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Radiocommunication Sector of ITU

Recommendation ITU-R P.1853-2
(08/2019)

**Time series synthesis of
tropospheric impairments**

P Series
Radiowave propagation

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Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

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RECOMMENDATION ITU-R P.1853-2

Time series synthesis of tropospheric impairments

(2009-2011-2019)

Scope

This Recommendation provides methods to synthesize time series of single-site and multi-site tropospheric impairments (gaseous, cloud, rain attenuation, and scintillation fading and enhancement) on Earth-space paths and a method to synthesize time series of rain attenuation on single terrestrial paths.

Keywords

Oxygen attenuation, water vapour attenuation, cloud attenuation, rain attenuation, scintillation, single slant path attenuation, multi-site, slant path attenuation, terrestrial single link rain attenuation total, attenuation, time series synthesizer, multi-link

The ITU Radiocommunication Assembly,

considering

- a) that for the proper planning of terrestrial and Earth-space systems it is necessary to have appropriate methods to synthesize the spatial variation and time dynamics of the propagation channel;
- b) that methods have been developed to synthesize the spatial variation and time dynamics of the propagation channel with sufficient accuracy,

recommends

1 that the methods given in Annex 1 should be used to synthesize the time series of the single-site and multi-site individual tropospheric attenuation components and of the unit-variance tropospheric scintillation on Earth-space paths. In particular:

- a) Sections 2.2 and 2.3 should be used to synthesize the time series of oxygen gaseous attenuation for single-site and multi-site paths, respectively;
- b) Sections 3.1 and 3.2 should be used to synthesize the time series of water vapour attenuation for single-site and multi-site paths, respectively;
- c) Sections 4.1 and 4.2 should be used to synthesize the time series of cloud attenuation for single-site and multi-site paths, respectively;
- d) Sections 5.1 and 5.2 should be used to synthesize the time series of rain attenuation for single-site and multi-site paths, respectively;
- e) Section 6 should be used to synthesize the unit-variance tropospheric scintillation,

2 that the methods given in Annex 2 should be used to synthesize the time series of the single-site and multi-site total tropospheric impairments on Earth-space paths. In particular:

- a) Section 2 should be used to synthesize single-site total impairments time series;
- b) Section 3 should be used to synthesize multi-site total impairments time series;

3 that the method given in Annex 3 should be used to synthesize the time series of rain attenuation on single terrestrial paths.

Annex 1

Time series synthesis of single-site and multi-site individual tropospheric impairments for Earth-space paths

1 Introduction

The methodology presented in this Annex provides techniques to synthesize single-site and multi-site individual and composite tropospheric impairments (gaseous, cloud, and rain attenuation, and unit-variance tropospheric scintillation fading and enhancement) on Earth-space paths.

2 Oxygen attenuation

The methods described below should be used to synthesize a constant value of oxygen gaseous attenuation.

2.1 Digital maps

The annual mean surface pressure data, P_{annual} (hPa), and the annual mean surface water vapour density data, WV_{annual} (g/m^3) are integral parts of this Recommendation and are available as digital maps in the zip file [R-REC-P.1853-2-201908-I!!ZIP-E.zip](#). The latitude grid is from -90° N to $+90^\circ$ N in 0.75° steps, and the longitude grid is from -180° E to $+180^\circ$ E in 0.75° steps.

The annual mean surface pressure and the annual mean surface water vapour density at any desired location at the surface of the Earth can be derived according to the following steps:

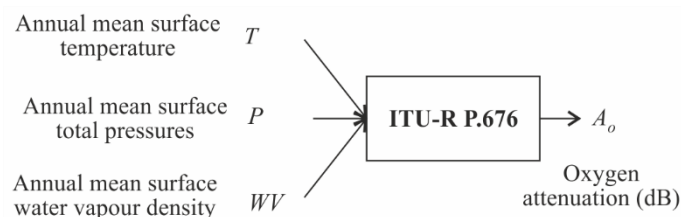
- determine the four grid points (Lat_1, Lon_1) , (Lat_2, Lon_2) , (Lat_3, Lon_3) and (Lat_4, Lon_4) surrounding the desired location (Lat, Lon) ;
- determine the annual mean surface pressures, P_1 , P_2 , P_3 , and P_4 , or the annual mean surface water vapour densities, WV_1 , WV_2 , WV_3 , and WV_4 at the surface of the Earth at the four surrounding grid points;
- determine P or WV at the desired location (Lat, Lon) by performing a bi-linear interpolation using the four surrounding grid points as described in Recommendation ITU-R P.1144.

The annual mean surface pressure and the annual mean surface water vapour density maps have been derived from 36 years (1979-2014) of European Centre of Medium-range Weather Forecast (ECMWF) ERA Interim data.

2.2 Single-site configuration

FIGURE 1

Block diagram of single-site oxygen gaseous attenuation time series synthesizer



Step SS_OX_1: Calculate the annual mean surface temperature T for the location of interest using local data. If local data is not available, the method provided in Recommendation ITU-R P.1510 can be used to predict T .

Step SS_OX_2: Calculate the annual mean surface pressure P for the location of interest using local data. If local data is not available, the method provided in § 2.1 can be used to predict P .

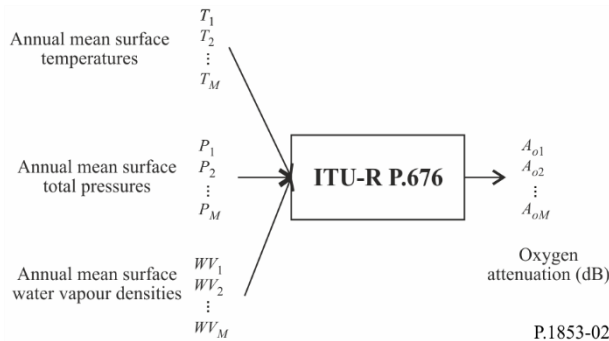
Step SS_OX_3: Calculate the annual mean surface water vapour density WV for the location of interest using local data. If local data is not available, the method provided in § 2.1 of this Annex can be used to predict WV .

Step SS_OX_4: Convert the annual mean temperature T , the annual mean surface pressure P , and the annual mean surface water vapour density WV into annual mean oxygen attenuation $A_O = \frac{h_O \gamma_O}{\sin \phi}$ according to the method “Approximate estimation of gaseous attenuation in the frequency range 1-350 GHz” described in Recommendation ITU-R P.676 for the frequency, elevation angle, and altitude of interest.

2.3 Multi-site configuration

FIGURE 2

Block diagram of multi-site oxygen gaseous attenuation time series synthesizer



Step MS_OX_1: Calculate the annual mean surface temperatures T_i , where $i = \{1, 2, \dots M\}$, for the M locations of interest using local data. If local data is not available, the method provided in Recommendation ITU-R P.1510 can be used to predict T_i .

Step MS_OX_2: Calculate the annual mean surface pressures P_i , where $i = \{1, 2, \dots M\}$, for the M locations of interest using local data. If local data is not available, the method provided in § 2.1 of this Annex can be used to predict P_i .

Step MS_OX_3: Calculate the annual mean surface water vapour densities WV_i , where $i = \{1, 2, \dots M\}$, for the M locations of interest using local data. If local data is not available, the method provided in § 2.1 of this Annex can be used to predict WV_i .

Step MS_OX_4: Convert the annual mean temperature T_i , the annual mean surface pressure P_i , and the annual mean surface water vapour density WV_i into annual mean oxygen attenuation $A_{O_i} = \frac{h_{O_i} \gamma_{O_i}}{\sin \phi_i}$ according to the method “Approximate estimation of gaseous attenuation in the frequency range 1-350 GHz” described in Recommendation ITU-R P.676 for the frequency, elevation angles, and altitudes of interest.

3 Water vapour attenuation

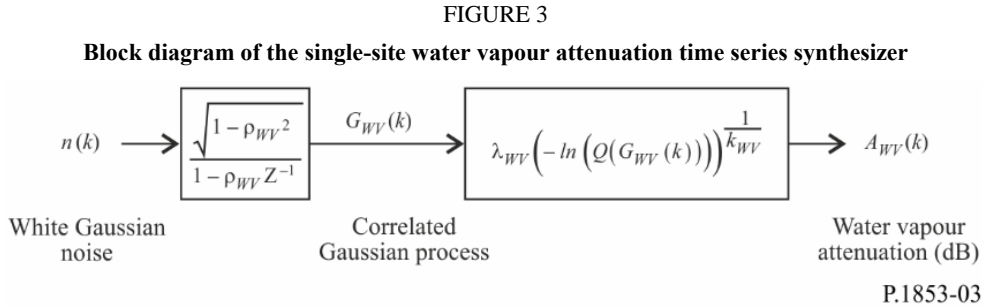
3.1 Single-site configuration

3.1.1 Overview

The single-site time series synthesis method assumes that the long-term statistics of water vapour attenuation (A_{wv}) is a Weibull distribution.

The single-site time series synthesis method synthesizes a time series that reproduces the spectral characteristics and probability distribution of water vapour attenuation.

As shown in Fig. 3, the water vapour attenuation discrete time series, $A_{wv}(t)=A_{wv}(kT_s)$, where T_s is the desired sampling time, is synthesized from the discrete white Gaussian noise process, $n(t)=n(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noise is low-pass filtered and transformed from a normal distribution to a Weibull distribution in a memoryless non-linearity.



The time series synthesizer is defined by three parameters:

- k_{wv} : shape parameter of the Weibull water vapour attenuation distribution
- λ_{wv} : scale parameter of the Weibull water vapour attenuation distribution
- β_{wv} : parameter that describes the time dynamics (s^{-1}).

3.1.2 Step-by-step method

The following step-by-step method is used to synthesize the single-site water vapour attenuation time series $A_{wv}(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, and k is the index of each sample.

A Estimation of k_{wv} and λ_{wv}

The parameters k_{wv} and λ_{wv} are determined from the cumulative distribution of water vapour attenuation, vs. probability of occurrence. Water vapour attenuation statistics can be determined from local measured data, or, in the absence of measured data, the water vapour attenuation prediction method based on integrated water vapour content in Recommendation ITU-R P.676 can be used.

For the location of interest, perform a Weibull fit of water vapour attenuation vs. probability of occurrence as follows:

Step SS_WV_1: Construct the set of pairs $[P_i, A_{wvi}]$ where P_i (% of time) is the probability the water vapour attenuation A_{wvi} (dB) is exceeded. The specific values of P_i should consider the probability range of interest; however, a suggested set of time percentages is 0.1, 0.2, 0.3, 0.5, 1, 2, 3, 5, 10, 20, 30 and 50%.

