Computational complexity-constrained spectral efficiency analysis for 6G waveforms

***Saulo Queiroz*[1], *João P. Vilela*[2, 4], *Benjamin Koon Kei Ng*[3], *Chan-Tong Lam*[3], *Edmundo Monteiro*[4]**

[1] Federal University of Technology (UTFPR), Ponta Grossa, PR, Brazil, [2] CRACS/INESCTEC, and Department of Computer Science, Faculty of Sciences, University of Porto, Porto, Portugal, [4] CISUC and Department of Informatics Engineering, University of Coimbra, Coimbra, Portugal, [3] Faculty of Applied Sciences, Macao Polytechnic University, Macau, China

Corresponding author: Saulo Queiroz, sauloqueiroz@utfpr.edu.br

In this work, we present a tutorial on how to account for the computational time complexity overhead of signal processing in the Spectral Efficiency (SE) analysis of wireless waveforms. Our methodology is particularly relevant in scenarios where achieving higher SE entails a penalty in complexity, a common trade-off present in 6G candidate waveforms. We consider that SE derives from the bit rate, which is impacted by time-dependent overheads. Thus, neglecting the computational complexity overhead in the SE analysis grants an unfair advantage to more computationally complex waveforms, as they require larger computational resources to meet a signal processing runtime below the symbol period. We demonstrate our points with two case studies. In the first, we refer to IEEE 802.11a-compliant baseband processors from the literature to show that their runtime significantly impacts the SE perceived by upper layers. In the second case study, we show that waveforms considered less efficient in terms of SE can outperform their more computationally expensive counterparts, if provided with equivalent high-performance computational resources. Based on these cases, we believe our tutorial can address the comparative SE analysis of waveforms that operate under different computational resource constraints.

Keywords: Computational complexity, OFDM, index modulation, signal processing, spectral efficiency

1. INTRODUCTION

Novel wireless Physical (PHY) layer waveforms are expected to reach unprecedented levels of bit rate and Spectral Efficiency (SE) to meet the traffic demands of beyond-5G scenarios. Towards that goal, it is usual for novel solutions to come at the cost of larger computational complexity overheads in comparison to the classic Orthogonal Frequency Division Multiplexing (OFDM) waveform, thereby impacting other key performance indicators such as manufacturing cost, chip area (portability), and power consumption. Since computational complexity and bit rate/SE often fall under distinct fields of expertise, comparing signal processing techniques that enhance one while compromising the other becomes increasingly difficult. This difficulty hinders the ability to determine whether a certain penalty in computational complexity is worthwhile for a gain in bit rate and SE.

The complexity-SE trade-off is evident, for example, in Index Modulation (IM), a signal processing technique expected to prominently outperform the SE of 5G [1]. IM can be further combined with other 6G candidate physical layer techniques and waveforms like Orthogonal Time Frequency Space (OTFS) [2], metasurface modulation [3], Non-Orthogonal Multiple Access (NOMA) [4], faster-than-Nyquist signaling [5], etc. With such broad flexibility to operate along with other signal processing techniques, IM is a good representative of the complexity/SE trade-off we are concerned about in this work.

By reviewing the literature, one can identify relatively modest progress towards the joint complexity-SE study of waveforms.

In [6], for example, the authors introduce the 'complexity-constrained capacity-achieving NOMA principle' to identify design targets in terms of capacity and complexity for a NOMA communication system. In [7], [8], [9], the authors solve the complexity/SE trade-off for the combinatorial mapper of the original OFDM-IM waveform [10]. Through theoretical and experimental tests, they demonstrate how to achieve the optimal balance between time complexity and SE for IM waveforms. In [11], the authors correlate the complexity and throughput of the Fast Fourier Transform (FFT) algorithm in the context of OFDM waveforms. They show that the FFT complexity nullifies the throughput of OFDM as the number of subcarriers grows. In [12], authors discuss some conditions under which the number of complex multiplications performed by the N-point FFT algorithm can be $O(\sqrt{N} \log \sqrt{N})$, thereby avoiding the throughput nullification of FFT for those cases. These aforementioned references concern the interplay between complexity and throughput motivated by trade-offs in specific waveforms and signal processing algorithms.

Some theoretical proposals employ the term "complexity" to study aspects of communication other than the relation between SE and computational resources, as is the case of this work. This is the case of the communication complexity theory and the Kolmogorov complexity. The communication complexity [13] concerns the minimum number of message exchanges to solve a problem, whose input parameters are distributed among entities of a network. Thus, although that theory concerns communication, the term "complexity" stands for the *number of messages* transmitted in the network.

In turn, the algorithmic information theory [14], [15], [16], also known as Kolmogorov complexity, relates the Shannon's information theory to the Turing's computational model aiming to identify the irreducible form of information, i.e., the shortest computer program that produces the information.

In this case, the term "complexity" stands for the *shortest length of a string* that represents the information. The string can be either the transmission information or the algorithmic source code of a computer program that generates the information. For example, assuming a computer model in which the string "xˆy" returns the result of the real number "x" raised to the real number power "y", the 4-length algorithmic string "10ˆ6" shortens the 7-length string representation "1000000" of the decimal number 'one million'.

A remarkable step towards a generic model of capacity and complexity is due to [17], in which the authors build on [7], [8], [9], and [11] to formalize the Spectro-Computational (SC) analysis. SC analysis is a theoretical framework that enhances classic performance indicators of information theory, such as capacity, bit rate, and SE, to account for the computational complexity overhead of signal processing. In SC terminology, the enhanced version of each classic performance indicator is designated by the prefix 'SC', e.g., SC capacity. These new definitions, however, were mostly intended to enable the formalization of a novel capacity regime referred to as comp-limited signals, in which the growth of capacity is conditioned by the available amount of computational (rather than spectrum or power) resources.

In this work, we rely on the SC framework to present a tutorial about the computational complexity-constrained SE analysis of signal waveforms. Such joint analysis plays a key role in fairer comparative analyses of waveforms constrained by different amounts of computational resources. In these cases, neglecting the impact of complexity can lead to unfair comparisons, because the superior SE performance of faster signals is conditioned by the allocation of more computational resources. If constrained by the same amount of computational resources as a slower signal, the fast signal can experience longer signal processing delays, thereby hindering its claimed SE if the complexity overhead is not neglected.

We illustrate our point through different examples. In one case study, we demonstrate how to compare the SC efficiency of waveforms that require different amounts of computational resources due to different time complexity constraints. We show that waveforms considered less efficient in terms of SE can outperform their more computationally expensive counterparts, if provided with equivalent high-performance computational resources. In another case study, we refer to the performance of some OFDM-baseband processors to illustrate how the SE perceived by upper layers can be significantly impacted if the time complexity of the physical layer is not considered. This can occur even when the runtime overhead meets the signal processing requirements of IEEE 802.11a for real-time communications.

The remainder of this work is organized as follows. In Section 2, we present the background of this work. In Section 3, we provide a tutorial on reflecting signal processing computational complexity in bit rate and SE analyses. Additionally, we explain how to assess these performance indicators under equitable computational resource constraints. In Section 4, we present case studies of our tutorial for the OFDM, OFDM-IM, and Dual-Mode OFDM-IM (DM-OFDM) waveforms and, in Section 5, we present comparative analyses of them based on numerical results. In Section 6, we present the conclusion of this work. For the reader's convenience, Table 1 lists the acronyms and abbreviations used throughout this work, while Table 2 describes the adopted symbols and notation.

