RECOMMENDATION ITU-R P.1240*

# ITU-R METHODS OF BASIC MUF, OPERATIONAL MUF AND RAY-PATH PREDICTION**

(Questions ITU-R 212/3 and ITU-R 223/3)

(1997)

The ITU Radiocommunication Assembly,

*considering*

a) that long-term reference ionospheric data and propagation prediction methods are needed for HF radio-circuit design, service planning and frequency band selection;

b) that maps of ionospheric characteristics are given in Recommendation ITU-R P.1239,

*recommends*

**1** that for the prediction of basic and operational MUFs, use should be made of the formulations contained in Annex 1 (for definitions, see Recommendation ITU-R P.373);

**2** that for the prediction of ray paths, use should be made of the formulations contained in Annex 2.

ANNEX 1

## Prediction of basic and operational MUFs

### 1 Introduction

Empirical formulae are presented for the evaluation of the monthly median basic MUF for the propagation path.

This MUF is estimated as the greatest of the basic MUF values for the propagation modes appropriate to the path length being considered.

The relationship between the operational MUF and basic MUF is given and a computer program is described leading to estimates of path basic MUF, operational MUF and optimum working frequency on a point-to-point propagation path of any length.

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* Radiocommunication Study Group 3 made editorial amendments to this Recommendation in 2000 in accordance with Resolution ITU-R 44.

** Computer programs associated with the prediction procedures and data described in this Recommendation are available from that part of the ITU-R website dealing with Radiocommunication Study Group 3.

## 2 Mode consideration

The modes considered are:

| | |
|---|---|
| 1F2 | 0 to $d_{max}$ |
| Higher order F2 modes | beyond $d_{max}$ |
| 1F1 | 2 000-3 400 km |
| 1E | 0-2 000 km |
| 2E | 2 000-4 000 km |

where the maximum ground range $d_{max}$ (km) for a single hop F2 mode is given by:

$$d_{max} = 4\,780 + (12\,610 + 2\,140 / x^2 - 49\,720 / x^4 - 688\,900 / x^6)\,(1 / B - 0.303)$$

with:

$$B = \text{M(3000)F2} - 0.124 + \left[ \left[\text{M(3000)F2}\right]^2 - 4 \right] \cdot \left[ 0.0215 + 0.005 \sin\left( \frac{7.854}{x} - 1.9635 \right) \right]$$

and $x$ = foF2/foE, or 2 whichever is the larger.

Ionospheric characteristics for the mid-point of the great-circle path are used.

## 3 Prediction of F2-layer basic MUF

### 3.1 Ground distance *D* up to $d_{max}$

The F2-layer basic MUF is given by:

$$\text{F2}(D)\text{MUF} = \left[ 1 + \left( \frac{C_D}{C_{3000}} \right) \left( B - 1 \right) \right] \cdot \text{foF2} + \frac{f_H}{2} \left( 1 - \frac{D}{d_{max}} \right)$$

where:

$f_H$: appropriate gyrofrequency (see Recommendation ITU-R P.1239)

and:

$$C_D = 0.74 - 0.591\,Z - 0.424\,Z^2 - 0.090\,Z^3 + 0.088\,Z^4 + 0.181\,Z^5 + 0.096\,Z^6$$

with $Z = 1 - 2D / d_{max}$

$C_{3000}$: value of $C_D$ for $D = 3\,000$ km where $D$ is the great-circle distance (km).

The above formulae apply for the basic MUF for the x-wave at zero distance, for the o-wave at $d_{max}$ and beyond and for some composite waves at intermediate distances. The corresponding o-wave basic MUF is given for all distances by deleting the last term in $f_H$ from the first formula.

### 3.2 Ground distance *D* greater than $d_{max}$

Values of F2($d_{max}$)MUF are determined for two control-point locations at $d_0/2$ from each terminal along the connecting great-circle path where $d_0$ is the hop-length of the lowest order F2 mode. The path MUF is the lower of the two values.

