International Telecommunication Union

Guidelines for the Application of Statistical Models to Telecommunication Market Regulation

Practical applications

These guidelines for the application of statistical models to the regulation of telecommunication markets were prepared by Ms Karla Montero Víquez, economist, under the direction of BDT's Regulatory and Market Environment Division (RME). The comments and suggestions made by ITU officials, in particular Ms Carmen Prado-Wagner, were very useful in preparing the guidelines.

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Application of Statistical Models to the Regulation of Telecommunication Markets

Introduction

The science of statistics is grounded in mathematics. It consists in collecting, analysing and interpreting data with a view to providing impartial explanations for different phenomena.

Although it is based on mathematics, the science of statistics is applicable to a broad range of disciplines that includes the social sciences, quality control and cost administration. Statistical tools can be applied in those disciplines to manage risk in business or government decision-making and to establish relations of cause and effect between different variables for the purpose of situation analysis.

The science of statistics is today an effective means of accurately describing the values of economic, political and social data. Thus, an expert in the use of statistical tools is able to interpret results for the benefit of the company, the government and society and thereby facilitate decision-making, and to infer the future behaviour of events similar to those analysed.

The application of statistical analysis tools makes it possible to take pertinent and timely decisions and to forecast the behaviour of one or a group of variables.

In conclusion, the science of statistics can be applied to almost any discipline and any situation, no matter what the level of complexity, although it is true that the more complex the situation, the more useful statistics are for analysis.

These guidelines discuss the application of statistical tools in specific situations related to telecommunication market regulation.

1. The context of project evaluation

What is a project?

"Projects" can be defined in many different ways. From the administrative point of view, a project is a set of activities organized in logical and orderly fashion for the purpose of fulfilling an objective and meeting specific targets; it has a start and an end. The best approach is that which defines the project as the search for an intelligent solution to a specific problem, a response to one of many specific needs that adds economic value.

From the economic perspective, human beings have an unlimited number of needs, from basic and fundamental necessities such as food and clothing, to wants stimulated by marketing and advertising tactics that exploit the need to feel a sense of belonging and the other determinants of demand. If the ultimate aim of the economy in an organized society is understood to be to satisfy the greatest number of needs with the limited amount of existing resources, then a well-conceived project will by definition optimize this concept.

From the perspective of the regulator, the objective of universal service/access projects is to guarantee the availability of a minimum set of good quality services, accessible to all users at an affordable price and without distorting competition. To meet that objective, the regulator establishes economic obligations for contributions in order to be able to build up a kind of investment fund for the development of universal service/ access projects.

It is the regulator's responsibility not only to guarantee proper use of the fund's financial resources, but also the effective and efficient implementation of the projects to which those resources are allocated. Project implementation should ensure fulfillment of objectives, the well-being of the target communities and a return for the project implementer, be it a telecommunication operator or another economic agent. This approach adds value and makes projects financially attractive that are by their nature considered to be high financial risks by those implementing them.

There are different kinds of projects depending on their objective and applicability. This study looks specifically at investment projects, which, if allocated a specific amount of capital and a proportion of varied inputs, can provide useful solutions and a return on investment, i.e. they are able to generate added economic value.

The objective of evaluating an investment project is to measure its economic return, i.e. to determine how much wealth it is able to generate. From the social perspective, the aim is to quantify the benefit of the investment to society.

The implementation of projects to develop or strengthen universal access/ service should be assessed from those perspectives, in order to ascertain that the solution put forward will satisfy the unmet need efficiently, safely and profitably, allowing scarce resources to be allocated to the best option, for the benefit of both the users and the operator or company implementing the project.

Analysing projects of this type in that way helps ease the reluctance of operators and businesspeople to take part in the regulator's comparative bids for universal service/access solutions, insofar as, from the business perspective, the ultimate aim of implementing an investment project is to generate added economic value for the company in the form of operational profitability.

The process of examining a project

The end purpose of examining a project is to determine as closely as possible the levels of investment, revenues, costs and benefits that a specific project implies for the investor, so that various options can be compared before a decision is made.

The process starts with the **formulation and preparation of the project**. Formulation is the stage at which the project is conceptually defined. Preparation is the stage during which all levels of project investment, revenues and cost are estimated. The outcome is a net cash flow that serves to forecast the potential behaviour of project revenues and cost, which will be examined with respect to the initial investment.

Project evaluation

Project evaluation is fundamental when it comes to deciding on the optimum use of resources, be it within a company or by an individual investor. Projects are evaluated by being submitted to multidisciplinary analysis by various specialists.