Table 1 – List of acronyms and abbreviations

b/s	bits per second
b/Hz/s	bits per Hertz per second
BER	Bit Error Rate
CP	Cyclic Prefix
DM	Dual Mode
DM-OFDM	Dual Mode OFDM with IM
FFT	Fast Fourier Transform
Hz	Hertz
IFFT	Inverse FFT
IM	Index Modulation
inst.	computational instructions
inst/s	computational instructions per second
IxS	Index Selector
LLR	Log-Likelihood Ratio
MTU	Maximum Transmission Unit
NOMA	Non-Orthogonal Multiple Access
OFDM	Orthogonal Frequency Division Multiplexing
OFDM-IM	OFDM with IM
OTFS	Orthogonal Time Frequency Space
PHY	Physical
Rx	Receiver
SC	Spectro-Computational
SE	Spectral Efficiency
SNR	Signal-to-Noise Ratio
THz	TeraHertz
Tx	Transmitter

2. BACKGROUND

In this section, we review the IM waveforms considered in this work and the SC framework.

2.1 Index modulated waveforms

The OFDM-IM waveform [10] divides an N-subcarrier OFDM signal into $g > 0$ sub-blocks having $n = N/g$ subcarriers each. In each sub-block, only $k > 0$ out of n subcarriers are active, yielding a total of $C(n,k) = \binom{n}{k} = n!/(n-k)!$ different waveform patterns. A total of $\lfloor \log_2 C(n,k) \rfloor$ bits are mapped (demapped) to these patterns following the so-called Index Selector (IxS) algorithm of OFDM-IM, which adds a complexity of $O(gnk)$ in comparison to the classic OFDM waveform. Moreover, each active subcarrier can be modulated as usual by an M-ary constellation diagram (M is a power of two), yielding a total of $k \log_2 M + \lfloor \log_2 C(n,k) \rfloor$ per sub-block. Aiming to increase this number of bits, the Dual Mode OFDM-IM (DM-OFDM) transmitter [18] works just like OFDM-IM unless no subcarrier is turned off. Instead, two different constellation diagrams, we denote as A and B, are employed to differentiate between active and non-active subcarriers, respectively. Considering that A and B have M_A and M_B points (both powers of two), respectively, the total number of bits in the DM-OFDM signal is $k \log_2 M_A + (n-k) \log_2 M_B + \lfloor \log_2 C(n,k) \rfloor$ per sub-block.

Under the 'ideal OFDM-IM setup' (which results from setting $g = 1$, $k = N/2$, and $M = 2$), the OFDM-IM signal modulates $N/2 + \lfloor \log_2 C(N, N/2) \rfloor$ bits across all N subcarriers, reaching its maximum SE gain over OFDM. However, this entails an IxS time complexity of $O(N^2)$ [9], [8]. Similarly, assuming $M = M_A = M_B = 2$ for the sake of convenience, the DM-OFDM symbol conveys a total of $N + \lfloor \log_2 C(N, N/2) \rfloor$ bits. As will be shown later in Section 4, this is the highest complexity along the OFDM-IM block diagram, outing the limits of the complexity-SE trade-off of the IM waveforms. For this reason, we will assume the ideal IM setup throughout this work, unless otherwise stated. For other details about the IM waveforms refer to the cited references.

2.2 SC analysis

In this section, we review the performance metrics of [17] to support the computational complexity-constrained SE analysis of Section 4.

The SC efficiency $\mathsf{SC_{SE}}$ of a signal as wide as W Hertz (Hz) is for the SC bit rate $\mathsf{SC_R}$ (also referred to as SC bit rate, SC throughput) just as the classic SE SE is for the classical bit rate R. In other words, considering a symbol

Table 2 – Symbols and notation

B	Bits per symbol
$C(n,k)$	Binomial coefficient $\binom{n}{k} = n!/(n-k)!$
g	Number of sub-blocks (IM waveforms)
n	Number of subcarriers per sub-block (IM waveforms)
k	Number of active subcarriers (IM waveforms)
M	Constellation order
M_A	Constellation order of DM-OFDM (mode A)
M_B	Constellation order of DM-OFDM (mode B)
N	Number of subcarriers
N_{cp}	Length of cyclic prefix
R	Bit rate
$T_{\text{comp-tx}}$	Signal processing runtime overhead of PHY transmitter
$T_{\text{comp-txofdm}}$	Signal processing runtime overhead of OFDM transmitter
$T_{\text{comp-txdm}}$	Signal processing runtime overhead of DM-OFDM transmitter
$T_{\text{comp-txim}}$	Signal processing runtime overhead of OFDM-IM transmitter
$T_{\text{comp-rx}}$	Signal processing runtime overhead of PHY receiver
$T_{\text{comp-rxofdm}}$	Signal processing runtime overhead of OFDM receiver
$T_{\text{comp-rxdm}}$	Signal processing runtime overhead of DM-OFDM receiver
$T_{\text{comp-rxim}}$	Signal processing runtime overhead of OFDM-IM receiver
$T_{\text{rx}}(N)$	Complexity function of an N-subcarrier PHY receiver
$T_{\text{rxdm}}(N)$	Complexity function of N-subcarrier DM-OFDM receiver
$T_{\text{rxdm2}}(N)$	Complexity function of optimized DM-OFDM receiver [9], [8]
$T_{\text{rxim}}(N)$	Complexity function of N-subcarrier OFDM-IM receiver
$T_{\text{rxim2}}(N)$	Complexity function of optimized OFDM-IM receiver [9], [8]
$T_{\text{rxofdm}}(N)$	Complexity function of N-subcarrier OFDM receiver
T_{sym}	Symbol duration
$T_{\text{tx}}(N)$	Complexity function of an N-subcarrier PHY transmitter
$T_{\text{txdm}}(N)$	Complexity function of N-subcarrier DM-OFDM transmitter
$T_{\text{txdm2}}(N)$	Complexity function of optimized DM-OFDM transmitter [9], [8]
$T_{\text{txim}}(N)$	Complexity function of N-subcarrier OFDM-IM transmitter
$T_{\text{txim2}}(N)$	Complexity function of optimized OFDM-IM transmitter [9], [8]
$T_{\text{txofdm}}(N)$	Complexity function of N-subcarrier OFDM transmitter
Δf	Subcarrier spacing
W	Bandwidth
$\mathcal{I}_{\text{rx}}$	Minimum required processing capability of the PHY receiver
$\mathcal{I}_{\text{rx-dm}}$	Minimum required processing capability for the DM-OFDM receiver (inst/s)
$\mathcal{I}_{\text{rx-im}}$	Minimum required processing capability for the OFDM-IM receiver (inst/s)
$\mathcal{I}_{\text{rx-ofdm}}$	Minimum required processing capability for the OFDM receiver (inst/s)
$\mathcal{I}_{\text{tx}}$	Minimum required processing capability of the PHY transmitter (inst/s)
$\mathcal{I}_{\text{tx-dm}}$	Minimum required processing capability for the DM-OFDM transmitter (inst/s)
$\mathcal{I}_{\text{tx-im}}$	Minimum required processing capability for the OFDM-IM transmitter (inst/s)
$\mathcal{I}_{\text{tx-ofdm}}$	Minimum required processing capability for the OFDM transmitter (inst/s)
SC_{SE}	SE accounting for time complexity
SC_{R}	Bit rate accounting for time complexity
$\text{SC}_{\text{R-dm}}$	DM-OFDM bit rate accounting for time complexity
$\text{SC}_{\text{R-im}}$	OFDM-IM bit rate accounting for time complexity
$\text{SC}_{\text{R-ofdm}}$	OFDM bit rate accounting for time complexity
$\text{SC}_{\text{SE-dm}}$	DM-OFDM SE accounting for time complexity
$\text{SC}_{\text{SE-im}}$	OFDM-IM SE accounting for time complexity
$\text{SC}_{\text{SE-ofdm}}$	OFDM-IM SE accounting for time complexity
SE	Spectral efficiency
SE_{ofdm}	Classic spectral efficiency of OFDM

