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| Fascicle on hydrometeor scatter interference | |

Scope

This fascicle provides mathematical foundations and validation of the hydrometeor scatter interference prediction method utilized in Recommendation ITU-R P.452.

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# 1 Introduction

This fascicle provides mathematical foundations and validation of the hydrometeor scatter interference prediction method. The fascicle starts by introducing the parameters impacting hydrometeor scatter (Section 2). There are three types of those parameters: meteorological parameters (Section 2.1), geometrical parameters (Section 2.2), and electromagnetic parameters (Section 2.3). Plane Earth representation and off axis squint angles are the only two geometric parameters addressed in this fascicle. Rain specific attenuation and raindrop bi-static cross section are the two electromagnetic parameters impacting hydrometeor scatter. General formulation for the specific attenuation and the bi-static cross section are considered in Section 2.3, and their polarization representation is given in Annex A. Three types of bi-static cross section for raindrops are considered: one formulation based on the Rayleigh approximation (Section 2.3.1), another formulation based on Mie scattering theorem (Section 2.3.2, Annex B, Annex C), and a new formulation derived in Section 3. Mathematical formulations relating bi-static cross section and specific attenuation are to rainfall rate and frequency are derived in Annex D and Annex E respectively. Those formulations are derived at two different temperatures: 0 and 30° C.

The hydrometeor scatter transmission loss involving the new bi-static cross section formulation as well as the new specific attenuation formulation found in Section 5. The transmission loss formulation encompasses a scatter transfer function which involves three-dimensional integration. Section 6 provides the parameters of the hydrometeor scattering geometry required for constructing the scatter transfer function. The integration of the scatter transfer function is performed numerically in [Document [3M/295](https://www.itu.int/md/R15-WP3M-C-0295/en) (WP 3M Chairman’s Report [3M/343 (Annex 10)](https://www.itu.int/md/R15-WP3M-C-0343/en) (revision to Rec. ITU-R P.452, Section 5)] using Gauss‑Legendre quadrature. An algorithm for calculating the nodes and weights required by Gauss‑Legendre quadrature integration is given in Annex F. An analytic solution for the scatter transfer function is reported in Annex G.

# 2 Parameters controlling hydrometeor scatter interference

## 2.1 Meteorological parameters

There are two types of meteorological parameters impacting hydrometeor scatter:

– One type related to the background atmosphere, and

– Another type related to the hydrometeor particles.

The meteorological parameters related to the background atmosphere are the atmospheric temperature profile, and water vapour (humidity) profile. These two profiles determine the attenuation due to atmospheric gases attenuating the hydrometeor scatter signals, and can be obtained from Recommendation ITU-R P.676-13.

The meteorological parameters related to the hydrometeor particles are the hydrometeor temperature, and rainfall rate. The hydrometeor temperature, depending on the operating frequencies, determines the hydrometeor complex relative permittivity which determines scattering amplitudes of raindrops, which in turn determine the bi-static scattering cross sections and the specific attenuation of raindrops controlling level and spatial distributions of scattered signals by the hydrometeor particles. On the other hand, rainfall rate is a function of rainfall terminal velocity, and it determines the raindrop size distribution (DSD) and rain cell size which are required for getting raindrop scattering amplitudes, and hence both bi-static scattering cross section and specific attenuation of raindrops. Also at lower frequencies, where the Rayleigh approximation can be used, rainfall rate can be used for getting radar reflectivity used in section 5 of Recommendation ITU-R P.452‑16.

### 2.1.1 Rainfall rate

The rainfall rate mathematical formulation is:

(1)

where, is the raindrop terminal fall velocity in , is raindrop diameter in , and is raindrop number concentration (rain size distribution) in .

### 2.1.2 Drop terminal fall velocity

Three formulas are widely used in representing the drop terminal fall velocity in terms of raindrop diameter:

(2a)

(2b)

(2c)

Numerical calculations showed that terminal fall velocity values based on (2b) and (2c) are in agreement with each other. This implies that terminal fall velocity values based on either (2b) or (2c) are more accurate than the corresponding values based on (2a). Accordingly, either (2b) or (2c) could be used in rain fall formulation (1b). However, instead of so doing, one of them is used in deriving the following fall velocity formulation that enables getting a closed form solution for the integral of (1).

(3)

(3a)

(3b)

(3c)

The above terminal fall velocity formulation is derived from simulated data through a multi-variant linear regression technique. A comparison between the predictions of (3) for terminal fall velocity, and the corresponding predictions of either (2b) or (2c) showed a good agreement between terminal velocity values based on (3) and the corresponding values based on either (2b) or (2c). Accordingly, (3) is used in rain fall integral (1).

### 2.1.3 Rain drop size distribution

Several types of formulations are usually used in formulating rain drop size distribution DSD. Among those types are negative exponential, gamma, normalized gamma, lognormal, and Weibull distributions. In this fascicle only the negative exponential, the gamma, Weibull, and lognormal distributions are considered. The negative exponential distribution functions could be captured into this formulation:

(4)

is a number concentration parameter, and 𝞚 is a slope factor. The coefficients and 𝞚 may be constants or functions of rainfall rate *R*. Quite a number of variants of the negative exponential distribution have appeared in the literature, of which perhaps the most widely used is that derived by Marshall and Palmer in 1948. Table 1 lists values for the coefficients of two of those functions.

Table 1

Coefficients of negative exponential rain size distribution functions

| Reference | Code |  | 𝞚 ( |
| --- | --- | --- | --- |
| Marshall and Palmer (1948) | MP |  |  |
| Ulbrich (Laws & Parsons) (1983) | ULP |  |  |

The gamma DSD is a generalized function of which the negative exponential is a special case. This DSD function was first introduced in the form:

(5)

This function has been employed by a number of authors, and that which is used in this study is listed in Table 2.

Table 2

Coefficients of gamma rain size distribution functions

| Reference | Code | ( | | 𝜦( |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ajayi and Olsen (1985) | AO |  | 260 |  | 1.43 | 2.6 |
|  | 210 |  | 1.43 | 3.1 |

The Weibull DSD is given by

(6)

Table 3 gives expressions for the coefficients for Weibull DSD functions used in this fascicle.

Table 3

Coefficients in Weibull DSD function

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Reference | Code |  |  | 𝜦 |  |
| Åsen and Gibbins [2002] | AGW1 | 1 180 | 1.69 | 1 | 0.65 |
| AGW1 | 421 | 2.15 | 1 | 0.99 |

The lognormal DSD function is given by:

(7)

Table 4 gives the coefficients of the lognormal DSD used in this fascicle.

