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Radiocommunication Sector of ITU

Recommendation ITU-R P.1407-4
(10/2009)

**Multipath propagation
and parameterization
of its characteristics**

P Series
Radiowave propagation



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Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

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RECOMMENDATION ITU-R P.1407-4

**Multipath propagation and parameterization
of its characteristics**

(Question ITU-R 203/3)

(1999-2003-2005-2007-2009)

Scope

Recommendation ITU-R P.1407 describes the nature of multipath propagation and defines the appropriate parameters for the statistical description of multipath effects, and provides examples of correlation effects among multiple propagation paths and their computation.

The ITU Radiocommunication Assembly,

considering

- a) the necessity of estimating the effects of multipath on services employing digital systems;
- b) that it is desirable to standardize the terminology and expressions used to characterize multipath,

recommends

- 1 that, to describe the concepts of multipath in a consistent manner, the terms and definitions given in Annex 1 should be employed;
- 2 that the correlation concepts of Annex 2 should be used to analyse the effects of multiple-input, multiple-output (MIMO) systems.

Annex 1**1 Introduction**

In radio systems with low antenna heights, there are often multiple indirect paths between the transmitter and receiver due to reflections from surrounding objects, in addition to the direct path when there is line-of-sight. Such multipath propagation is particularly significant in urban environments, where the sides of buildings and paved road surfaces provide strong reflections. As a result, the received signal consists of the summation of several components having various amplitudes, phase angles and directions of arrival.

The resulting spatial variability of signal strength can be viewed as having two regimes:

- a) rapid fading which varies over distances of the order of a wavelength due primarily to changes in phase angles of different signal components;
- b) slow fading which varies over larger distances due primarily to changes in shadowing loss by surrounding objects.

In addition, the various signal components can be Doppler shifted by different amounts due to the movement of the mobile or of reflecting objects such as vehicles.

The multipath mobile channel can be characterized in terms of its impulse response which varies at a rate dependent on the speed of the mobile and/or the scatterers. Therefore, a receiver has to be able to cope with the signal distortion arising from echoes in the channel as well as the rapid changes in the nature of this distortion. Such characteristics of the mobile radio channel are described by the power delay profiles and the Doppler spectra which are obtained from wideband channel sounding measurements.

Signals transmitted to and from moving vehicles in urban or forested environments exhibit extreme variations in amplitude due to multiple scattering. Fades of 30 dB or more below the mean level are common. The instantaneous field strength when measured over distances of a few tens of wavelengths is approximately Rayleigh-distributed. The mean values of these small sector distributions vary widely from area to area, depending on the height, density and distribution of hills, trees, buildings and other structures.

Multipath propagation characteristics are a major factor in controlling the quality of digital mobile communications. Physically, multipath propagation characteristics imply multipath number, amplitude, path-length difference (delay), and arrival angle. These can be characterized from the power delay profile. Alternatively, the Fourier transform of the complex impulse response results in the complex transfer function whose amplitude versus frequency characteristics give the frequency selectivity of the multipath and is related to the correlation bandwidth.

Definitions of small sector (or small-scale) channel parameters are given in § 2 and 3. Statistics of small-scale parameters are subsequently used to produce cumulative distribution functions (CDF). Medium-scale CDF covers a particular route of measurement, which is of the order of tens to hundreds of metres. The combined data set from a number of medium-scale routes is considered to be large-scale or global characterization which is representative of the surveyed environment e.g. hilly terrain, urban, suburban, indoor large rooms, corridors, etc.

A time-varying linear channel can be characterized by a linear transversal filter. The output of this filter contains a sum of delayed, attenuated and Doppler shifted versions of the input signal. The channel is then represented by the delay-Doppler-spread function, sometimes referred to as the scattering function. This function represents the multipath phenomenon in the three dimensions of excess delay, Doppler frequency and power density. This formulation is particularly suitable for realizing a hardware simulator in the form of a dynamic transversal filter.

2 Parameters of delay profiles

2.1 Definitions of power delay profiles

The appropriate parameters for the statistical description of regarding multipath delay time can be computed from any of three types of power delay profiles: the instantaneous power delay profile; short-term power delay profile; or long-term power delay profile, which are either time averages obtained when the receiver is stationary and represent variations in the environment, or spatial averages obtained when the receiver is in motion.