duration of T_{sym} seconds of a signal carrying B bits, the classic bit rate (in b/second, b/s) and SE are defined as

$$R = B/T_{\text{sym}} \quad \text{b/s}, \tag{1}$$

$$\text{SE} = R/W \quad \text{b/s/Hz}. \tag{2}$$

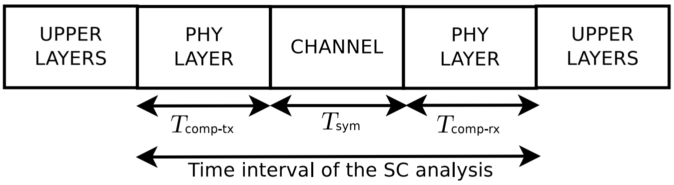

Figure 1 – By accounting for the PHY time complexity overheads at both transmitter ($T_{\text{comp-tx}}$) and receiver ($T_{\text{comp-rx}}$), the SC analysis captures the worst-case bit rate of a PHY symbol as perceived by upper layers.

Similarly, in the SC framework the quantities SC_R and SC_SE denote the enhanced versions of the classic bit rate R and SE SC_SE, respectively. These quantities are related as follows:

$$\text{SC}_\text{SE} \;=\; \text{SC}_\text{R}/W \quad \text{b/s/Hz}. \tag{3}$$

To define SC_R, let us denote $T_{\text{comp-tx}}$ and $T_{\text{comp-rx}}$ as the signal processing runtime at the transmitter and receiver basedband processors, respectively. From this, the SC bit rate SC_R is

$$\text{SC}_\text{R} \;=\; \frac{B}{T_{\text{comp-tx}} + T_{\text{sym}} + T_{\text{comp-rx}}} \quad \text{b/s}. \tag{4}$$

Eq. *(4)* is the base to study whether a signal operates under the comp-limited capacity regime. That regime concerns whether the number of bits per unit of transmission (i.e., symbol) grows asymptotically faster than the lower bound complexity required to process the symbol. It derives from the fixed Signal-to-Noise Ratio (SNR) capacity regime of Shannon which assumes that capacity grows linearly with the bandwidth. Assuming packets of bounded length, i.e., the Maximum Transmission Unit (MTU) of above-PHY layer, the number of symbol in a PHY layer burst rapidly decreases in the fixed SNR regime, since more bits can be conveyed per symbol as spectrum widens. Such a condition meets the scenario expected for 6G and beyond wireless networks, in which the exploitation of Terahertz (THz) and sub-THz frequency bands might enable extremely fast signals because of the abundant spectrum resources [19].

The study of comp-limited signals considers the maximum delay a symbol can experience. This matches the delay experienced by the first symbol in a burst transmission. This largest delay stems not only from the symbol duration time but also from all signal processing delays imposed by the physical layer before delivering the bits to the above layers. This condition is captured by *(4)* and is illustrated in Fig. 1. Therefore, unless otherwise stated, the analyses we present throughout this work consider the system premises of the comp-limited signal regime.

The signal processing runtime depends on two main factors. The first is the total number of computational instructions (inst.) the physical layer must execute on the baseband processor, determined by the signal processing algorithms and their input sizes. We denote these quantities as $T_\text{tx}(W)$ and $T_\text{rx}(W)$ for the transmitter and receiver, respectively. The second factor is the amount of computational resources of the baseband processors denoted as $\mathcal{I}_\text{tx}$ and $\mathcal{I}_\text{rx}$ for the transmitter and receiver, respectively. These quantities represent the minimum number of instructions per second (inst/s) required to process all computational instructions of a signal in a runtime no higher than the symbol period. Accordingly, the signal processing runtime overheads at the transmitter and receiver are defined as follows:

$$T_{\text{comp-tx}} \;=\; \frac{T_\text{tx}(W)}{\mathcal{I}_\text{tx}} \quad \text{seconds}, \tag{5}$$

$$T_{\text{comp-rx}} \;=\; \frac{T_\text{rx}(W)}{\mathcal{I}_\text{rx}} \quad \text{seconds}. \tag{6}$$

Note that the parameters $\mathcal{I}_\text{tx}$ and $\mathcal{I}_\text{rx}$ capture the computational resources (e.g., clock, memory, chip area) allocated to the baseband signal processor.

3. COMPLEXITY-CONSTRAINED BIT RATE ANALYSIS OF SIGNALS

In this section, we present a generic tutorial on how to account for the signal processing computational complexity overhead when assessing the bit rate and SE of a signal. We also show how to ensure fairer comparative analyses of signals constrained by different quantities of computational resources.

Bit rate and its derived metrics, such as SE, are time-sensitive indicators of waveform performance. Comparative analyses of different waveforms typically assume the same symbol duration to ensure equitable channel time allocation. This assumption directly impacts the computational resources assigned to the baseband signal processor for each waveform.

A key requirement for baseband processors is to complete the signal processing within the symbol period to maintain real-time communication capabilities [20],[21],[22]. This means that signals with higher computational complexity, requiring more instructions to execute, need larger computational resources to keep the processing runtime within the symbol duration. Neglecting computational complexity and resources in the analysis can introduce bias in comparative rankings based on classic bit rate and SE metrics, favoring more complex waveforms. The computational resources for these signals would exceed the processing requirements of low-complexity waveforms. These excess resources could allow low-complexity signals to adopt more robust PHY layer settings or advanced signal processing techniques (like stronger error correction codes and more precise channel estimators), ultimately improving bit rate or SE, but at the cost of increased complexity [7]. Therefore, larger computational resources can enable higher bit rates for low-complexity signals, while high-complexity signals would face longer processing delays (and slower bit rates) if constrained by the hardware limitations of simpler signals..

Our point is that all waveforms should benefit from the same processing capability required by the most computationally complex signal in a comparative study. To achieve this, we adopt the SC framework [17] and propose a methodology consisting of the following steps for comparative performance evaluation of signals constrained by different requisites of computational resources:

1. **Set the key variables and parameters of the symbol**
   (a) **Symbol duration $T_{\mathbf{sym}}$**: As previously discussed, T_{sym} establishes an upper-bound requirement for the signal processing runtime of any waveform analyzed in this work. The value of T_{sym} is determined based on channel measurements taken from scenarios that reflect the typical environments where the signals are expected to propagate (e.g., indoor, outdoor, local area, wide area, etc.). Without loss of generality, we adopt a symbol duration of 4 μs in our case studies, as defined by the IEEE 802.11 standards for indoor wireless local area networks.
   (b) **Number of bits per symbol B, bandwidth W, and subcarrier spacing Δf**: As is well known from channel capacity regimes [17], the number of bits that can be reliably transmitted through a symbol primarily depends on the received power and the spectrum bandwidth. For most modern waveforms, B is a function of the number of subcarriers per symbol, N, and the constellation order, M, which determines the number of bits per subcarrier. For simplicity, one may adopt a single-variable asymptotic analysis. In our case studies, for example, we treat M as a constant parameter, implying that the number of bits per subcarrier is at most $\log_2 M$, and B grows solely with N. For OFDM-based waveforms (as considered in the case studies of this work), the symbol bandwidth is given by $W = \Delta f \cdot N$, where Δf denotes the subcarrier spacing in Hz. We assume Δf to be constant for different values of N, which is consistent with practical wireless communication standards (e.g., Δf = 312.5 kHz in the IEEE 802.11a standard)
2. **Obtain the complexity function required to process a symbol for each waveform.**
   Following the assumptions used to define B, we also express the computational complexities of all signals as functions of N, treating M as a constant. As is common in algorithm analysis, one may account only for the most computationally intensive algorithm or computational instruction to define the complexity functions $T_{\text{tx}}(N)$ and $T_{\text{rx}}(N)$, representing the computational complexities at the transmitter and receiver, respectively. For instance, the overall complexity of a basic N-subcarrier OFDM transmitter (or receiver) can be simplified to the FFT's complexity (approximately $N \log_2 N$), as it asymptotically dominates the computational cost of the waveform as N grows. Alternatively, one may consider the complexities of all signal processing algorithms involved in the waveform (examples are provided later). It is important to highlight that any of these assumptions can be modified while preserving the analytical methodology presented in this work.
3. **Calculate the required processing capability for the most complex signal.**
   To identify the most demanding signal in terms of required processing capability, compute the quantities $\mathcal{I}_{\text{tx}}$ and $\mathcal{I}_{\text{rx}}$ for each signal, recalling the constraints $T_{\text{comp-tx}} \leq T_{\text{sym}}$ and $T_{\text{comp-rx}} \leq T_{\text{sym}}$. By allowing each baseband signal processor to operate at the maximum permitted processing delay (i.e., $T_{\text{comp-tx}} = T_{\text{comp-rx}} = T_{\text{sym}}$) and considering equations *(5)* and *(6)* in the context of the most complex signal, the required computational resources for the baseband processors of the transmitters and receivers of all waveforms in a comparative study are given by:

$$\mathcal{I}_{\text{tx}} = \frac{T_{\text{tx}}(N)}{T_{\text{sym}}}, \quad \text{inst/s} \tag{7}$$

$$\mathcal{I}_{\text{rx}} = \frac{T_{\text{rx}}(N)}{T_{\text{sym}}}, \quad \text{inst/s} \tag{8}$$

   respectively.
4. **Calculate the signal processing runtime for all waveforms assuming the required processing capability of the most complex signal.**
   With the quantities $\mathcal{I}_{\text{tx}}$ and $\mathcal{I}_{\text{rx}}$ derived from the prior step and the complexity functions of each signal, compute the signal processing runtime of each waveform according to *(5)* and *(6)*.
5. **Compute the complexity-constrained bit rate and SE of each signal.**
   Finally, based on the number of bits per symbol of each waveform design and the signal processing runtime obtained in the prior step, compute the complexity-constrained bit rates and SEs of all waveforms by referring to the *(4)* and *(3)*, respectively.

Fig. 2 provides a flow diagram summarizing the methodology. It highlights the required inputs and the quantities produced at each step of the process. Note that this methodology is applicable to any waveform, as long as its number of bits, symbol duration, and computational complexity per symbol are defined.

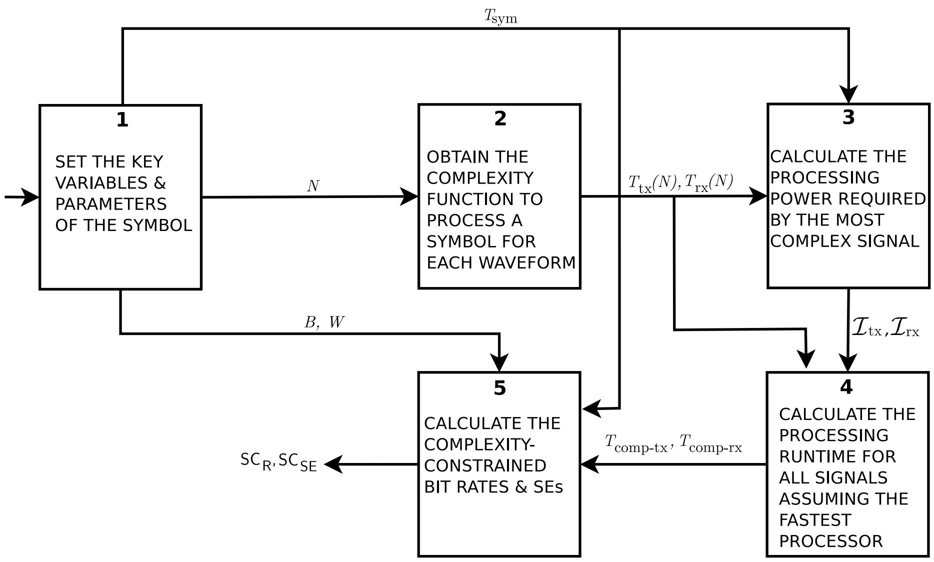

Figure 2 – Flow diagram to assess (and compare) the complexity-constrained bit rates SC_R and SEs SC_{SE} of signals constrained by different processing requirements. Considering an N-subcarrier transmitter that modulates B bits in a W-Hz signal, the baseband signal processor takes $T_{comp\text{-}tx}$ seconds to execute $T_{tx}(N)$ computational instructions if empowered by the same processing capability $\mathcal{I}_{tx}$ (inst/s) required by the most complex signal of the comparative study. This runtime overhead adds to the symbol duration T_{sym} and the analogous runtime at the receiver $T_{comp\text{-}rx}$ to compute SC_R (b/s) and SC_{SE} (b/s/Hz).

4. CASE STUDIES

In this section, we exemplify the tutorial of Section 3 by introducing case studies for the classic OFDM waveform and the IM variants OFDM-IM and DM-OFDM.

4.1 Waveforms time complexity

OFDM and IM waveforms have the same computational complexity except for the IxS and the detection algorithms. As we discussed in Section 2, the IxS of the IM waveforms add an extra complexity of $O(gnk)$ in comparison to OFDM. This overhead verifies at both the transmitter and the receiver. Thus, considering the ideal IM setup described in Section 2, the complexity overhead introduced by each IxS is $O(1 \cdot N \cdot N/2) = O(N^2)$. Recall that the IxS complexity is zero for OFDM. Therefore, assuming as negligible both the time complexities of the look-up table (de)mapping and the Cyclic Prefix (CP) addition/remotion, and omitting the implementation-dependent constants, the overall number of computational instructions performed by the OFDM, OFDM-IM, and DM-OFDM transmitters under the ideal IM setup can be given by *(9)*, *(10)*, and *(11)*, respectively.

$$T_{\text{txofdm}}(N) = \overbrace{N \log_2 N}^{\text{IFFT}} \quad \text{inst.} \tag{9}$$

$$T_{\text{txim}}(N) = N \log_2 N + \overbrace{N^2}^{\text{IxS}} \quad \text{inst.} \tag{10}$$

$$T_{\text{txdm}}(N) = N \log_2 N + N^2 \quad \text{inst.} \tag{11}$$

Recalling that IM waveforms split N subcarriers into g groups of n subcarriers each, i.e., $N = gn$, and the ideal setup implies in $M = M_A = M_B = 2$, the growth of the signal detection complexity solely depends on the number of subcarriers rather than on the constellation length M. This is due to the Log-Likelihood Ratio (LLR)-based detection strategy [10]. In this context, the complexity of detection for OFDM, OFDM-IM, and DM-OFDM become $O(N \log_2 M) = O(N)$, $O(gnM) = O(2N)$[1] and $O(gn(M_A + M_B)) = O(4N)$, respectively. Thus, the resulting time complexity at their respective receivers are

$$T_{\text{rxofdm}}(N) = \overbrace{N \log_2 N}^{\text{FFT}} + \overbrace{N}^{\text{LLR}} \quad \text{inst.} \tag{12}$$

$$T_{\text{rxim}}(N) = N \log_2 N + 2N + \overbrace{N^2}^{\text{de-IxS}} \quad \text{inst.} \tag{13}$$

$$T_{\text{rxdm}}(N) = N \log_2 N + 4N + N^2 \quad \text{inst.} \tag{14}$$

It is remarkable to note that the original IxS leads to a computational complexity higher than the detection and FFT procedures if SE maximizes (i.e., $k = N/2$, $g = 1$). From the general $O(gnk)$ time complexity of the original IxS algorithm, it is easy to conclude that *the originally proposed IxS does not run in linear time complexity unless* $k = O(1)$. In this case, one might give up the optimal SE, since the optimal balance between time complexity and SE is achieved if the IM mapper runs in exact $\Theta(N)$ time complexity (though faster algorithms are possible)