Table 4

Coefficients in Weibull DSD function

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Reference | Code |  |  |  |
| Veyrunes | Ve | 136 | 0.3573 + 0.0051lnR | 0.0679lnR-0.103 |

### 2.1.4 Rain cell structure

This section presents only the rain cell structure because the model deals with scattering from one cell only. The extensive literature on the rain cell topic suggests that the exponential rain cell and the mixed Gaussian-exponential rain cell (Luini and Capsoni, 2011, Owolawi, and Walingo, 2015) represent at best the main integral parameters of real rain cells (e.g. coverage area, mean rain rate, rain accumulation, etc.). In this fascicle, the exponential rain cell is considered. Such a cell has a rotational symmetry within the horizontal cross-section, where the rainfall rate is assumed to vary exponentially and can be expressed as (Owolawi, and Walingo, 2015)

(8)

where is the radial distance from the centre of the rain cell, is the peak rain rate at the centre, and is the equivalent radius of the cell:

(9)

To reduce computer running time (Owolawi and Walingo, 2015), the model in (9) linking *Rm* and **0 employs a simplified/approximated version of the corresponding original model of Luini and Capsoni (2011). On the other side, the implementation of the latter is expected to enhance the representativeness of the synthetic rain environment used for hydrometeor scattering calculations, especially by taking into account the local probability of occurrence of each rain cell.

## 2.2 Geometrical parameters

### 2.2.1 Conversion to plane-earth representation

Figure 1 depicts the geometry of two stations on the curved Earth surface, separated by a distance *d* (km) along Earth surface and by an angle at the Earth centre.

(10)

In (10), is the effective (median) radius of the Earth, *k*50 (as determined in equation (5) of ITU‑R P.452-17) is the median effective Earth radius factor, and is the average Earth radius (as determined in page 8 of ITU-R P.452-17). The local elevation angles of each station antenna are, and ; and the azimuthal offsets of the antenna main-beam axes (boresights) for each station from the direction of the other station are and . The azimuth angles are defined as positive in the clockwise sense. Station 1 coordinate is described by with the horizontal plane as the *x-y* plane, the *x*-axis pointing in the direction of Station 2, the -axis is pointed inside the page, and the *z*-axis pointing vertically upwards. Station 2 coordinate is described by with the -axis is taken to be parallel to the earth surface and pointing toward Station 1. The -axis is pointing out of the page, and the -axis is along the vertical of Station 2.

Converting a curved Earth to a plane Earth representation can be achieved through the following steps:

– Choosing Station 1 coordinate as the reference coordinate;

– Transferring Station 2 coordinate into the reference coordinate .

FIGURE 1

Geometry of two stations on curved Earth



Transferring Station 2 coordinate into the reference coordinate can be achieved as follows:

– Titling the vertical of Station 2 by the angle around the axis to make the vertical of Station 2 parallel to the vertical of the reference frame; and

– Rotating the horizontal coordinate by angle to make the plane parallel to the plane.

FIGURE 2

Rotation of the coordinate around the yielding

Tilting the vertical coordinate by the angle yields a coordinate . The unit vectors , , and are related to the unit vectors , , and as follow

(11a)

(11b)

(11c)

Furthermore, rotating the horizontal coordinate by the angle brings the coordinate to the reference coordinate .

(12a)

(12b)

(12c)

Introducing (12) into (11) transfers the coordinate into the reference coordinate

(13a)

(13b)

(13c)

In the reference coordinate, Station 1 antenna has a height of null value () and Station 2 antenna has a height with:

                km (14)

Moreover, the elevation and azimuth angles describing the boresight of Station 1 antenna in the reference frame are equal to the corresponding local angles.

                rad (15)

As for the formulations of the elevation angle and the azimuth angle describing the boresight axis of Station 2 antenna in the reference frame they can be derived as follows. First the boresight axis of Station 2 antenna is represented in the local frame of Station 2 as in (16).

(16)

Then it is represented by in the reference frame as in (17).

(17)

Introducing (13) into (16) yields (18):

(18)

Comparing (18) against (17) leads to the following identities:

(19)

(20)

From (19) the elevation angle of Station 2 in the plane Earth representation is:

(21)

And from (20), the azimuthal offset of Station 2 from Station 1 is:

(22)

### 2.2.2 Off axis squint angles

The off axis squint angles are required to determine if hydrometeor scatter can cause interference. These angles are formulated for the rain scatter geometry depicted in Figure 3.

FIGURE 3

Schematic of rain scatter geometry for the general case of side scattering

Chart, radar chart

Description automatically generated

In getting the off axis squint angles, the following vector notations are used:

– a vector in three-dimensional space is represented by a three-element single‑column matrix of the lengths of the projections of the line onto the Cartesian *x*, *y* and *z* axes;

– a vector is represented by a symbol in bold typeface. Unit vector is represented by the symbol **V**; and

– a general vector (i.e. including magnitude) is represented by another, appropriate symbol, for example **R**.

The interference path in Figure 3 may be from the Station 2 side-lobes into the Station 1 main beam, or vice versa. Moreover, centre of the rain cell is located along the main beam antenna axis of Station 1 at the point of closest approach between the two antenna beams. The geometry is established in vector notation as follows. The vector from Station 1 to Station 2 is defined as:

(23)

In the above, is the distance between the two stations along the Earth surface, and is height of Station 2 relative to the height of Station 1. The height accounts for Earth curvature if it is required as described in Section 2.2.1. The unit-length vector **V10** in the direction of the Station 1 antenna main beam is:

(24)

and the unit-length vector **V20** in the direction of the Station 2 antenna main beam with taking Earth surface curvature into account (section 2.2.1) is:

(25)

To determine the off axis squint angles the vectors **R12**, *r*2**V20**, *rS***VS0** and *r*1**V10** are considered with the vector **VS0** perpendicular to both **V10** and **V20**.

(26)

Since the four vectors **R12**, *r*2**V20**, *rS***VS0** and *r*1**V10** form a closed three-dimensional polygon, the identity (27) can be written:

 (27)

The identity (27) can be converted into a matrix equation that can be solved for the unknown distances as in (28).

(28)

The operatorin equations (29) is the inverse matrix operator.

Getting the unknown distances enables calculating the off-axis squint angle at Station 1 or 2 of the point of closest approach on the other station main beam axis as in (29).

(29)

## 2.3 Electromagnetic parameters

Raindrop electromagnetic parameters represented by the bi-static cross section per unit volume, and the specific attenuation of raindrops are required for hydrometeor scatter study. For a field with polarization illuminating an integration element in direction the specific attenuation is:

(30)

and the bi-static cross section in a scattering direction for a polarization is:

(31)

In (31), is the element of the scattering amplitude matrix. Details about the scattering amplitude and field polarization representations are given in Annex A. In (30), is the scattering amplitude in the forward direction, and is the imaginary part operator.

The scattering amplitude of a single raindrop can be obtained through either matching the boundary conditions at the raindrop surface or using the following integrand:

(32)

In (32), is the field inside the raindrop at a position vector when the incident field has polarization, and () is complex relative permittivity of raindrop. Moreover, the integral in (32) is carried over the raindrop volume. The field inside the raindrop can be obtained through either numerical techniques or approximate techniques. The most widely used approximate techniques are the Rayleigh approximation, and the Rayleigh - Gans approximation. The Rayleigh approximation, which is used in section 5 of Recommendation ITU-R P.452-16, is valid only for raindrops with diameters very small compared to the operational wavelength. This is because in the Rayleigh approximation the inner field is estimated by the corresponding static field.