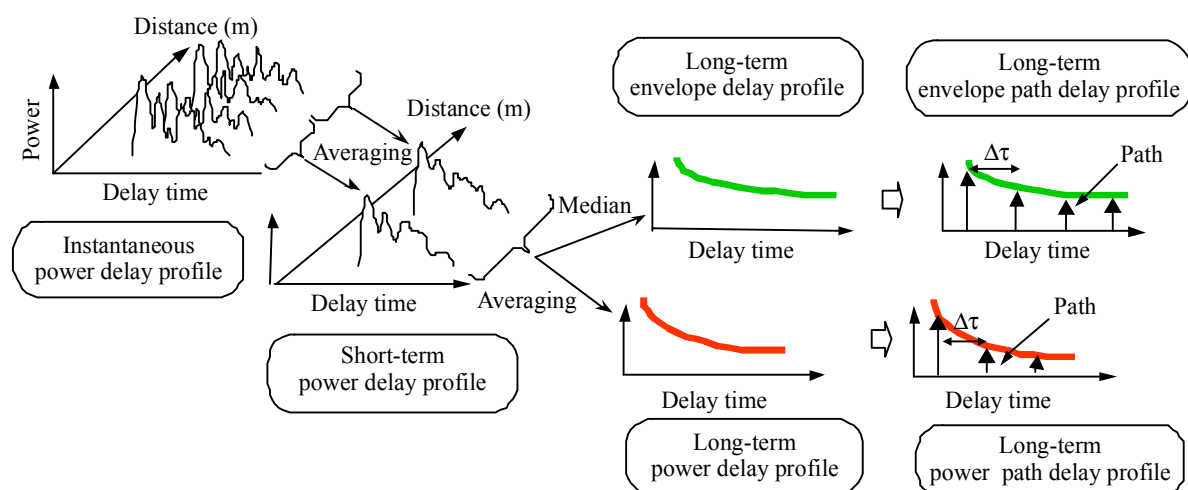
Definitions of power delay profiles are given as shown in Fig. 1.

The instantaneous power delay profile is the power density of the impulse response at one moment at one point.

The short-term power delay profile is obtained by spatially averaging the instantaneous power delay profiles over several tens of wavelengths within the range where the same multipath components are maintained in order to suppress the variation of rapid fading.

FIGURE 1

Definition of power delay profiles



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The long-term power delay profile is obtained by spatially averaging the short-term power delay profiles at approximately the same distance from the base station (BS) in order to suppress the variations due to shadowing.

Long-term power delay profiles with a discrete excess delay time normalized by time resolution $1/B$, where B is the bandwidth, are defined as long-term power path delay profiles, instead of continuous power delay profiles.

On the other hand, the long-term envelope delay profile is the median value of the short-term power delay profiles at approximately the same distance from the base station; it expresses the shape of the delay profile at the area considered.

2.2 Definitions of statistical parameters

The appropriate parameters for the statistical description regarding multipath delay time are given below. The *average delay* is the power weighted-average of the excess delays measured and is given by the first moment of the power delay profile (the square of the amplitude of the impulse response).

The *r.m.s. delay spread* is the power weighted standard deviation of the excess delays and is given by the second moment of the power delay profile. It provides a measure of the variability of the mean delay.

The *delay window* is the length of the middle portion of the power delay profile containing a certain percentage (typically 90%) of the total energy found in that impulse response.

The *delay interval* is defined as the length of the impulse response between two values of excess delay which mark the first time the amplitude of the impulse response exceeds a given threshold, and the last time it falls below it. The threshold used depends on the dynamic range of the measuring equipment: a typical value is 20 dB below the peak level of the delay profile.

The *correlation bandwidth* is defined as the band of frequencies for which the autocorrelation function of the transfer function is above a given threshold; a typical threshold value is 0.5.

The *number of multipath or signal components* is the number of peaks in a power delay profile whose amplitude are within A dB of the highest peak and above the noise floor.

2.2.1 Total energy

The *total energy*, P_m , of the impulse response is:

$$P_m = \int_{t_0}^{t_3} P(t) dt \quad (1)$$

where:

- $P(t)$: power density of the impulse response
- t : delay with respect to a time reference
- t_0 : instant when $P(t)$ exceeds the cut-off level for the first time
- t_3 : instant when $P(t)$ exceeds the cut-off level for the last time.

2.2.2 Average delay time

The average delay, T_D , is given by the first moment of the power delay profile:

$$T_D = \frac{\int_0^{\tau_e} \tau P(\tau) d\tau}{\int_0^{\tau_e} P(\tau) d\tau} - \tau_a \quad (2a)$$

where:

- τ : excess time delay variable and is equal to $t - t_0$
- τ_a : arrival time of the first received multipath component (first peak in the profile)
- $\tau_e = t_3 - t_0$.

In discrete form with time resolution $\Delta\tau$ ($= 1/B$), equation (2a) becomes:

$$T_D = \frac{\sum_{i=1}^N \tau_i P(\tau_i)}{\sum_{i=1}^N P(\tau_i)} - \tau_M \quad (2b)$$

$$\tau_i = (i - 1) \Delta\tau = (i - 1)/B \quad (i = 1, 2, \dots, N)$$

where $i = 1$ and N are the indices of the first and the last samples of the delay profile above the threshold level, respectively, and M is the index of the first received multipath component (first peak in the profile).

The delays may be determined from the following relationship:

$$t_i(\mu s) = 3.3 r_i \quad \text{km} \quad (3)$$

where r_i is the sum of the distances from the transmitter to the multipath reflector, and from the reflector to the receiver, or is the total distance from the transmitter to receiver for t_{LOS} .