Step SS_WV_2: Transform the set of pairs $[P_i, A_{wvi}]$ to $\left[\ln\left(-\ln\frac{P_i}{100}\right), \ln A_{wvi} \right]$.

Step SS_WV_3: Determine the variables a and b by performing a least squares fit to the linear function for the n set of pairs:

$$\ln A_{WVi} = a \ln \left(-\ln \frac{P_i}{100} \right) + b \quad (1)$$

as follows:

$$\left\{ \begin{array}{l} a = \frac{n \sum_{i=1}^n \ln A_{WVi} \ln \left(-\ln \frac{P_i}{100} \right) - \sum_{i=1}^n \ln A_{WVi} \sum_{i=1}^n \ln \left(-\ln \frac{P_i}{100} \right)}{n \sum_{i=1}^n \left[\ln \left(-\ln \frac{P_i}{100} \right) \right]^2 - \left[\sum_{i=1}^n \ln \left(-\ln \frac{P_i}{100} \right) \right]^2} \\ b = \frac{\sum_{i=1}^n \ln(A_{WVi}) - a \sum_{i=1}^n \ln \left(-\ln \frac{P_i}{100} \right)}{n} \end{array} \right. \quad (2)$$

Step SS_WV_4: Calculate the parameters k_{WV} and λ_{WV} as follows:

$$\left\{ \begin{array}{l} k_{WV} = \frac{1}{a} \\ \lambda_{WV} = \exp(b) \end{array} \right. \quad (3)$$

B Low-pass filter parameter

Step SS_WV_5: Set $\beta_{WV} = 3.65 \times 10^{-6}$ (s⁻¹).

C Time series synthesis

The time series, $A_{WV}(kT_s)$, $k = 1, 2, 3, \dots$ is synthesized as follows:

Step SS_WV_6: Synthesize a white Gaussian noise time series, $n(kT_s)$, where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step SS_WV_7: Set $G_{WV}(0) = 0$

Step SS_WV_8: Filter the noise time series, $n(kT_s)$, with a recursive low-pass filter defined by:

$$G_{WV}(kT_s) = \rho_{WV} \times G_{WV}((k-1)T_s) + \sqrt{1-\rho_{WV}^2} \times n(kT_s) \quad \text{for } k = 1, 2, 3, \dots \quad (4)$$

where:

$$\rho_{WV} = e^{-\beta_{WV} T_s} \quad (5)$$

Step SS_WV_9: Calculate $A_{WV}(kT_s)$, for $k = 1, 2, 3, \dots$ as follows:

$$A_{WV}(kT_s) = \lambda_{WV} \left(-\log \left[Q(G_{WV}(kT_s)) \right] \right)^{1/k_{WV}} \quad (6)$$

where (see Recommendation ITU-R P.1057):

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \quad (7)$$

Step SS_WV_10: Discard the first 5 000 000 samples from the synthesized time series.

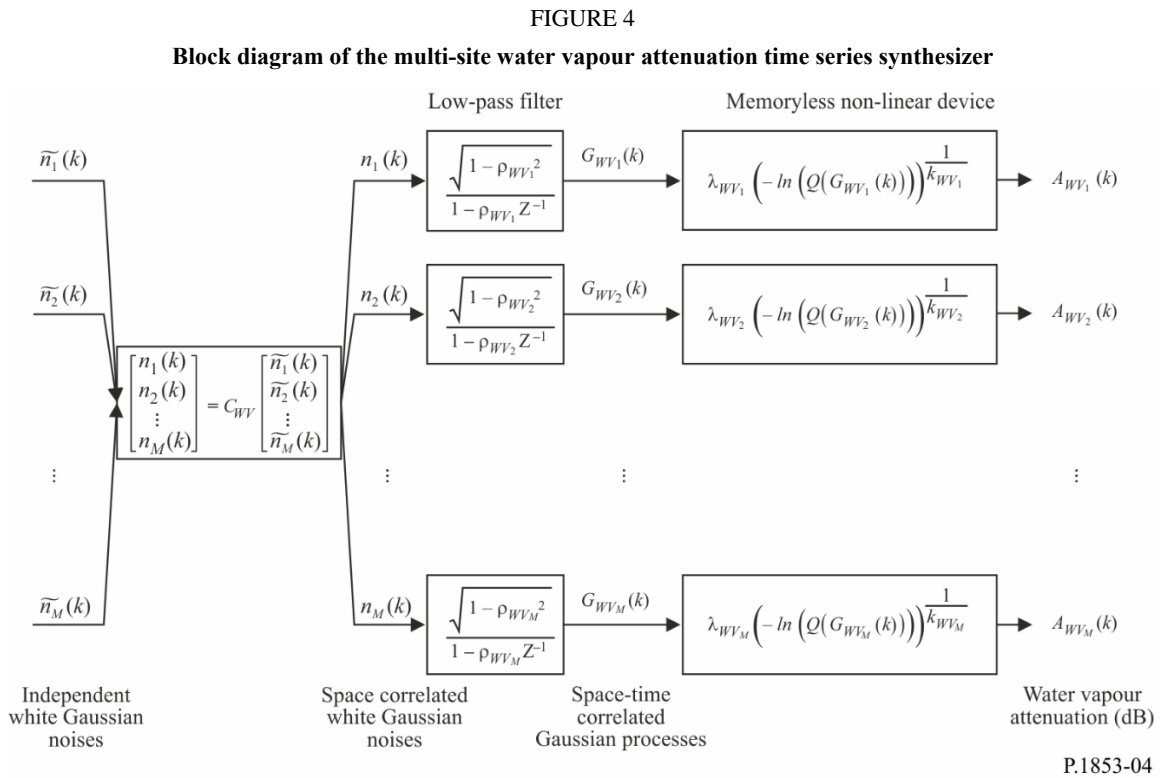
3.2 Multi-site configuration

3.2.1 Overview

The multi-site time series synthesis method assumes that the long-term statistics of water vapour attenuation for each site (A_{wvi}), where $i = \{1, 2, \dots, M\}$ (M is the total number of sites), is a Weibull distribution.

The time series synthesis method synthesizes M correlated time series that reproduce the spatial variation, spectral characteristics and probability distribution of water vapour attenuation.

As shown in Fig. 4, the multi-site water vapour attenuation time series, $A_{wvi}(t)=A_{wvi}(kT_s)$, where T_s is the sampling time, are synthesized from the discrete white Gaussian noise processes, $n_i(t)=n_i(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noises are low-pass filtered and transformed from a normal distribution to a Weibull distribution in a memoryless non-linearity.



The time series synthesizer is defined by the following parameters for site i :

- k_{wvi} : shape parameter of the Weibull water vapour attenuation distribution
- λ_{wvi} : scale parameter of the Weibull water vapour attenuation distribution
- β_{wvi} : parameter that describes the time dynamics (s^{-1})
- D_{ij} : distance between site i and site j (km)

3.2.2 Step-by-step method

The following step-by-step method is used to synthesize the multi-site water vapour attenuation time series $A_{wvi}(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, k is the index of each sample, and $i=1,2,\dots, M$ (M being the total number of sites).

A Estimation of k_{wvi} and λ_{wvi}

Step MS_WV_1: For each site number $i = \{1, 2, \dots, M\}$, perform the steps of Paragraph A of the single-site water vapour attenuation time series synthesizer specified in § 3.1.2.

B Low-pass filter parameters

Step MS_WV_2: For each site number $i = \{1, 2, \dots, M\}$, set $\beta_{wvi} = 3.65 \times 10^{-6} \text{ (s}^{-1}\text{)}$

C Time series synthesis

Step MS_WV_3: For each site number $i = \{1, 2, \dots, M\}$, synthesize a white Gaussian noise time series, $\tilde{n}_i(kT_s)$ where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step MS_WV_4: Calculate the $M \times M$ matrix $\mathbf{R}_n = [r_{n_{ij}}]$ for $i = \{1, 2, \dots, M\}$ and $j = \{1, 2, \dots, M\}$ as:

$$r_{n_{ij}} = \frac{1 - \rho_{wvi} \rho_{wvj}}{\sqrt{1 - \rho_{wvi}^2} \sqrt{1 - \rho_{wvj}^2}} r_{G_{wvij}} \quad (8)$$

where $\rho_{wvi} = e^{-\beta_{wvi} T_s}$ and $\rho_{wvj} = e^{-\beta_{wvj} T_s}$

and

$$r_{G_{wvij}} = r_{G_{wv}}(D_{ij}) = 0.29 \cdot \exp\left(-\frac{D_{ij}}{38}\right) + 0.71 \cdot \exp\left(-\frac{D_{ij}}{900}\right) \quad (9)$$

Step MS_WV_5: Calculate the Cholesky factorization¹ of the matrix \mathbf{R}_n to determine the lower triangular matrix $\mathbf{C}_{wv} = [c_{wvij}]$; i.e.:

$$\mathbf{R}_n = \mathbf{C}_{wv} \mathbf{C}_{wv}^T \quad (10)$$

Step MS_WV_6: For each time step kT_s calculate $\mathbf{n}(kT_s) = [n_1(kT_s) \ n_2(kT_s) \ \dots \ n_M(kT_s)]^T$ where:

$$\mathbf{n}(kT_s) = \mathbf{C}_{wv} \cdot \tilde{\mathbf{n}}(kT_s) \quad (11)$$

and $\tilde{\mathbf{n}}(kT_s) = [\tilde{n}_1(kT_s) \ \tilde{n}_2(kT_s) \ \dots \ \tilde{n}_M(kT_s)]$

For reference:

$$\begin{cases} n_1(kT_s) = \tilde{n}_1(kT_s) \\ n_2(kT_s) = c_{wv21} \tilde{n}_1(kT_s) + c_{wv22} \tilde{n}_2(kT_s) \\ \vdots \\ n_M(kT_s) = c_{wvM1} \tilde{n}_1(kT_s) + c_{wvM2} \tilde{n}_2(kT_s) + \dots + c_{wvMM} \tilde{n}_M(kT_s) \end{cases} \quad (12)$$

Step MS_WV_7: For each site number $i = \{1, 2, \dots, M\}$ and each time step, perform Steps SS_WV_7 to SS_WV_10 of Paragraph C of the single-site water vapour attenuation time series synthesizer specified in § 3.1.2.

¹ Matlab, Octave, and Python have Cholesky factorization functions.

4 Cloud attenuation

4.1 Single-site configuration

4.1.1 Overview

As suggested in Recommendation ITU-R P.840, the statistics of the long-term integrated liquid water content (ILWC) can be approximated by a log-normal distribution conditioned on the probability of non-zero cloud attenuation. Since the relationship between cloud attenuation (A_C) and ILWC is linear (see Recommendation ITU-R P.840), the statistics of the long-term cloud attenuation (A_C) can similarly be approximated by a conditioned log-normal distribution.