The Rayleigh – Gans approximation, in which the inner field is estimated by the incident field, is applicable only to tenuous particles with . Th kind of particle cannot be used to model raindrop particles. This is because raindrops have complex relative permittivity with values that could reach 80 or 90 depending on frequency. On the other hand, matching the boundary conditions at the raindrop surface yields more rigorous formulation to the scattering amplitude element such as Mie scattering formulation which is reported in section 2.3.2 of this fascicle.

### 2.3.1 Rayleigh approximation

For a spherical raindrop with diameter very small compared to the wavelength, the inner normalized field amplitude with a unit incident electric field can be approximated as:

(33)

Introducing (33) into (32) yields the following scattering amplitude formulation:

(34)

with is the raindrop volume. Substituting for the volume and the wavenumber (, 𝛌 is wavelength) into (34) yields:

(35)

Based on (31) and (35), the radar cross section per unit volume under the Rayleigh approximation can be written as:

(36)

(37)

(38)

The factor is introduced into (36) to make the units of radar reflectivity in . Dropping the last term of (36) and introducing the resultant into (73) of Recommendation ITU-R P.452-16 yields (74) of Recommendation ITU-R P.452-16 except of the high frequency correction term. The radar reflectivity is related to the rainfall rate through the following relationship:

(39)

Table 5 provides some typical values for the coefficients and of equation (39).

Table 5

Coefficients for usage of relationship (39)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Usage |  |  | Optimum for | Recommended for |
| Marshall – Palmer | 200 | 1.6 | General stratiform precipitation |  |
| East – Cool Stratiform | 130 | 2.0 | Winter stratiform precipitation east of continental divide | Orographic rain – East |
| West – Cool Stratiform | 75 | 2.0 | Winter stratiform precipitation west of continental divide | Orographic rain – West |
| WSR-88D Convective | 300 | 1.4 | Summer Deep convection | Other non-tropical convection |
| Rosenfeld Tropical | 250 | 1.2 | Tropical convective systems |  |

It is worth mentioning that in (36) and (38) are the polarization loss factors that were not accounted for by ITU-R P.452-16. The explicit expressions of these factors can be written through using formulations reported in Equations (A-3)-(A-5), and the corresponding formulations in the scattered directions are as follows.

(40a)

(40b)

(40c)

(40d)

In the above, and are the elevation angles of the incident and scattered directions respectively (Figure A-1). The angles and are the corresponding azimuth angles.

### 2.3.2 Mie scattering theorem

Based on the Mie scattering theorem the bi-static scattering cross section per unit volume due to scattering from a spherical raindrop is:

(41)

Where can be written after substituting for the scattering amplitudes in (31) from (A-24) – (A‑27) yielding:

(42a)

(42b)

(42c)

(42d)

In the above, is the scattering angle (A-10). In addition, is the angle that rotates from the incident vertical polarization vector anticlockwise to the polarization vector (Figure A-2 and A-3). Furthermore, is the angle that rotates from scattered vertical polarization anticlockwise to (Figure A-2 and A-3). and are Mie’s scattering amplitude in the scattering plane system, which are called local scattering amplitudes:

(43a)

(43b)

In (43a) and (43b), and are Mie scattering coefficients.

(44)

, and . In addition, and are the Ricatti Bessel and Hankel functions; and and are the corresponding first order derivatives with respect to the argument. Details about Ricatti Bessel and Hankel functions; and their first order derivatives are given in Annex B. Furthermore, and in (43) are the associated Legendre polynomials and their derivatives are given in Annex C.

# 3 Rain bi-static scattering cross section

The proposed rain bi-static scattering cross section per unit volume is taken to be the bi-static cross section reported in (41) based on Mie scattering. These bi-static cross section formulations are used because they yield more rigorous values. Those values can also be used to investigate the validity domains of the corresponding values based on the Rayleigh approximation.

Introducing (42a)-(42d) into (41) and performing the integration yields the following formulations for the rain bi-static cross section per unit volume.

(45)

with , , and are the local bi-static scattering cross sections per unit volume

(46)

The angles and in (45) provide the dependence of bi-static cross sections on the geometric factors and polarizations. Moreover, numerical and analytical analysis showed that for frequencies up to 100 GHz, the following identities hold.

(47)

Figure 4 and Figure 5 are presented to justify the above identities. In both figures, values of two bi-static cross section ratios are calculated:

– Like polarized ratio (), and

– Cross polarized ratio (

The values of like polarized ratio and cross polarized ratio are compared against the corresponding values of and respectively. In calculating Figures 4 and 5, Marshall and Palmer drop size distribution of Table 1 is used for with = 0.1 mm, and = 5 mm. The integration in (46) is carried out using trapezoidal rule with equal diameter steps of 0.4 mm. Moreover, the complex relative permittivity of the raindrop is equated to the corresponding complex relative permittivity of pure water given by equations (5) – (13) of ITU-R P. 527-4.

Figure 4

Like () and cross () polarized bi-static cross section at a frequency of 20 GHz (cosine stands for , and cosine square stands for , Rainfall rate = 100 mm/hr)



Figure 5

Like () and cross () polarized bi-static cross section at a frequency of 100 GHz (cosine stands for , and cosine square stands for , Rainfall rate = 100 mm/hr)



The trends of the like and cross polarized ratios represented by (47) and depicted in Figure 4 and Figure 5 are accepted. This is because due to the symmetry of the raindrop at both forward scattering direction () and backscattering direction ( 180°), the bi-static cross sections and should have equal values. At scattering angle of 90 degrees, the two polarization vectors associated with ( and in Figure A-3) are perpendicular to each other as it can be inferred from (A-8) and (A-9). Moreover, (47) agrees with Equation (1.2.10), page 11 of Tsang et al (2000).

Based on (47), the bi-static cross section formulation given in (45) can be simplified into

(48a)

(48b)

(48c)

(48d)

From (48a) – (48d), it is obvious that the local bi-static cross section provides the dependence on rainfall rate, rain temperature, and frequency as well as scattering angle. This is illustrated in Figure 6 and Figure 7 where the horizontal local bi-static cross is calculated as a function of scattering angle at two different temperatures and two different frequencies. Figures 6 and 7 are calculated using the same parameters used in calculating Figures 4 and 5.

Figure 6

Local bi-static cross section () at a frequency of 20 GHz



Figure 7

Local bi-static cross section () at a frequency of 100 GHz



The explicit dependence of the bi-static scattering section on the scattering angle can be written as.

(49)

The above formulation is derived through applying multi-variant linear regression on bi-static scattering cross simulated. The simulated data are generated through performing the integration of the first equation of (47) using different values for the scattering angle.

Equations (50) provide the dependence of the coefficients () of (49) on rainfall rate.

