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International Telecommunication Union
Radiocommunication Sector

Recommendation ITU-R P.1511-3

(08/2024)

P Series: Radiowave propagation

Topography for Earth-space propagation modelling

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Series	Title
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BR	Recording for production, archival and play-out; film for television
BS	Broadcasting service (sound)
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F	Fixed service
M	Mobile, radiodetermination, amateur and related satellite services
P	**Radiowave propagation**
RA	Radio astronomy
RS	Remote sensing systems
S	Fixed-satellite service
SA	Space applications and meteorology
SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
TF	Time signals and frequency standards emissions
V	Vocabulary and related subjects

***Note**: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R P.1511-3

Topography for Earth-space propagation modelling

(Question ITU-R 214/3)

(2001-2015-2019-2024)

Scope

This Recommendation provides global topographic data, information on geographic coordinates, and topographic height data for the prediction of propagation effects for Earth-space paths in ITU-R P-series Recommendations.

Keywords

Topography, geographic coordinates, geodetic height, orthometric height, ellipsoid, geoid, geoid undulation

The ITU Radiocommunication Assembly,

considering

a) that information on ground topography is needed for the prediction of attenuation and scattering;

b) that the information is needed for all locations of the globe, especially when regional or continental calculations are necessary,

recommends

1 that the data in Annex 1 should be used to obtain the topographic height of the surface of the Earth above mean sea level when no local data or data with better spatial resolution are available;

2 that the method in Annex 1 should be used to convert the height above the WGS 84 ellipsoid to the height above the WGS 84 geoid (i.e. the height above mean sea level), or *vice versa*, when no local data or data with better resolution are available;

3 that the method in Annex 1 should be used to calculate the look angles and range between two stations in geodetic coordinates.

Acronyms/Abbreviations

EGM Earth Gravitational Model

WGS World Geodetic System

Related ITU-R Recommendations

Recommendation ITU-R P.618

Recommendation ITU-R P.676

Recommendation ITU-R P.836

Recommendation ITU-R P.1144

Recommendation ITU-R P.2145

NOTE – The latest revision/edition of the Recommendation should be used.

List of symbols

a	semi-major (equatorial) radius
b	semi-minor (polar) radius
f	flattening factor
Φ	geocentric latitude
φ	geodetic latitude
$R(\varphi)$	geocentric Earth radius as a function of geodetic latitude
R_1	average Earth radius
R_2	radius of a sphere of equal area of the Earth
R_3	radius of a sphere of equal volume of the Earth
$h_{EGM2008}$	geoid undulation (height of the WGS 84 geoid relative to the WGS 84 ellipsoid)
$h_{ellipsoid}$	geodetic height above the WGS 84 ellipsoid
h_{geoid}	orthometric height above the WGS 84 geoid
R_{xyz}	3-dimensional rotation matrix
N_s	radius of curvature in the prime vertical at the target geodetic latitude
N_g	radius of curvature in the prime vertical at the origin geodetic latitude
λ	longitude (position relative to the prime reference meridian)
α	geodetic azimuth angle
υ	geodetic elevation angle
R	free-space slant range

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Annex 1

1 Topography

The following sections provide methods to predict or calculate the heights of three different Earth surfaces:

Topographic height: Topographic height, described in § 1.1, is the height of the physical surface of the Earth above mean sea level, which may be highly irregular. For land, this is the landmass topography, and, for water (e.g. ocean, lake, sea), this is the surface of the water.

WGS 84 reference ellipsoid: The WGS 84 reference ellipsoid, described in § 2, is a simple approximation of the figure and gravity field of the Earth. The WGS 84 ellipsoid approximates mean sea level to within ±100 m. Typical radionavigation system receivers report heights relative to the WGS 84 reference ellipsoid.

WGS 84 reference geoid: The WGS 84 reference geoid, described in § 2, is a combination of the WGS 84 reference ellipsoid and the Earth gravitational model EGM2008, which characterizes the undulation of the equipotential surface of the gravity field of the Earth. The WGS 84 reference geoid is the standard reference for mean sea level.

