not known for the location under study, it is possible to derive P_0 from the world maps in Figs. 7 to 10 which give the percentage of time during which the refractivity gradient over the lowest 100 m of the atmosphere is less than or equal to –100 N-units/km.

Where more reliable local data are not available, N_s may be derived from the global sea level refractivity N_0 maps of Figs. 1 and 2 and equation (9).

For D_n ≤ Med, the cumulative probability P_1 of D_n may be obtained from:

$$P_1 = \frac{1}{1 + \left[\left(\frac{|D_n - Med|}{B} + k_2\right)k_3\right]^{E_1}} \tag{12}$$

where:

$$B = \left|\frac{0.3\,Med - N_s + 210}{2}\right|$$

$$E_1 = \log_{10}(F + 1)$$

$$F = \frac{2 \times |D_n - Med|}{\left(\frac{B}{67}\right)^{6.5} + 1}$$

$$k_2 = \frac{1.6B}{120}$$

$$k_3 = \frac{120}{B}$$

Equation (12) is valid for values of *Med* > –120 N-units/km and
for the interval –300 N-units/km < D_n < 50 N-units/km.

For $D_n > Med$, the cumulative probability P_2 of D_n is computed from:

$$P_2 = 1 - \frac{1}{1 + \left[\left(\frac{|D_n - Med|}{B} + k_2\right)k_4\right]^{E_1}} \tag{13}$$

where:

$$B = \left|\frac{0.3\,Med - N_s + 210}{2}\right|$$

$$E_1 = \log_{10}(F + 1)$$

$$F = \frac{2 \times |D_n - Med|}{\left(\frac{B}{67}\right)^{6.5} + 1}$$

$$k_4 = \left[\frac{100}{B}\right]^{2.4}$$

Equation (13) is valid for values of $Med > -120$ N-units/km and for the interval -300 N-units/km $< D_n <$ 50 N-units/km.