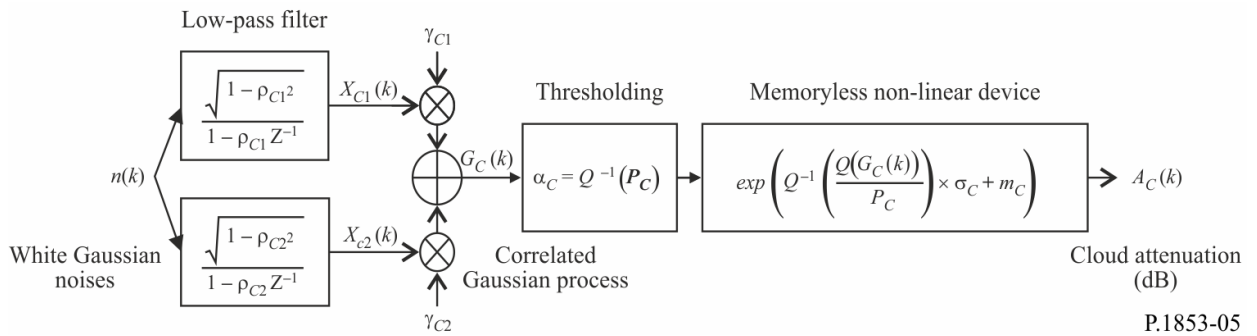
Consequently, the single-site time series synthesis method approximates the statistics of the long-term cloud attenuation (A_C) by a conditioned log-normal distribution.

The single-site time series synthesis method synthesizes a time series that reproduces the spectral characteristics and the probability distribution of cloud attenuation.

As shown in Fig. 5, the single-site cloud attenuation time series, $A_C(t)=A_C(kT_s)$, where T_s is the sampling time, is synthesized from the discrete white Gaussian noise process, $n(t)=n(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noise is low-pass filtered, truncated to match the desired cloud probability of occurrence, and transformed from a truncated normal distribution to a conditioned log-normal distribution in a memoryless non-linearity.

FIGURE 5

Block diagram of the cloud attenuation time series synthesizer



The time series synthesizer is defined by eight parameters:

- m_C : mean of the conditioned log-normal cloud attenuation distribution
- σ_C : standard deviation of the conditioned log-normal cloud attenuation distribution
- P_C : probability of cloud attenuation (%)
- α_C : truncation threshold of the correlated Gaussian noise
- β_{C1} : parameter that describes the time dynamics of the process fast component (s^{-1})
- β_{C2} : parameter that describes the time dynamics of the process slow component (s^{-1})
- γ_{C1} : parameter that describes the weight of the process fast component
- γ_{C2} : parameter that describes the weight of the process slow component

4.1.2 Step-by-step method

The following step-by-step method is used to synthesize the cloud attenuation time series $A_C(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, and k is the index of each sample.

A Estimation of μ_C , σ_C and P_C

The mean, m_C , standard deviation, σ_C , and probability of non-zero cloud attenuation, P_C , parameters of the conditioned log-normal distribution can be calculated from local data. If local data is not available, these parameters can be determined from the digital maps of integrated liquid water content reduced to 0 °C in Recommendation ITU-R P.840.

If no local measured data are available for the location of interest, determine the conditional log-normal parameters as follows:

Step SS_CL_1: Determine the parameters, m_{ILWC1} , m_{ILWC2} , m_{ILWC3} , m_{ILWC4} , σ_{ILWC1} , σ_{ILWC2} , σ_{ILWC3} , σ_{ILWC4} , P_{ILWC1} , P_{ILWC2} , P_{ILWC3} and P_{ILWC4} at the four closest grid points of the digital maps provided in Recommendation ITU-R P.840.

Step SS_CL_2: Determine the m_{ILWC} , σ_{ILWC} and P_{ILWC} parameters value at the desired location by performing a bi-linear interpolation of the four values of each parameter at the four grid points as described in Recommendation ITU-R P.1144.

Step SS_CL_3: Calculate the cloud liquid water specific attenuation coefficient $K_l(f,T)$ from the method described in Recommendation ITU-R P.840 for f , the frequency of interest, and $T=273.15$ K.

Step SS_CL_4: Set

$$\begin{cases} m_C = m_{ILWC} + \ln\left(\frac{K_l}{\sin \varphi}\right) \\ \sigma_C = \sigma_{ILWC} \\ P_C = P_{ILWC} \end{cases} \quad (13)$$

Where φ is the elevation angle of the slant path of interest.

B Low-pass filter parameters

Step SS_CL_5: Set:

$$\begin{aligned} \beta_{C1} &= 5.7643 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{C2} &= 1.7663 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{C1} &= 0.4394 \\ \gamma_{C2} &= 0.7613 \end{aligned}$$

C Truncation threshold

Step SS_CL_6: The truncation threshold α_C is:

$$\alpha_C = Q^{-1}\left(\frac{P_C}{100}\right) \quad (14)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

D Time series synthesis

The time series, $A_C(kT_s)$, $k = 1, 2, 3, \dots$ is synthesized as follows:

Step SS_CL_7: Synthesize a white Gaussian noise time series, $n(kT_s)$, where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step SS_CL_8: Set $X_{C1}(0) = 0$; $X_{C2}(0) = 0$

Step SS_CL_9: Filter the noise time series, $n(kT_s)$, with two recursive low-pass filters defined by:

$$\begin{cases} X_{c1}(kT_s) = \rho_{c1} \times X_{c1}((k-1)T_s) + \sqrt{1-\rho_{c1}^2} \times n(kT_s) \\ X_{c2}(kT_s) = \rho_{c2} \times X_{c2}((k-1)T_s) + \sqrt{1-\rho_{c2}^2} \times n(kT_s) \end{cases} \quad \text{for } k = 1, 2, 3, \quad (15)$$

where:

$$\begin{cases} \rho_{C1} = e^{-\beta_{C1}T_s} \\ \rho_{C2} = e^{-\beta_{C2}T_s} \end{cases} \quad (16)$$

Step SS_CL_10: Calculate $G_C(kT_s)$, for $k = 1, 2, 3, \dots$ as follows:

$$G_C(kT_s) = \gamma_{c1} \times X_{c1}(kT_s) + \gamma_{c2} \times X_{c2}(kT_s) \quad (17)$$

Step SS_CL_11: Calculate $A_C(kT_s)$ (dB), for $k = 1, 2, 3, \dots$ as follows:

$$A_C(kT_s) = \begin{cases} \exp\left(Q^{-1}\left[\frac{100}{P_C}Q(G_C(kT_s))\right] \times \sigma_C + m_C\right) & \text{for } G_C(kT_s) > \alpha_C \\ 0 & \text{for } G_C(kT_s) \leq \alpha_C \end{cases} \quad (18)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

Step SS_CL_12: Discard the first 5 000 000 samples from the synthesized time series. Cloud attenuation events are represented by sequences whose values are greater than 0 dB for a consecutive number of samples.

4.2 Multi-site configuration

4.2.1 Overview

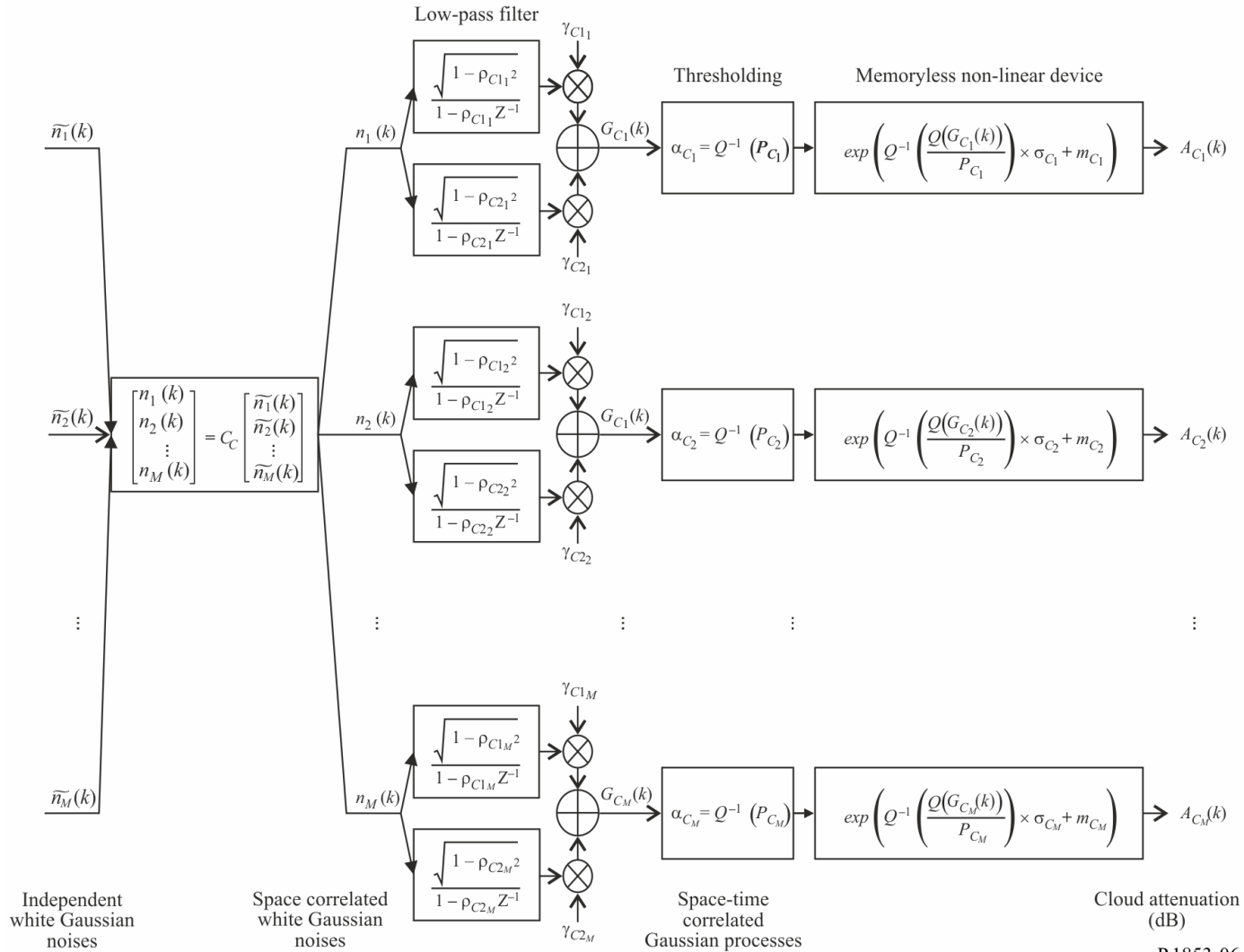
The multi-site time series synthesis method assumes that the long-term statistics of cloud attenuation for each site (A_{Ci}), where $i = \{1, 2, \dots, M\}$ (M is the total number of sites), is a conditioned log-normal distribution.