(50)

The dependence of the regression coefficients (, and ) of (50) on frequency, and temperature is provided in Annex D.

In concluding this section it is worth mentioning that the quantities within the curly brackets of (48a) – (48d) can be considered as polarization loss factors in analogy to those reported in (40a) – (40b) for the Rayleigh approximation.

# 4 Rain specific attenuation

The rain specific attenuation prediction model in this recommendation is different from that in Recommendation ITU-R P. 838-3 and should only be used for the purposes as required in § 5 for hydrometeor-scatter interference prediction. The reason for this warning is because the rain specific attenuation prediction model of Recommendation ITU-R P. 838-3 is developed for oblate spheroidal raindrops, while the hydrometeor scatter model is developed for spherical raindrops. This difference has an impact on formulations that include the specific rain attenuation formulations of Equations (82) – (87), the raindrop bistatic cross section formulations of Equations (123a) – (123d). Furthermore, the rain specific attenuation model of Recommendation ITU-R P.838-3 has no temperature dependence, while both the rain specific attenuation and the bistatic cross section in this section have temperature dependence.

Based on Mie scattering theorem, the rain specific attenuation can be obtained through introducing (A-29) into (30) yielding:

(51)

Figure 8

Rain specific attenuation based on (51) as a function of frequency at two different rainfall values and two different temperature values



Based on (51), the specific attenuation due to spherical raindrops is independent on either polarization or the scattering angle. It depends only on rainfall rate, frequency, and temperature as shown in Figure 8 which is calculated using the same parameters used in calculating Figure 4 and Figure 5.

The dependence of rain specific attenuation on rainfall rate is given in (52):

(52)

and explicit dependence of the above regression coefficients and on frequency, and temperature is given in Annex E.

# 5 Hydrometeor scatter transmission loss formulation

To derive the hydrometeor scatter transmission loss between two stations located on the Earth surface equation (73) of Recommendation ITU-R P.452-16 is recalled.

(53)

where:

: transmitted power (W)

: received power (W)

𝛌: wavelength (m)

: antenna 1 and antenna 2 gain (linear)

: raindrop bi-static cross section per unit volume (m2/m3)

: differential volume at the scattering point (m3)

: distances from scattering volume to antenna 1 and antenna 2 respectively (m)

: attenuation due to atmospheric gases (dB)

: attenuation due to rain (dB)

: a constant for converting dB to Nepers (

: height dependence of the radar reflectivity:

Equation (53) can be rewritten as:

(54)

where

: distances in km ()

: scattering volume in km3 ().

Substituting for the wavelength in terms of the frequency reduces (53) into:

(55)

with is the frequency in GHz ().

On the other hand, the transmission loss in dB unit is defined as:

(56)

Accordingly, based on (54)-(55) the scattering loss transmission loss can be written as:

(57)

With is the scatter transfer function:

(58)

Equations (57) and (58) provide the proposed transmission loss formulation for updating section 5 of Recommendation ITU-R P.452-16.

The calculation of the path attenuation relies on the single rain cell illustrated in Section 2.1.4: the prediction of path attenuation could be enhanced by employing full synthetic rain fields resulting from the aggregation of multiple rain cells (Luini and Capsoni, 2011).

# 6 Hydrometeor scatter geometry

In constructing the hydrometeor scatter geometry, the centre of the rain cell may be varied depending on the scenarios that minimize values of transmission losses, and hence maximize the interfering scattered power. Then a cylindrical coordinate system is established to facilitate the integration of the scatter transfer function (58). The characteristics of both station antennas in the cylindrical coordinate are derived along with the integration limits. The station characteristics are represented by position vectors from the station to the integration elements, the polarization vectors associated with each position vector, the polarization transformation, and the off boresight angle between each position vector and the boresight of the corresponding antenna.

## 6.1 Cylindrical integration coordinate

The cylindrical coordinate system ( is constructed through selecting a rain cell with its vertical axis intersecting the main beam axis of Station 1 at point 0 () as shown in Figure 9.

(59)

The distance in (59) can be obtained from (28). By selecting the point 0 () as the centre of the cylindrical coordinate, the Cartesian coordinate of an arbitrary point A (*x*, *y*, *h*) within the cell can be transformed into a cylindrical coordinate () as in (60).

(60)

Figure 9

Hydrometeor scatter link geometry

Chart, radar chart

Description automatically generated

## 6.2 Station 1 characteristics

The vector is the magnitude, and is the unit length vector) is the position vector extending from Station 1 to the integration element A (*x*, *y*, *h*). The magnitude of this vector can be expressed as follows:

(61)

Introducing (60) into (61) yields:

(62)

The direction of the vector is given by the unit vector :

(63)

(64)

with:

(65)

(66)

The off-boresight angle at the point (*x*, *y*, *h*) for the Station 1 antenna is:

(67)

Equations (24) and (63) are used to obtain the vector scalar product in (67).

The horizontal , and vertical polarization vectors associated with the unit vector , and hence the transmitting antenna (Station 1) are given by:

(68)

(69)

(70)

In getting (68)-(70), the unit vector in the incident direction in (A-4) is replaced by the unit vector given by (63). Then the resultant is introduced into (A-3) and (A-5) respectively.

## 6.3 Station 2 characteristics

The position vector (, is the magnitude and is unit vector) extends from Station 2 to the integration element at the point A. The magnitude of this vector is the distance from Station 2 to the integration element at :

(71)

Equation (71) can be reduced through using (60) yielding:

(72)

(73)

The direction of the vector is given by the unit vector :

(74)

with:

(75)

(76)

The off-boresight angle between the point (*x*, *y*, *h*) for the Station 2 antenna the angle:

(77)

In (77), and are the elevation angle and the azimuth angle of boresight of Station 2 antenna, and they can be obtained from (25).

For the receiving antenna, the associated horizontal and vertical polarization vectors are:

(78)

(79)

(80)

(81)

In getting (78)-(81), the unit vector in the scattering direction in (A-4) is replaced by the unit vector given by (73). Then the resultant is introduced into (A-3) and (A-5) respectively.

## 6.4 Polarization transformation

At the integration volume, () the propagating signals undergoes through two polarization transformations. The first transformation is for the signal emitted by Station 1 antenna and illuminating the integration volume, and the second transformation is for the signal scattered by the integration volume toward Station 2 antenna. As shown in Annex A, to establish these two transformations the following parameters are required:

– the scattering angle ;

– the angle , and

– the angle .

The angles , and are the rotation angles of the incident and scattered vertical polarizations anticlockwise to the polarization vector (Figure A-2).