## 4 Prediction of F1-layer basic MUF

Ionospheric propagation via the F1-layer is important for transmission distances in the 2 000-3 400 km range at mid and high latitudes during the summer months. For these transmission distances the F1-layer basic MUF is taken as the product of the mid-path value of foF1 (see Recommendation ITU-R P.1239) and the *M* factor $M_{F1}$. This *M* factor was derived from ray-tracing calculations on electron density versus height profiles obtained from representative noon ionograms recorded at mid and high latitudes. It is assumed that these factors are applicable for all solar zenith angles. The $M$ factor can be determined from the following numerical expressions:

$$M_{F1} = J_0 - 0.01\,(J_0 - J_{100})\,R_{12}$$

where:

$$J_0 = 0.16 + 2.64 \times 10^{-3}\,D - 0.40 \times 10^{-6}\,D^2$$
$$J_{100} = -0.52 + 2.69 \times 10^{-3}\,D - 0.39 \times 10^{-6}\,D^2$$

and where $D$ represents the great-circle distance (km) in the range of 2 000-3 400 km.

# 5 Prediction of E-layer basic MUF

## 5.1 Ground distance up to 2 000 km

Ionospheric propagation via a single reflection from the E-layer is important over distances up to 2 000 km. The E-layer basic MUF of a particular mode may be estimated as the product of the mid-path value of foE (see Recommendation ITU-R P.1239) and the $M$ factor $M_E$. This $M$ factor based on ray-path calculations for a parabolic model E-layer with hmE = 110 km, ymE = 20 km, when effects of the Earth's magnetic field are neglected, is given by:

$$M_E = 3.94 + 2.80\,x - 1.70\,x^2 - 0.60\,x^3 + 0.96\,x^4$$

where:

$$x = \frac{D - 1\,150}{1\,150}$$

and $D$ represents the great-circle distance (km).

## 5.2 Ground distance between 2 000 and 4 000 km

The 2E MUF, for ranges between 2 000 and 4 000 km, is taken as E(2000)MUF expressed in terms of the mid-path foE.

# 6 Prediction of the operational MUF

For prediction purposes the operational MUF (see Recommendation ITU-R P.373) when determined by an F2-mode is expressed in terms of the basic MUF for different seasons, times of day and transmitter radiated power as shown in Table 1. Use of entries appropriate to mid-path conditions is suggested. When the operational MUF is determined by an E or F1 mode it is taken equal to the corresponding basic MUF.

TABLE 1

**Ratio of the median operational MUF to the median basic MUF for an F2-mode, $R_{op}$**

| | Summer | | Equinox | | Winter | |
|---|---|---|---|---|---|---|
| Equivalent isotropically radiated power (dBW) | Night | Day | Night | Day | Night | Day |
| ≤ 30 | 1.20 | 1.10 | 1.25 | 1.15 | 1.30 | 1.20 |
| > 30 | 1.25 | 1.15 | 1.30 | 1.20 | 1.35 | 1.25 |

# 7 Prediction of the optimum working frequency

The FOT (Recommendation ITU-R P.373) is estimated in terms of the operational MUF using the conversion factor $F_l$ set equal to 0.95 if the path basic MUF is determined by an E or F1 mode and as given in Table 2 if the path basic MUF is determined by an F2 mode.

TABLE 2

**Ratio $F_l$ of FOT to operational MUF when determined by an F2-mode**

a) $R_{12}$ less than 50 as a function of season, mid-path local time *t* and mid-path geographic latitude λ (North or South of equator)