Every project evaluation is unique and special, but there is a methodology that can be applied in all cases. That methodology has the advantage of being sufficiently flexible to be adapted to any kind of project, so long as the information needed for the evaluation is available.

Generally speaking, and in terms of process, projects are evaluated at three levels that are distinguished by the depth of analysis each involves.

The first level, and hence the simplest, is the **project profile or idea**. It is in this phase that the existence of an unmet need and/or a business opportunity is identified. The analysis draws on existing information, common sense, or expert opinion based on professional experience. At this level, the project's objective and scope are defined in general terms, and the existence of associated projects and the need for a prior or subsequent, additional project determined.

The more general items of information required at this level include:

1. the project objectives, both general and specific;

2. the project scope:

- its raison d'être;
- its motivation;
- its importance for the company and the national community;
- its general strategy and estimated timetable for implementation;
- how the proposed product/service is to be produced/provided.

The second level is **prefeasibility**, which goes into greater depth. At this level, the information available from secondary and primary sources is analysed. In the case of market research, detailed consideration is given to the technology to be used, and the project's total costs and economic profitability are estimated. Investors base their decisions on these parameters.

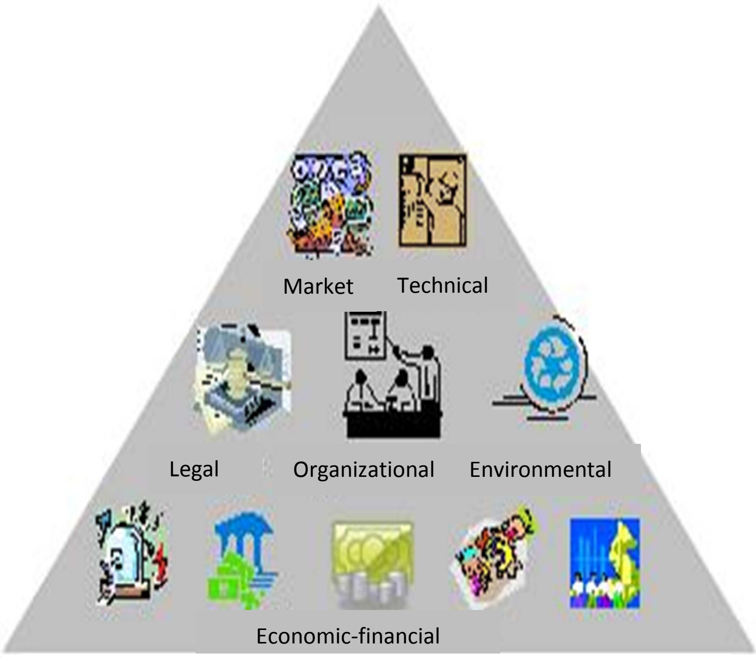


Figure 1.1 Preliminary studies for project evaluation

At the prefeasibility level, preliminary studies are conducted that will subsequently be incorporated into the project feasibility study. Such studies are required in order to determine the existence of prior conditions that may generate the possibility of developing a project. As a rule, six specific studies have to be conducted during the prefeasibility phase in order to evaluate the project (see Figure 1.1).

The market study

The existence of an unmet market need, or of an opportunity to supply a new service or product or to improve an existing one, must be confirmed. The first thing to do, therefore, is to define the good or service that is to be supplied, in order to estimate the quantity that customers will be willing to acquire at a given price. This, in short, is the demand analysis. It implies studying trends in demand, if historical data are available, or the behaviour of demand in countries or among competitors with experience in the same field.

The study of supply, i.e. the analysis of the conditions in which the economy can make the good or service available to the market, is key. It includes obtaining data on the number of suppliers of the good or service on the market, the location, existing and used capacity, the quality and price of the goods or services, expansion plans, fixed investment and number of employees.

It is essential to analyse the price of the good or service in order to calculate future revenues. The type of price and the way in which it will fluctuate as the context changes must be clearly defined.