[1] We deliberately don't omit from our complexity analyses the constants associated to the waveform-related parameters (e.g., M, M_A, and M_B) because of their impact on the 'real-time constraint' of the signal processing.

and $k = N/2$, as observed in [7]. This optimized solution was proposed in [9], [8], based on which the overall computational complexity of the OFDM-IM and DM-OFDM transmitters *(10)*, *(11)*, and receivers *(13)*, *(14)*, respectively, improve to

$$T_{\text{txim2}}(N) = N\log_2 N + \overbrace{N}^{\substack{\text{Optimized}\\ \text{IxS[9],[8]}}} \quad \text{inst.} \tag{15}$$

$$T_{\text{txdm2}}(N) = N\log_2 N + N \quad \text{inst.} \tag{16}$$

$$T_{\text{rxim2}}(N) = N\log_2 N + 2N + \overbrace{N}^{\substack{\text{Optimized}\\ \text{de-IxS}}} \quad \text{inst.} \tag{17}$$

$$T_{\text{rxdm2}}(N) = N\log_2 N + 4N + N \quad \text{inst.} \tag{18}$$

Despite that, we have observed that the IM literature is mostly focused on the "complexity vs. communication" trade-off of the detection problem and they usually refer to an IxS procedure reminiscent to the non-optimized original IM proposal e.g., [23], [1]. Due to this, in addition to the "optimized" IM, our analyses throughout this work will also consider the time complexity of the original IM mapper popularly referred to by the IM literature as given in *(10)*, *(11)*, *(13)*, and *(14)*.

4.2 Baseband signal computation runtime

The computational resources required by the design of a given digital baseband processor are constrained by several factors like budget, portability, power consumption, runtime constraints, etc. We abstract these resources by modeling the required processing capability as a quantity given in algorithmic instructions per second (inst/s). Thus, let us denote $\mathcal{I}_{\text{tx-ofdm}}$, $\mathcal{I}_{\text{tx-im}}$, and $\mathcal{I}_{\text{tx-dm}}$ the minimum number of inst/s required by the baseband signal processors of the OFDM, OFDM-IM, and DM-OFDM transmitters, respectively. Besides, denote $\mathcal{I}_{\text{rx-ofdm}}$, $\mathcal{I}_{\text{rx-im}}$, and $\mathcal{I}_{\text{rx-dm}}$ as the analogous performance indicators for the corresponding receivers, respectively. Considering the computational complexities of the OFDM, OFDM-IM, and DM-OFDM transmitters as given by *(9)*, *(10)*, and *(11)*, respectively, the corresponding baseband signal computation time of the OFDM, OFDM-IM and DM-OFDM transmitters are given as follows:

$$T_{\text{comp-txofdm}}(N) = \frac{T_{\text{txofdm}}(N)}{\mathcal{I}_{\text{tx-ofdm}}} \quad \text{seconds,} \tag{19}$$

$$T_{\text{comp-txim}}(N) = \frac{T_{\text{txim}}(N)}{\mathcal{I}_{\text{tx-im}}} \quad \text{seconds,} \tag{20}$$

$$T_{\text{comp-txdm}}(N) = \frac{T_{\text{txdm}}(N)}{\mathcal{I}_{\text{tx-dm}}} \quad \text{seconds.} \tag{21}$$

Similarly, based on the computational complexities of the OFDM, OFDM-IM, and DM-OFDM receivers, given by *(12)*, *(13)*, and *(14)*, respectively, the baseband signal computation time at the OFDM, OFDM-IM and DM-OFDM receivers result

$$T_{\text{comp-rxofdm}}(N) = \frac{T_{\text{rxofdm}}(N)}{\mathcal{I}_{\text{rx-ofdm}}} \quad \text{seconds,} \tag{22}$$

$$T_{\text{comp-rxim}}(N) = \frac{T_{\text{rxim}}(N)}{\mathcal{I}_{\text{rx-im}}} \quad \text{seconds,} \tag{23}$$

$$T_{\text{comp-rxdm}}(N) = \frac{T_{\text{rxdm}}(N)}{\mathcal{I}_{\text{rx-dm}}} \quad \text{seconds.} \tag{24}$$

In the above equations, we decouple computational complexity (numerator) from the processing capability of the baseband processor (denominator). Hence, hardware techniques that speed up computation (e.g., pipelining, LUTs) can decrease the wall-clock runtime but cannot decrease time complexity, since this latter quantity solely depends on the signal processing algorithms.

4.3 SC throughput

The total number of bits entering the digital baseband processors of the OFDM, OFDM-IM, and DM-OFDM waveforms are $N\log_2 M$, $N/2\log_2 M + \lfloor\log_2\binom{N}{N/2}\rfloor$, and $N/2\log_2 M_A + N/2\log_2 M_B + \lfloor\log_2\binom{N}{N/2}\rfloor$, respectively. The algorithmic throughput (also referred to as SC throughput) of OFDM, OFDM-IM and DM-OFDM are respectively defined as

$$\text{SC}_{\text{R-ofdm}}(N) = \frac{N\log_2 M}{T_{\text{comp-txofdm}}(N) + T_{\text{sym}} + T_{\text{comp-rxofdm}}(N)} \quad \text{b/s,} \tag{25}$$

$$\text{SC}_{\text{R-im}}(N) = \frac{N/2\log_2 M + \lfloor\log_2\binom{N}{N/2}\rfloor}{T_{\text{comp-txim}}(N) + T_{\text{sym}} + T_{\text{comp-rxim}}(N)} \quad \text{b/s,} \tag{26}$$

$$\text{SC}_{\text{R-dm}}(N) = \frac{N/2(\log_2 M_A + \log_2 M_B) + \lfloor\log_2\binom{N}{N/2}\rfloor}{T_{\text{comp-txdm}}(N) + T_{\text{sym}} + T_{\text{comp-rxdm}}(N)} \quad \text{b/s.} \tag{27}$$

Under the "ideal setup" assumed throughout this work, one gets $M = M_A = M_B = 2$, yielding a total number of bits of N, $N/2 + \lfloor\log_2\binom{N}{N/2}\rfloor$, and $N + \lfloor\log_2\binom{N}{N/2}\rfloor$, for OFDM, OFDM-IM, and DM-OFDM, respectively.