| λ \ *t* | 22-02 | 02-06 | 06-10 | 10-14 | 14-18 | 18-22 | |
|---|---|---|---|---|---|---|---|
| > 75° | 0.60 | 0.65 | 0.69 | 0.72 | 0.68 | 0.67 | Winter |
| 65-75° | 0.68 | 0.71 | 0.75 | 0.76 | 0.75 | 0.70 | |
| 55-65° | 0.74 | 0.76 | 0.80 | 0.80 | 0.82 | 0.73 | |
| 45-55° | 0.79 | 0.78 | 0.83 | 0.85 | 0.84 | 0.76 | |
| 35-45° | 0.81 | 0.79 | 0.85 | 0.87 | 0.89 | 0.77 | |
| 25-35° | 0.81 | 0.74 | 0.86 | 0.82 | 0.85 | 0.78 | |
| 15-25° | 0.78 | 0.67 | 0.87 | 0.75 | 0.77 | 0.79 | |
| < 15° | 0.71 | 0.70 | 0.88 | 0.86 | 0.87 | 0.79 | |
| > 75° | 0.67 | 0.72 | 0.74 | 0.73 | 0.80 | 0.65 | Equinox |
| 65-75° | 0.70 | 0.75 | 0.76 | 0.74 | 0.82 | 0.69 | |
| 55-65° | 0.73 | 0.78 | 0.80 | 0.75 | 0.81 | 0.73 | |
| 45-55° | 0.75 | 0.80 | 0.81 | 0.76 | 0.81 | 0.76 | |
| 35-45° | 0.77 | 0.81 | 0.81 | 0.77 | 0.80 | 0.78 | |
| 25-35° | 0.78 | 0.80 | 0.82 | 0.78 | 0.81 | 0.74 | |
| 15-25° | 0.77 | 0.75 | 0.83 | 0.81 | 0.83 | 0.69 | |
| < 15° | 0.76 | 0.66 | 0.86 | 0.89 | 0.86 | 0.75 | |
| > 75° | 0.68 | 0.79 | 0.84 | 0.87 | 0.85 | 0.76 | Summer |
| 65-75° | 0.70 | 0.81 | 0.83 | 0.86 | 0.86 | 0.77 | |
| 55-65° | 0.72 | 0.84 | 0.83 | 0.84 | 0.86 | 0.81 | |
| 45-55° | 0.75 | 0.85 | 0.82 | 0.83 | 0.85 | 0.84 | |
| 35-45° | 0.79 | 0.85 | 0.80 | 0.82 | 0.83 | 0.85 | |
| 25-35° | 0.79 | 0.82 | 0.78 | 0.80 | 0.81 | 0.80 | |
| 15-25° | 0.77 | 0.78 | 0.77 | 0.79 | 0.79 | 0.73 | |
| < 15° | 0.74 | 0.75 | 0.80 | 0.83 | 0.82 | 0.69 | |

b) $R_{12}$ greater than or equal to 50 and less than or equal to 100 as a function of season, mid-path local time *t* and mid-path geographic latitude λ (North or South of equator)

| λ \ *t* | 22-02 | 02-06 | 06-10 | 10-14 | 14-18 | 18-22 | |
|---|---|---|---|---|---|---|---|
| > 75° | 0.76 | 0.78 | 0.68 | 0.67 | 0.62 | 0.70 | Winter |
| 65-75° | 0.79 | 0.81 | 0.74 | 0.70 | 0.73 | 0.73 | |
| 55-65° | 0.82 | 0.83 | 0.79 | 0.75 | 0.80 | 0.76 | |
| 45-55° | 0.84 | 0.82 | 0.83 | 0.81 | 0.84 | 0.78 | |
| 35-45° | 0.83 | 0.81 | 0.85 | 0.86 | 0.86 | 0.79 | |
| 25-35° | 0.78 | 0.76 | 0.85 | 0.85 | 0.85 | 0.78 | |
| 15-25° | 0.74 | 0.71 | 0.85 | 0.83 | 0.82 | 0.76 | |
| < 15° | 0.77 | 0.69 | 0.87 | 0.86 | 0.85 | 0.78 | |
| > 75° | 0.64 | 0.61 | 0.73 | 0.74 | 0.74 | 0.67 | Equinox |
| 65-75° | 0.68 | 0.71 | 0.77 | 0.74 | 0.78 | 0.70 | |
| 55-65° | 0.70 | 0.75 | 0.80 | 0.72 | 0.78 | 0.73 | |
| 45-55° | 0.73 | 0.77 | 0.81 | 0.74 | 0.76 | 0.75 | |
| 35-45° | 0.75 | 0.78 | 0.82 | 0.78 | 0.76 | 0.76 | |
| 25-35° | 0.77 | 0.76 | 0.82 | 0.83 | 0.78 | 0.72 | |
| 15-25° | 0.75 | 0.73 | 0.84 | 0.87 | 0.81 | 0.69 | |
| < 15° | 0.79 | 0.68 | 0.86 | 0.89 | 0.84 | 0.80 | |
| > 75° | 0.82 | 0.80 | 0.82 | 0.85 | 0.80 | 0.79 | Summer |
| 65-75° | 0.83 | 0.82 | 0.79 | 0.82 | 0.82 | 0.82 | |
| 55-65° | 0.83 | 0.82 | 0.77 | 0.79 | 0.82 | 0.83 | |
| 45-55° | 0.81 | 0.81 | 0.76 | 0.77 | 0.81 | 0.82 | |
| 35-45° | 0.78 | 0.78 | 0.75 | 0.78 | 0.78 | 0.78 | |
| 25-35° | 0.77 | 0.83 | 0.75 | 0.79 | 0.77 | 0.74 | |
| 15-25° | 0.77 | 0.69 | 0.78 | 0.82 | 0.78 | 0.73 | |
| < 15° | 0.79 | 0.63 | 0.84 | 0.85 | 0.81 | 0.77 | |