1.1 Topographic height

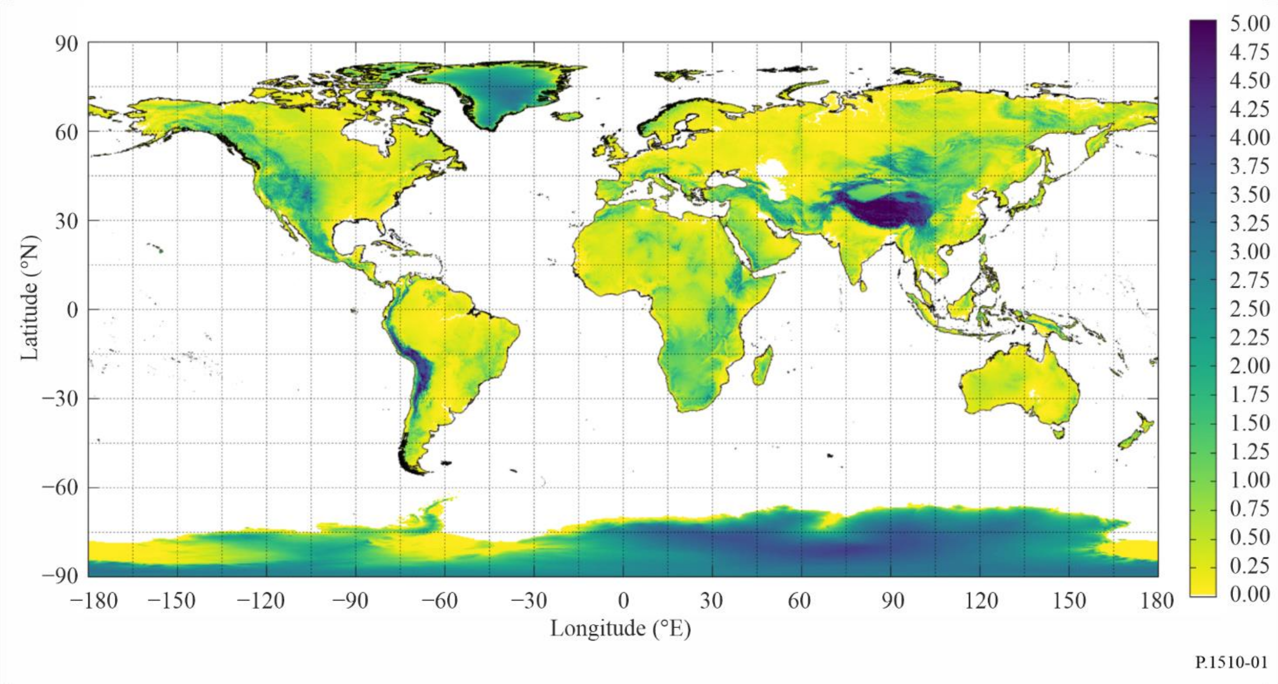
Topographic height is defined as the height of the surface of the Earth above mean sea level. The values of topographic height (m) are an integral part of this Recommendation and are available in the form of a digital map provided in the file R-REC-P1511-3-1.zip within the supplement file R-REC-P.1511-3-202408-I!!ZIP-E.

Topographic height data are provided on a 1/12° grid in both latitude and longitude. The topographic height of a location different from a grid point, can be obtained by performing a bi-cubic interpolation of the values at the sixteen closest grid points, as described in Annex 1 of Recommendation ITU-R P.1144.

A global map of the topographic height of surface of the Earth above mean sea level is shown in Fig. 1.

FIGURE 1

Topographic height of the surface of the Earth above mean sea level (km)



Information on coastlines and country borders can be obtained from the ITU Digitized World Map that is available from the BR.

2 WGS 84 Earth coordinate system

The WGS 84 Earth coordinate system represents the surface of the Earth as a geoid, which is an equipotential gravitational surface that approximates mean sea level. The geoid is a combination of a reference ellipsoid and an Earth Gravitation Model (EGM).

Unless otherwise specified, the latitudes and longitudes in ITU-R P-series Recommendations are geodetic rather than geocentric; i.e. the latitudes and longitudes defined by the WGS 84 ellipsoid.

2.1 WGS 84 reference ellipsoid

The WGS 84 reference ellipsoid is defined by its (equatorial) semi-major axis, a, where a = 6 378.137 km, and a flattening factor, $1/f$, where f = 298.257 223 563.

The (polar) semi-minor axis, b, is defined as $b = a\,(1 - f)$, in which case, $b \approx$ 6 356.752 314 245 km.