4.4 Time complexity-constrained SE

The SE constrained by the time complexity overhead (also referred to as SC efficiency, SCE) readily results from the SC throughput. For OFDM, OFDM-IM, and

DM-OFDM it is respectively defined as,

$$\mathrm{SC}_{\text{SE-ofdm}}(N) = \frac{\mathrm{SC}_{\text{R-ofdm}}(N)}{W} \quad \text{b/s/Hz}, \qquad (28)$$

$$\mathrm{SC}_{\text{SE-im}}(N) = \frac{\mathrm{SC}_{\text{R-im}}(N)}{W} \quad \text{b/s/Hz}, \qquad (29)$$

$$\mathrm{SC}_{\text{SE-dm}}(N) = \frac{\mathrm{SC}_{\text{R-dm}}(N)}{W} \quad \text{b/s/Hz}. \qquad (30)$$

As discussed in Section 3, the computation time (of either the transmitter or the receiver) must not be larger than the symbol duration to ensure a real-time capable physical layer implementation. Thus, the largest tolerable value for the computation time is the symbol duration. Considering this case, the SCEs of *(28)*, *(29)*, and *(30)* respectively simplify as follows:

$$\mathrm{SC}_{\text{SE-ofdm}}(N) = \frac{\mathrm{SC}_{\text{R-ofdm}}(N)}{3(N+N_{cp})} \quad \text{b/s/Hz}, \qquad (31)$$

$$\mathrm{SC}_{\text{SE-im}}(N) = \frac{\mathrm{SC}_{\text{R-im}}(N)}{3(N+N_{cp})} \quad \text{b/s/Hz}, \qquad (32)$$

$$\mathrm{SC}_{\text{SE-dm}}(N) = \frac{\mathrm{SC}_{\text{R-dm}}(N)}{3(N+N_{cp})} \quad \text{b/s/Hz}. \qquad (33)$$

The aforementioned simplification stems from the fact that the denominator of the SC throughput rewrites to $3T_{\text{sym}}$ for each waveform if the computation time is equal to the symbol duration. From this, it results $W \cdot 3T_{\text{sym}} = N\Delta f \cdot 3(1/\Delta f + T_{\text{cp}}) = 3(N + N_{\text{cp}})$.

5. NUMERICAL RESULTS

In this section, we analyse the OFDM bit rate and SE under the time computational overheads of specific baseband processors (Section 5.1). In Section 5.2, we investigate whether the superior SE claimed by IM waveforms do hold if its time complexity overhead is accounted for the bit rate analysis.

5.1 Impact of baseband runtime on the OFDM performance

In this subsection, we refer to the OFDM baseband processors of [21] and [22] to demonstrate how SE can be severely impacted if the signal processing time overhead is considered akin to the CP time overhead. For the sake of discussion, let us label these baseband processors as A and B, respectively. As reported in [21], processor A achieves a computation time of $T_{\text{comp-txA}} = 0.55\ \mu$s and $T_{\text{comp-rxA}} = 3.95\ \mu$s at the transmitter and receiver, respectively. These results are reported for an IEEE 802.11a signal of 54 Mb/s (i.e., $M = 64$, $N = 64$, $T_{\text{cp}} = 0.8\ \mu$s, and $T_{\text{sym}} = 4\ \mu$s). Under the same bit rate, processor B achieves a runtime of $T_{\text{comp-rxB}} = 3.29\ \mu$s at the receiver. The runtime $T_{\text{comp-txB}}$ for the transmitter A is not reported by [22]. For this case, in what follows we assume a transmitter empowered by the same baseband processor of A, i.e., $T_{\text{comp-txB}} = T_{\text{comp-txA}} = 0.55\ \mu$s. These time parameters and the SC throughput we discuss next are summarized in Table 3.

In all cases, the real-time signal processing requisite $T_{\text{comp}} \leq T_{\text{sym}}$ is satisfied by the transmitters and the receivers. However, the runtime at the receivers A and B are ≈ 4.93× and ≈ 4.11× higher than the standard CP overhead of IEEE 802.11a. Moreover, the respective signal computation runtime of the processors correspond to circa 53% and 49% of the overall time overheads introduced by the physical layer, i.e., $T_{\text{comp-tx}} + T_{\text{sym}} + T_{\text{comp-rx}}$. To assess the complexity-constrained bit rate perceived by the upper layers, we refer to the SC throughput. By recalling that only 48 (out of the $N = 64$) subcarriers convey useful data bits in IEEE 802.11a and that six bits are sent per subcarrier by 64-QAM points ($M = 64$) in this case, one gets an SC bit rate of

$$\mathrm{SC}_{\text{R-A}}(48) = \frac{48 \cdot 6}{0.55 + 4 + 3.95} \approx 33.8 \quad \text{b}/\mu\text{s}, \qquad (34)$$

for a communication empowered by baseband processor A. Analogously, the baseband processor B achieves an SC throughput

$$\mathrm{SC}_{\text{R-B}}(48) = \frac{48 \cdot 6}{0.55 + 4 + 3.29} \approx 36.7 \quad \text{b}/\mu\text{s}. \qquad (35)$$

In both cases, the OFDM bit rate without the computational overheads is

$$R(48) = \frac{48 \cdot 6}{4} = 72 \quad \text{b}/\mu\text{s}, \qquad (36)$$

nearly half of obtained if the baseband processor runtime overhead is not neglected. *Therefore, similar to the CP overhead, the signal processing computational complexity overhead can dramatically impair the waveform bit rate even when the baseband signal processor can run below the symbol period, as required for real-time communication performance.* Consequently, (complexity-constrained) SE impairs accordingly.

Given the IEEE 802.11a standard bandwidth of W = 20 MHz, the expected OFDM SE of our case study impairs from

$$\begin{aligned}\mathrm{SE}_{\text{ofdm}} &= \frac{R(48)}{20\ \text{MHz}} = \frac{72\ \text{b}/\mu\text{s}}{20\ \text{MHz}} \\ &= 3.60 \quad \text{b/s/Hz}, \qquad (37)\end{aligned}$$

to

$$\begin{aligned}\mathrm{SC}_{\text{SE-A}} &= \frac{\mathrm{SC}_{\text{R-A}}(48)}{20\ \text{MHz}} = \frac{33.8\ \text{b}/\mu\text{s}}{20\ \text{MHz}} \\ &= 1.69 \quad \text{b/s/Hz}, \qquad (38)\end{aligned}$$

$$\begin{aligned}\mathrm{SC}_{\text{SE-B}} &= \frac{\mathrm{SC}_{\text{R-B}}(48)}{20\ \text{MHz}} = \frac{36.7\ \text{b}/\mu\text{s}}{20\ \text{MHz}} \\ &\approx 1.83 \quad \text{b/s/Hz}, \qquad (39)\end{aligned}$$

Table 3 – Complexity-constrained throughput of an IEEE 802.11a WiFi symbol considering the signal processing overhead of different baseband processors (N = 48, M = 64).

Comparison	Processing delay (μs)		Symbol duration (μs)	Throughput (b/μs)
	Transmitter	Receiver		
Processor A [21]	0.55	3.59	4	33.8
Processor B [22]	0.55	3.29	4	36.7
Ideal processor, Eq. *(1)*	0	0	4	72

for OFDM being processed on baseband processors A and B, respectively.

5.2 SE constrained by time complexity: IM vs. OFDM

In this section, we present a comparative time complexity-constrained SE analysis among OFDM, OFDM-IM, and DM-OFDM waveforms. The analysis follows the steps of Section 3 assuming the 'ideal IM setup', i.e., $M = 2$ (BPSK modulation), $k = N/2$, and $g = 1$ for the IM waveforms. We assume the PHY layer parameters of the IEEE 802.11a standard for all waveforms, $T_{\text{sym}} = 4$ μs and $N = 64$.