Winter: November, December, January, February in the Northern Hemisphere and May, June, July, August in the Southern Hemisphere.

Summer: May, June, July, August in the Northern Hemisphere and November, December, January, February in the Southern Hemisphere.

Equinox: March, April, September, October in both hemispheres.

TABLE 2 *(continued)*

c) $R_{12}$ greater than 100 as a function of season, mid-path local time $t$ and mid-path geographic latitude λ (North or South of equator)

| λ \ $t$ | 22-02 | 02-06 | 06-10 | 10-14 | 14-18 | 18-22 | |
|---|---|---|---|---|---|---|---|
| > 75° | 0.62 | 0.70 | 0.74 | 0.67 | 0.64 | 0.73 | Winter |
| 65-75° | 0.69 | 0.74 | 0.77 | 0.72 | 0.72 | 0.78 | |
| 55-65° | 0.77 | 0.78 | 0.81 | 0.80 | 0.79 | 0.82 | |
| 45-55° | 0.83 | 0.80 | 0.84 | 0.87 | 0.84 | 0.86 | |
| 35-45° | 0.86 | 0.81 | 0.87 | 0.90 | 0.87 | 0.87 | |
| 25-35° | 0.83 | 0.76 | 0.89 | 0.90 | 0.88 | 0.86 | |
| 15-25° | 0.78 | 0.70 | 0.89 | 0.89 | 0.89 | 0.83 | |
| < 15° | 0.83 | 0.76 | 0.89 | 0.90 | 0.89 | 0.84 | |
| > 75° | 0.66 | 0.67 | 0.75 | 0.66 | 0.70 | 0.72 | Equinox |
| 65-75° | 0.67 | 0.71 | 0.73 | 0.70 | 0.70 | 0.72 | |
| 55-65° | 0.69 | 0.75 | 0.71 | 0.71 | 0.71 | 0.72 | |
| 45-55° | 0.73 | 0.78 | 0.70 | 0.72 | 0.74 | 0.73 | |
| 35-45° | 0.79 | 0.82 | 0.75 | 0.78 | 0.80 | 0.84 | |
| 25-35° | 0.81 | 0.82 | 0.87 | 0.87 | 0.87 | 0.86 | |
| 15-25° | 0.81 | 0.77 | 0.89 | 0.92 | 0.90 | 0.85 | |
| < 15° | 0.80 | 0.79 | 0.86 | 0.90 | 0.90 | 0.82 | |
| > 75° | 0.73 | 0.74 | 0.82 | 0.83 | 0.79 | 0.75 | Summer |
| 65-75° | 0.75 | 0.75 | 0.77 | 0.80 | 0.80 | 0.77 | |
| 55-65° | 0.77 | 0.76 | 0.74 | 0.77 | 0.80 | 0.80 | |
| 45-55° | 0.79 | 0.76 | 0.73 | 0.75 | 0.80 | 0.84 | |
| 35-45° | 0.80 | 0.76 | 0.75 | 0.75 | 0.79 | 0.84 | |
| 25-35° | 0.81 | 0.76 | 0.82 | 0.81 | 0.79 | 0.83 | |
| 15-25° | 0.81 | 0.77 | 0.85 | 0.86 | 0.81 | 0.80 | |
| < 15° | 0.80 | 0.79 | 0.86 | 0.89 | 0.85 | 0.78 | |

Winter: November, December, January, February in the Northern Hemisphere and May, June, July, August in the Southern Hemisphere.