2.2.3 r.m.s. delay spread

The root mean square (r.m.s.) delay spread, S , is defined by the square root of the second central moment:

$$S = \sqrt{\frac{\int_0^{\tau_e} (\tau - T_D - \tau_a)^2 P(\tau) d\tau}{\int_0^{\tau_e} P(\tau) d\tau}} \quad (4a)$$

In discrete form with time resolution $\Delta\tau$, equation (4a) becomes:

$$S = \sqrt{\frac{\sum_{i=1}^N (\tau_i - T_D - \tau_M)^2 P(\tau_i)}{\sum_{i=1}^N P(\tau_i)}} \quad (4b)$$

2.2.4 Delay window

The delay window, W_q , is the length of the middle portion of the power delay profile containing a certain percentage, q , of the total power:

$$W_q = (t_2 - t_1) \quad (5)$$

whereby the boundaries t_1 and t_2 are defined by:

$$\int_{t_1}^{t_2} P(t) dt = \frac{q}{100} \int_{t_0}^{t_3} P(t) dt = \frac{q}{100} P_m \quad (6)$$

and the energy outside of the window is split into two equal parts $\left(\frac{100-q}{200}\right)P_m$.

2.2.5 Delay interval

The delay interval, I_{th} , is defined as the time difference between the instant t_4 when the amplitude of the power delay profile first exceeds a given threshold P_{th} , and the instant t_5 when it falls below that threshold for the last time:

$$I_{th} = (t_5 - t_4) \quad (7)$$

2.2.6 Frequency correlation bandwidth

The Fourier transform of the power density of the impulse response provides the autocorrelation $C(f)$ of the transfer function:

$$C(f) = \int_0^{\tau_e} P(\tau) \exp(-j 2 \pi f \tau) d\tau \quad (8)$$

For a Rician channel, equation (8) underestimates the correlation bandwidth. For such channels it is more accurate to estimate the correlation bandwidth from the spaced frequency correlation function, which is obtained from the time variant complex transfer function by computing the correlation coefficient for different frequency spacings.

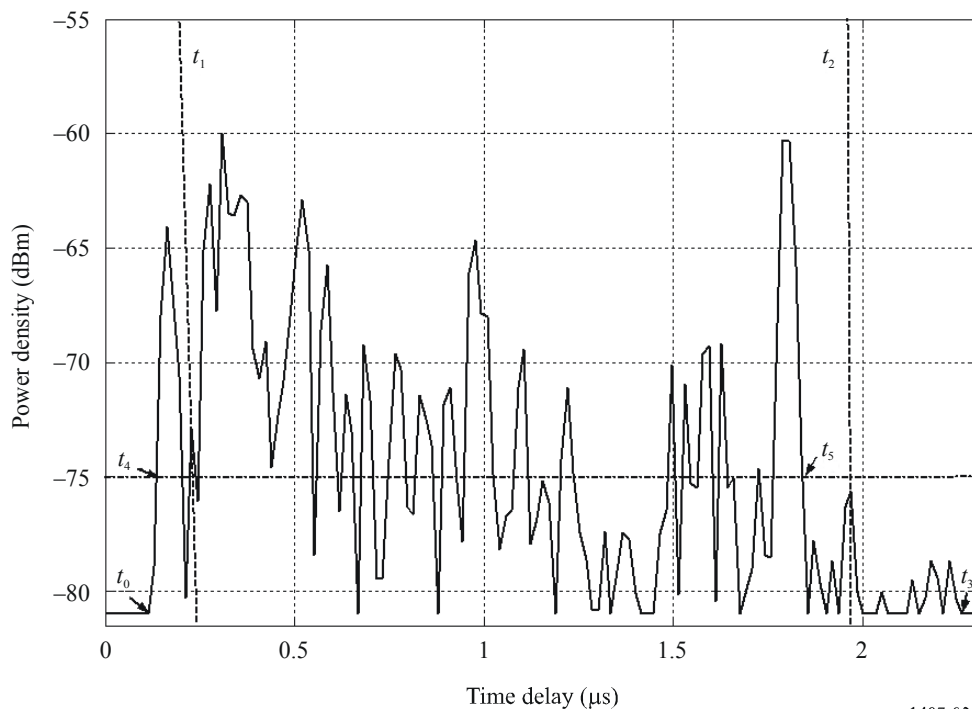
The correlation bandwidth B_x is defined as the frequency for which $|C(f)|$ equals $x\%$ of $C(f=0)$.

2.2.7 Recommended values of parameters

Delay windows for 50%, 75% and 90% power, delay intervals for thresholds of 9, 12 and 15 dB below the peak and correlation bandwidth for 50% and 90% of correlation are recommended when analysing data. It is worth noting that the effects of noise and spurious signals in the system (from RF to data processing) can be very significant. Therefore, it is important to determine the noise and/or spurious threshold of the systems accurately and to allow a safety margin on top of that. A safety margin of 3 dB is recommended, and in order to ensure the integrity of results, it is recommended that a minimum peak-to-spurious ratio of, for example, 15 dB (excluding the 3 dB safety margin) is used as an acceptance criterion before an impulse response is included in the statistics.

An example of the use of some of these terms is given in Fig. 2.

FIGURE 2



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Power delay profile illustrating the following parameters: the delay window, W_{90} , containing 90% of the received power is marked between the two vertical dashed lines (t_1 and t_2), the delay interval, I_{15} , containing the signal above the level 15 dB below the peak, lies between t_4 and t_5 . t_0 and t_3 indicate the start and the end of the profile above the noise floor.