The time series synthesis method synthesizes a time series that reproduces the spatial variation, spectral characteristics and the probability distribution of cloud attenuation.

As shown in Fig. 6, the multi-site cloud attenuation time series, $A_{Ci}(t) = A_{Ci}(kT_s)$, where T_s is the sampling time, are synthesized from the discrete white Gaussian noise processes, $n_i(t) = n_i(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noises are low-pass filtered, truncated to match the desired cloud probability of occurrence, and transformed from a truncated normal distribution to a conditioned log-normal distribution in a memoryless non-linearity.

FIGURE 6

Block diagram of the multi-site cloud attenuation time series synthesizer



The time series synthesizer is defined by the following parameters for site i :

- m_{Ci} : mean of the conditioned log-normal cloud attenuation distribution
- σ_{Ci} : standard deviation of the conditioned log-normal cloud attenuation distribution
- P_{Ci} : probability of cloud attenuation (%)
- α_{Ci} : truncation threshold of the correlated Gaussian noise
- β_{C1i} : parameter that describes the time dynamics of the process fast component (s^{-1})
- β_{C2i} : parameter that describes the time dynamics of the process slow component (s^{-1})
- γ_{C1i} : parameter that describes the weight of the process fast component
- γ_{C2i} : parameter that describes the weight of the process slow component
- D_{ij} : distance between site i and site j (km)

4.2.2 Step-by-step method

The following step-by-step method is used to synthesize the multi-site cloud attenuation time series $A_{Ci}(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, k is the index of each sample, and $i=1,2,\dots, M$ (M being the total number of sites).

A Estimation of m_{Ci} , σ_{Ci} and P_{Ci}

Step MS_CL_1: For each site number $i = \{1, 2, \dots, M\}$, calculate m_{Ci} , σ_{Ci} and P_{Ci} by performing the steps in Paragraph A of the single-site cloud attenuation time series synthesizer specified in § 4.1.2.

B Low-pass filter parameters

Step MS_CL_2: For each site number $i = \{1, 2, \dots, M\}$, Set:

$$\begin{aligned}\beta_{C1i} &= 5.7643 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{C2i} &= 1.7663 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{C1i} &= 0.4394 \\ \gamma_{C2i} &= 0.7613\end{aligned}$$

C Truncation thresholds

Step MS_CL_3: The truncation thresholds α_{Ci} are:

$$\alpha_{Ci} = Q^{-1}\left(\frac{P_{Ci}}{100}\right) \quad (19)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

D Time series synthesis

Step MS_CL_4: For each site number $i = \{1, 2, \dots, M\}$, synthesize a white Gaussian noise time series, $\tilde{n}_i(kT_s)$ where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step MS_CL_5: Calculate the $M \times M$ matrix $\mathbf{R}_n = [r_{n_{ij}}]$ for $i = \{1, 2, \dots, M\}$ and $j = \{1, 2, \dots, M\}$ as:

$$r_{n_{ij}} = \frac{1}{\gamma_{c_{1i}}\gamma_{c_{1j}} \frac{\sqrt{1-\rho_{c_{1i}}^2}\sqrt{1-\rho_{c_{1j}}^2}}{1-\rho_{c_{1i}}\rho_{c_{1j}}} + \gamma_{c_{2i}}\gamma_{c_{2j}} \frac{\sqrt{1-\rho_{c_{2i}}^2}\sqrt{1-\rho_{c_{2j}}^2}}{1-\rho_{c_{2i}}\rho_{c_{2j}}} + \gamma_{c_{1i}}\gamma_{c_{2j}} \frac{\sqrt{1-\rho_{c_{1i}}^2}\sqrt{1-\rho_{c_{2j}}^2}}{1-\rho_{c_{1i}}\rho_{c_{2j}}} + \gamma_{c_{2i}}\gamma_{c_{1j}} \frac{\sqrt{1-\rho_{c_{2i}}^2}\sqrt{1-\rho_{c_{1j}}^2}}{1-\rho_{c_{2i}}\rho_{c_{1j}}}} r_{G_{G_{ij}}} \quad (20)$$

where $\rho_{c_{1i}} = e^{-\beta_{c_{1i}}T_s}$, $\rho_{c_{2i}} = e^{-\beta_{c_{2i}}T_s}$, $\rho_{c_{1j}} = e^{-\beta_{c_{1j}}T_s}$, and $\rho_{c_{2j}} = e^{-\beta_{c_{2j}}T_s}$

$$\text{and } r_{G_{G_{ij}}} = r_{G_c}(D_{ij}) = 0.55 \cdot \exp\left(-\frac{D_{ij}}{24}\right) + 0.45 \cdot \exp\left(-\frac{D_{ij}}{700}\right) \quad (21)$$

Step MS_CL_6: Calculate the Cholesky¹ factorization of the matrix \mathbf{R}_n to determine the lower triangular matrix $\mathbf{C}_c = [c_{c_{ij}}]$ where:

$$\mathbf{R}_n = \mathbf{C}_c \mathbf{C}_c^T \quad (22)$$

Step MS_CL_7: Calculate $\mathbf{n}(kT_s) = [n_1(kT_s) \ n_2(kT_s) \ \dots \ n_M(kT_s)]^T$ as:

$$\mathbf{n}(kT_s) = \mathbf{C}_c \cdot \tilde{\mathbf{n}}(kT_s) \quad (23)$$

where $\tilde{\mathbf{n}}(kT_s) = [\tilde{n}_1(kT_s) \ \tilde{n}_2(kT_s) \ \dots \ \tilde{n}_M(kT_s)]^T$

For reference:

$$\begin{cases} n_1(kT_s) = \tilde{n}_1(kT_s) \\ n_2(kT_s) = c_{c_{21}} \tilde{n}_1(kT_s) + c_{c_{22}} \tilde{n}_2(kT_s) \\ \vdots \\ n_M(kT_s) = c_{c_{M1}} \tilde{n}_1(kT_s) + c_{c_{M2}} \tilde{n}_2(kT_s) + \dots + c_{c_{MM}} \tilde{n}_M(kT_s) \end{cases} \quad (24)$$

Step MS_CL_8: For each site number $i = \{1, 2, \dots, M\}$, perform Steps SS_CL_8 to SS_CL_12 of Section D of the single-site cloud attenuation time series synthesizer given in § 4.1.2.

5 Rain attenuation

5.1 Single-site configuration

5.1.1 Overview

The single-site rain attenuation time series synthesis method assumes the long-term statistics of rain attenuation is a log-normal distribution conditioned on the probability of non-zero rain attenuation. The ITU-R rain attenuation prediction method in Recommendation ITU-R P.618 is well-approximated by a conditioned log-normal distribution over the most significant range of exceedance probabilities. Since the Earth-space rain attenuation prediction method predicts non-zero rain attenuation for exceedance probabilities greater than the probability of rain attenuation, the time series synthesis method adjusts the attenuation time series so the rain attenuation corresponding to exceedance probabilities greater than the probability of rain attenuation is 0 dB.

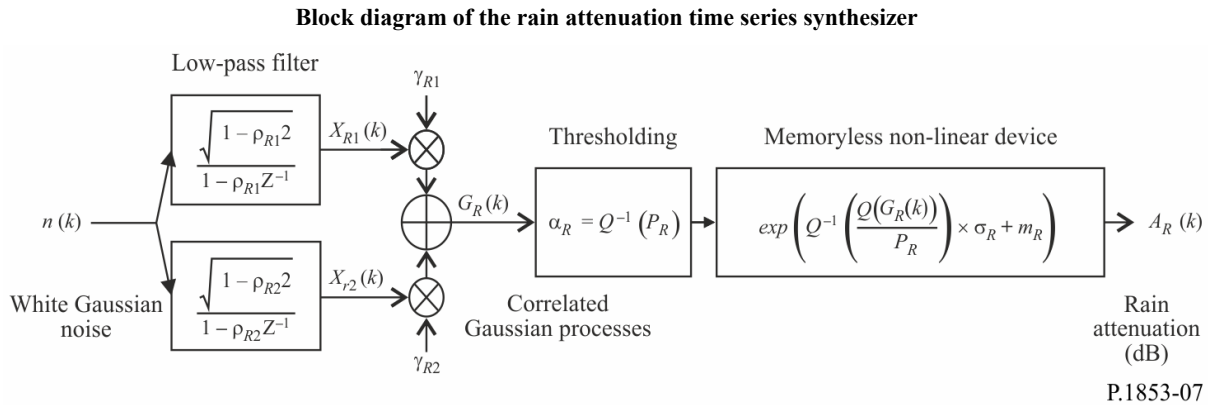
This method is valid for frequencies between 4 GHz and 55 GHz and elevation angles between 5° and 90°.

The single-site rain attenuation time series synthesis method synthesizes a time series that reproduces the spectral characteristics, fade slope and fade duration statistics, and probability distribution of rain

attenuation events. Interfade duration statistics are also reproduced, but only within individual attenuation events.

As shown in Fig. 7, the single-site rain attenuation time series, $A_R(t)=A_R(kT_s)$, where T_s is the sampling time, is synthesized from the discrete white Gaussian noise process, $n(t)=n(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noise is low-pass filtered, truncated to match the desired rain attenuation probability of occurrence, and transformed from a truncated normal distribution to a conditioned log-normal distribution in a memoryless non-linearity.

FIGURE 7



The time series synthesizer is defined by eight parameters:

- m_R : mean of the conditioned log-normal rain attenuation distribution
- σ_R : standard deviation of the conditioned log-normal rain attenuation distribution
- P_R : probability of rain attenuation (%)
- α_R : truncation threshold of the correlated Gaussian noise
- β_{R1} : parameter that describes the time dynamics of the process fast component (s^{-1})
- β_{R2} : parameter that describes the time dynamics of the process slow component (s^{-1})
- γ_{R1} : parameter that describes the weight of the process fast component
- γ_{R2} : parameter that describes the weight of the process slow component

5.1.2 Step-by-step method

The following step-by-step method is used to synthesize the rain attenuation time series $A_R(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, and k is the index of each sample.

