# BAYESIAN ONLINE LEARNING-BASED SPECTRUM OCCUPANCY PREDICTION IN COGNITIVE RADIO NETWORKS

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Abstract – Predicting the near future of primary user (PU) channel state availability (i.e., spectrum occupancy) is quite important in cognitive radio networks in order to avoid interfering its transmission by a cognitive spectrum user (i.e., secondary user (SU)). This paper introduces a new simple method for predicting PU channel state based on energy detection. In this method, we model the PU channel state detection sequence (i.e., "PU channel idle" and "PU channel occupied") as a time series represented by two different random variable distributions. We then introduce Bayesian online learning (BOL) to predict in advance the changes in time series (i.e., PU channel state.), so that the secondary user can adjust its transmission strategies accordingly. A simulation result proves the efficiency of the new approach in predicting PU channel state availability.

Keywords – Bayesian online learning, cognitive radio, primary user, spectrum occupancy prediction.

## **1. INTRODUCTION**

In cognitive radio networks a secondary user is allowed to opportunistically utilize the vacant spectrum channels left by the primary user without interfering with their transmission. One of the key challenges for secondary users in cognitive radio networks is how to know when to occupy or leave the spectrum (i.e., the channels) for primary users' transmission. To tackle this problem, the secondary user must be capable of predicting in advance the channel availability of the primary user (i.e., whether the PU channels' status are "idle" or "busy") so that it can occupy or leave the channels for PU transmission.

The spectrum occupancy prediction problem has been widely investigated, for example, the idea of predictive spectrum access was first introduced in [1], in which the authors utilize Hidden Markov Model (HMM) to solve the spectrum occupancy prediction problem. Later on, the HMM-based spectrum prediction model received great attention in the literature [2-4]. And, due to the fact that HMM-based approaches require a priori knowledge of the PUs' traffic pattern, other machine learning approaches such as neural network [5], Bayesian inference [6] and online support vector regression (SVR) [7] have been adopted for the prediction of PU channel availability. However, these prediction techniques consider only time-invariant PU model behaviors. While in real-world cognitive radio systems, PU traffic patterns can also exhibit time-variant traffic patterns, which is hard to characterize using the above-mentioned machine learning algorithms. On the other hand, the Bayesian online learning algorithm (BOL) [8] has a capability to track both time-variant and time-invariant dual-states switching time series behaviors. Motivated by the fact that the nature of the PUs channel state availability can be also modeled as dual-states switching time series, we propose a new spectrum occupancy (PUs channel state) prediction technique that utilizes BOL to perform PU channel availability prediction in cognitive radio network. In more details, we captured the PU channel state energy detection sequence using a time series that switches over the time between two different random distributions representing the PU channel state (i.e., PU idle or PU occupied). We then fed this time series as an observations sequence into a BOL prediction algorithm to estimate or predict in advance the point of the time when the change will occur between the two states of the time series so that SUs can adjust their transmission strategies accordingly. The experimental results show the effectiveness of the BOL algorithm in predicting the changing points of the time series that were generated to capture PU channel availability.

The paper is organized as follows: first, we introduce the system model for energy detection and time series generation. Second, we present the BOL method to predict the time series that is generated to capture the PU channel state, followed by the simulation results and the overall work conclusion.

### 2. SYSTEM MODEL

The system model for predicting the PU channel state is illustrated by the block diagram Fig. 1. This diagram contains the energy detection block model that detects the PU channel state (PU signal present or absent), the time series generation block model to capture the PU channel state detection sequence followed by the Bayesian online learning algorithm block to predict the near future of the PU channel state (i.e., to detection changing points or the switching point of the time series) by utilizing the previously detected channel state information.