5.2.1 *Required computational resources of the waveforms*

By referring to *(7)* for a symbol duration $T_{\text{sym}} = 4$ μs, and considering the computational complexities given in *(9)*, *(10)*, and *(11)* for $N = 64$-subcarrier signals, the minimum required processing capability for the baseband signal processors of the OFDM, OFDM-IM, and DM-OFDM transmitters are respectively given by

$$\mathcal{I}_{\text{tx-ofdm}} = \frac{T_{\text{txofdm}}(64)}{4} = \frac{64\log_2 64}{4} = 96 \quad \text{inst}/\mu\text{s}, \tag{40}$$

$$\mathcal{I}_{\text{tx-im}} = \mathcal{I}_{\text{tx-dm}} = \frac{T_{\text{txim}}(64)}{4} = \frac{64\log_2 64 + 64^2}{4} = 1120 \quad \text{inst}/\mu\text{s}. \tag{41}$$

As expected, the IM transmitters require more powerful baseband signal processors with 1120 inst/μs to meet a maximum processing runtime of 4 μs. This stems from the fact that they rely on the same IxS procedure at the transmitter. As will be shown later, the difference between the complexities of OFDM-IM and DM-OFDM lies in the receiver.

Following a similar approach, we refer to *(12)*, *(13)*, and *(14)* to obtain the required processing capabilities for the OFDM, OFDM-IM, and DM-OFDM receivers, namely,

$$\mathcal{I}_{\text{rx-ofdm}} = \frac{T_{\text{rxofdm}}(64)}{4} = \frac{64\log_2 64 + 64}{4} = 112 \quad \text{inst}/\mu\text{s}, \tag{42}$$

$$\mathcal{I}_{\text{rx-im}} = \frac{T_{\text{rxim}}(64)}{4} = \frac{64\log_2 64 + 2\cdot 64 + 64^2}{4} = 1152 \quad \text{inst}/\mu\text{s}, \tag{43}$$

$$\mathcal{I}_{\text{rx-dm}} = \frac{T_{\text{rxdm}}(64)}{4} = \frac{64\log_2 64 + 4\cdot 64 + 64^2}{4} = 1184 \quad \text{inst}/\mu\text{s}. \tag{44}$$

Based on these results, we will assess SC bit rates and SEs considering the processing capabilities of 1120 inst/μs and 1184 inst/μs for all transmitters and receivers, respectively.

5.2.2 *Baseband signal runtime under equitable computational resources*

To ensure a fairer bit rate analysis, we consider the computation time of each waveform assuming equitable computational resources of the most complex signal. According to our prior analysis, both OFDM-IM and DM-OFDM are the most complex transmitters, requiring 1120 inst/μs each *(41)*. If empowered by this superior amount of computational resources of DM-OFDM instead of the minimum required by OFDM (thereby, the denominator of *(19)* modifies accordingly), the baseband processor of the OFDM transmitter achieves a runtime performance of

$$T_{\text{comp-txofdm}} = \frac{64\log_2 64}{1120} \approx 0.34 \quad \mu\text{s}, \tag{45}$$

whereas the IM transmitters reach a runtime of 4 μs, as previously dimensioned.

Applying similar methodology to the receivers, each waveform must be empowered by a baseband processor capable of performing 1184 inst/μs, which corresponds to the required processing capability demanded by DM-OFDM, the most complex receiver. Under such superior processing capability, the computation runtime of the OFDM *(22)*, OFDM-IM *(23)*, and DM-OFDM *(24)* at the

Table 4 – SC performance indicators of OFDM, OFDM-IM, and DM-OFDM under IEEE 802.11a parameters (N=64, T_{sym} = 4 μs, W=20 MHz).

	Performance Metric	OFDM	OFDM-IM	DM-OFDM
Tx.	Computational Complexity $T_{\text{tx}}(64)$ (inst)	384	4480	4480
	Minimum Required Processing Capability $\mathcal{I}_{\text{tx}}$ (inst/μs)	96	1120	1120
	Signal processing runtime under equitable computational resources $T_{\text{comp-tx}}$ (μs)	0.34	4	4
Rx.	Computational Complexity $T_{\text{rx}}(64)$ (inst)	448	4608	4736
	Minimum Required Processing Capability $\mathcal{I}_{\text{rx}}$ (inst/μs)	112	1152	1184
	Signal processing runtime under equitable computational resources $T_{\text{comp-rx}}$ (μs)	0.37	3.89	4
LINK	Complexity-Constrained Bit rate SC_{R} (b/μs)	10.19	5.71	7.6
	Complexity-Constrained SE SC_{SE} (b/s/Hz)	0.51	0.28	0.38

receivers are respectively given by

$$T_{\text{comp-rxofdm}} = \frac{64\log_2 64 + 64}{1184} \approx 0.37 \quad \mu\text{s}, \tag{46}$$

$$T_{\text{comp-rxim}} = \frac{64\log_2 64 + 2\cdot 64 + 64^2}{1184} \approx 3.89 \quad \mu\text{s}, \tag{47}$$

$$T_{\text{comp-rxdm}} = \frac{64\log_2 + 4\cdot 64 + 64^2}{1184} = 4 \quad \mu\text{s}. \tag{48}$$

As one can see, OFDM has a runtime efficiency one order of magnitude better than its IM counterparts if provided the same computational resources.

5.2.3 Discussion

Assuming each waveform is allowed a maximum signal processing runtime equal to the symbol period (4 μs), the results presented in the previous section indicate that the computational resources of IM waveforms are overdimensioned if considered for the processing needs of OFDM. Nevertheless, an OFDM chipset designer should take these resources into account for fair comparison with the considered IM waveforms.

With these excess computational resources, the OFDM chipset could accommodate more robust configurations or advanced signal processing algorithms, ultimately improving the system's overall bit rate. For example, a larger budget for the baseband processor chipset could enable the implementation of a more efficient bit encoder to enhance error recovery at the receiver. In the case of convolutional encoders, this could involve increasing the constraint length. Similarly, the receiver could adopt more complex algorithms, such as improved channel estimators, equalizers, and decoders, to demodulate more bits efficiently.

5.2.4 Fairer bit rate and SE analyses

Given the excessive computational resources of DM-OFDM, an OFDM designer may consider improving the runtime performance rather than accommodating more robust algorithms, as previously discussed. This would reflect in superior SC bit rates and SE. Indeed, recalling that only 48 (out of 64 subcarriers) convey data bits in the IEEE 802.11a standard, under the improved processing runtime discussed in the prior section, the SC bit rates of OFDM *(25)*, OFDM-IM *(26)*, and DM-OFDM *(27)* are

$$\text{SC}_{\text{R-ofdm}}(48) = \frac{48}{0.34+4+0.37} \approx 10.19 \quad \text{b}/\mu\text{s}, \tag{49}$$

$$\text{SC}_{\text{R-im}}(48) = \frac{48/2 + \lfloor \log_2 \binom{48}{24} \rfloor}{4+4+3.89} \approx 5.71 \quad \text{b}/\mu\text{s}, \tag{50}$$

$$\text{SC}_{\text{R-dm}}(48) = \frac{48 + \lfloor \log_2 \binom{48}{24} \rfloor)}{4+4+4} \approx 7.6 \quad \text{b}/\mu\text{s}, \tag{51}$$

respectively. This yields respective complexity-constrained SEs of

$$\text{SC}_{\text{SE-ofdm}}(48) = \frac{10.19 \text{ b}/\mu\text{s}}{20 \text{ MHz}} \approx 0.51 \quad \text{b/s/Hz}, \tag{52}$$

$$\text{SC}_{\text{SE-im}}(48) = \frac{5.71 \text{ b}/\mu\text{s}}{20 \text{ MHz}} \approx 0.28 \quad \text{b/s/Hz}, \tag{53}$$

$$\text{SC}_{\text{SE-dm}}(48) = \frac{7.6 \text{ b}/\mu\text{s}}{20 \text{ MHz}} \approx 0.38 \quad \text{b/s/Hz}. \tag{54}$$

These results, along with their previously discussed components, are summarized in Table 4.