3 Parameters of direction of arrival

3.1 Definition of angle of arrival power profiles

The appropriate parameters for the statistical description regarding the angle of arrival of the multipath can be computed from any of three types of angle of arrival power profiles: instantaneous angle of arrival power profile; short-term angle of arrival power profile; or long-term angle of arrival power profile, which are either time averages obtained when the receiver is stationary and represent variations in the environment, or spatial averages obtained when the receiver is in motion.

Definitions of angle of arrival power profiles are given as shown in Fig. 4.

The instantaneous angle of arrival power profile is the power density of the impulse response at one moment at one point.

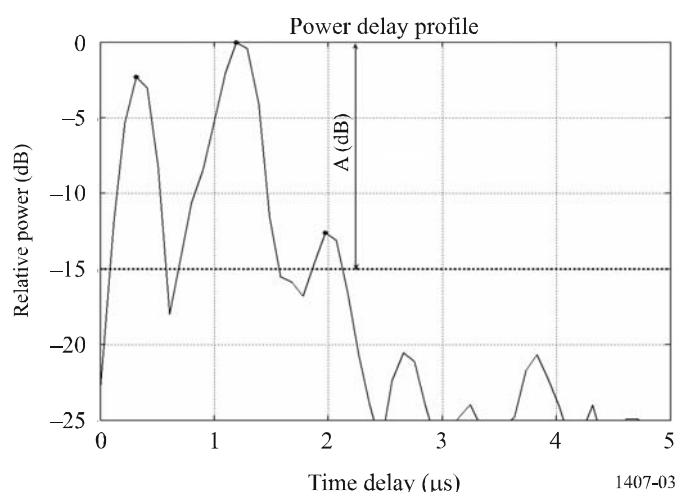
The short-term angle of arrival power profile is obtained by spatially averaging the instantaneous angle of arrival power profiles over several tens of wavelengths within the range where the same multipath components are maintained in order to suppress the variations due to rapid fading.

The long-term angle of arrival power profile is obtained by spatially averaging the short-term angle of arrival power profiles at approximately the same distance from the base station (BS) in order to suppress the variation due to shadowing.

Long-term angle of arrival power profiles with a discrete angle normalized by the angular resolution of the antenna are defined as long-term angle of arrival power path profiles, instead of continuous angle of arrival power profiles.

On the other hand, the long-term angle of arrival envelope profile is the median value of the short-term angle of arrival power path profiles at approximately the same distance from the base station, and characterizes the shape of the angle of arrival power profile at the area considered.

FIGURE 3
Power delay profile indicating multipath components above threshold level



3.2 Definitions of statistical parameters

The definitions of the appropriate parameters for the statistical description regarding the multipath angle of arrival are given below:

The *average angle* of arrival is the power-weighted average of the measured directions of arrival and is given by the first moment of the power azimuth spectrum. (It can also be called the power angular profile.)

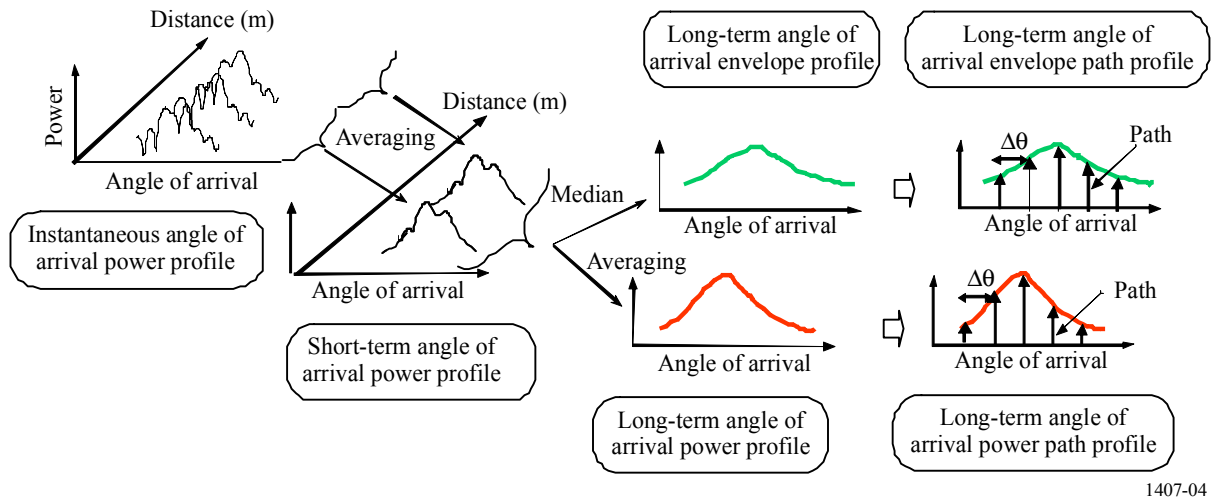
The angle of arrival *power profile* is the angular power characteristic within the azimuth/horizontal plane.

The *r.m.s. angular spread* is the power-weighted standard deviation of the direction of arrival and is given by the second moment of the power angular profile. It provides a measure of the variability of the mean angle of arrival.