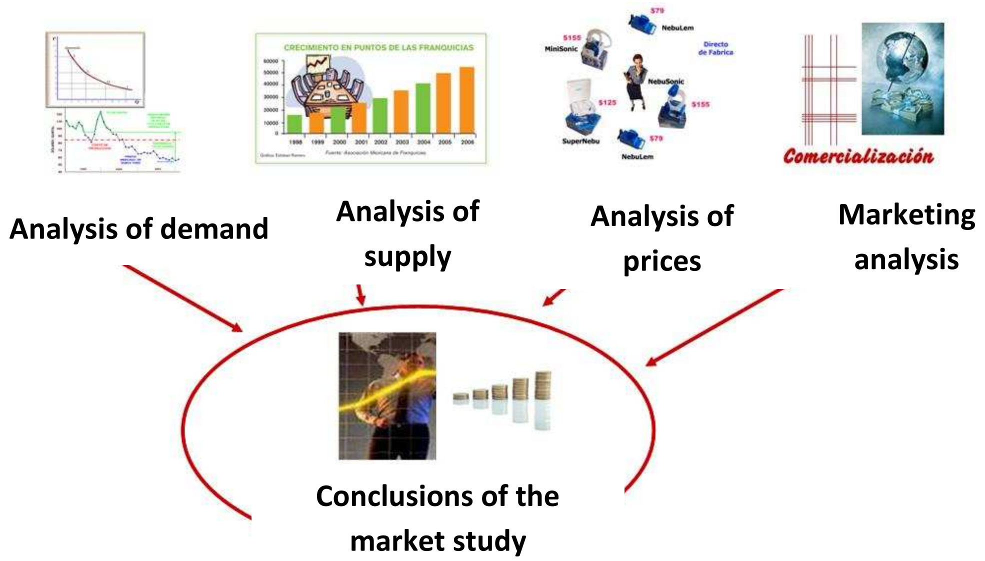


Figure 1.2 Structure of the market study

The technical study

The main objective of the technical study is to obtain the information needed to quantify the amount to be invested and the associated operating costs. From the technical perspective, various productive processes may exist. It is generally believed that the most modern procedures and technologies should be applied; this may be the best option from the technical point of view, but not necessarily from the financial perspective.

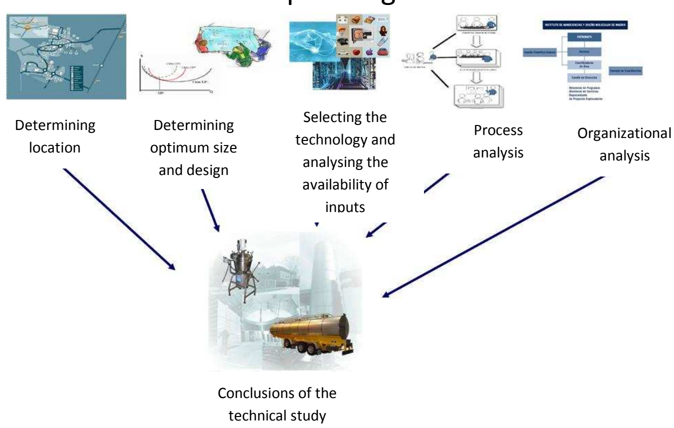


Figure 1.3 Structure of the technical study

One of the study's main conclusions should be the optimal production function for using the available resources to manufacture or develop the good or service.

Some of the basic items to be covered by the study are:

- justification of the technology used;
- location and amount of equipment to use;
- investment components;
- periods (times) at which to make the investment;
- cost of related operations;

- method used;
- project capacity, i.e. the amount of goods or services that can be produced or supplied;
- the factors determining capacity;
- expansion and improvement for future capacity;
- method used to determine optimum capacity.

The study of legal viability

This implies a review of the legislation that is binding on both the company and the project and which, in one way or another, may influence decision-making as the project evolves.

The organizational and administrative study

This study includes an explanation of the project's administrative and organizational arrangements, so that the operating and maintenance costs and the administrative and marketing expenses, for example, can be more easily identified.

As indicated earlier, the project operations must be simulated. For this, it is important to know the administrative procedures that may be used, etc. The systems and procedures that define each project specifically also determine the investment in physical infrastructure, and thus serve to define needs for physical space, offices, parking spaces, and so on.

Suffice it to conduct a simple analysis that determines the influence of administrative procedures on the amount invested and the project costs. Accounting-financial procedures and systems, systems for information, planning and staff budgeting, procurement, credit, invoicing and many more are associated with specific operating costs.

The environmental impact study

This study is related to the tendency to include in the analysis the approach to procedures aimed at ensuring that the end consumers obtain constant environmental improvements in respect of the products and services to be provided thanks to the investment, and that any damages to the environment can be repaired at a lower cost in the future. This part of the study should incorporate, to the extent possible, the ISO 14000 standard.

The evaluators must consider the complete production cycle that the investment will generate, and determine the environmental impact of the supplier of inputs for extraction, production, transportation and packaging of the raw material, the product distribution system, waste management, visual impacts, increased noise levels (noise pollution), etc. These factors should not only be considered, they should also be measured and any mitigating and compensatory mechanisms required established.

The mitigating, compensatory and impact-reducing mechanisms have costs that should be quantified and included in the project's spending stream.