The geocentric Earth radius $R(\varphi)$ at geodetic latitude φ is:

$$R(\varphi) = \sqrt{\frac{(a^2 \cos\varphi)^2 + (b^2 \sin\varphi)^2}{(a \cos\varphi)^2 + (b \sin\varphi)^2}} \qquad (1)$$

For a location on the surface of the WGS 84 reference ellipsoid, the relation between geocentric latitude Φ and geodetic latitude φ is:

$$\tan\Phi = (1 - f)^2 \tan\varphi \qquad (2)$$

In addition, three Earth radii measures are defined as follows:

Term	Notation	Value (km)
Average Earth radius	R_1	6 371.008 771 4
Radius of a sphere of equal area	R_2	6 371.007 181 0
Radius of a sphere of equal volume	R_3	6 371.000 790 0

The average Earth radius is defined as the average of the three semi-axes; i.e. $(2a + b)/3$. R_1, R_2, and R_3 are all well-approximated by 6 371.0 km.

2.2 WGS 84 reference geoid

Unless otherwise specified, the height in ITU-R P-series Recommendations is the height above mean sea level defined by the WGS 84 geoid. The height difference between the WGS 84 geoid and the WGS 84 ellipsoid is the geoid undulation, $h_{EGM2008}$, defined by the 2008 version of the U.S. National Geospatial-Intelligence Agency (NGA) Earth Gravitational Model, EGM2008.

For any location, the height above the ellipsoid, $h_{ellipsoid}$, the height above the geoid, h_{geoid}, and $h_{EGM2008}$ are related by:

$$h_{ellipsoid} = h_{EGM2008} + h_{geoid} \tag{3}$$

or

$$h_{geoid} = h_{ellipsoid} - h_{EGM2008} \tag{4}$$

For practical simplicity, typical radionavigation system receivers report heights relative to the WGS 84 ellipsoid, $h_{ellipsoid}$. Equation (4) can be used to convert a height above the WGS 84 ellipsoid to a height above the WGS 84 geoid.

The values of $h_{EGM2008}$ (m) in a tide-free[1] system are an integral part of this Recommendation and are available in the form of a digital map provided in the file R-REC-P1511-3-2.zip contained in the supplement file R-REC-P.1511-3-202408-I!!ZIP-E.

The data are provided on a 1/12° grid in both latitude and longitude. For a location different from the grid points, $h_{EGM2008}$ at the desired location can be obtained by performing a bi-cubic interpolation of the values at the sixteen closest grid points, as described in Annex 1 of Recommendation ITU-R P.1144.

A global map of $h_{EGM2008}$ is shown in Fig. 2, and the maximum absolute value of $h_{EGM2008}$ is ~100 m.

Some P-series integral data products that reference heights above mean sea level have been derived using EGM96, the 1996 version of the U.S. National Geospatial-Intelligence Agency (NGA) Earth Gravitational Model rather than EGM2008. While EGM2008 represents significant improvements relative to EGM96 in both spatial resolution and accuracy, the global rms difference between EGM2008 and EGM96 is less than 1 m.

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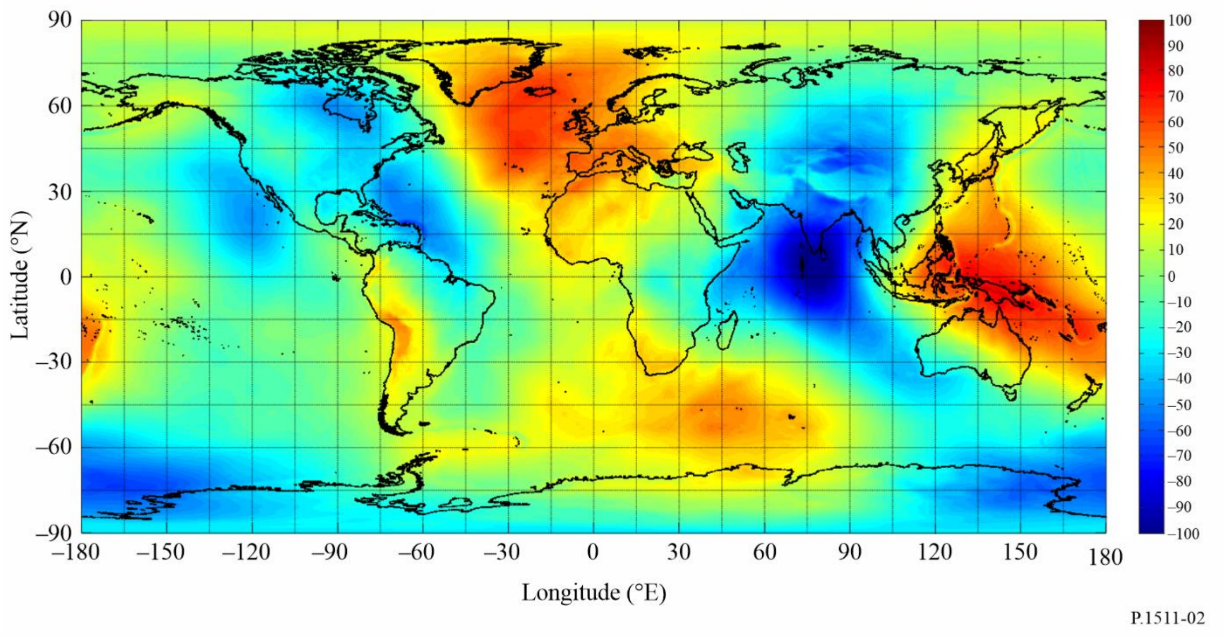
[1] A tide-free system ignores the deformation of the Earth and the tidal effects of the Sun and the Moon.