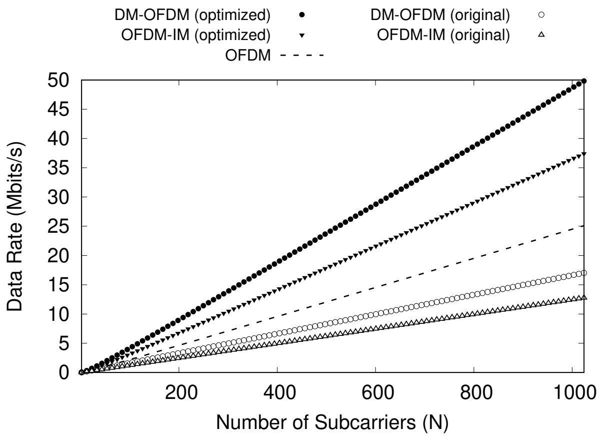

Figure 3 – Computational complexity-constrained bit rate comparison of original DM-OFDM and OFDM-IM (blank circle and triangle points, respectively) and OFDM (dashed line). By providing all baseband processors with the same computational resources of DM-OFDM (the most complex waveform), the original IM waveforms can be outperformed by OFDM if the optimal complexity-SE balance of DM-OFDM and OFDM-IM (black circle and triangle points, respectively) is not considered.

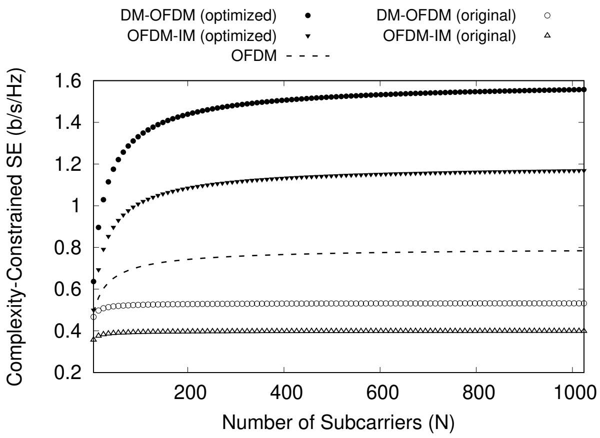

Figure 4 – Equivalent time complexity-constrained SE performance of bit rates shown in Fig. 3.

Note that the increase in complexity for achieving a higher bit rate benefits DM-OFDM more than OFDM-IM. Nonetheless, the low complexity of OFDM ensures robust overall performance, provided that computational complexity is not overlooked and computational resources are equitably considered for all waveforms. To revert these results, IM waveforms should definitely adopt more efficient procedures. For example, by considering the optimized IxS mapper of [8], [9] rather than those originally proposed for the waveforms, both OFDM-IM and DM-OFDM can achieve better SC bit rates and SE, as shown in plots on Fig. 3 and Fig. 4 for varying N, respectively.

6. CONCLUSION

In this work, we presented a tutorial on the computational complexity-constrained SE analysis of wireless signal waveforms. Our tutorial can be applied to any waveform design, regardless of the baseband processor technology, as long as the signal's SE, computational complexity, and symbol period are known. Through step-by-step examples, we demonstrated how the computational complexity overhead of the physical layer can effectively impact the SE delivered to the upper layers. This makes a strong case to reconsider the assumption of negligible signal processing runtime in the SE analysis, particularly when comparing waveforms that present different computational resource constraints to ensure a baseband processor runtime below the symbol period. We demonstrated this point through a case study in which waveforms considered less efficient in terms of SE can outperform their more computationally expensive counterparts when provided with equivalent high-performance computational resources.

Future work can study the complexity-SE trade-off in the context of other waveforms such as NOMA and multiple-mode index modulation. Also, the SC efficiency analysis can be enhanced to account for the power consumption of the signal computational complexity. Finally, future work may build upon our methodology to quantitatively analyze the relationship between computational complexity and the useful bit rate (i.e., after decoding) for a given error correction coding scheme. Traditionally, computational complexity and Bit Error Rate (BER) have been treated as heterogeneous performance indicators in such contexts. Our unified methodology can help quantify the impact of one indicator on the other.

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AUTHORS

SAULO QUEIROZ is an associate professor at the Department of Computer Science of the Federal University of Technology (UTFPR) in Brazil. He completed his Ph.D. with distinction and honors at the University of Coimbra (Portugal). During his academic graduation, he has contributed to open source projects in the field of networking, having participated in initiatives such as Google Summer of Code. Over the last decade, he has lectured disciplines on computer science such as design and analysis of algorithms, data structures and communication signal processing. His current research interest comprises networking and signal processing for wireless communications.

JOÃO P. VILELA is an assistant professor at the Department of Computer Science of the University of Porto, Portugal, and a senior researcher at CISUC and INESC TEC. He was a professor at the University of Coimbra after receiving his Ph.D. in Computer Science from the University of Porto in 2011, and a visiting researcher at Georgia Tech and MIT, USA. In recent years, Dr. Vilela has been coordinator and team member of several national, bilateral, and European-funded projects in security and privacy. His main research interests are in security and privacy of computer and communication systems, with applications such as wireless networks, Internet of Things and mobile devices. Specific research topics include wireless physical-layer security, security of next-generation networks, privacy-preserving data mining, location privacy and automated privacy protection.

Benjamin Koon Kei Ng received B.A.Sc., M.A.Sc., and Ph.D. degrees in engineering science and electrical engineering from the University of Toronto, in 1996, 1998, and 2002, respectively. From 2005 to 2009, he was with Radiospire Networks Inc., Boston, MA, USA, where he was a senior communications engineer focusing on the UWB and millimeter wave technologies. He joined Macao Polytechnic University, Macau, China, in 2010, where he is currently an associate professor with the Faculty of Applied Sciences. His research interests include wireless communications and signal processing, with an emphasis on MIMO, NOMA, and machine learning technologies.

Chan-Tong Lam received B.Sc. (Eng.) and M.Sc. (Eng.) degrees from Queen's University, Kingston, ON, Canada, in 1998 and 2000, respectively, and a Ph.D. degree from Carleton University, Ottawa, ON, Canada, in 2007. He is currently an Associate Professor with the Faculty of Applied Sciences, Macao Polytechnic University, Macau, China. From 2004 to 2007, he participated with the European Wireless World Initiative New Radio (WINNER) Project. His research interests include mobile wireless communications, machine learning in communications, and computer vision in smart city..

Edmundo Monteiro is currently a full professor with the University of Coimbra, Portugal. He has more than 30 years of research experience in the field of computer communications, wireless networks, quality of service and experience, network and service management, and computer and network security. He participated in many Portuguese, European, and international research projects and initiatives. His publication list includes over 200 publications in journals, books, and international refereed conferences. He has co-authored nine international patents. He is a member of the editorial board of Wireless Networks (Springer) journal and is involved in the organization of many national and international conferences and workshops. He is also a senior member of the IEEE Communications Society and the ACM Special Interest Group on Communications. He is also a Portuguese representative in IFIP TC6 (Communication Systems).