The scattering angle can be written as:

(82)

The unit vectors and in (82) are substituted for from (63) and (74) respectively. Further reduction for (82) yields:

(83)

Then subsisting for and from (60) leads to the following scattering angle formulation:

(84)

The rotation angle can be obtained from (A-12) which recalled in (85) for convenience:

(85)

Subsisting for the unit vectors , , and from (70), (68), and (75) reduces (85) into:

(86)

Then using (60) yields the required formulation for the rotation angle :

(87)

The rotation angle can be written from (A-15) as:

(88)

with:

(89)

and

(90)

with stands for matrix transpose. A further reduction for (89) and (90) yields:

(91)

and

(92)

In getting the vector scalar product in (91) and (92), equations (67), (68), and (73) are used.

## 6.5 Integration volume

The common volume between the two antenna beams is usually the integration volume. However, determining the common volume involves solution of several algebraic equations which is difficult to achieve. Accordingly, instead an alternate volume enclosing the common volume is used for performing the integration. The alternate volume is a cylindrical volume with a centre located at the point 0 () where the rain cell axis intersects with Station 1 antenna main beam as shown in Figure 10. This volume has a symmetry with the azimuth angle . Due to this symmetry the integration over extends from 0 to . On the other hand, the vertical dimensions of the alternate volume are determined by the points where main beams of either Station 1 antenna or Station 2 antenna, intersect with the axis of rain cell. The points associated with farther distances for the integration centre are selected.

FIGURE 10

Geometry for determining vertical dimensions of the integration volume



The geometry for obtaining the points of intersection of Station 1 antenna main beam with rain cell axis is given in Figure 10. From the triangle depicted in Figure 10, the distance between the point 0 and point where the upper portion of the antenna beam intersects with rain cell axis is:

(93)

From the triangle the distance between the point 0 and the lower portion of antenna beam intersects with rain cell axis is:

(94)

Similar to (93)-(94), the corresponding distances for Station 2 antenna main beam be written as:

(95)

(96)

By selecting the distances of maximum values, the vertical dimensions of the integration volume, and hence the limits of integration over the vertical variable can be written as:

(97)

(98)

Figure 11

Geometry for determining radius of the integration volume



The geometry for getting the radius , of the integration volume is given in Figure 11. From the triangle S1QS2, in Figure 12, the distance can be written as:

(99)

(100)

Subtracting the distance from the horizontal projection of the distance gives the radius :

(101)

Incorporating (99) into (101) gives the explicit expression for the radius of the integration volume.

(102)

(103)

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Annex A  
  
Raindrop scattered field representation

The far zone scattered field in arbitrary direction from a single raindrop illuminated by a plane wave propagating in an arbitrary direction is:

(A-1)

In above, is the distance between the raindrop center and the observation point. In addition, and are two orthonormal unit vectors and each is orthogonal to the scattered direction as shown in Figure A-1. Furthermore, is the free space wavenumber (**,** 𝛌 is the wavelength).One should keep in mind that there is a time factor of ( is frequency in Hz, and is time in second) is assumed and suppressed.

Figure A-1

Polarization vectors of incident and scattered fields

and in (A-1), are the scattered field components which are related to the vertical and horizontal components of the incident field through the scattering amplitude matrix elements ‘s.

(A-2)

There are two common choices of the orthonormal unit systems ( and that can be used to describe scattering by a raindrop:

– horizontal and vertical polarization system, and

– scattering plane system.

Horizontal – vertical polarization system

In this system, which is depicted in Figure A-2, the horizontal and vertical polarization vectors are defined with respect to the vertical on the Earth surface . The horizontal polarization is orthogonal to the incident plane defined by the vertical on Earth surface ( axis) and the incident direction .

(A-3)

In obtaining (A-3), the following formulation for the incident direction is used:

(A-4)

FIGURE A-2

The vertical - horizontal polarization system )

The vertical polarization is parallel to the incident plane and orthogonal to the horizontal polarization vector .

(A-5)

Similar relations to (A-3) – (A-5) can be written in the scattering direction through replacing the subscript with the subscript .

Other names for the horizontal polarization are TE polarization, perpendicular polarization, and polarization. Other names for the vertical polarization are TM polarization, parallel polarization, and polarization. Furthermore, this type of system coincides with spherical coordinate unit vectors of the antenna. Accordingly, it is used in this fascicle to describe the polarizations of station antennas. In this system the scattering amplitude matrix in (A-2) obeys this relation

(A-6)

The scattering plane system

As shown in Figure A-3, in this system the polarization vectors and are taken to be similar and to be perpendicular to the scattering plane formed by the incident direction and the scattered direction .

(A-7)

The angle is the angle that rotates from the incident vertical polarization vector anticlockwise to the polarization vector .

The polarization vectors and are taken to be parallel to the scattering plane, with:

(A-8)

and

(A-9)

with is the scattering angle which is the angle between the incident and scattered directions:

(A-10)

In obtaining (A-7) – (A-9), the unit vectors , , and are considered as the coordinate system, and the scattering direction is represented in that system as follows:

(A-11)

Based on (A-11), the angle can be written as:

(A-12)

FIGURE A-3

The scattering plane systems ) and (

In this system the scattering amplitude matrix of (A-2) obeys this relation:

(A-13)

The advantage of this system is that the scattering amplitude takes a simple form for a particle of high symmetry such as raindrops. The disadvantage for this system is that the directions of the polarization vectors and depend on the scattered direction.

For a spherical raindrop, the scattering amplitude matrix elements are

(A-14)

with and are Mie scattering amplitudes given in (34a) and (34b).

Based on (A-13) and (A-14) the scattering amplitude matrix for a spherical raindrop can reduces into

(A-15)

Since the antenna polarizations are given in the vertical and horizontal polarization system, the scattering matrix elements given in (A-15) need to be transferred to the horizontal-vertical polarization system. This can be achieved using the tensor representation of the scattering amplitude matrix.

(A-16)

Then applying the scalar vector product from left by the polarization vector and from right by the polarization vector () gives the element of the scattering amplitude with the horizontal – vertical polarization system

Introducing (A-16) into (A-14) yields

(A-17)

(A-18)

(A-19)

(A-20)

To get explicit expressions for scalar vector products in (A-17) – (A-20) the following equations are constructed.

(A-21)

(A-22)

with

(A-23)

Equations (A-21) and (A-22) are constructed by analogy to (A-7) and (A-8). Equivalently, the unit vectors , , and are considered as a coordinate system, the unit polarization vectors and are represented in that coordinate system.

Upon exploiting (A-7), (A-8), (A-21), and (A-22) in getting explicit expressions for the scalar vector products, (A-17) – (A-20) can be simplified as follows

(A-24)

(A-25)

(A-26)

(A-27)

The scattering amplitudes in the forward direction = are required as shown in (31). In the forward direction the scattering angle has null value leading to equal values for Mie scattering amplitudes:

(A-28)

Equation (A-28) is obtained by introducing (C-6) into (44a) and (44b). Introducing (A-28) into (A-24) – (A-27) with keeping in mind that in the forward direction , yield the following formulation for the scattering amplitudes in the forward direction:

(A-29)

(A-30)

Explicit formulation of Mie scattering coefficients, ’s and ’s are given in (45).

