REPORT 944

THEORETICAL NETWORK PLANNING

(Question 46/10, Study Programme 46L/10)

(1982)

1. Introduction

Broadcasting-transmitter networks should be planned in such a way that the required coverage of the area is provided using the minimum number of frequencies. From the purely technical standpoint, the coverage area of each transmitter depends upon a number of factors, for example: transmitter power, minimum usable field strength, radio-frequency protection ratio, the distance between transmitters sharing the same or adjacent channels, channel spacing, bandwidth of emission and other factors influencing wave propagation. It also depends on the channel distribution scheme.

When a large number of channels are to be planned or replanned for a particular AM or FM sound or television service, it has been found that effecting an efficient use of the spectrum can prove difficult when employing empirical methods only. For this reason, a theory of uniform transmitter networks was developed during the late 1950s and early 1960s [EBU, 1960]. This method can be applied with success when some uniformity of standards exists for the service to be planned. Furthermore, the frequency band to be planned should be constrained as little as possible, i.e. there should ideally be complete freedom in assigning any frequency to any transmitter.

The theory is not only useful in designing new or remodelling actual transmitter networks, but also provides a powerful tool for determining optimal technical parameters such as channel spacing, transmitter characteristics, etc., and identifying the best attainable coverage possible.

The method described below has already been used during the VHF/UHF European Broadcasting conference, Stockholm 1961, during the African VHF/UHF Broadcasting Conference, 1963, and helped in preliminary studies for the Geneva 1975 LF/MF Broadcasting Conference.

2. The theory of regular networks

It should be noted that the networks which will be studied in the present section are purely theoretical in the following sense:

- all the transmitters are identical: their power and antenna height are the same;
- they are equipped with non-directional antennas;
- propagation is isotropic and independent of frequency, at least within the band to be planned.
- for the purpose of calculating distances, the planning area is assumed to be flat and the population evenly spread over its surface; there are neither political nor natural boundaries.

In those conditions, and provided interference is negligible, the coverage area, i.e., the region where a good reception is achievable with a normal domestic receiver, is limited by a contour within which the electromagnetic field strength is greater than or equal to the value necessary to obtain a given signal-to-noise ratio. In the ideal situation described above, this contour is a circle whose radius depends on the type of service and the propagation laws valid for the frequency range considered. If it is desired to cover the whole of the planning area using such circular coverage areas, it is quite obvious that the number of transmitters per unit surface will be minimized if they are situated at the vertices of a lattice of equilateral triangles (Fig. 1). It should be noted that there is not one transmitter per triangle, but rather one transmitter for each pair of triangles forming a rhombus (shaded area of Fig. 1). This remark is of importance when computing the network efficiency.

With this configuration, every location on the planning area is served by at least one transmitter and overlapping of service areas is minimized. In what follows, we will take as a unit of length the side of the elementary equilateral triangle. The transmitter density is then one per elementary rhombus, the area of which is $\sqrt{3}/2$. The network efficiency can be expressed as the ratio of the total area where the service is acceptable (the area of the elementary rhombus) to the sum of the (overlapping) coverage areas. A regular lattice of equilateral triangles gives the optimum value, i.e., 0.83 (each transmitter has a coverage area of $\pi/3$ with a radius of $\sqrt{3}/3$). The ideal value would obviously be 1, but it is not attainable as it is impossible to cover a surface with circles without overlapping areas. A regular lattice of squares (or of isosceles right triangles) only achieves a ratio of 0.64 (each transmitter has a coverage area of $\pi/2$ with a radius of $\sqrt{2}/2$).

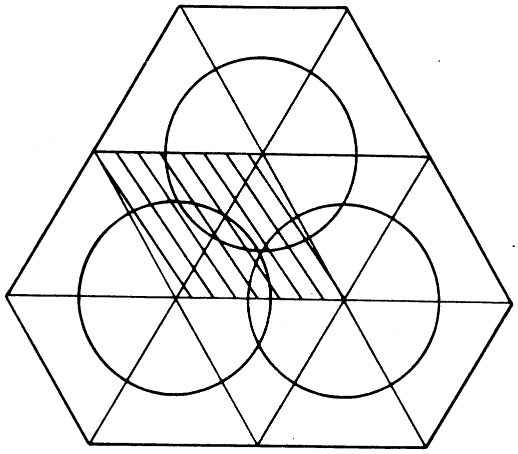

FIGURE 1 – *Location and coverage areas of identical broadcasting transmitters in a regular network*

It should be noted that actual transmitter networks are neither geometrically regular, nor do their technical characteristics correspond to those of the theoretical network. Deviations from geometrical regularity and from the power and antenna height of the theoretical network towards lower values will inevitably result in lower network efficiency. Nevertheless it is possible to derive, from the results of studies concerning theoretical networks, a fairly clear picture of the relationship between these factors and network efficiency.

3. An example of theoretical network

It is assumed first of all that the available frequency band has been divided, in an appropriate manner, into channels sufficiently wide for a sound or television programme to be located within its limits and adequately spaced (9 kHz for AM sound broadcasting; 100 or 200 kHz for FM sound broadcasting; 6 or 8 MHz for television (200 kHz and 6 MHz correspond to the American channels)) to make co-channel interference predominant vis-à-vis adjacent-channel interference.

It is then easy to derive, from this channel spacing, the number of channels that can be used when planning. As this number is obviously finite, it is inevitable that, at some more or less large distance from the starting point, the first channel assigned has to be re-used. The smaller the distance D, between two transmitters operating on the same channel, the smaller the number of channels needed to cover the whole planning area. But it is necessary that D be large enough for the two co-channel transmitters not to interfere unduly with each other.