A Estimation of m_R , σ_R and P_R

The parameters m_R and σ_R are determined from the cumulative distribution of rain attenuation vs. probability of occurrence. These parameters can be determined from local data. If local data is not available, m_R and σ_R can be determined from the rain attenuation prediction method in Recommendation ITU-R P.618.

For the Earth-space elevation angle and frequency of interest, perform a log-normal fit of rain attenuation vs. probability of occurrence as follows:

Step SS_RA_1: Determine P_R (% of time), the probability of rain attenuation on the path from local measured data, or, in the absence of measured data, the prediction method “Probability of rain attenuation on a slant path” of Recommendation ITU-R P.618 can be used.

Step SS_RA_2: Construct the set of pairs $[P_i/P_R, A_{Ri}]$ where P_i (% of time) is the probability the rain attenuation A_{Ri} (dB) is exceeded where $P_i \leq P_R$. The specific values of P_i should consider the probability range of interest; however, a suggested set of time percentages is 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, 0.3, 0.5, 1, 2, 3, 5, and 10%, with the constraint that $P_i \leq P_R$.

Step SS_RA_3: Transform the set of pairs $[P_i/P_R, A_{Ri}]$ to $\left[Q^{-1}\left(\frac{P_i}{P_R}\right), \ln A_{Ri} \right]$,

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

Step SS_RA_4: Determine the variables m_R and σ_R by performing a least-squares fit to $\ln A_{Ri} = \sigma_R Q^{-1}\left(\frac{P_i}{P_R}\right) + m_R$ for all i . The least-squares fit can be determined using the “Step-by-step procedure to approximate a complementary cumulative distribution by a log-normal complementary cumulative distribution” described in Recommendation ITU-R P.1057.

B Low-pass filter parameters

Step SS_RA_5: Set:

$$\beta_{R1} = 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)}$$

$$\beta_{R2} = 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)}$$

$$\gamma_{R1} = 0.3746$$

$$\gamma_{R2} = 0.7738$$

C Truncation threshold

Step SS_RA_6: The truncation threshold α_R is:

$$\alpha_R = Q^{-1}\left(\frac{P_R}{100}\right) \quad (25)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

D Time series synthesis

The rain attenuation time series, $A_R(kT_s)$, $k = 1, 2, 3, \dots$ is synthesized as follows:

Step SS_RA_7: Synthesize a white Gaussian noise time series, $n(kT_s)$, where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step SS_RA_8: Set $X_{R1}(0) = 0$; $X_{R2}(0) = 0$

Step SS_RA_9: Filter the noise time series, $n(kT_s)$, with two recursive low-pass filters defined by:

$$\begin{cases} X_{R1}(kT_s) = \rho_{R1} \times X_{R1}((k-1)T_s) + \sqrt{1-\rho_{R1}^2} \times n(kT_s) \\ X_{R2}(kT_s) = \rho_{R2} \times X_{R2}((k-1)T_s) + \sqrt{1-\rho_{R2}^2} \times n(kT_s) \end{cases} \quad \text{for } k = 1, 2, 3, \dots \quad (26)$$

where:

$$\begin{cases} \rho_{R1} = e^{-\beta_{R1}T_s} \\ \rho_{R2} = e^{-\beta_{R2}T_s} \end{cases} \quad (27)$$

Step SS_RA_10: Calculate $G_R(kT_s)$, for $k = 1, 2, 3, \dots$ as follows:

$$G_R(kT_s) = \gamma_{R1} \times X_{R1}(kT_s) + \gamma_{R2} \times X_{R2}(kT_s) \quad (28)$$

Step SS_CL_11: Calculate $A_R(kT_s)$ (dB), for $k = 1, 2, 3, \dots$ as follows:

$$A_R(kT_s) = \begin{cases} \exp\left(Q^{-1}\left[\frac{100}{P_R}Q(G_R(kT_s))\right] \times \sigma_R + m_R\right) & \text{for } G_R(kT_s) > \alpha_R \\ 0 & \text{for } G_R(kT_s) \leq \alpha_R \end{cases} \quad (29)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

Step SS_CL_12: Discard the first 5 000 000 samples from the synthesized time series. Rain attenuation events are represented by sequences whose values are above 0 dB for a consecutive number of samples.

5.2 Multi-site configuration

5.2.1 Overview

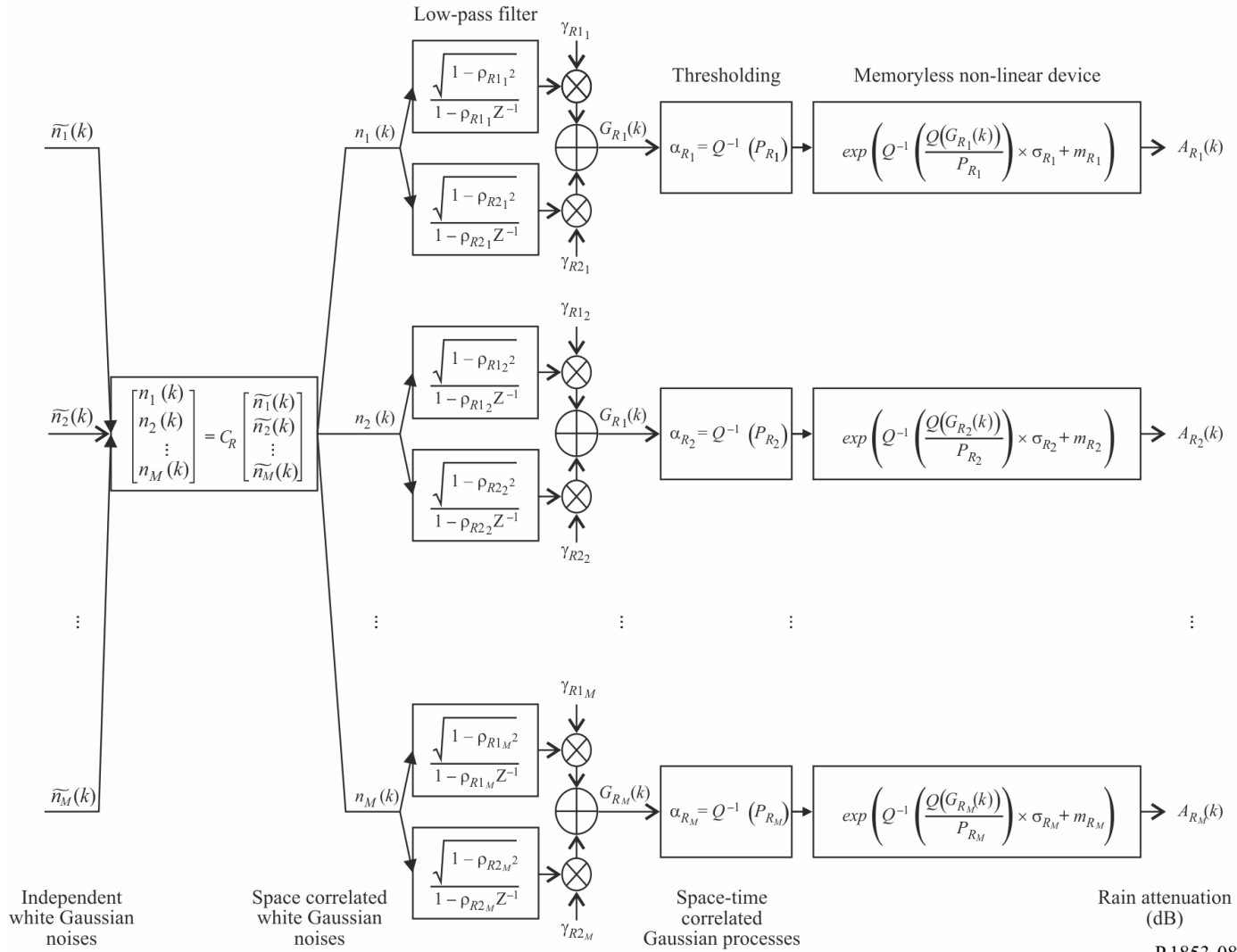
The multi-site rain attenuation time series synthesis method assumes that the long-term statistics of rain attenuation for each site (A_{Ri}), where $i = \{1, 2, \dots, M\}$ (M is the total number of sites), is a conditioned log-normal distribution.

The multi-site rain attenuation time series synthesis method synthesizes M time series that reproduce the spatial variation, spectral characteristics and probability distribution of rain attenuation.

As shown in Fig. 8, the multi-site rain attenuation time series, $A_{Ri}(t) = A_{Ri}(kT_s)$, where T_s is the sampling time, are synthesized from the discrete white Gaussian noise processes, $n_i(t) = n_i(kT_s)$. For convenience, in the Figure, kT_s is only denoted k . The white Gaussian noises are low-pass filtered, truncated to match the desired cloud probability of occurrence, and transformed from a truncated normal distribution to a conditioned log-normal distribution in a memoryless non-linearity.

FIGURE 8

Block diagram of the multi-site rain attenuation time series synthesizer



The time series synthesizer is defined by the following parameters for site i :

- m_{Ri} : mean of the conditioned log-normal rain attenuation distribution
- σ_{Ri} : standard deviation of the conditioned log-normal rain attenuation distribution
- P_{Ri} : probability of rain attenuation (%)
- α_{Ri} : truncation threshold of the correlated Gaussian noise
- β_{R1i} : parameter that describes the time dynamics of the process fast component (s^{-1})
- β_{R2i} : parameter that describes the time dynamics of the process slow component (s^{-1})
- γ_{R1i} : parameter that describes the weight of the process fast component
- γ_{R2i} : parameter that describes the weight of the process slow component
- D_{ij} : distance between site i and site j (km)

5.2.2 Step-by-step method

The following step-by-step method is used to synthesize the multi-site rain attenuation time series $A_{Ci}(kT_s)$, $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, k is the index of each sample, and $i=1,2,\dots,M$ (M being the total number of sites).

A Estimation of m_{Ri} , σ_{Ri} and P_{Ri}

Step MS_RA_1: For each site number $i = \{1, 2, \dots, M\}$, perform the steps specified in Paragraph A of the single-site rain attenuation time series synthesizer given in § 5.1.2.