The *angular window* is the width of the middle portion of the angle of arrival power profile containing the defined certain percentage of the total energy found in that angle of arrival power profile measurement.

The *angle interval* (or *angular spacing*) is defined as the width of the impulse response (or width of the angular profile) between two values of direction of arrival. It marks the first angle at which the amplitude of the angle of arrival power profile exceeds a given threshold, and the last angle at which it falls below that threshold. The threshold used depends on the dynamic range of the measuring equipment: a typical value is 20 dB below the peak level of the angle of arrival power profile.

FIGURE 4
Definition of the angle of arrival power profiles



3.2.1 Total energy

Let the received power in the direction θ be $P(\theta)$.

The *total energy*, P_0 , of the angle of arrival power profile is defined as the power beyond the threshold level L_0 which is set to separate the signal from noise, as shown in Fig. 5:

$$P_0 = \int_{\theta_0}^{\theta_3} P(\theta) d\theta \quad (9a)$$

where:

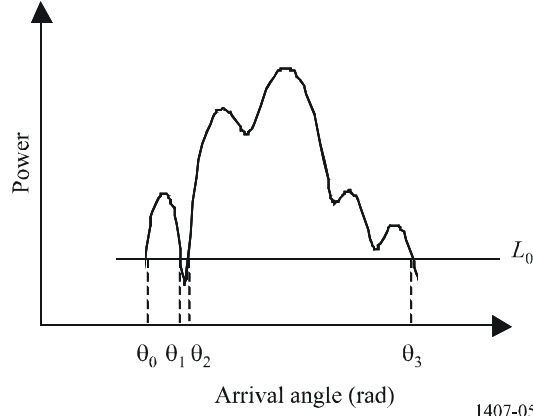
- | | |
|---------------|---|
| θ : | measured from the direction of the principal signal (assumed to be stationary within the duration of the measurement) (rad) |
| $P(\theta)$: | angle of arrival power profile above the threshold level L_0 ; below L_0 , $P(\theta) = 0$ |
| L_0 : | level with some margin (3 dB recommended) over the noise floor |
| θ_0 : | angle of arrival when $P(\theta)$ exceeds the threshold level L_0 for the first time in $\theta_{max}(-\pi, \pi)$ |
| θ_3 : | angle of arrival when $P(\theta)$ exceeds the threshold level L_0 for the last time in $\theta_{max}(-\pi, \pi)$. |

In discrete form equation (9a) becomes:

$$P_0 = \sum_{i=1}^N P(\theta_i) \quad (9b)$$

where $i = 1$ and N are the indices of the first and the last samples of the angle of arrival power profile above the threshold level, respectively.

FIGURE 5
Total energy



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3.2.2 Average arrival angle

The average angle of arrival, T_A , is given by the first moment of the power angular profile:

$$T_A = \frac{1}{P_0} \int_{\theta_0}^{\theta_3} \theta P(\theta) d\theta \quad (10a)$$

In discrete form with angular resolution $\Delta\theta$, equation (10a) becomes:

$$T_A = \frac{\sum_{i=1}^N \theta_i P(\theta_i)}{\sum_{i=1}^N P(\theta_i)} \quad (10b)$$

$$(\theta_i = (i - 1) \Delta\theta \quad (i = 1, 2, \dots, N))$$

where $i = 1$ and N are the indices of the first and the last samples of the angle of arrival power profile above the threshold level, respectively.

3.2.3 r.m.s. angular spread

The r.m.s. angular spread S_A of the direction of arrival is defined as follows:

$$S_A = \sqrt{\frac{1}{P_0} \int_{\theta_0}^{\theta_3} (\theta - T_A)^2 P(\theta) d\theta} \quad (11a)$$

In discrete form with angular resolution $\Delta\theta$, equation (11a) becomes:

$$S_A = \sqrt{\frac{\sum_{i=1}^N (\theta_i - T_A)^2 P(\theta_i)}{\sum_{i=1}^N P(\theta_i)}} \quad (11b)$$

where $i = 1$ and N are the indices of the first and the last samples of the angle of arrival power profile above the threshold level, respectively.

3.2.4 Angular window

The angular window, θ_w , is the width of the middle portion of the angle of arrival power profile containing a percentage q , of the total power as shown in Fig. 6:

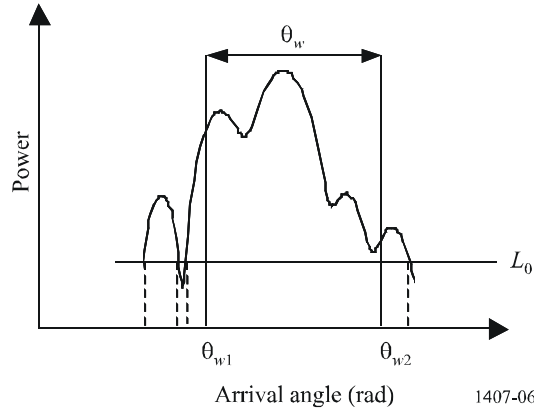
$$\theta_w = \theta_{w2} - \theta_{w1} \quad (12)$$

whereby the boundaries θ_{w1} and θ_{w2} are defined by:

$$\int_{\theta_{w1}}^{\theta_{w2}} P(\theta) d\theta = \frac{q}{100} \int_{\theta_0}^{\theta_3} P(\theta) d\theta = \frac{q}{100} P_0 \quad (13)$$

and the energy outside of the window is split into two equal parts $\left(\frac{100-q}{200}\right)P_0$.