It is not difficult to evaluate D. With the help of suitable propagation curves, field strengths of the wanted transmitter and of the interferer are computed, for given percentages of time (see Recommendations 368 and 370), on the circle defined in § 2, limiting the coverage area of the wanted transmitter. D is chosen sufficiently large so that the difference, expressed in dB, between the two field strengths is larger than the protection ratio which is to be found in the relevant Recommendations 560, 412, 418 and Report 306. The case of multiple interference will be dealt with later.

With N channels, it is possible to cover N elementary rhombi, that is an area equal to $N\sqrt{3}/2$, taking as unit area the surface of a square whose side is equal to the side of the elementary triangle. So another kind of network arises using rhombi consisting of N elementary rhombi, and it is logical to stick to the conclusion found in § 2: a given channel will be repeated at the nodes of a lattice of rhombi whose shorter diagonal is equal to the side, the area of which is $N\sqrt{3}/2$. This rhombus will be called the "co-channel rhombus" and the length D of its side is $\sqrt{N}$ ($\sqrt{N}$ times the side of the elementary triangle). The characteristics of rhombic lattices will be shown in a simple example, where $N = 13$ (see Fig. 2).

- *Period:* all the co-channel rhombi are identical. A two-dimensional periodicity appears. It may remind the reader of the theory of elliptical functions.
- *Channel distribution:* within a co-channel rhombus, channels might be distributed at random, but in every rhombus their locations should be identical. However, a random distribution is not suitable if it is necessary to take into account interference other than co-channel interference, which is usually the case. Assuming that a homogenous network leads to the most efficient utilization of the spectrum, it is necessary for the interfering signals on any channel to have the same strength, wherever the transmitter is.

Hence if, taking account of the protection ratio between adjacent channel transmitters (referred to here as 1-channel transmitters; likewise 2-channel transmitters are spaced by 2 channels, etc.), channel 1 is placed at a point which is sufficiently far from channel 0, this defines for channel 1 a co-channel rhombic which is offset from the co-channel rhombic for channel 0, by the vector $\overline{01}$. To meet the requirement referred to above, for homogeneity, channel 2 will be placed such that $\overline{12} = \overline{01}$, and so on for the other channels. Channel N falls on channel 0. If we consider the 13 channel network in Fig. 2, it is seen that someone moving along the straight lines of the elementary triangle lattice will find the channel numbers are in an arithmetic progression, modulo 13. Starting from channel 0 and heading west, he will encounter channels: 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 0, etc. It is therefore very easy to define the entire network on the basis of the progression step in two distinct directions: + 2 towards the left and + 5 towards the top righthand corner.

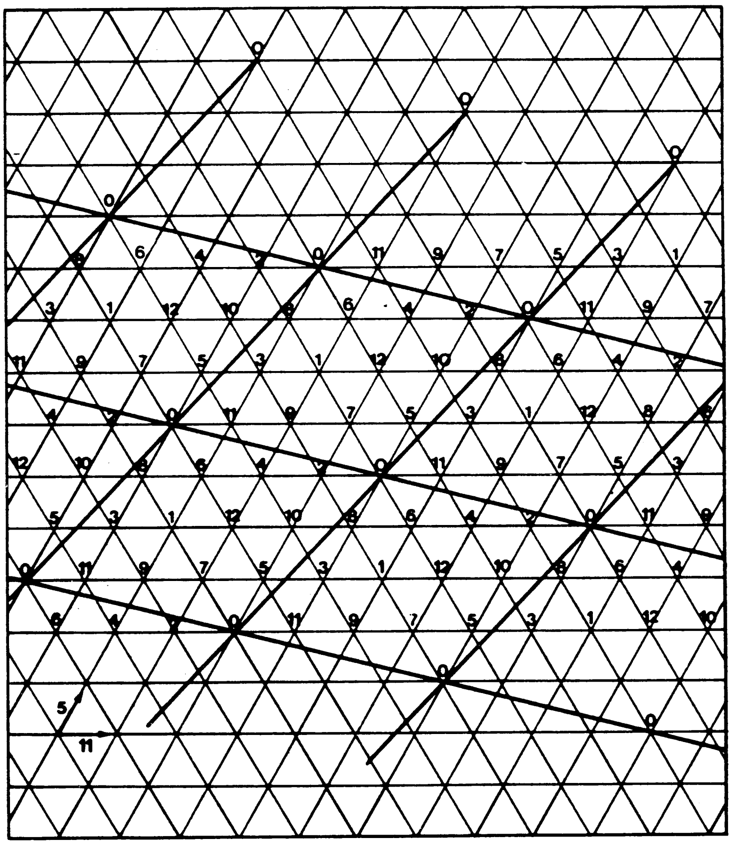

FIGURE 2 – *Example of regular lattice for 13 channels*

4. Ideal and non-ideal networks

To make the calculations easier, it is convenient to refer the plan to a system of two coordinate axes, bearing equal scales but making an angle of 60° rather than the conventional 90° (see Fig. 3).

We already know that the side of the co-channel rhombus has a length equal to $\sqrt{N}$, but it is necessary for the rhombus to coincide with the intersections of the elementary lattice. In the tilted coordinate system, the distance of any point (x,y) from the origin is:

$$(x^2 + xy + y^2)^{1/2}\,*$$

If two integers a and b can be found such that $a^2 + ab + b^2 = N$ (number of channels), then there is a co-channel rhombus whose vertices coincide with the intersections of the elementary lattice. $N = 13$ corresponds to the case where $a = 3$ and $b = 1$. In the following discussion, such numbers will be called "rhombic numbers". Some of them are given on Fig. 4.

* All the distances in this section are calculated on the base of this formula and expressed in the multiples of units equal to the length of a side of an elementary triangle.