B Low-pass filter parameters

Step MS_RA_2: For each site number $i = \{1, 2, \dots, M\}$, Set:

$$\begin{aligned}\beta_{R1i} &= 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{R2i} &= 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{R1i} &= 0.3746 \\ \gamma_{R2i} &= 0.7738\end{aligned}$$

C Truncation thresholds

Step MS_RA_3: The truncation thresholds α_{Ri} are:

$$\alpha_{Ri} = Q^{-1} \left(\frac{P_{Ri}}{100} \right) \quad (30)$$

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

D Time series synthesis

Step MS_RA_4: For each site number $i = \{1, 2, \dots, M\}$, synthesize a white Gaussian noise time series, $\tilde{n}_i(kT_s)$ where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step MS_RA_5: Calculate the matrix $\mathbf{R}_n = [r_{n_{ij}}]$ for $i = \{1, 2, \dots, M\}$ and $j = \{1, 2, \dots, M\}$ as:

$$r_{n_{ij}} = \frac{1}{\gamma_{R1i}\gamma_{R1j} \frac{\sqrt{1-\rho_{R1i}^2}\sqrt{1-\rho_{R1j}^2}}{1-\rho_{R1i}\rho_{R1j}} + \gamma_{R2i}\gamma_{R2j} \frac{\sqrt{1-\rho_{R2i}^2}\sqrt{1-\rho_{R2j}^2}}{1-\rho_{R2i}\rho_{R2j}} + \gamma_{R1i}\gamma_{R2j} \frac{\sqrt{1-\rho_{R1i}^2}\sqrt{1-\rho_{R2j}^2}}{1-\rho_{R1i}\rho_{R2j}} + \gamma_{R2i}\gamma_{R1j} \frac{\sqrt{1-\rho_{R2i}^2}\sqrt{1-\rho_{R1j}^2}}{1-\rho_{R2i}\rho_{R1j}}} r_{G_{Rij}} \quad (31)$$

where $\rho_{R1_i} = e^{-\beta_{R1_i} T_s}$, $\rho_{R2_i} = e^{-\beta_{R2_i} T_s}$, $\rho_{R1_j} = e^{-\beta_{R1_j} T_s}$, and $\rho_{R2_j} = e^{-\beta_{R2_j} T_s}$

and $r_{GR_{ij}} = r_{GR}(D_{ij}) = 0.59 \cdot \exp\left(-\frac{D_{ij}}{31}\right) + 0.41 \cdot \exp\left(-\frac{D_{ij}}{800}\right)$

Step MS_RA_6: Use the Cholesky¹ factorization of the matrix \mathbf{R}_n to define the lower triangular matrix $\mathbf{C}_R = [c_{R_{ij}}]$ where:

$$\mathbf{R}_n = \mathbf{C}_R \mathbf{C}_R^T \tag{32}$$

Step MS_RA_7: Calculate $\mathbf{n}(kT_s) = [n_1(kT_s) \ n_2(kT_s) \ \dots \ n_M(kT_s)]^T$ as:

$$\mathbf{n}(kT_s) = \mathbf{C}_R \cdot \tilde{\mathbf{n}}(kT_s) \tag{33}$$

where $\tilde{\mathbf{n}}(kT_s) = [\tilde{n}_1(kT_s) \ \tilde{n}_2(kT_s) \ \dots \ \tilde{n}_M(kT_s)]^T$

For reference:

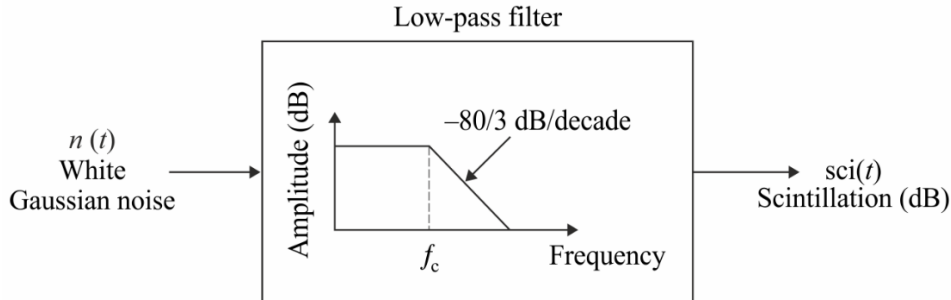
$$\begin{cases} n_1(kT_s) = \tilde{n}_1(kT_s) \\ n_2(kT_s) = c_{R_{21}} \tilde{n}_1(kT_s) + c_{R_{22}} \tilde{n}_2(kT_s) \\ \vdots \\ n_M(kT_s) = c_{R_{M1}} \tilde{n}_1(kT_s) + c_{R_{M2}} \tilde{n}_2(kT_s) + \dots + c_{R_{MM}} \tilde{n}_M(kT_s) \end{cases} \tag{34}$$

Step MS_RA_8: For each site number $i = \{1, 2, \dots, M\}$, perform Steps SS_RA_8 to SS_RA_12 of Section D of the single-site rain attenuation time series synthesizer given in § 5.1.2.

6 Unit-variance tropospheric scintillation

As shown in Fig. 9, a scintillation time series, $sci(t)$, can be generated by filtering white Gaussian noise, $n(t)$, such that the asymptotic power spectrum of the filtered time series has an $f^{-8/3}$ roll-off and a cut-off frequency, f_c , of 0.1 Hz. Note that the standard deviation of the scintillation increases as the rain attenuation increases.

FIGURE 9
Block diagram of the unit-variance scintillation time series synthesizer



Annex 2

Time series synthesis of single-site and multi-site total tropospheric impairments for Earth-space paths

1 Introduction

The methodology presented in this Annex provides a technique to synthesize single-site and multi-site total tropospheric impairments (gaseous, cloud, and rain attenuation; and scintillation fading and enhancement) and is applicable to Earth-space paths.

2 Single-site configuration

2.1 Overview

The time series due to the combination of single-site gaseous attenuation, cloud attenuation, rain attenuation and scintillation fading and enhancement are synthesized using the method illustrated in Fig. 10 using the methods described in the Annex 1. An appropriate correlation between cloud attenuation and rain attenuation has been introduced using the same temporal correlation and underlying Gaussian process. The temporal correlation of rain attenuation has been favoured as the rain attenuation is the dominant impairment.

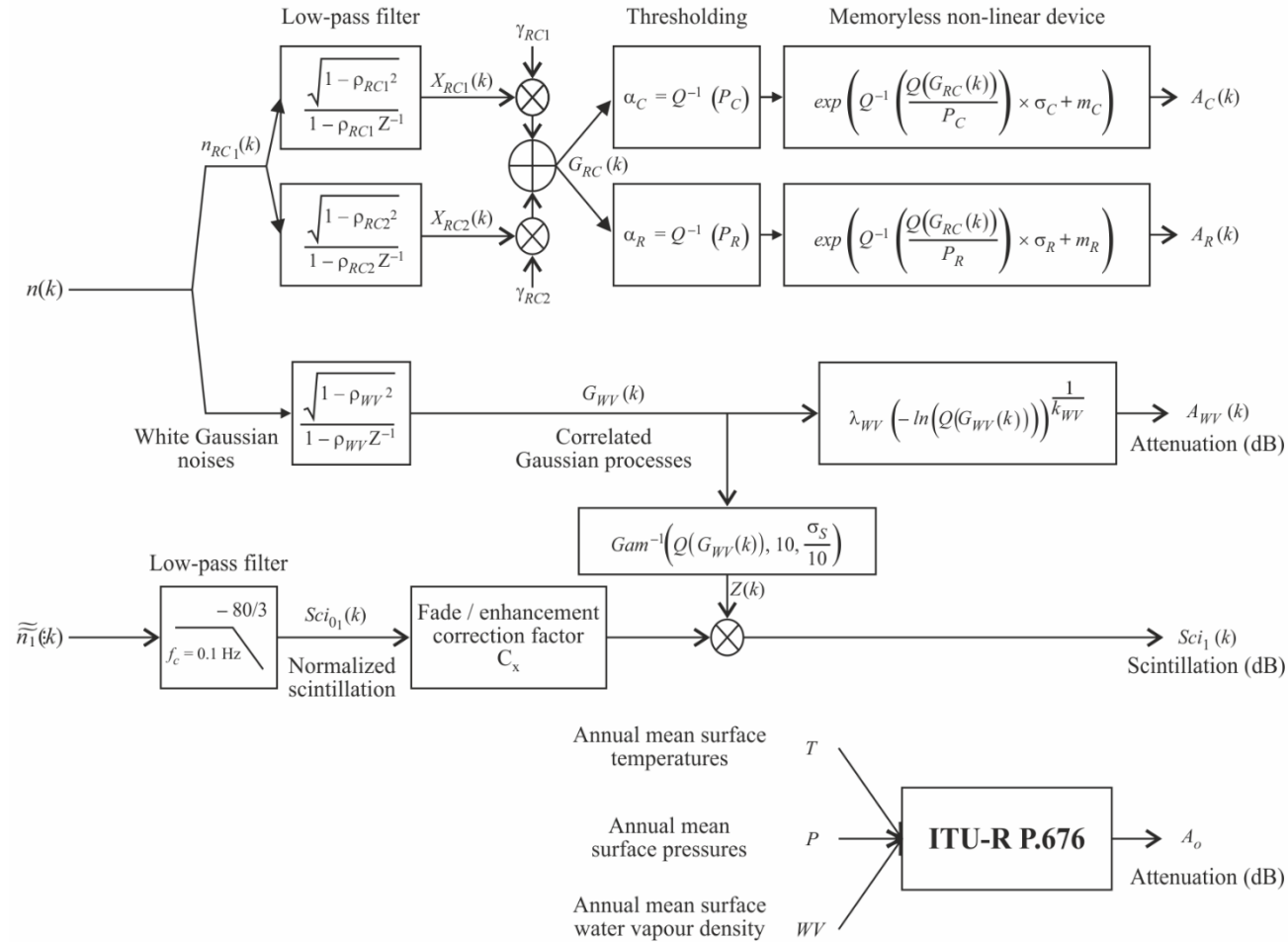
Cloud attenuation is interpolated if: a) a rain event is synthesized with a value of attenuation greater than 0 dB, and b) the cloud attenuation exceeds a threshold $A_{c_thresh} = \frac{K_l}{\sin \varphi}$ where φ is the elevation angle and K_l is given by Step SS_CL_3 of Annex 1.

For Earth-space paths, the time series synthesis method is valid for frequencies between 4 GHz and 55 GHz and elevation angles between 5° and 90°. For low frequencies, moderate to high elevation angles, and temperate regions, total attenuation may be well-approximated with sufficient accuracy by rain attenuation only.

This time series synthesis method synthesizes a time series that reproduces the spectral characteristics, fade slope and fade duration statistics of total attenuation events. Interfade duration statistics are also reproduced, but only within individual attenuation events.