Fig. 1. Block diagram of PU channel state perdition based on BOL algorithm

In the diagram depicted in Fig.1,  $Y_t$ : is the instantaneous energy statistic,  $\lambda$ : is the detection threshold,  $x_t$ : is the time series that is generated to capture the detection sequence or PU channel states (i.e., PU present or PU absent) over the time. The time series  $x_t$  is represented using two different random distributions for each state (i.e.,  $x_t \in \{v_1, v_2 \dots v_L\}$  for PU signal absent state, and  $x_t \in \{v_{L+1} \dots v_M\}$  for PU signal present state).

## **3. ENERGY DETECTION MODEL**

In energy detection based spectrum sensing the SU sensor observes the licensed spectrum to determine whether the primary user signal is present or absent by doing the binary hypothesis test on the received signal over the time  $1 \le t \le T$ , as follows:

$$y_{t}(n) = \begin{cases} w_{t}(n) & \text{H0 (absent)} \\ \sqrt{\gamma_{t}} s_{t}(n) + w_{t}(n) & \text{H1(present)} \end{cases}$$
(1)

where  $y_t(n)$  the observed received signal  $s_t(n)$  the primary user's signal, and it is assumed to be Gaussian i.i.d random process with zero mean and variance  $\sigma_t^2$ ,  $w_i(n)$  is the noise, and assumed to be Gaussian i.i.d random process with zero mean and

variance  $\sigma_t^2$ ,  $\gamma_t$ : the SNR at time t. The instantaneous energy statistic at the secondary user sensing node  $Y_t$  can be represented by

$$Y_t = \frac{1}{N} \sum_{n=1}^{N} |(y_t(n))|^2$$
,  $1 \le t \le T$  (2)

where, N: is the number of samples used by the sensing node SU for energy detection, T: the overall system simulation time and  $Y_t$  is a random variable whose probability density function (PDF) is chi-square distribution with 2N degrees of freedom for the complexed value ( $y_t(n)$ ,and N degrees of freedom for the real value case. For the value of N  $\ge$  200,  $Y_t$  can be approximated using the Gaussian distribution. Therefore, the distribution of the power test  $Y_t$  for wide band signal follows:

$$Y_{t} \sim \begin{cases} (\sigma_{t}^{2}, 2\sigma_{t}^{4}/N) & H0\\ (\sigma_{t}^{2}(1+\gamma_{t}), 2\sigma_{t}^{4}(1+\gamma_{t})^{2}/N & H1 \end{cases}$$
(3)

If we assume that the noise variance and SNR at every sensing node remains unchanged during the observation time t, then  $\sigma_t^2$  and can be written as  $\gamma_t \sigma_t^2 = \sigma_u^2$ ,  $\gamma_t = \gamma_u$ . Thus, for a chosen threshold  $\lambda_t = \lambda$  the probability of false alarm  $P_f$  can be written as:

$$P_{f}(\lambda) = Pr(Y_{t} > \lambda | H0)$$

$$= \frac{1}{\sqrt{2\pi\sigma_{u}}} \int_{\lambda}^{\infty} e^{-(\lambda - \sigma_{u})^{2}/\sqrt{2}\sigma_{u}^{2}}$$

$$= Q\left(\frac{\lambda}{\sigma_{u}^{2}} - 1\right)$$
(4)

where Q (.) is the complementary distribution function of Gaussian distribution with zero mean and unit variance. From equation (4) and for a given probability of false alarm  $P_f$ , the single user decision threshold can be written as:

$$\lambda = \left(\sqrt{\frac{2}{N}}Q^{-1} + 1\right)\sigma_{u}^{2} \tag{5}$$

where  $Q^{-1}(.)$  is the inverse of the Q (.) function. And the instantaneous primary user channel state detection sequences the sensing results over the time can be written as a function of the decision threshold as follows:

$$D_{t} = \begin{cases} 0 & \text{PU signal absent} \quad Y_{t} < \lambda \\ 1 & \text{PU signal present} \quad Y_{t} \ge \lambda \\ ,1 \le t \le T. \end{cases}$$
(6)