There are other conditions for the choice of a and b: they shall not have any common divisor, either between themselves or with N, and neither of them can be equal to zero. For instance, if they are both even numbers, only the even-numbered channels will appear. The other restriction will be given later. The theory still works if N is not exactly a rhombic number. The co-channel rhombus becomes a co-channel parallelogram, with an area equal to N. In this case, the sides of the parallelogram are not equal; one of them is shorter than $\sqrt{N}$ and as a consequence this co-channel distance is slightly smaller than for the rhombus.

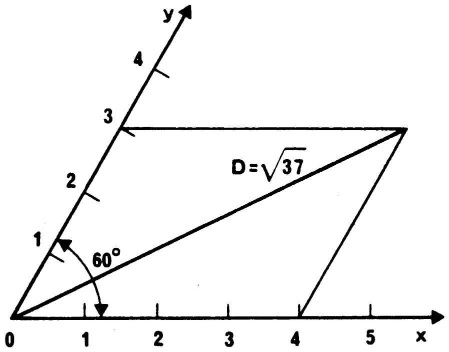

FIGURE 3 – *Reference coordinates of regular lattices*

b \ a	1	2	3	4	5	6	7	8	9	10	11	12
1	3	7	13	21	31	43	57	73	91	111	133	157
2			19		39		67		103		147	
3				37	49		79	97		139		
4					61		93		133			
5						91	109	129	151			
6							127					

FIGURE 4 – *Table of rhombic numbers*

Figure 5 shows a network with 26 channels, 26 not being a rhombic number. However, in this particular case, it is seen that the shortest co-channel distance is the shorter side of the parallelogram whose length equals 4.358. This is not much less than the distance $\sqrt{N}$ = 5.099 which would have been obtained if 26 were a rhombic number. The lengths of other co-channel distances, e.g. the longer side of the parallelogram or the shorter diagonal of it, are 5.291 and 6.082 respectively. The next rhombic number is 31 an example of which is given in Fig. 6.

Figure 7 shows a network with 120 channels, 120 not being a rhombic number. However, in this particular case, it is seen that the shortest co-channel distance is the diagonal whose length equals 10.58. The lengths of the sides are 11.53 and 10.82, respectively. None of these numbers is much less than the distance $\sqrt{N}$ = 10.95, which would have been obtained if 120 were a rhombic number. This lattice may not be optimum for VHF planning.

In the case of N not being a rhombic number, there are three possibilities:

– not to take note of this value N and consider the nearest rhombic number which could be used instead (e.g. 111; see Fig. 4);

– to consider the resulting regular lattice despite its consisting of co-channel parallelograms;

– to distort the lattice so as to transform co-channel parallelograms to rhombi. In these circumstances the elementary rhombi would become parallelograms, and network efficiency would decrease. This decrease will only be slight when N is fairly large.

The best choice among the three options depends on the circumstances to be taken into account.

It is not always easy to find the progression steps such that the channel assigned to the origin will reappear at the vertices of the co-channel rhombus or parallelogram. Referring Fig. 2 to the slant coordinate system of Fig. 3, the steps p along the x-axis and q along the y-axis must satisfy the following equations:

$$1 \cdot p + 3 \cdot q = kN \qquad \text{and} \qquad 4 \cdot p - 1 \cdot q = k'N$$

k and k' are integers.

In Fig. 2, $p = 11$, $q = 5$, $k = 2$, and $k' = 3$, for $N = 13$. These steps are indicated in Fig. 2. In the case of a non-rhombic number, there is more than one parallelogram and many attempts are necessary to find the best one [Arnaud, 1962]. Even in the case of rhombic numbers it may happen that two different co-channel rhombi exist, as it appears on Fig. 4 for 91 (9 and 1 or 6 and 5).

The case where a or $b = 0$ is a trivial one. If, for example, $b = 0$, the co-channel rhombus sides coincide with the x and y axes, and $a = \sqrt{N}$. The sum of steps along the axes from one vertex to the next must be N; consequently their value is $\sqrt{N}$ and it is then only possible to write on the lattice channels that are multiples of $\sqrt{N}$.