The economic – financial study

This is the last step in the prefeasibility study. The aims are to compile and systematize the monetary information obtained in the previous stages, prepare analytical tables and collect additional data so as to assess the project with a view to determining its profitability.

The project evaluation is based on the estimated cash flow resulting from revenues and costs. This is measured using various criteria that complement each other. Because it is impossible to confirm the assumptions established for the purposes of financial forecast and project design, there is a risk inherent in the investment.

Lastly, an important variable is that related to financing. When its effect on cash flow is included, the return on the resources invested in the project are measured. At this point there is sufficient information to decide whether to accept or reject the project, in the light of the project and the investor cash flow.

The final level of analysis is that of **feasibility**. This is the most in-depth level. It is based on all the considerations established in the prefeasibility analysis, which are scrutinized down to the smallest detail. This process of detailing and refining the information in no way modifies the decision taken, insofar, of course, as the data and calculations used in the prefeasibility analysis are reliable and have been properly evaluated.
Figure 1.4 sums up the project evaluation process.

Figure 1.4 Project evaluation process

Objectives of project evaluation

Project evaluation has three basic objectives:

- to ascertain that a potentially unsatisfied market exists and that it is viable from the operational point of view to enter that market and produce or supply the good or service being studied;
- to demonstrate that it is technologically possible to meet the need detected;
- to demonstrate that it makes economic sense to implement the project in the conditions established.

The key thing to bear in mind is that every project starts as an idea and responds to a need. If there is no need, there is no project. Once it has been demonstrated that the project is feasible, the next step is to start implementing it. It is nevertheless important to remember that the efforts being made in connection with project administration will also serve that purpose.

2. Project evaluation and predictive analysis

The main instrument used for project evaluation is the net cash flow. The cash flow is based on estimations, made in the light of a series of assumptions, of how the variables it comprises will behave in the future. The estimations are therefore never accurate, because they depend on whether or not those assumptions are met.

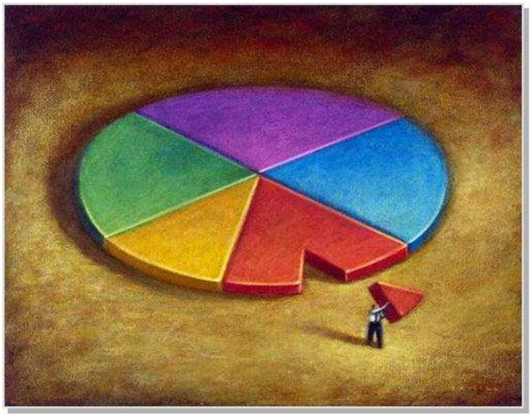
The term "predictive" refers to the rational analysis of what will happen. Often this concept is associated with that of "forecast". Strictly speaking, a forecast is the prediction of an economic variable, such as gross domestic product (GDP); this may depend on the values given to the analysis variables, in which case the forecast is said to be conditional, because it depends on those values being met.

From the point of view of the investor, forecasting the future serves to enhance the decision-making process, while for the business's administration, it helps anticipate future conditions and acts as a starting point for planning operations that will influence the course of future events. This same principle applies to regulatory bodies: predictive analysis allows them to evaluate how key variables such as demand will respond to changes in the market that may be the outcome of regulatory measures such as adjustments to the range of ceiling prices.

Predictive analysis implicitly encompasses risk and associated return. Risk is the possibility that losses will be incurred when the investment is made. Different investments have different levels of risk. For example, bank deposits and government bonds are considered to be low-risk. In this sense, there is a direct relationship between risk and return: the higher the investment risk, the greater the potential anticipated return, to compensate the investor. Predicative analysis is therefore very useful for evaluating projects considered to be high-risk, including those relating to universal access/service. The latter are usually associated with regions that are geographically hard to reach and have low population densities, impeding a return on the investment.

Predictive analysis is based on statistical analysis, and therefore, before any further discussion of the former and its application, it is important to review a number of basic statistical concepts.

Descriptive statistics provide a quantitative abstraction of a phenomenon so that its characteristics can be known. Data series are analysed in order to establish conclusions pertaining to the behaviour of those variables. They present information in user-friendly, usable and understandable form. Descriptive statistics paint the situation as it is, and thus serve to describe and sum up observations on a matter, a phenomenon or a research problem. They are calculated using data from a sample or a population.



Inferential statistics provide a quantitative appreciation of a phenomenon. The deduction or inference must be proven to be accepted as reliable and valid. What happens is that a conclusion is inferred based on what has been determined for the sample and is then extrapolated to the entire population.