## 4. TIME-SERIES GENERATION BASED ON ENERGY PRIMARY USER DETECTION SEQUENCE

After the detection of the instantaneous primary user channel state D<sub>t</sub> (i.e., PU signal present or PU absent) based on the energy detector as explained in the previous section (other spectrum-sensing algorithms such as data fusion based cooperative sensing approach could be also used here ). Our next goal is to predict the near future of the primary user channel state. In order to do so, we denote the period that the primary user signal is absent as "idle state" and the period that the primary user signal present as "occupied state". And to estimate the time when the channel state will switch from idle to occupied or vice versa (i.e., the change-points time) we generate a time series  $x_t$  to capture the instantaneous detection sequence signal D<sub>t</sub> .Then we transfer the two states of the time series ("PU present" or "absent",) into observations using two different random distributions for each state (i.e.,  $x_t \in$ idle  $\{v_1, v_2 \dots v_L\}$ for state and xt ∈  $\{v_{L+1} \dots v_M\}$  for occupied state. The equation that capture PU detection sequence xt can be written as follows:

$$x_t \in \begin{cases} \{ v_1, v_2, \dots, v_L \} & Y_t < \lambda \\ \{ v_{L+1}, \dots, v_M \} & Y_t \ge \lambda \\ 1 \le t \le T \end{cases}$$
(7)

We can note that  $x_t$  formulates a time series of two non-overlapping states over the time T. The effectiveness of the Bayesian online learning algorithm in predicting the change-points in such a time series has been shown in many applications such as finance, biometrics, and robotics [8].

## 5. TIME-SERIES PREDICTION BASED ON BAYESIAN ONLINE LEARNING ALGORITHM

We assume a sequence of observations  $x_t = x_1, x_2, x_3, ...$  with two non-overlapping states, as denoted by equation (7). The delineations between the two non-overlapping states are called the changepoints. To determine these change-points we need to estimate the posterior distribution over the current "run length=:  $r_t$ " or the time since the last changepoint, given the data so far observed. Under the assumption that change-points occur as a stochastic process, the data between change-points are i.i.d distributed, the parameters are independent across the change-points, and when the change-point has

occurred the run length  $r_t$  will drop to zero; otherwise, the run length is increased by one. We use the BOL algorithm to calculate the posterior run length at time t, i.e. P ( $r_t|x_{1:t}$ ), sequentially. This posterior is used to make online predictions robust to underlying regime changes through marginalization of the run length variable as follows:

$$P(x_{t+1}|x_{1:t}) = \sum_{r_t} P(x_{t+1}|x_{1:t}, r_t) P(r_t|x_{1:t})$$
  
=  $\sum_{r_t} P(x_{t+1}|x_r) P(r_t|x_{1:t})$  (8)

where  $x_r$  refers to the set of observations  $x_t$  associated with run length  $r_t$ , and  $P(x_{t+1}|x_r)$  is computed using the underlying predictive model UPM (the training model set), to find  $P(r_t, x_{1:t})$ , we estimate the run length distribution  $P(r_{ti}, x_{1:t})$ , for i = 1, 2, ..., tof run length  $r_t$ . For each time step t, the run length distribution contains i-elements of probabilities such that  $\sum_{i=1}^t r_{ti} = 1$ . The run length posterior is found by normalizing the joint likelihood as:

$$P(r_t|x_{1:t}) = \frac{P(r_{ti}, x_{1:t})}{\sum_{r_{ti}} P(r_{ti}, x_{1:t})}$$
(9)

If we denote the joint likelihood distribution of the run length  $r_t$  at time t for the observed data  $x_{1:t}$ ,  $P(r_{ti}, x_{1:t})$  as  $\varphi_t$ : =  $P(r_{ti}, x_{1:t})$  we can update the joint likelihood online recursively using:

$$\begin{split} \varphi_{t} &= P(r_{ti}, x_{1:t}) = P(x|r)P(r) \\ &= \sum_{(r_{t}-1)i} P(r_{ti}, x_{1:t}| r_{(t-1)i}, x_{1:t-1})P(r_{t-1}, x_{1:t-1}) \\ &= \sum_{(r_{t}-1)i} P(r_{ti}|, r_{(t-1)i})P(x_{t}|r_{(t-1)i}, x_{r}) \end{split}$$
(10)

where  $P(r_{ti}|, r_{(t-1)i})$  is the change-point prior or hazard function and  $P(x_t|r_{(t-1)i}, x_r)$  is the likelihood or the underlying predictive model UPM (the model training data set). All the distributions mentioned so far are implicitly conditioned on the set of hyperparameters  $\theta$ .