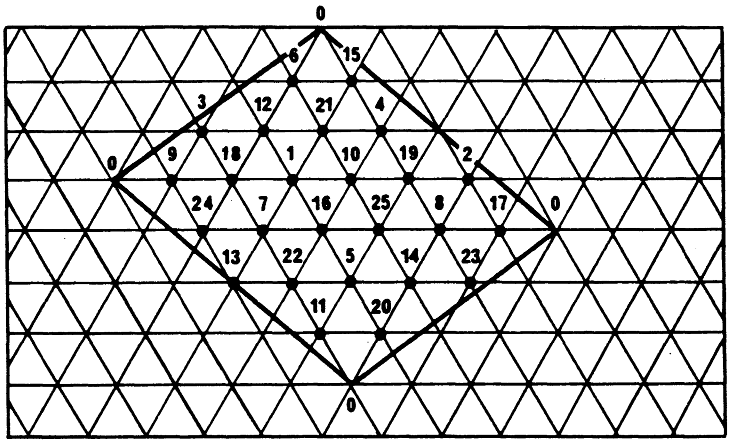

FIGURE 5 – *Network with 26 channels*

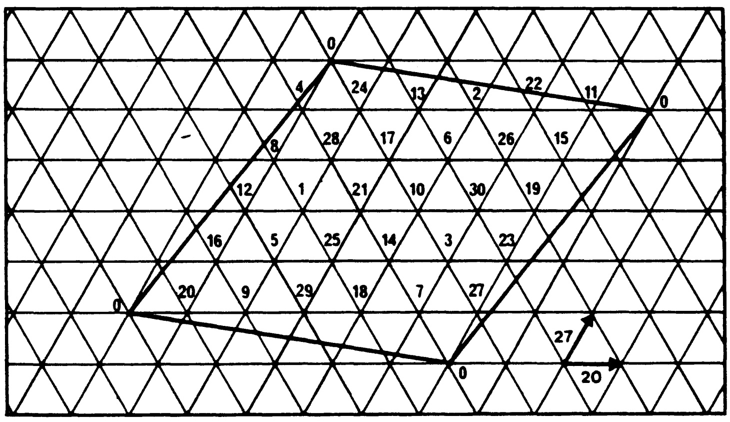

FIGURE 6 – *Example of an optimum regular lattice for 31 channels*

5. Evaluation of interference

5.1 *Co-channel interference*

It has been explained in § 3 how the minimum necessary number of channels derives from the consideration of propagation curves and protection ratios, but only one interfering transmitter has been taken into account. On Fig. 8 are drawn a few co-channel rhombi for a co-channel distance D, the wanted transmitter being at the centre and the eighteen others surrounding it are the interferers. In the case of VHF or UHF broadcasting, the coverage area is evaluated on the basis of the 50% or 1% of the time propagation curves of Recommendation 370 (for the wanted or interfering transmitter, respectively), taking due account of the characteristics of the transmitter (effective radiated power, antenna height, frequency) and the minimum usable field strength. (See Recommendation 412). In § 2, the coverage area radius has been found to be $\sqrt{3}/3$ times the side of the elementary rhombus. So the scale of the lattice is known. The first six interfering transmitters are at a distance $D = \sqrt{N}$, and as a first approximation it will be assumed here that D is large enough and that the interfering field strength is almost the same at the wanted transmitter location and on the fringe of its coverage area. In a more accurate study, the ratio of wanted/interfering signals should be calculated at the limit of the coverage area, at least at the six locations shown by roman numerals I to VI on Fig. 8.

D has to be long enough for the difference (in decibels) between the wanted field strength exceeded for 50% of the time and the combined interfering field strengths exceeded for 1% of the time, to be greater or equal to the co-channel protection ratio. Then it should be checked whether the twelve transmitters located on the second hexagon (see Fig. 8) have any influence, but in general this is not the case unless D is very small.