Annex B  
  
Ricatti-Bessel and Hankel functions in Mie scattering

Ricatti-Bessel and Hankel functions within Mie’s scattering coefficients (44) are related to the corresponding spherical Bessel and spherical Hankel functions in (B-1).

(B-1)

with is Bessel function of second kind. The Ricatti-Bessel and Hankel functions and the corresponding spherical functions are governed by the following recurrence relation:

(B-2)

with is either or . is the first order derivative is governed by (B-3):

(B-3)

Equations (B-4a) – (B-4e) give explicit expressions of for :

(B-4a)

(B-4b)

(B-4c)

(B-4d)

(B-4e)

Equations (B-5a) – (B-5e) give for :

(B-5a)

(B-5b)

(B-5c)

(B-5d)

(B-5e)

The recurrence relation (B-2) can be used in building an algorithm for getting the spherical Bessel function of the second kind ’s with:

(B-6)

The recurrence relation (B-2) cannot be used to get either the spherical or the Ricatti-Bessel functions of first kind. In building an algorithm for getting those functions, a backward recurrence relation for those functions can be used. In getting the backward recurrence relation (B-2) is divided by yielding:

(B-7)

with

(B-8)

(B-7) can be rewritten as:

(B-9)

Equation (B-9) is the required backward relation with starting n value of:

(B-10)

The corresponding initial value is given the following continued fraction:

(B-11)

The above continued fraction is terminated at a value of:

(B-12)

Annex C  
  
Associated Legendre polynomial in Mie scattering

The associated Legendre polynomials and their derivatives are required for getting Mie scattering amplitude (44) are governed by the following recurrence relations:

(C-1)

(C-2)

With , and .

(C-3a)

(C-3b)

(C-3c)

(C-3d)

and

(C-4a)

(C-4b)

= (C-4c)

(C-4d)

Furthermore, and are alternately even and odd function of :

(C-5a)

(C-5b)

Moreover, for , , and:

(C-6)

Annex D  
  
Bi-static cross section regression coefficients

In getting the explicit expressions for multi-variant linear regression shows that the dependence of on the scattering angle can be captured by (49) which is recalled here as (D-1)

(D-1)

The coefficients () are related to rainfall rate as in (D-2)

(D-2)

The dependence of any of the coefficients of (D-2) on the frequency can be written as in (D‑3)

(D-3)

Moreover, the dependence of each of the coefficients of (D-3) on temperature is

(D-4)

Values of the coefficients, and for each of the coefficients of (D-4) are provided in Tables D-1 through D-14 below. The coefficients are obtained through performing linear regression over simulated bi-static scattering cross data. The simulated data are generated from the local bistatic scattering cross section of Equation (46) through changing the values of the following parameters:

– Raindrop diameter from 0.1 to 5 mm in step of 0.2 mm

– Frequency from 5 to 100 GHz in step of 0.5 GHz

– Rainfall rate from 1 to 150 mm/hr in step of 5 mm/hr

– Temperature from 0 to 35° C in step of 2° C, and

– Scattering angle from 0 to 180 degrees in step of 1 degree.

The integration over the raindrop diameter in (46) is performed numerically through trapezoidal integration technique

Table D-1

Values of coefficients , and forof (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −23.033 | −0.019039 | −1.3511e-07 |
| 1 | 1.0988 | 0.0057909 | −1.8732e-05 |
| 2 | −0.053826 | −0.00051258 | 2.8893e-06 |
| 3 | 0.0017167 | 2.0326e-05 | −1.5242e-07 |
| 4 | −3.3231e-05 | −4.2625e-07 | 3.8089e-09 |
| 5 | 3.7396e-07 | 4.9297e-09 | −4.969e-11 |
| 6 | −2.2438e-09 | −2.9763e-11 | 3.2787e-13 |
| 7 | 5.5409e-12 | 7.3317e-14 | −8.6497e-16 |

Table D-2

Values of coefficients, and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | 1.758 | 0.0078061 | −0.00019642 |
| 1 | −0.034774 | −0.0010409 | 4.8582e-05 |
| 2 | 0.0031934 | 1.2441e-05 | −3.9051e-06 |
| 3 | −0.00014758 | 1.6661e-06 | 1.5195e-07 |
| 4 | 3.3014e-06 | −7.0142e-08 | −3.2085e-09 |
| 5 | −3.8772e-08 | 1.1439e-09 | 3.7652e-11 |
| 6 | 2.3188e-10 | −8.5799e-12 | −2.3101e-13 |
| 7 | −5.5887e-13 | 2.4612e-14 | 5.779e-16 |

Table D-3

Values of coefficients, and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.051224 | 0.00081531 | 1.1534e-05 |
| 1 | 0.0011587 | −0.00031961 | −2.3173e-06 |
| 2 | −8.8754e-05 | 3.5484e-05 | 1.4933e-07 |
| 3 | 9.6328e-07 | −1.6609e-06 | −4.7112e-09 |
| 4 | 5.927e-08 | 3.9523e-08 | 8.0972e-11 |
| 5 | −1.6618e-09 | −5.0408e-10 | −7.7464e-13 |
| 6 | 1.5626e-11 | 3.2862e-12 | 3.8749e-15 |
| 7 | −5.0972e-14 | −8.6057e-15 | −7.8859e-18 |

Table D-4

Values of coefficients , and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | 0.28927 | 0.037271 | −0.00010078 |
| 1 | −0.11742 | −0.011059 | 6.6665e-05 |
| 2 | 0.010231 | 0.00093297 | −8.6068e-06 |
| 3 | −0.00041831 | −3.5477e-05 | 4.2065e-07 |
| 4 | 8.8529e-06 | 7.2358e-07 | −1.0192e-08 |
| 5 | −1.0313e-07 | −8.2014e-09 | 1.3111e-10 |
| 6 | 6.2591e-10 | 4.8736e-11 | −8.5865e-13 |
| 7 | −1.5469e-12 | −1.1849e-13 | 2.2546e-15 |

Table D-5

Values of coefficients , and forof (D-3) in case of of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.2317 | −0.005093 | 0.00045796 |
| 1 | 0.020016 | −0.00076558 | −0.00011298 |
| 2 | 0.00060157 | 0.00020785 | 9.1237e-06 |
| 3 | −9.7303e-05 | −1.2384e-05 | −3.5392e-07 |
| 4 | 3.2711e-06 | 3.3064e-07 | 7.4324e-09 |
| 5 | −5.0187e-08 | −4.5084e-09 | −8.6694e-11 |
| 6 | 3.6714e-10 | 3.0694e-11 | 5.2868e-13 |
| 7 | −1.0386e-12 | −8.2853e-14 | −1.3148e-15 |