Summer: May, June, July, August in the Northern Hemisphere and November, December, January, February in the Southern Hemisphere.

Equinox: March, April, September, October in both hemispheres.

## 8 Computer program

The procedures described in this Annex are implemented in the computer program MUFFY, which predicts basic MUF, operational MUF and optimum working frequency as a function of time of day, for given propagation path, month and sunspot number.

ANNEX 2

# Prediction of ray path

For a simplified estimation of oblique ray paths, reflection may be assumed to take place from an effective plane mirror located at height $h_r$.

In the following:

$$x = \text{foF2} / \text{foE} \qquad \text{and} \qquad H = \frac{1\,490}{\text{M(3000)F2} + \Delta M} - 316$$

with:

$$\Delta M = \frac{0.18}{y - 1.4} + \frac{0.096\,(R_{12} - 25)}{150}$$

and $y = x$ or 1.8, whichever is the larger.

a) For $x > 3.33$ and $x_r = f / \text{foF2} \geq 1$, where $f$ is the wave frequency

$h_r = h$ or 800 km, whichever is the smaller

where: $h = A_1 + B_1\, 2.4^{-a}$ for $B_1$ and $a \geq 0$

$= A_1 + B_1$ otherwise

with: $A_1 = 140 + (H - 47)\, E_1$

$B_1 = 150 + (H - 17)\, F_1 - A_1$

$E_1 = -\, 0.09707\, x_r^3 + 0.6870\, x_r^2 - 0.7506\, x_r + 0.6$

$F_1$ is such that:

$F_1 = -\, 1.862\, x_r^4 + 12.95\, x_r^3 - 32.03\, x_r^2 + 33.50\, x_r - 10.91$ for $x_r \leq 1.71$
$F_1 = 1.21 + 0.2\, x_r$ for $x_r > 1.71$

and: $a$ varies with distance $d$ and skip distance $d_s$ as

$a = (d - d_s)/(H + 140)$

where: $d_s = 160 + (H + 43)\, G$

$G = -\, 2.102\, x_r^4 + 19.50\, x_r^3 - 63.15\, x_r^2 + 90.47\, x_r - 44.73$ for $x_r \leq 3.7$
$G = 19.25$ for $x_r > 3.7$

b) For $x > 3.33$ and $x_r < 1$

$h_r = h$ or 800 km, whichever is the smaller

where: $h = A_2 + B_2\, b$ for $B_2 \geq 0$

$= A_2 + B_2$ otherwise

with: $A_2 = 151 + (H - 47)\, E_2$

$B_2 = 141 + (H - 24)\, F_2 - A_2$

$E_2 = 0.1906\, Z^2 + 0.00583\, Z + 0.1936$

$F_2 = 0.645\, Z^2 + 0.883\, Z + 0.162$

where: $Z = x_r$ or 0.1, whichever is the larger and $b$ varies with normalized distance $d_f$, $Z$ and $H$ as follows:

$b = -\, 7.535\, d_f^4 + 15.75\, d_f^3 - 8.834\, d_f^2 - 0.378\, d_f + 1$

where: $d_f = \dfrac{0.115\, d}{Z(H + 140)}$ or 0.65, whichever is the smaller

c) For $x \leq 3.33$

$h_r = 115 + H\, J + U\, d$ or 800 km, whichever is the smaller

with: $J = -0.7126\, y^3 + 5.863\, y^2 - 16.13\, y + 16.07$

and: $U = 8 \times 10^{-5}\, (H - 80)\, (1 + 11\, y^{-2.2}) + 1.2 \times 10^{-3}\, H\, y^{-3.6}$