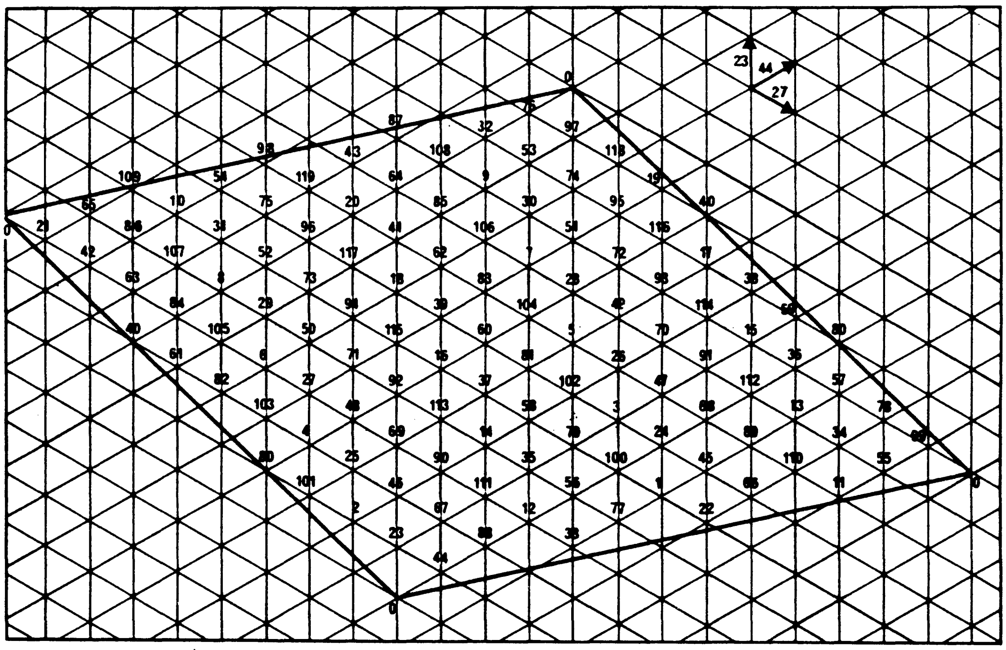

FIGURE 7 – *Network with 120 channels*

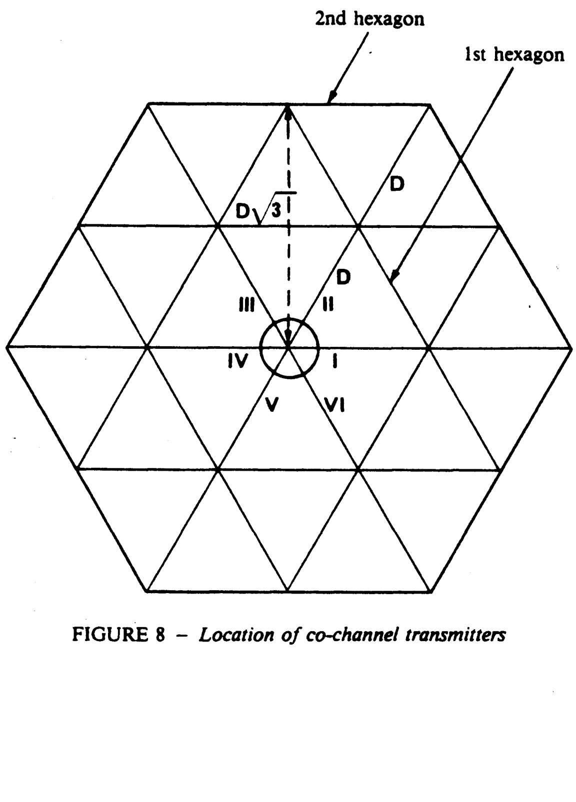

FIGURE 8 – *Location of co-channel transmitters*

5.2 *Adjacent channel interference*

The network of Fig. 2 will serve as a practical example. D is assumed to have been found to be equal to $\sqrt{13}$. By definition, the network is regular and any given channel can be studied. Take for instance channel 7. The adjacent channel distances are $\sqrt{3}$ and 2. There are 4 adjacent channel interfering transmitters. As in the case of co-channel interference, their total field strength will be evaluated at the transmitter location first, and then, if the wanted/interfering signal ratio is only slightly different from the protection ratio, at the most critical points at the limit of the coverage area. Other adjacent channel transmitters are much more remote. Their distances are $\sqrt{7}$, $\sqrt{12}$, $\sqrt{13}$, etc.

For a wanted transmitter on channel C, there are in each co-channel rhombus at least two adjacent channel interfering transmitters on channels $C + 1$ and $C - 1$. The further the transmitters $C + 1$ and $C - 1$ are from the corners of the co-channel rhombus of C, the better will be the network; the ideal positions will be the centres of gravity of the two equilateral triangles making up the rhombus. Hence if transmitter $C + 1$ is exactly at one of these centres of gravity, $C + 2$ will be exactly at the other and $C + 3$ will coincide with C which is absurd. This occurs with the network with $N = 21$ channels. If the coincidence of a point in the network with the centres of gravity is only approximate, channel $C + 2$ will be twice as far from the centre of gravity of the triangle, however channel $C + 3$ will be still fairly close to channel C which is unfavourable in FM sound broadcasting as this corresponds to a carrier spacing of 300 kHz (in parts of Europe) for which the protection ratio is only slightly negative (-7 dB).

5.3 *Multiple interference*

Both in theory and in practice more than one interfering transmitter has to be respected. At the site of the wanted transmitter in the network of Fig. 2 there are co-channel transmitters interfering with near identical signal strength and in addition there are four interfering adjacent-channel transmitters. It should not be overlooked that at least 2-channel and 3-channel transmitters will also need to be taken into account. In practice it has proved useful to compute the usable field strength on the basis of at least those interferers which cause the strongest interference potential. It cannot however be a matter for this description of a planning method to advocate one or the other method of computing multiple interference effects.

6. Planning constraints

When planning a particular type of service, some constraints have to be kept in mind and most of them result from the receiver design. It is useful to distinguish between internal and external constraints. Such constraints and their generating mechanisms are discussed in Report 946.

7. Use of theoretical networks

Theoretical networks may be of some help in trying to find the optimum characteristics of transmitters [Sauvet-Goichon, 1980] or the most suitable type of modulation parameters or channel spacing. There is a good probability that a solution found to be optimum in a regular network would be the best one for an actual network as well and in any event it will be far easier to study the former with the help of computerized methods.

7.1 *Simple network*

The allocated band, 16 MHz wide, is divided into 159 channels. Below 159 the largest rhombic number is 157 (12 and 1, see Fig. 4). Two channels are then left empty. They may replace channels whose use is precluded by some local constraints (see § 6) or fill gaps in mountainous areas or near borders. As a general rule, more than one programme is broadcast from a given location and some geographical neighbouring assignments will be moved to the same point. Figures 9 and 10 show two examples of channel clustering. Assignments made at the vertices of the shaded areas are concentrated to the centre of gravity of each area. It should be checked that this does not involve an unacceptable increase in interference.

Figure 11 gives the steps to build a 157 channel network. Their derivation is given in Annex I. The 1961 Stockholm Plan for UHF television has been based on a channel clustering C, $C + 3$, $C + 6$.

7.2 *Multiple networks*

When there is a large number of channels (more than about 50), and if, as is usual, each centre consists of several transmitters, it is possible to split the band into sub-bands, each of them including the same number of channels and possibly reserved for a separate programme. It should be noted that the African Plan has been designed on a channelling of 86 kHz, i.e. 185 channels in 16 MHz, for 6 networks of 31 channels or 5 networks of 37 channels (37 is a rhombic number).