FIGURE 10

Block diagram of the single-site total attenuation and scintillation time series synthesizer



2.2 Step-by-step method

The following step-by-step method synthesizes the time series of single-site total tropospheric impairments (gaseous attenuation, cloud attenuation, rain attenuation, and scintillation fading and enhancement) $A_{TOT}(kT_s)$, with $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, and k is the index of each sample.

Step SS_TOT_1: Synthesize a white Gaussian noise time series, $n(kT_s)$, where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step SS_TOT_2: Calculate the annual mean oxygen gaseous attenuation time series A_O following the procedure in § 2.2 of Annex 1.

Step SS_TOT_3: Calculate the water vapour attenuation time series $A_{WV}(kT_s)$ starting from the Gaussian noise time series, $n(kT_s)$, following the procedure in § 3.1.2 of Annex 1.

Step SS_TOT_4: Calculate the cloud attenuation time series $A_C(kT_s)$ starting from the Gaussian noise time series, $n(kT_s)$, following the procedure in § 4.1.2 of Annex 1 and replace *Step SS_CL_5* by:

Set:

$$\begin{aligned}\beta_{C1} &= \beta_{R1} = \beta_{RC1} = 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{C2} &= \beta_{R2} = \beta_{RC2} = 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{C1} &= \gamma_{R1} = \gamma_{RC1} = 0.3746 \\ \gamma_{C2} &= \gamma_{R2} = \gamma_{RC2} = 0.7738\end{aligned}$$

Step SS_TOT_5: Calculate the rain attenuation time series $A_R(kT_s)$ starting from the Gaussian noise time series, $n(kT_s)$, following the procedure recommended in § 5.1.2 of Annex 1 and replace *Step SS_RA_5* by:

Set:

$$\begin{aligned}\beta_{R1} &= \beta_{RC1} = 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{R2} &= \beta_{RC2} = 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{R1} &= \gamma_{RC1} = 0.3746 \\ \gamma_{R2} &= \gamma_{RC2} = 0.7738\end{aligned}$$

Step SS_TOT_6: For each time step kT_s : if $A_R(kT_s) > 0$ and $A_C(kT_s) > A_{C_{thresh}} = \frac{K_l}{\sin \phi}$, then

$$\text{set } A_C(kT_s) = \frac{K_l}{\sin \phi}.$$

Step SS_TOT_7: Define the scintillation fading and enhancement polynomials as:

$$\begin{aligned}a_{Fade}(P) &= -0.061 \times (\log_{10}(P))^3 + 0.072 \times (\log_{10}(P))^2 - 1.71 \times \log_{10}(P) + 3.0 \\ a_{Enhance}(P) &= -0.0597 \times (\log_{10}(P))^3 - 0.0835 \times (\log_{10}(P))^2 - 1.258 \times \log_{10}(P) + 2.672\end{aligned}$$

Step SS_TOT_8: Synthesize the unit variance scintillation time series $Sci_0(kT_s)$ following the method recommended in § 6 of Annex 1.

Step SS_TOT_9: Calculate the correction coefficient time series $C_x(kT_s)$ in order to distinguish between scintillation fades and scintillation enhancements:

$$C_x(kT_s) = \begin{cases} \frac{a_{Fade}(100 \times Q[Sci_0(kT_s)])}{a_{Enhance}(100 \times Q[Sci_0(kT_s)])} & \text{for } Sci_0(kT_s) > 0 \\ 1 & \text{for } Sci_0(kT_s) \leq 0 \end{cases} \quad (35)$$

where the Q function is defined in § 3.1.2 of Annex 1 of this Recommendation and specified in Recommendation ITU-R P.1057.

Step SS_TOT_10: If $C_x(kT_s) < 1$ or, $100 \times Q[Sci_0(kT_s)] > 45$ then set $C_x(kT_s) = 1$

Step SS_TOT_11: Calculate the scintillation standard deviation σ_S according to the method “Calculation of monthly and long-term statistics of amplitude scintillations at elevation angles greater than 5 degrees” of Recommendation ITU-R P.618 (for information, σ_S is named σ in Recommendation ITU-R P.618).

Step SS_TOT_12: Transform the intermediate underlying Gaussian process $G_{WV}(kT_s)$ from step *SS_TOT_3* into the Gamma² distributed time series $Z(kT_s)$ as follows:

$$Z(kT_s) = Gam^{-1} \left[Q(G_{WV}(kT_s)), 10, \frac{\sigma_S}{10} \right] \quad (36)$$

where the function Gam is the Gamma complementary cumulative distribution function specified in Recommendation ITU-R P.1057 and defined as:

$$Gam(x, k, \theta) = \int_x^{\infty} \frac{x^{k-1} \exp(-x/\theta)}{\Gamma(k)\theta^k} dt \quad (37)$$

Note that $Q(G_{WV}(kT_s)) = \exp \left(- \left(\frac{A_{WV}(kT_s)}{\lambda_{WV}} \right)^{k_{WV}} \right)$

Step SS_TOT_13: Calculate the scintillation time series $Sci(kT_s)$ as follows:

$$Sci(kT_s) = \begin{cases} Sci_0(kT_s) \times C_x(kT_s) \times Z(kT_s) \times [A_R(kT_s)]^{5/12} & \text{for } A_R(kT_s) > 1 \\ Sci_0(kT_s) \times C_x(kT_s) \times Z(kT_s) & \text{for } A_R(kT_s) \leq 1 \end{cases} \quad (38)$$

Step SS_TOT_14: Calculate total tropospheric impairments time series $A_{TOT}(kT_s)$ as follows:

$$A_{TOT}(kT_s) = A_R(kT_s) + A_C(kT_s) + A_V(kT_s) + A_O + Sci(kT_s) \quad (39)$$

3 Multi-site configuration

3.1 Overview

The time series of multi-site total attenuation and scintillation fading and enhancement are synthesized using the method illustrated in Fig. 11 using the methods described in the previous sections. The appropriate correlation between cloud attenuation and rain attenuation are introduced

² Matlab, Octave, and Python have Gamma and inverse Gamma distribution functions.

by using the same temporal correlation and the same underlying Gaussian process. The temporal correlation of rain attenuation has been favoured as the rain attenuation is the dominant impairment. For the same reason, the spatial correlation of rain attenuation has been favoured.

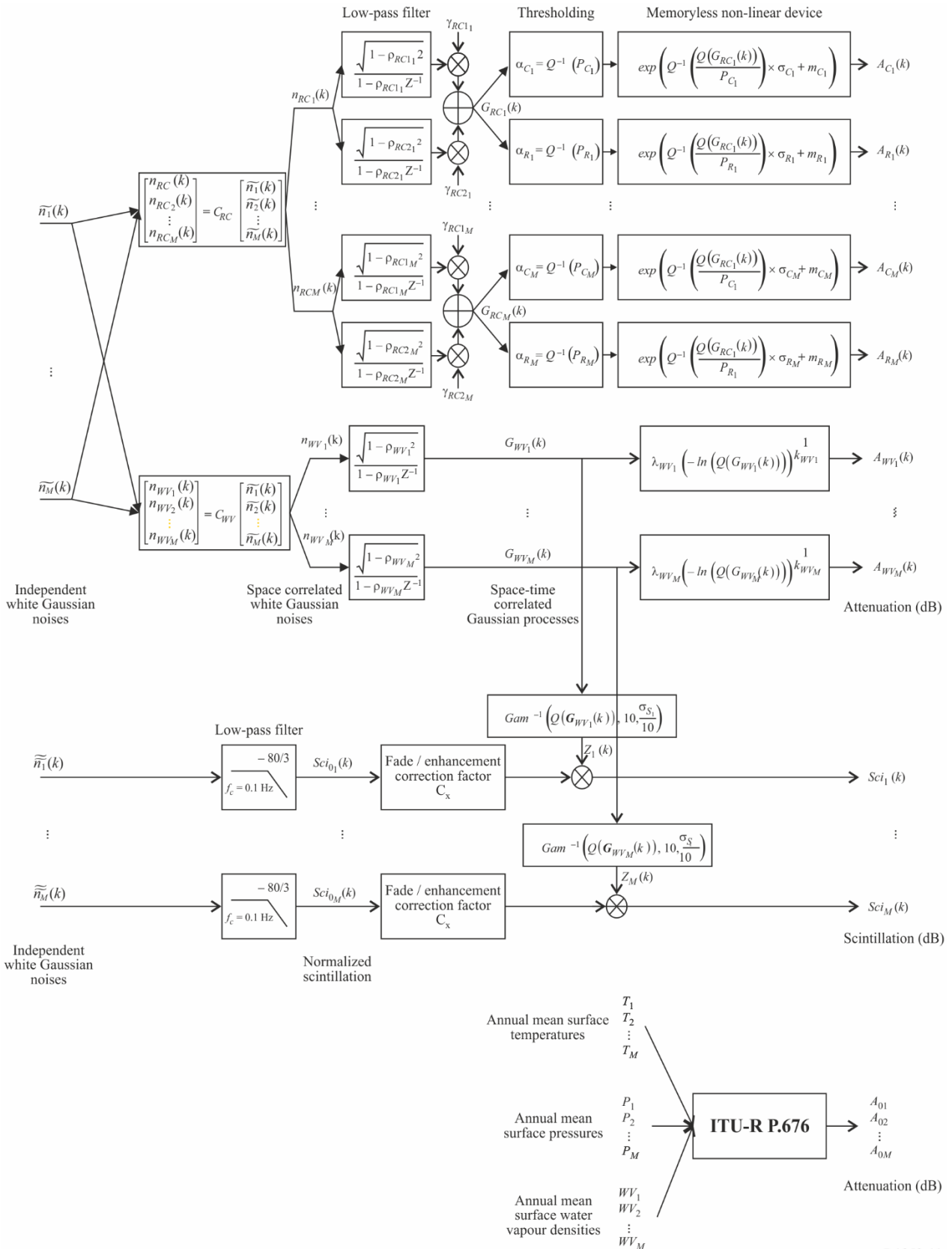
If: a) a rain event is generated (synthetic rain attenuation is greater than 0 dB) and b) cloud attenuation exceeds threshold $A_{c_{thresh}} = \frac{K_l}{\sin \varphi}$ where φ is the elevation angle and K_l is given in Step SS_CL_3, then cloud attenuation is interpolated.

For Earth-space paths, the time series synthesis method is valid for frequencies between 4 GHz and 55 GHz and elevation angles between 5° and 90°. For some circumstances (e.g. low frequencies, moderate to high elevation angles, temperate areas), the total attenuation may be approximated by only the rain attenuation with sufficient accuracy.

The time series synthesis method generates a time series that reproduces spatial variation, spectral characteristics, fade slope and fade duration statistics of total attenuation events. The interfade duration statistics are also reproduced but only within individual attenuation events.