FIGURE 6
Angular window



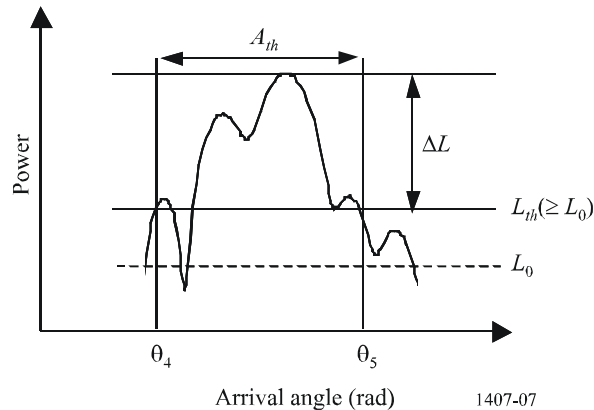
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3.2.5 Angle interval (angular spacing)

The angle interval A_{th} , is defined as the angle difference between the angle θ_4 when the amplitude of the angle of arrival power profile first exceeds a given threshold L_{th} , and the angle θ_5 when it falls below that threshold for the last time as shown in Fig. 7:

$$A_{th} = \theta_5 - \theta_4 \quad (14)$$

FIGURE 7
Angle interval



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3.2.6 Spatial correlation distance

In particular for multiple-output multiple-input (MIMO) channels, the spatial correlation coefficient for different spacing d is obtained from the angle variant complex transfer function of the power angular profile. The spatial correlation coefficient $R(d)$, is defined as follows:

$$R(d) = \frac{\int_{\theta_0}^{\theta_3} P(\theta) \exp(-j2\pi d \sin \theta / \lambda) d\theta}{\int_{\theta_0}^{\theta_3} P(\theta) d\theta} \quad (15)$$

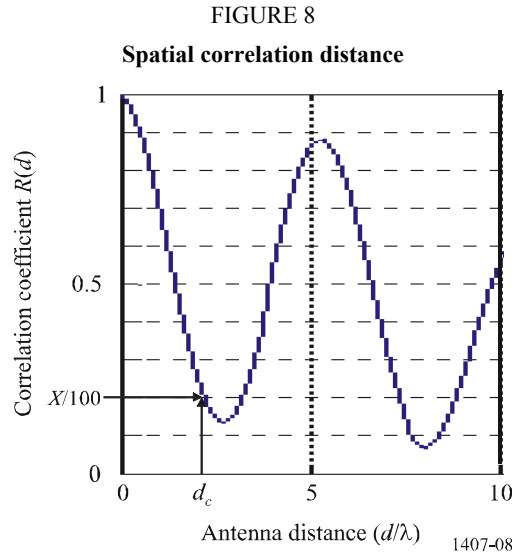
where:

d : distance for different spacing

λ : wavelength.

As shown in Fig. 8, the spatial correlation distance d_c is defined as the first cut off distance at which $|R(d)|$ equals $x\%$ of $|R(d=0)|$.

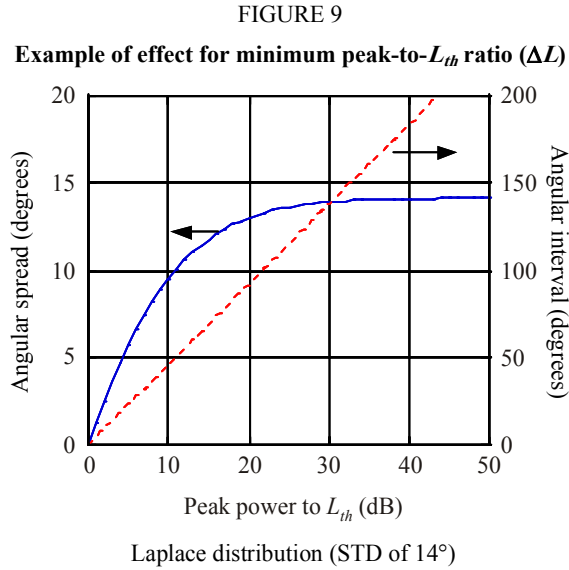
$$|R(d_c)| / |R(0)| = x/100 \quad (16)$$



3.2.7 Recommended parameters

Angular windows for 50%, 75% and 90% power, angle intervals for thresholds of 9, 12 and 15 dB below the peak, and correlation distances for 50% and 90% of correlation are recommended to permit a detailed analysis of data. Furthermore, it is worth noting that the effects of noise and spurious signals in the system (from RF to data processing) can be very significant. Therefore, it is important to determine the noise and/or spurious threshold of the systems accurately and to provide a safety margin on top of that. A safety margin of 3 dB is recommended, and in order to ensure the integrity of results, it is recommended that a minimum peak-to-spurious ratio of, for example, 15 dB (excluding the 3 dB safety margin) be used as an acceptance criterion limiting the angle of arrival power profiles included in the statistics. Figure 9 shows an example of the effect of setting the magnitude of minimum peak-to- L_{th} ratio (ΔL). In this figure, the angle of arrival power profile is assumed to be a Laplace distribution (double exponential distribution) with angular spread of 14° ; angular spread and angular interval are calculated as functions of the peak power-to- L_{th} ratio. This

figure shows that these parameters undergo significant changes even for essentially identical values. Therefore the value used as ΔL in the statistical evaluation should be specified.