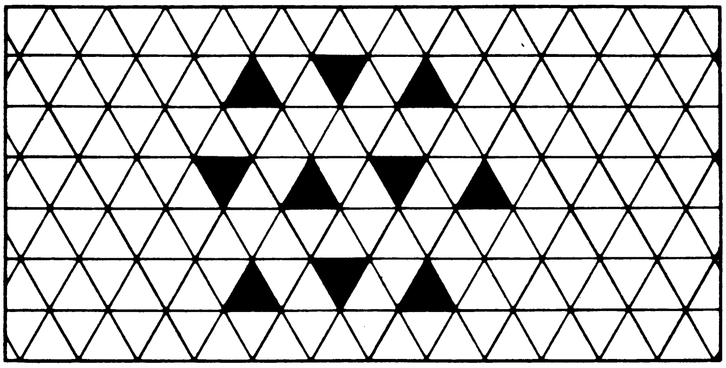

FIGURE 9 – *Clustering of neighbouring channels*

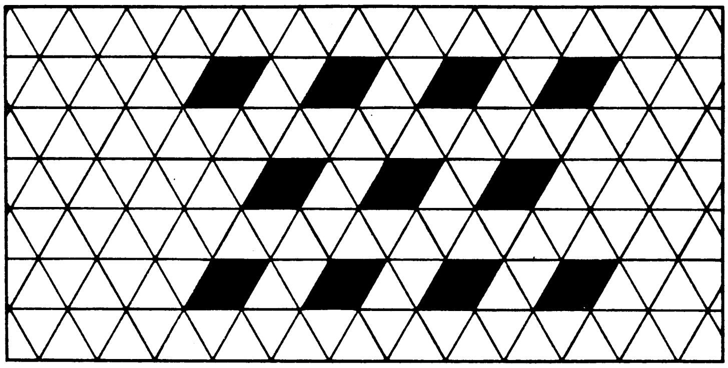

FIGURE 10 – *Clustering of neighbouring channels*

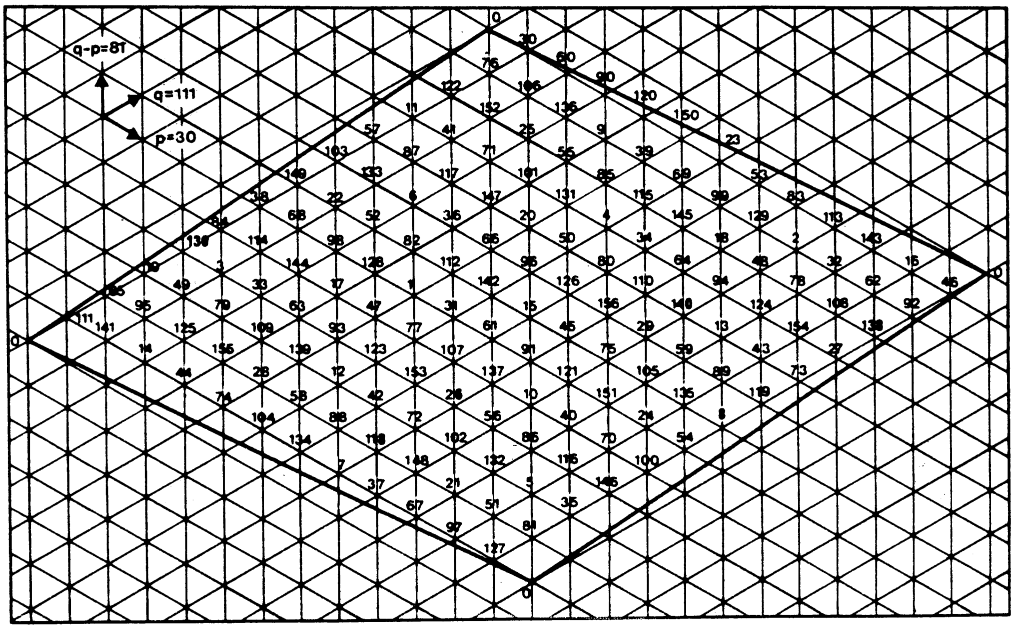

FIGURE 11 – *Example of an optimum regular lattice for 157 channels*

8. Number of channels needed for full area coverage with one programme in VHF sound broadcasting

8.1 *General considerations*

Based on the previously discussed method the number of channels needed for the dissemination of one programme in VHF sound broadcasting to provide full coverage was evaluated.

The investigations are made for different average distances between transmitters, two antenna heights, (300 m and 600 m), two coverage factors (90% and 100%) and an effective radiated power of 100 kW. Monophonic and stereophonic services were taken into account. The calculation of the interferences was carried out using the simplified multiplication method (see Report 945).

This method had previously been used during:

- the European Broadcasting Conference, Stockholm, 1961,
- the African Broadcasting Conference, Geneva, 1963.

8.2 *Performance of the calculations*

8.2.1 *Basic assumptions*

The investigations are based on idealized networks with a regular lattice of transmitters, linear distribution of channels and equilateral co-channel triangles. The so-called elementary triangles which are formed by three neighbouring transmitters are in this case not equilateral in general, but only for certain numbers of channels [EBU, 1960].

The calculations are based on the following conditions:

- field strength E (50,50) (Recommendation 370),
- field strength E (50,1) (Recommendation 370),
- protection ratios (Recommendation 412),
- local variation of field strength: 8.3 dB,
- receiving antenna,
  monophony : omnidirectional,
  stereophony : directional with 12 dB front-to-back ratio,
- minimum usable field strength (Recommendation 412)
  monophony : 48 dB(μV/m),
  stereophony : 54 dB(μV/m),
- channel spacing : 100 kHz.

8.2.2 *Calculation of the usable field strength*

8.2.2.1 *Method of calculation*

The calculation of these interferences was carried out using the simplified multiplication method.

Experience in some countries has shown that the coverage situation found in actual practice is, on average, in acceptable agreement with computation results obtained with the simplified multiplication method. In the subsequent sections only this method has been used for the determination of the required number of channels.

8.2.2.2 *Number of interfering transmitters*

For the calculation of interferences in the theoretical lattice, the 18 strongest interfering co-channel transmitters and the 40 strongest interfering adjacent channel transmitters having frequency separations up to 400 kHz were taken into account. (The relatively large number of interfering adjacent channel transmitters is only relevant with close channel spacings.)

8.2.3 *Variation of parameters*

The average distance of transmitters was varied between 40 and 120 km in steps of 10 km. For the effective height of the transmitter antenna, 300 and 600 m were chosen, because the average height values in many European countries fall into that range. The effective radiated power was set to 100 kW to ensure that interference rather than noise will limit the coverage area; this being a condition for efficient spectrum utilization (see Report 414 (Kyoto, 1978)). For small distances between transmitters however, considerably less power would provide equal percentage of coverage with the same number of channels.

8.2.4 *Results*

The number of channels needed in a theoretical network under different conditions can be found in Figs. 12 to 15. For a monophonic service with an omnidirectional receiving antenna and for a stereophonic service with a directional receiving antenna (12 dB front-to-back ratio), the number of channels required is almost equal.