Table D-6

Values of coefficients , and for of (D-3) in case of of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.036841 | −0.0025519 | −2.6162e-05 |
| 1 | 0.012953 | 0.00084793 | 5.2688e-06 |
| 2 | −0.001305 | −8.5265e-05 | −3.3134e-07 |
| 3 | 5.9518e-05 | 3.7912e-06 | 9.6604e-09 |
| 4 | −1.454e-06 | −8.7204e-08 | −1.4711e-10 |
| 5 | 1.91e-08 | 1.085e-09 | 1.1752e-12 |
| 6 | −1.2719e-10 | −6.9419e-12 | −4.3973e-15 |
| 7 | 3.3748e-13 | 1.7914e-14 | 5.0804e-18 |

Table D-7

Values of coefficients , and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.0022144 | −0.0014792 | 0.00030493 |
| 1 | −0.008123 | −0.00055742 | −8.8598e-05 |
| 2 | 0.0018507 | 0.00015755 | 8.3245e-06 |
| 3 | −8.484e-05 | −8.944e-06 | −3.572e-07 |
| 4 | 1.9127e-06 | 2.323e-07 | 7.9876e-09 |
| 5 | −2.2827e-08 | −3.1221e-09 | −9.6893e-11 |
| 6 | 1.4148e-10 | 2.1074e-11 | 6.0585e-13 |
| 7 | −3.5797e-13 | −5.6545e-14 | −1.5317e-15 |

Table D-8

Values of coefficients, and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.048207 | −0.019603 | 3.8001e-06 |
| 1 | −0.00041118 | 0.0045669 | −1.0444e-05 |
| 2 | 0.0016887 | −0.00031651 | 1.7242e-06 |
| 3 | −0.00011195 | 1.0267e-05 | −1.0534e-07 |
| 4 | 3.0478e-06 | −1.776e-07 | 2.9975e-09 |
| 5 | −4.1397e-08 | 1.6755e-09 | −4.3294e-11 |
| 6 | 2.8014e-10 | −8.0718e-12 | 3.0902e-13 |
| 7 | −7.5246e-13 | 1.5353e-14 | −8.6683e-16 |

Table D-9

Values of coefficients, and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.1208 | −0.0018073 | −7.7431e-05 |
| 1 | 0.039712 | 0.00099445 | 2.2037e-05 |
| 2 | −0.0039312 | −0.00012817 | −2.1388e-06 |
| 3 | 0.00017579 | 6.5734e-06 | 9.7426e-08 |
| 4 | −4.0495e-06 | −1.669e-07 | −2.3253e-09 |
| 5 | 5.0716e-08 | 2.2341e-09 | 3.0006e-11 |
| 6 | −3.2704e-10 | −1.512e-11 | −1.9832e-13 |
| 7 | 8.496e-13 | 4.0796e-14 | 5.2636e-16 |

Table D-10

Values of coefficients, and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | 0.02176 | 0.0005804 | 9.4104e-06 |
| 1 | −0.0067089 | −0.00020457 | −2.5829e-06 |
| 2 | 0.0006556 | 2.156e-05 | 2.4395e-07 |
| 3 | −2.966e-05 | −9.9711e-07 | −1.0876e-08 |
| 4 | 6.9633e-07 | 2.3723e-08 | 2.5604e-10 |
| 5 | −8.7919e-09 | −3.0371e-10 | −3.2742e-12 |
| 6 | 5.6633e-11 | 1.9905e-12 | 2.15e-14 |
| 7 | −1.462e-13 | −5.2428e-15 | −5.6776e-17 |

Table D-11

Values of coefficients , and for of D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.046298 | −0.0057663 | −0.00018642 |
| 1 | 0.01272 | 0.0017156 | 4.8813e-05 |
| 2 | −0.0010278 | −0.0001572 | −4.0434e-06 |
| 3 | 3.4667e-05 | 6.2343e-06 | 1.5655e-07 |
| 4 | −6.1228e-07 | −1.3209e-07 | −3.2237e-09 |
| 5 | 5.8573e-09 | 1.5529e-09 | 3.65e-11 |
| 6 | −2.9595e-11 | −9.5349e-12 | −2.1505e-13 |
| 7 | 6.2019e-14 | 2.3843e-14 | 5.1597e-16 |

Table D-12

Values of coefficients , and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.15558 | −0.0042536 | −9.7633e-05 |
| 1 | 0.047606 | 0.0017754 | 3.2832e-05 |
| 2 | −0.0046542 | −0.00020881 | −3.4242e-06 |
| 3 | 0.00020312 | 1.0088e-05 | 1.6314e-07 |
| 4 | −4.6089e-06 | −2.4592e-07 | −4.0092e-09 |
| 5 | 5.6717e-08 | 3.2014e-09 | 5.2747e-11 |
| 6 | −3.5983e-10 | −2.1241e-11 | −3.5329e-13 |
| 7 | 9.2209e-13 | 5.6467e-14 | 9.4664e-16 |

Table D-13

Values of coefficients , and for of (D-3) in case of of (D-2)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | 0.11087 | 0.0061252 | 5.7319e-05 |
| 1 | −0.029622 | −0.0017355 | −1.5993e-05 |
| 2 | 0.0025942 | 0.00016036 | 1.5228e-06 |
| 3 | −0.00010795 | −6.8539e-06 | −6.7714e-08 |
| 4 | 2.3816e-06 | 1.5498e-07 | 1.5828e-09 |
| 5 | −2.8856e-08 | −1.9149e-09 | −2.0088e-11 |
| 6 | 1.8081e-10 | 1.2223e-11 | 1.3102e-13 |
| 7 | −4.5761e-13 | −3.1539e-14 | −3.4402e-16 |

Table D-14

**Values of coefficients , and for of (D-3) in case of of (D-2)**

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −0.015838 | −0.00071563 | −6.1847e-06 |
| 1 | 0.0042926 | 0.00019142 | 1.6456e-06 |
| 2 | −0.00039171 | −1.6809e-05 | −1.5321e-07 |
| 3 | 1.6946e-05 | 6.9046e-07 | 6.7237e-09 |
| 4 | −3.8371e-07 | −1.5112e-08 | −1.5611e-10 |
| 5 | 4.6824e-09 | 1.8183e-10 | 1.9738e-12 |
| 6 | −2.9185e-11 | −1.1361e-12 | −1.2841e-14 |
| 7 | 7.3004e-14 | 2.8813e-15 | 3.3644e-17 |

Validation

Figures D-1 and D-2 below are calculated as a validation for the regression formulations of (D-1) – (D-4) and the associated regression coefficients reported in Tables D-1 through Table D-14.

Each figure provides a comparison between bi-static cross section values based on the regression formulations of (D-1) through (D-4), and the corresponding values based on Mie scattering. In each figure the bi-static cross section values are calculated as a function of scattering angle at two different temperatures: 0° C, and 35° C. In Figure D-1, bi-static cross section values are calculated at 20 GHz, and in Figure D-2, the bi-static cross section values are calculated at 100 GHz.