Annex 2

1 Introduction

This Annex illustrates some results of computing the correlation coefficients from a power angular profile and the effect of the correlation coefficients on MIMO capacity.

2 Computing the spatial correlation coefficients

The definition in equation (15) of Annex 1 has been used to compute the spatial correlation. This Annex briefly introduces a result and illustrates how the correlation is affected by antenna spacing.

Figure 10 shows an ideal truncated Laplacian power-azimuth spectrum (PAS) such as:

$$PAS_L(\varphi) = \sum_{k=1}^{N_c} \frac{Q_{L,k}}{\sigma_{L,k} \sqrt{2}} \exp \left[-\frac{\sqrt{2} |\varphi - \varphi_{0,k}|}{\sigma_{L,k}} \right] \left\{ \varepsilon[\varphi - (\varphi_{0,k} - \Delta\varphi_k)] - \varepsilon[\varphi - (\varphi_{0,k} + \Delta\varphi_k)] \right\} \quad (17)$$

where $\varepsilon(\varphi)$ is the step function and N_c the number of clusters, $\varphi_{0,k}$ is the mean angle of incidence of k -th cluster, $\sigma_{L,k}$ is the angular spread. PAS is defined over $[\varphi_0 - \Delta\varphi, \varphi_0 + \Delta\varphi]$. The power normalization condition is assumed as:

$$\sum_{k=1}^{N_c} Q_{L,k} \left[1 - \exp \left(-\frac{\sqrt{2} \Delta\varphi_k}{\sigma_{L,k}} \right) \right] = 1 \quad (18)$$

Then the envelope correlation coefficient is given by:

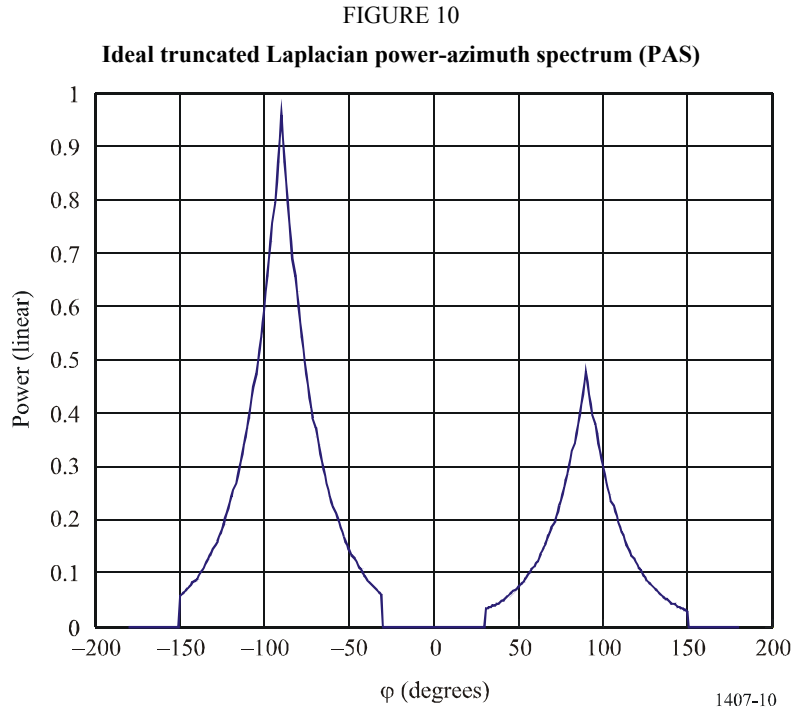
$$\rho_e(D) = |R_{XX}(D) + jR_{XY}(D)|^2 \quad (19)$$

where:

$$D = 2\pi d/\lambda$$

d : antenna spacing
 λ : wavelength,

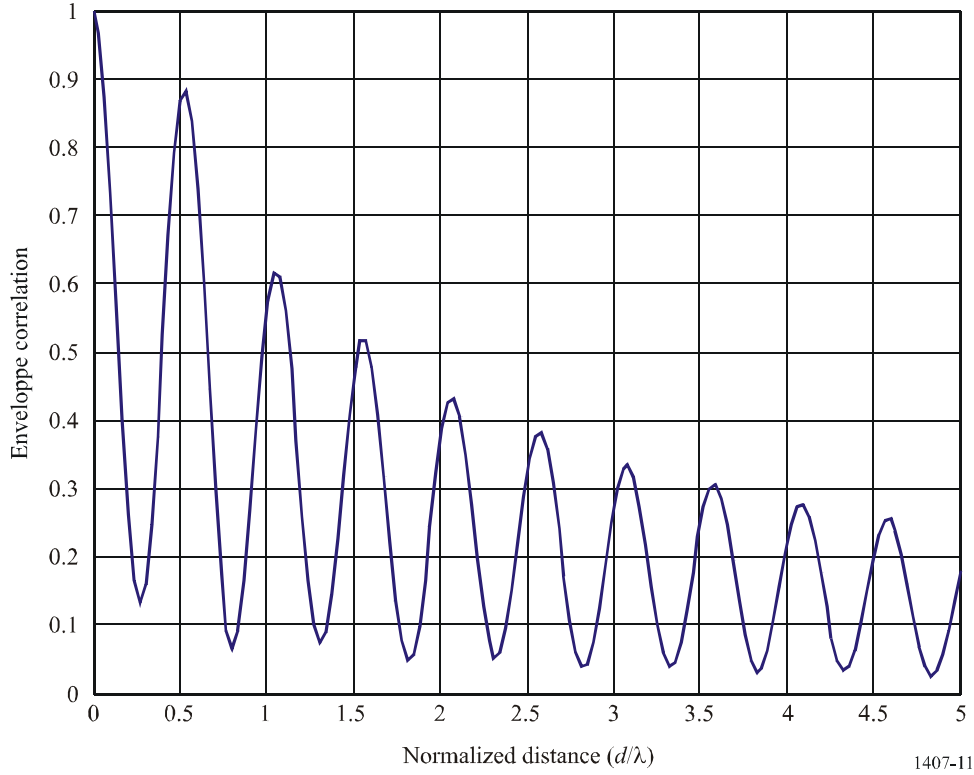
and the cross-correlation functions $R_{XX}(D)$ and $R_{XY}(D)$ are defined in equation (16).



Normalized Laplacian PAS for a two-cluster case. $AS = 30^\circ$, $\varphi_0 \in [-90^\circ, +90^\circ]$.
 Additionally, the $+90^\circ$ cluster has half the power of the -90° case.

Figure 11 illustrates the resulting spatial correlation.

FIGURE 11

Resulting spatial correlation

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Envelope correlation coefficient versus the normalized distance $= d/\lambda$ for the two-cluster case shown in Fig. 10.

3 Effect of the correlation coefficients on MIMO capacity

For Rayleigh fading channels, the ergodic MIMO capacity without channel knowledge at the transmitter is:

$$C = \log_2 \det \left(I_{n_R} + \frac{P}{n_T \sigma^2} R_R^{1/2} H_w R_T H_w^H (R_R^{1/2})^H \right) = \log_2 \det \left(I_{n_R} + \frac{P}{n_T \sigma^2} H_w R_T H_w^H R_R^H \right) \quad (20)$$

where:

n_R and n_T : number of receiver and transmitter antennas, respectively

P : total transmit power

I_{n_R} : $n_R \times n_R$ identity matrix

$(\cdot)^H$ and $\det(\cdot)$: Hermitian and determinant operation, respectively

H_w : matrix whose elements are independent identically-distributed complex Gaussian Random variables with zero mean and unit variance

$(\cdot)^{1/2}$: Hermitian square root of a matrix.

The matrices R_R and R_T determine the spatial correlations between the receivers and the transmitters, respectively, where the channel matrix H is defined by $H = R_R^{1/2} H_w R_T^{1/2}$, $R_R^{1/2}$, and $R_T^{1/2}$ are positive definite Hermitian matrices, and finally they are assumed to be normalized such that $[R_R]_{j,j}$ for $j = 1, K, n_R$ and $[R_T]_{i,i}$ for $i = 1, K, n_T$.

By assuming that R_R and R_T have full rank and $n_R = n_T = n$, then at high S/N the capacity can be approximated as:

$$C \approx \log_2 \det \left(\frac{P}{n_T \sigma^2} H_w H_w^H \right) + \log_2 \det(R_R) + \log_2 \det(R_T) \quad (21)$$

If we denote the eigenvalues of R_R by λ_i , $i = 1, K, n$, then $\sum_{i=1}^n \lambda_i = n$. From the arithmetic mean-geometric mean inequality:

$$\prod_{i=1}^n \lambda_i \leq 1 \quad (22)$$

Since $\det(R_R) = \prod_{i=1}^n \lambda_i$, it follows that $\log_2 \det(R_R) \leq 0$, and is zero only if all eigenvalues of R_R are equal, i.e. $R_R = I_n$. Thus, the correlation determines the MIMO capacity and the loss in the ergodic capacity at high S/N is given by $(\log_2 \det(R_R) + \log_2 \det(R_T))$ bit/s/Hz.

Figure 12 illustrates the effect of spatial correlations on the ergodic capacity of a MIMO channel with $n_R = n_T = 2$. In the figure $R_T = I_2$, is assumed. The receiver correlation matrix is chosen according to:

$$R_R = \begin{bmatrix} 1 & \rho_R \\ \rho_R^* & 1 \end{bmatrix} \quad (23)$$

where ρ_R denotes the spatial correlation between the receive antennas.