Assuming a simple BOL model represented using a constant hazard function H  $(r|\theta_h)$ : =  $\theta_h$ , this means P  $(r_t = 0|r_{t-1}, \theta_h)$  is independent of  $r_{t-1}$  and is constant. We can represent the underlying predictive model with a basic predictive model that model a scalar  $(x_t \in \mathcal{R})$  by placing a normal-inverse-gamma prior on i.i.d Gaussian observations [9]:

$$\mathbf{x}_{t} \sim (\boldsymbol{\mu}_{t}, \boldsymbol{\sigma}_{t}^{2}) \tag{11}$$

$$\mu_{t} \sim (\mu_{0}, \sigma_{t}^{2} / \kappa_{t}), \sigma_{t}^{-2} \sim \text{Gamma}(\alpha_{t}, \beta_{t})$$
(12)

For this model the parameters are  $\eta \coloneqq \{\mu_t, \sigma_t^2\}$  and model hyper-parameters the are  $\theta = {\mu_t, \sigma_t^2, \kappa_t, \alpha_t, \beta_t}$ . A new value for  $\mu_t$  and  $\sigma_t^2$ are sampled at each change-point. The posterior on  $\eta$  is updated at every new data point for each run length, for example for a model with a training set  $\{x_1, x_3, ..., x_L\}$  or  $x_{1:L}$  and initial mean and variance  $\mu_0, \sigma_0^2$  respectively the first update of  $\eta$  (the corrected prior mean and variance) is:  $\sigma_1^2 =$  $\mu_1 = (\mu_0 - E(x_{1:L}))/\text{std}(x_{1:L})$ and  $\sigma_0^2$ /std (x<sub>1:L</sub>) where std := standard deviation.

The BOL algorithm after the training is written as given in [10] is as follows:

1. Initialize or calculate from the training data set the corrected prior mean  $\mu_1$ , the corrected prior variance  $\sigma_1^2$ , the degree of freedom  $\beta_1$ , the run length distribution P ( $r_{(1)i}$ ) = 1,  $\alpha_1$  and  $\kappa_1$ .

2. While (new data  $x_t$  is available) do:

3. Compute the Gaussian prediction function by the student's t-distribution (which gives a posterior predictive distribution on  $x_t$  of)

$$\pi_{t}^{(r)} = P(x_{t}|x_{r},\theta) = P(x_{t}|\mu_{t},\sigma_{t}^{2},\beta_{t},\kappa_{t}) =$$
  
 
$$St_{2\alpha_{t}} \left(\mu_{t},\frac{\beta_{t}}{\sigma_{t}^{2}}\frac{\kappa_{t}}{\kappa_{t}+1}\right)$$
(13)

St =: Student's t-distribution probability density function

4. For i = 1 = 1 to t - 1, compute growth probabilities

$$P(r_{ti}, x_{1:t}) = P(r_{t-1}, x_{1:t-1}) \pi_t^{(r)} (1 - H(r_t))$$
(14)

where we assume that the hazard function  $H(r_t) = \lambda^{-1}$ , where  $\lambda$  is a timescale parameter.