FIGURE 2

Geoid undulation, $h_{EGM2008}$, from EGM2008 (m)



3 Calculation of look angles and range between two stations in geodetic coordinates

The free-space look angles and free-space slant range between two stations in geodetic coordinates can be calculated as follows, where:

- φ_g : Origin geodetic latitude
- λ_g : Origin longitude
- h_g : Origin height above mean sea level (e.g. above the WGS 84 reference ellipsoid) (km)
- φ_s : Target geodetic latitude
- λ_s : Target longitude
- h_s : Target height above mean sea level (e.g. above the WGS 84 reference ellipsoid) (km)
- α : free-space geodetic azimuth angle from the origin location to the target location
- ν : free-space geodetic elevation angle from the origin location to the target location
- R : free-space slant range between the origin location and target location

and a and f are defined in § 2.1.

Step 1: Calculate:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} - \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} \tag{5}$$

where:

$$\begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} = \begin{bmatrix} \left(N_g + h_g\right) \cos\varphi_g \cos\lambda_g \\ \left(N_g + h_g\right) \cos\varphi_g \sin\lambda_g \\ \left[N_g (1-f)^2 + h_g\right] \sin\varphi_g \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} (N_s + h_s)\cos\varphi_s\cos\lambda_s \\ (N_s + h_s)\cos\varphi_s\sin\lambda_s \\ [N_s(1-f)^2 + h_s]\sin\varphi_s \end{bmatrix} \tag{7}$$

and

$$N_g = \frac{a}{\sqrt{1-(2f-f^2)\sin^2(\varphi_g)}} \tag{8a}$$

$$N_s = \frac{a}{\sqrt{1-(2f-f^2)\sin^2(\varphi_s)}} \tag{8b}$$

Step 2: Calculate:

$$\begin{bmatrix} e \\ n \\ u \end{bmatrix} = [R_{xyz}] \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{9}$$

where:

$$[R_{xyz}] = \begin{bmatrix} -\sin\lambda_g & \cos\lambda_g & 0 \\ -\sin\varphi_g\cos\lambda_g & -\sin\varphi_g\sin\lambda_g & \cos\varphi_g \\ \cos\varphi_g\cos\lambda_g & \cos\varphi_g\sin\lambda_g & \sin\varphi_g \end{bmatrix} \tag{10}$$

Step 3: Then, ν , the elevation (i.e. the vertical) angle at the origin in the plane containing the geodetic vertical (i.e. normal to the ellipsoid) measured from the local geodetic horizon; α, the azimuth angle at the origin in the plane of the local geodetic horizon measured clockwise from geodetic north; and R, the free-space slant range between the origin and the target are

$$\nu = \tan^{-1}\frac{u}{\sqrt{e^2+n^2}} \tag{11}$$

$$\alpha = \tan^{-1}\frac{e}{n} \tag{12}$$

$$R = \sqrt{x^2 + y^2 + z^2} \quad \text{(km)} \tag{13}$$

$\tan^{-1}\frac{y}{x}$ should be calculated using the four-quadrant inverse tangent function $\mathrm{atan2}(y, x)$ which calculates the counter clockwise angle between the positive x-axis and the ray from the origin to the point (x, y) in the Cartesian plane. By convention, $\mathrm{atan2}(0,0) = 0$.

NOTE – Matlab and Octave implement these calculations using the function geodetic2aer.