In an idealized theoretical network based on the above assumptions with an effective transmitter antenna height of 300 m, an effective radiated power of 100 kW and an average distance between transmitters greater than 70 km:

- for 90% coverage about 25 channels are necessary (Fig. 12),
- for 100% coverage about 31 channels are necessary (Fig. 13).

Examples of possible channel distribution for corresponding numbers of channels are given in Figs. 5 and 6.

The number of channels needed for one programme in a theoretical network can only give an approximation to the number needed for a real network. The less the uniformity of the real network considered the more extra channels will be needed. As experience with existing networks in the range 87.5-100 MHz in continental Europe has shown, up to 4 programmes are possible in this range.

Based on theoretical study and practical experience, it may be assumed that for the entire frequency band 87.5-108 MHz should be practicable:

- for 90% coverage about 7 programmes,
- for 100% coverage about 6 programmes,

In practical networks, the power and the effective antenna height of any single transmitter should be set to the lowest possible value to obtain the intended coverage.

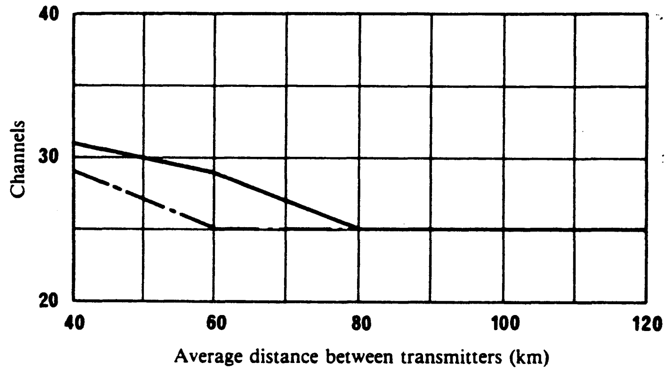

FIGURE 12 – *Number of channels needed in a theoretical network for coverage 90 % and antenna height 300 m*

—·— : monophony (0 dB - antenna gain)
——— : stereophony (12 dB - front-to-back ratio of the antenna)

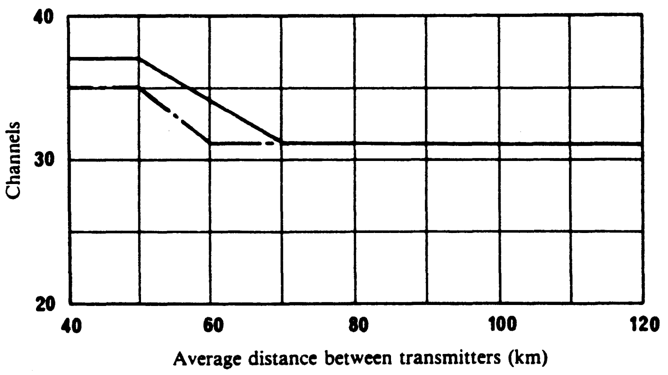

FIGURE 13 – *Number of channels needed in a theoretical network for coverage 100 % and antenna height 300 m*

—·— : monophony (0 dB - antenna gain)
——— : stereophony (12 dB - front-to-back ratio of the antenna)

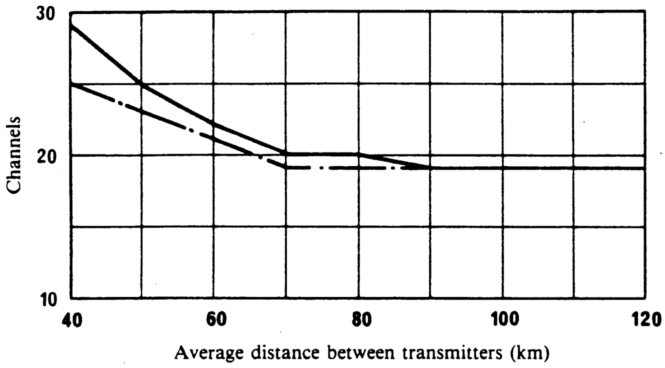

FIGURE 14 – *Number of channels needed in a theoretical network for coverage 90 % and antenna height 600 m*

—·— : monophony (0 dB - antenna gain)
——— : stereophony (12 dB - front-to-back ratio of the antenna)

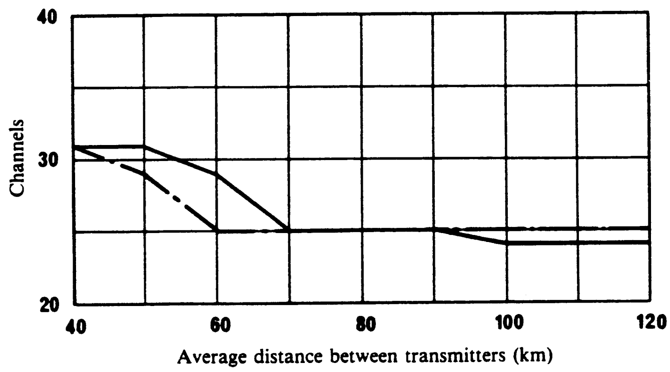

FIGURE 15 – *Number of channels needed in a theoretical network for coverage 100 % and antenna height 600 m*

—·— : monophony (0 dB - antenna gain)
——— : stereophony (12 dB - front-to-back ratio of the antenna)

9. Conclusions

When the number of channels is high, it seems difficult to achieve by purely empirical means the best use of the frequency spectrum. The theory of regular networks appears as an efficient tool to reach an optimum. Moreover, in an area where the surface of the earth is divided amongst many countries, it is a good way to assure them they have obtained a fair share of the spectrum. Consideration could also usefully be given to means of planning irregular networks in special cases where these may be more appropriate.