## 5. Compute change-point probabilities

$$P(r_{ti}, x_{1:t}) = \sum_{i=1}^{t-1} \frac{P(r_{ti}, x_{1:t})}{\lambda - 1}$$
(15)

#### 6. Compute run length distribution

$$P(r_{ti}|x_{1:t}) = \frac{P(r_{ti},x_{1:t})}{P(x_{1:t})} = \frac{P(r_{ti},x_{1:t})}{\sum_{i=1}^{t} P(r_{ti},x_{1:t})}$$
(16)

7. Update sufficient statistics

$$\mu_{t+1} = \frac{\kappa_t \mu_t + \kappa_t}{\kappa_t + 1} \tag{17}$$

$$\kappa_{t+1} = \kappa_t + 1 \tag{18}$$

$$\sigma_{t+1}^{2} = \frac{1}{\beta_{t}} \Big[ (\kappa_{t} + 1) + \frac{1}{2} (x_{t} - \mu_{t})^{2} \Big]$$
(19)

$$\beta_{t+1} = \beta_t + \frac{\kappa_t (x_t - \mu_t)}{2(\kappa_t + 1)}$$
(20)

$$\alpha_{t+1} = \alpha_t + \frac{1}{2} \tag{21}$$

8. If

$$\underset{i}{\operatorname{argmax}} \quad P(r_{ti}|x_{1:t}) \tag{22}$$

Then the change-point has occurred, reset run length to zero  $r_t = 0$ . If not, increment  $r_t = r_{t-1} + 1$ .

9. Perform prediction

$$P(x_{t+1}|x_t) = \sum_{i=1}^{t-1} P(x_{t+1}|x_t, r_t) P(r_{ti}, x_{1:t})$$
(23)

10. Go to step 2.

#### 6. SIMULATION RESULTS

To evaluate the performance of the BOL algorithm in predicting PU channel state availability we generate a simulated detection sequence considering single user cognitive radio system with noise variance  $\sigma_u^2 = 1$  and SNR  $\gamma_u = -22$ dB. The local energy detection decision is made after observing N =1000 samples. We employ equation (6) to generate the time series that captures the simulated detection sequence.

Figure 2 shows the performance of the BOL algorithm in predicting the time series that is generated to capture the primary user channel state detection sequence for single user "sensing node" cognitive radio network with a randomly distributed channel occupancy over T = 350 time points after training the algorithm offline over a set of training observations L = 150 time points (we use millisecond as time point here). For training the algorithm we use  $\mu_0 = 2$  and  $\sigma_0^2 = 1$ , the timescale parameter  $\lambda = 2000, \alpha_1 = 1$ , and  $\kappa_1 =$  $\beta_1 = 1/\sigma_1^2$ . In Fig.2 the top plot shows the simulated detection sequence signal, where 150 ms of the simulated detection sequence signal used for training the hyper-parameters  $\theta$ , and the remaining 350 ms is used for testing the algorithm. The middle plot shows the generated time series to capture the primary user channel state detection sequence with the random distribution for idle states represented by  $x_t \in \{1,2,3\}$  and  $x_t \in \{4,5,6\}$  for occupied states.



Fig. 2. The simulated detection sequence signal (150 time points used to train the hyper-parameters  $\theta$ ), the generated time series to capture the detection sequence and the performance of the BOL algorithm in predicting the change-points for 350 time points testing time series, (assuming the time point is in millisecond: = *ms*)

The bottom plot shows the posterior probability of the current run length  $P(r_t|x_{1:t})$  at each time step, using a logarithmic color scale; the lighter pixels indicate higher probability, the red crosses are the change-points and the darker pixels represent the current run length. As we can see run-length is dropped to zero immediately after the change-point. The time consumed for training the model is 5.223582 milliseconds while the testing time is only 0.204281 milliseconds.

#### 7. CONCLUSION

In this paper, we have studied the problem spectrum occupancy prediction for a single user cognitive radio network based on the Bayesian online learning model. We modeled the detection sequence of primary user channel state availability as a time series changing over the time between two states (PU idle and PU occupied). We introduced Bayesian online learning to predict in advance the changes in the states of the time series. Finally, we evaluated the performance of our algorithm using a simulated PU detection sequence. The simulation results have verified the effectiveness of the BOL model in predicting PU channel state availability.

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