Figure D-1

Comparison between two types of values of the local bi-static cross section () at a frequency of 20 GHz



Figure D-2

Comparison between two types of values of the local bi-static cross section () at a frequency of 100 GHz



As a conclusion to this annex, it is worth mentioning that as shown in Tables D-1 to D-14 the regression coefficients, , and associated with the regression coefficients of Equation (D-3) have very low values for higher orders of (. So introducing those coefficients into (D-4) and then introducing that resultants into (D-3) could leads to terms of very lower values. Those lower values terms could be overlooked in comparison to terms of higher values leading to error into the coefficients . To overcome such an issue the direct summations in (D-3) is turned into multiplications as follows

Rewrite Equation (D-3) as follows

(D-5)

The coefficients ( are the coefficients with dropping and for brevity. Then apply the software implementation on (D-5) instead of (D-3).

In a software code implementation of (D-5), the first summation performed by the code is the term . Such a term has two terms that of equal values. So no term of those two terms could be overlooked in comparison to the other term.

Similar approach can be applied into the software implementations of (D-1), (D-2), and (D-4), as well as into the software implementation of Equations (E-2), (E-3), (E5), and (E-6) of Annex E.

Annex E  
  
Specific attenuation regression coefficients

This Annex provides mathematical formulations for the regression coefficients of rain specific attenuation given in (52). This formulation is recalled as equation (E-1) for convenience.

(E-1)

Figures E-1 and E–2 depict comparisons between the coefficients and 𝛋 of (E-1) and the corresponding coefficients for vertical and horizontal polarizations reported in Recommendation ITU-R P.838-3. The comparisons are performed as a function of frequency and at a temperature of 30 °C. One should note that the coefficients reported in Recommendation ITU-R P.838-3 have no temperature dependence.

The dependence of the regression coefficients and 𝛋 of Equation (E-1) on frequency (GHz) can be written as

(E-2)

and

(E-3)

with

(E-4)

Each of the coefficients ( of equation (E-2) and the coefficients ( of equation (E-3) depends on the temperature (°C).

The dependence of each of the coefficients ( on temperature can be written as

(E-5)

and the dependence of each of the coefficients ( on temperature can be written as

(E-6)

The values of and ’s () are given in Table E-1 and E-2 respectively. The coefficients are obtained through performing linear regression over simulated rain specific attenuation data. The simulated data are generated from the rain specific attenuation formulation of Equation (51) through changing the values of the following parameters:

– Raindrop diameter from 0.1 to 5 mm in step of 0.2 mm,

– Frequency from 5 to 100 GHz in step of 0.2 GHz,

– Rainfall rate from 1 to 150 mm/hr in step of 5 mm/hr, and

– Temperature from 0 to 35° C in step of 2° C.

The integration over the raindrop diameter in (51) is performed numerically through trapezoidal integration technique.

Table E-1

Values of the coefficients () of (E-5)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | 0.86481 | 0.0025984 | −3.2727e-05 |
| 1 | −0.32507 | −0.025593 | 0.00040852 |
| 2 | 0.70075 | 0.041632 | −0.00084479 |
| 3 | −0.4162 | −0.023144 | 0.00063446 |
| 4 | 0.11971 | 0.0054147 | −0.00022071 |
| 5 | −0.018495 | −0.00049312 | 3.6339e-05 |
| 6 | 0.0012143 | 8.1571e-06 | −2.2949e-06 |

Table 2

Values of the coefficients () of (E-6)

|  |  |  |  |
| --- | --- | --- | --- |
| m |  |  |  |
| 0 | −9.2859 | −0.026677 | 7.4162e-05 |
| 1 | 1.5977 | −0.021172 | 0.001127 |
| 2 | 0.45627 | −0.0010862 | −0.0014558 |
| 3 | −0.15347 | 0.016763 | 0.00066036 |
| 4 | 0.040141 | −0.0062665 | −0.00012758 |
| 5 | −0.0049951 | 0.00064387 | 8.9007e-06 |

Figures E-1 and E-2 are calculated as a validation for the regression formulations of (E-1) – (E-6) and the associated regression coefficients reported in Tables E-1 and Table E-2. Each figure provides a comparison between rain specific attenuation values based on the regression formulations (E-1) – (E‑6), and the corresponding values based on Mie scattering. In each figure the specific attenuation is calculated as a function of frequency at two different temperatures: 0° C , and 35° C. In Figure E‑3, the specific attenuation is calculated at a rainfall rate of 5 mm/hr, and in Figure E-3 the specific attenuation is calculated at a rainfall rate of 100 mm/hr.

Figure E-1

Rain specific attenuation as a function of frequency (R = 5 mm/hr)



Figure E-2

Rain specific attenuation as a function of frequency (R = 100 mm/hr)



Annex F  
  
Analytic solution for the scatter transfer integral

The analytic approach addressed in this fascicle is based on the Steepest Descent Method (SDM). To set the scattering transfer integral into a form suitable for applying the SDM, both the transmitting and receiving antennas are taken to have narrow beams. Accordingly, for the transmitting antenna, the linear form of the gain could be written as:

(F-1)

is the linear gain along the antenna boresight axis, and are the vertical and horizontal angles measured from the boresight axis of the transmitting antenna; and and are the half-power beamwidths of the antenna in the vertical and horizontal direction respectively. Similarly, the receiving antenna the gain could be written as:

(F-2)

is the linear gain along the antenna boresight axis, and are the vertical and horizontal angles measured from the boresight axis of the receiving antenna; and and are the half-power beamwidths of the antenna.

To simplify the mathematical formulations of the angles and , a local Cartesian coordinates () is created. The coordinate centre is positioned at intersection point of the transmitter and receiver antenna boresights. The axis is aligned along boresight of the transmitter antenna , and is taken to be perpendicular on boresights of the transmitting and receiving antennas.

(F-3)

and axis is takento be perpendicular on both and .

(F-4)

Within the coordinate (), the angles and could be approximated as:

(F-5a)

(F-5b)

The angles and are expressed in terms of another local Cartesian coordinate () created through rotating the coordinate () around axis toward the transmitter by an angle equal the scattering angle .

(F-6a)

(F-6b)

Introducing (f-5) into (F-1) ; and (F-6) into (F-2), then introducing the resultants into (58) reduce the scattering transfer integral into:

(F-7)

(F-8)

(F-9)

Based on the SDM, the integral in (F-7) could be approximated as:

(F-10)

In the above, is the saddle point, and is the determinant of the Hess matrix calculated at the saddle point.

(F-11)

To get the saddle point, the first derivatives of are derived and then equated to zero.

(F-12a)

(F-12b)

(F-12c)

Equating each of the above derivatives to zero and solving the resultant equations imply that the saddle point occurs at the centre of two local coordinates (which is the intersection point of the two antenna boresights (.

The second order derivatives of at the saddle point are:

(F-13a)

(F-13b)

(F-13c)

(F-13d)

(F-13e)

At the saddle point from (F-9), (F-11), (F-12), and (F-13), we can write:

(F-14a)

(F-14b)

Introducing (F-14) into (F-10) yields the analytic approximation of the scatter transfer integral based on the SDM.

(F-15)

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