ANNEX I

CHOICE OF THE PROGRESSION STEPS IN AN OPTIMUM 157 CHANNEL LATTICE

A systematic study has been made of all the rhombic numbers less than 160, involving calculations of distance for 1-, 2- and 3- channel distances for all sets of values of p and q.

Those results that appear the most useful are shown below (Table I) (to simplify the presentation, the square of the distance is shown). The Table extends only to $N = 100$ to limit its size.

TABLE I

	p	q	Adjacent	2-channel	3-channel
$N = 19$ ($a = 3$, $b = 2$)	6	10	4	3	3
$N = 31$ ($a = 5$, $b = 1$)	7	27	7	7	3
$N = 37$ ($a = 4$, $b = 3$)	6	29	9	3	7
	9	25	9	7	3
$N = 39$ ($a = 5$, $b = 2$)	4	29	7	7	9
$N = 43$ ($a = 6$, $b = 1$)	5	13	12	7	3
$N = 49$ ($a = 5$, $b = 3$)	12	29	13	9	3
$N = 57$ ($a = 7$, $b = 1$)	11	37	13	7	9
$N = 61$ ($a = 5$, $b = 4$)	6	23	13	7	12
	15	27	16	13	3
$N = 67$ ($a = 7$, $b = 2$)	4	53	13	13	12
	8	39	19	13	3
	21	27	12	7	13
$N = 73$ ($a = 8$, $b = 1$)	5	33	19	7	12
	6	25	13	9	16
	7	17	19	19	3
	11	58	12	13	19
$N = 79$ ($a = 7$, $b = 3$)	4	17	13	19	21
	6	65	16	7	19
	11	27	19	9	12
	22	54	21	19	3
$N = 91$ ($a = 6$, $b = 5$)	4	68	16	19	21
	9	62	21	13	12
	15	73	25	21	3
$N = 91$ ($a = 9$, $b = 1$)	4	55	19	21	12
	6	37	19	9	21
	8	19	27	19	3
$N = 93$ ($a = 7$, $b = 4$)	5	61	25	13	9
	17	40	16	13	21
$N = 97$ ($a = 8$, $b = 3$)	6	81	19	13	21
	13	30	21	19	12
	26	60	28	21	3

As an example we give here the details of the calculations used to find the best progression steps for $N = 157$ channels.

As $157 = 12^2 + 12 \times 1 + 1^2$, the step p and q should be such that

$$12p + q = 0 \text{ modulo } 157$$

p and q must greater than 3 so that transmitter coverages on channels C and $C + 1$, $C + 2$ or $C + 3$ do not overlap.

The first values to be tested are $p = 4$ and $q = 109$. Then $q - p = 105$ (see Fig. 11 for definition of p, q, $p - q$). The question now is to find out where channel 1 (or 156) falls, as here the co-channel rhombus of channel 0 is considered. As the lattice is homogeneous and regular, we can begin with any channel. The ideal location for channel 1 is at the centre of gravity of any of the two triangles which join up to form the co-channel rhombus. The adjacent channel distance would then be

$$\sqrt{157} \times \sqrt{3}/3 = 7.23$$

For $p = 4$ and $q = 109$, channel 1 will be found at three $(q - p)$ steps: $q - p = 105$; $3 \times 105 = 315 = 1$ modulo 157. Adjacent channel distance is 3, which is far shorter than the maximum of 7.23.

For $p = 5$ and $q = 97$ $(q - p = 92)$ channel 156 will be found at five $(q - p)$ steps plus two p steps: $5 \times 92 + 2 \times 5 = 470 = 156$ modulo 157. Adjacent channel distance is $\sqrt{19}$. Hopefully a better solution exists.

For $p = 6$ and $q = 85$ $(q - p = 79)$ channel 1 will be found at two steps, as $2\ (q - p) = 158 = 1$ modulo 157. This is far too close.

For $p = 7$ and $q = 73$ $(q - p = 66)$ channel 156 will be found at three p steps plus four q steps: $3 \times 7 + 4 \times 73 = 313 = 156$ modulo 157. The 1-channel distance is $\sqrt{37} = 6.08$. Channels 156 and 1 are near the centres of gravity of the triangles as are channels 155 and 2; channels 154 and 3 at a distance of $\sqrt{12}$ from channel 0. A thorough search amongst all the pairs, p and q, shows that there are only a few sets of steps resulting in 1-channel distance greater than or equal to 6. They are given in the following Table II, together with the 2-channel and 3-channel distances.

TABLE II

p	q	Adjacent	2-channel	3-channel
7	73	$\sqrt{37}$	$\sqrt{39}$	$\sqrt{12}$
11	25	$\sqrt{48}$	$\sqrt{37}$	$\sqrt{3}$
20	74	$\sqrt{37}$	$\sqrt{21}$	$\sqrt{21}$
24	26	6	1 (1)	
30	111	$\sqrt{39}$	$\sqrt{19}$	$\sqrt{21}$
33	75	$\sqrt{39}$	$\sqrt{7}$	$\sqrt{48}$

(1) This eliminates the combination.

The final choice depends on the protection ratios for 2 and 3 channel interferences. The pair 24 and 26 will not be suitable.

If it were planned to cluster channels in groups of 3 or 4 (see Figs. 9 and 10) it would be more appropriate to have all steps greater than, say, 20 to avoid multiplexing problems (see [Arnaud, 1962]). Such a network is drawn on Fig. 11 ($p = 30$, $q = 111$). If no clustering is foreseen, or if a channel spacing of 20 is acceptable for colocated transmitters, then the network $p = 20$, $q = 74$ is slightly better.

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