FIGURE 11

Block diagram of the multi-site total attenuation and scintillation time series synthesizer



3.2 Step-by-step method

The following step-by-step method is used to synthesize the time series of multi-site total tropospheric impairments (gaseous attenuation, cloud attenuation, rain attenuation, and tropospheric scintillation fading and enhancement) $A_{TOTi}(kT_s)$, with $k = 1, 2, 3, \dots$, where T_s is the time interval between samples, k is the index of each sample, and $i=1,2,\dots, M$ (M being the total number of sites).

Step MS_TOT_1: For each site number $i = \{1, 2, \dots, M\}$, synthesize a white Gaussian noise time series, $\tilde{n}_i(kT_s)$ where $k = 1, 2, 3, \dots$ with zero mean and unit variance at a sampling period, T_s , of 1 s.

Step MS_TOT_2: Calculate the oxygen gaseous attenuation time series A_{O_i} following the procedure recommended in § 2.3 of Annex 1.

Step MS_TOT_3: Calculate the water vapour attenuation time series $A_{WV_i}(kT_s)$ starting from the Gaussian noise time series, $\tilde{n}_i(kT_s)$, following the procedure recommended in § 3.2 of Annex 1.

Step MS_TOT_4: Calculate the cloud attenuation time series $A_{C_i}(kT_s)$ starting from the Gaussian noise time series, $\tilde{n}_i(kT_s)$, following the procedure recommended in § 4.2 of Annex 1, and replace Step MS_CL_2 by:

Set:

$$\begin{aligned}\beta_{C1i} &= \beta_{R1i} = \beta_{RC1i} = 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{C2i} &= \beta_{R2i} = \beta_{RC2i} = 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{C1i} &= \gamma_{R1i} = \gamma_{RC1i} = 0.3746 \\ \gamma_{C2i} &= \gamma_{R2i} = \gamma_{RC2i} = 0.7738\end{aligned}$$

and replace the computation of $r_{G_{Cij}}$ in *MS_CL_5* by:

$$r_{G_{Cij}} = r_{G_{Rij}} = r_{G_{RCij}} = 0.59 \cdot \exp\left(-\frac{D_{ij}}{31}\right) + 0.41 \cdot \exp\left(-\frac{D_{ij}}{800}\right) \quad (40)$$

Step MS_TOT_5: Calculate the rain attenuation time series $A_{R_i}(kT_s)$ starting from the Gaussian noise time series, $\tilde{n}_i(kT_s)$, following the procedure recommended in § 5.2.2 of Annex 1, and replace *Step MS_RA_2* by:

Set:

$$\begin{aligned}\beta_{R1i} &= \beta_{RC1i} = 9.0186 \times 10^{-4} \text{ (s}^{-1}\text{)} \\ \beta_{R2i} &= \beta_{RC2i} = 5.0990 \times 10^{-5} \text{ (s}^{-1}\text{)} \\ \gamma_{R1i} &= \gamma_{RC1i} = 0.3746 \\ \gamma_{R2i} &= \gamma_{RC2i} = 0.7738\end{aligned}$$

and replace the computation of $r_{G_{Rij}}$ in *MS_RA_5* by:

$$r_{G_{Rij}} = r_{G_{RCij}} = 0.59 \cdot \exp\left(-\frac{D_{ij}}{31}\right) + 0.41 \cdot \exp\left(-\frac{D_{ij}}{800}\right) \quad (41)$$

Step MS_TOT_6: For each site number $i = \{1, 2, \dots, M\}$, and for each time stamp kT_s , if $A_{R_i}(kT_s) > 0$ and $A_{C_i}(kT_s) > A_{C_{i,thresh}} = \frac{K_l}{\sin \phi_i}$, then set $A_{C_i}(kT_s) = \frac{K_l}{\sin \phi_i}$.

Step MS_TOT_7: Define the scintillation fading and enhancement polynomials as:

$$a_{Fade}(P) = -0.061 \times (\log_{10}(P))^3 + 0.072 \times (\log_{10}(P))^2 - 1.71 \times \log_{10}(P) + 3.0$$

$$a_{Enhance}(P) = -0.0597 \times (\log_{10}(P))^3 - 0.0835 \times (\log_{10}(P))^2 - 1.258 \times \log_{10}(P) + 2.672$$

Step MS_TOT_8: For each site number $i = \{1, 2, \dots, M\}$, synthesize unit variance scintillation time series $Sci_{0i}(kT_s)$ following the method recommended in § 6 of Annex 1.

Step MS_TOT_9: For each site number $i = \{1, 2, \dots, M\}$, calculate the correction coefficient time series $C_{xi}(kT_s)$ in order to distinguish between scintillation fades and enhancements:

$$C_{x_i}(kT_s) = \begin{cases} \frac{a_{Fade}(100 \times Q[Sci_{0_i}(kT_s)])}{a_{Enhance}(100 \times Q[Sci_{0_i}(kT_s)])} & \text{for } Sci_{0_i}(kT_s) > 0 \\ 1 & \text{for } Sci_{0_i}(kT_s) \leq 0 \end{cases} \quad (42)$$

where the Q function is defined in § 3.1.2 of Annex 1 of this Recommendation and specified in Recommendation ITU-R P.1057.

Step MS_TOT_10: If $C_{x_i}(kT_s) < 1$ or $100 \times Q[Sci_{0_i}(kT_s)] > 45$ then $C_{x_i}(kT_s) = 1$ (43)

Step MS_TOT_11: For each site number $i = \{1, 2, \dots, M\}$, calculate the scintillation standard deviation σ_{Si} according to the method “Calculation of monthly and long-term statistics of amplitude scintillations at elevation angles greater than 5 degrees” Recommendation ITU-R P.618.

Step MS_TOT_12: For each site number $i = \{1, 2, \dots, M\}$, transform the intermediate underlying Gaussian process $G_{WV_i}(kT_s)$ from step *MS_TOT_3* into the Gamma distributed time series $Z_i(kT_s)$ as follows:

$$Z_i(kT_s) = Gam^{-1} \left[Q(G_{WV_i}(kT_s)), 10, \frac{\sigma_{S_i}}{10} \right] \quad (44)$$

where the function Gam is the Gamma complementary cumulative distribution function specified in Recommendation ITU-R P.1057 and defined as:

$$Gam(x, k, \theta) = \int_x^{\infty} \frac{t^{k-1} \exp(-t/\theta)}{\Gamma(k)\theta^k} dt \quad (45)$$

Note that $Q(G_{WV_i}(kT_s)) = \exp \left(- \left(\frac{A_{WV_i}(kT_s)}{\lambda_{WV_i}} \right)^{k_{WV_i}} \right)$.

Step MS_TOT_13: For each site number $i = \{1, 2, \dots, M\}$, calculate the scintillation time series $Sci_i(kT_s)$ as follows:

$$Sci_i(kT_s) = \begin{cases} Sci_{0_i}(kT_s) \times C_{x_i}(kT_s) \times Z_i(kT_s) \times [A_{R_i}(kT_s)]_{i2}^5 & \text{for } A_{R_i}(kT_s) > 1 \\ Sci_{0_i}(kT_s) \times C_{x_i}(kT_s) \times Z_i(kT_s) & \text{for } A_{R_i}(kT_s) \leq 1 \end{cases} \quad (46)$$

Step MS_TOT_14: For each site number $i = \{1, 2, \dots, M\}$, calculate total tropospheric impairments time series $A_{TOTi}(kT_s)$ as follows:

$$A_{TOTi}(kT_s) = A_{R_i}(kT_s) + A_{C_i}(kT_s) + A_{V_i}(kT_s) + A_{O_i} + S_{C_i}(kT_s) \quad (47)$$

Annex 3

Time series synthesis of rain attenuation on single terrestrial paths

The following method should be used to synthesize time series of rain attenuation on single terrestrial paths. The method to synthesize single-site rain attenuation time series on Earth-space paths described in § 5 of Annex 1 can be used for a single terrestrial path with the following changes.

Similar to Earth-space paths:

- It is also assumed that the long-term statistics of rain attenuation on terrestrial paths is a conditioned log-normal distribution.
- The ITU-R rain attenuation prediction method in Recommendation ITU-R P.530 is well-approximated by a conditioned log-normal distribution over the most significant range of exceedance probabilities.
- Since the terrestrial rain attenuation prediction method predicts non-zero rain attenuation for exceedance probabilities greater than the probability of rain, the time series synthesis method adjusts the attenuation time series so the rain attenuation corresponding to exceedance probabilities greater than the probability of rain is 0 dB.
- The time series synthesis method for terrestrial paths is valid for frequencies between 4 GHz and 40 GHz and path lengths between 2 km and 60 km.

To synthesize single-site rain attenuation time series on terrestrial paths, perform all of the steps in § 5.1.2 but replace paragraph A, by the following paragraph A':

A Estimation of m_R , σ_R and P_R

The parameters m_R and σ_R are determined from the cumulative distribution of rain attenuation vs. probability of occurrence. Rain attenuation statistics can be determined from local data, or, if local data is not available, the rain attenuation prediction method in Recommendation ITU-R P.530 can be used.

For the path and frequency of interest, perform a log-normal fit to the rain attenuation vs. probability of occurrence as follows:

Step SS_RA_1': Determine P_R (% of time), the probability of rain attenuation on the path from local measured data, or, in the absence of measured data, the probability of rain, $P_{0_{annual}}$, given in Recommendation ITU-R P.837 can be used..

Step SS_RA_2': Construct the set of pairs $[P_i/P_R, A_{Ri}]$ where P_i (% of time) is the probability the rain attenuation A_{Ri} (dB) is exceeded where $P_i \leq P_R$. The specific values of P_i should consider the probability range of interest; however, a suggested set of time percentages is 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, 0.3, 0.5, 1, 2, 3, 5, and 10%, with the constraint that $P_i \leq P_R$.

Step SS_RA_3': Transform the set of pairs $[P_i/P_R, A_{Ri}]$ to $\left[Q^{-1}\left(\frac{P_i}{P_R}\right), \ln A_{Ri} \right]$,

where the Q function is defined in § 3.1.2 and specified in Recommendation ITU-R P.1057.

Step SS_RA_4: Determine the variables m_R and σ_R by performing a least-squares fit to $\ln A_{Ri} = \sigma_R Q^{-1}\left(\frac{P_i}{P_R}\right) + m_R$ for all i . The least-squares fit can be determined using the “Step-by-step procedure to approximate a complementary cumulative distribution by a log-normal complementary cumulative distribution” described in Recommendation ITU-R P.1057.