Population is the total set of individuals, objects or measures that share several observable characteristics in one place and at a specific point in time.

A sample is a subset that is reliably representative of the population. There are different kinds of samples. The kind selected will depend on the desired quality and representativity of the population study.

- *Random*: when individuals are randomly selected and every individual has the same chance of being included.

- *Stratified:* when the population is divided into strata or subsets according to the variables or characteristics that are being researched. Each stratum has to be in proportion to the population.
- *Systematic*: when a pattern or criterion is used to select the sample. For example, one of every ten families identified will be interviewed.

Figure 2.1 illustrates the concepts of population and sampling.

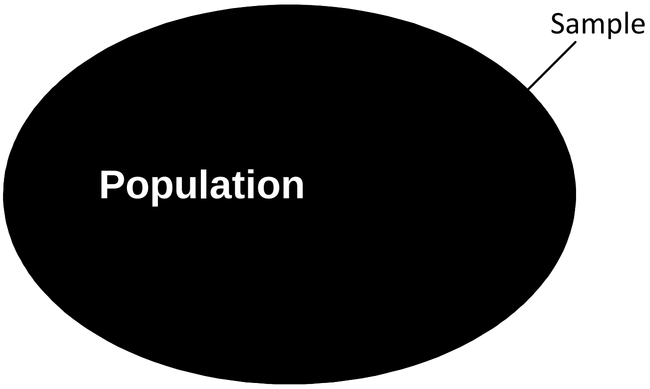


Figure 2.1 Population and statistical sample

Estimation and/or forecasting methods

The future behaviour of a variable, a group of variables or a specific situation, as in the case of project analysis, can be approximated in various ways. The most relevant are briefly discussed below.

Game theory

This method is used to predict economic outcomes on the basis of strategic movements by rival agents that foresee their opponents' reactions. The main objective of applying game theory is to determine the best strategy for each player, which will be the strategy that maximizes the anticipated benefit.

Scientists have studied games since time immemorial, developing mathematical theories and models. Statistics is a branch of mathematics that emerged from the very calculations used to devise winning strategies for games of chance. Concepts such as probability, weighted average and standard deviation were coined by mathematical statisticians and are applied to the analysis of games of chance or in the frequent social and economic situations in which decisions must be made and risks taken amid random components.

These concepts are used for forecast and control purposes, and serve to enhance understanding of how the real situation being studied will behave more effectively than direct observation.

One practical application of this method of prediction would be the outcome of what is known as constant sum games. In such games every combination of strategies results in the same sum of payments (or utilities) to every player. All situations of exchange that do not allow the creation or destruction of resources are constant sum games.

These games apply only in certain conditions:

- when there are many factors affecting not only price, but also quality and service;

- when it is not possible to determine the risks accurately beforehand;
- when it takes a long time to produce the articles that will be bought;
- when production is frequently interrupted because of many changes.

Most real examples in business and politics, like the prisoner's dilemma, are non-zero-sum games, because some of the outcomes have net results that are above or below zero. In other words, a win by one player does not necessarily means a loss by another. For example, a business contract ideally involves a positive-sum outcome in which every opponent ends up in a better position than if he had not entered into the contract.

Negotiation is appropriate whenever there is an area of agreement. That area exists when there are concomitant acceptable outcomes for both parties. This is why negotiation is appropriate in the case of the regulator intervening to set prices for interconnectivity services, when the operators cannot reach agreement within the statutory deadline.

One game the regulator can use to mediate conflicts between operators is the **trust game**. The trust game is similar in several ways to the prisoner's dilemma. It implies a sequential game, however, in which one player first decides the level of trust he has in the second player. In the case at hand each player is an operator.

The greater the trust, the higher the payout for the second player, who has to decide whether or not the trust corresponds to a decision that is mutually beneficial to both players.

A classic example has two players starting a game with a pot of 10 dollars each. Player 1 has to decide how much of his 10 dollars to send to player 2. The amount sent triples on its way to player 2. Once player 2 has received the triple amount, he has to decide how much to return to player 1. The amount returned is not tripled.

Clearly, one round of this game has a Nash equilibrium of (USD 10,USD 10) in which player 2 should keep the entire amount received, and yet player 1 has no incentive to send money to player 2. The social optimum of this game is achieved when player 1 sends his whole pot to player 2, generating a total payout of 3 x USD 10 + USD 10 = USD 40.

The aim is to generate an exercise that shows the operators that specifically with regard to interconnectivity, mutual cooperation is the best solution, because it allows both parties to maximize their benefits. If one betrays the other, there are no incentives for the future generation of mutually beneficial business dealings.

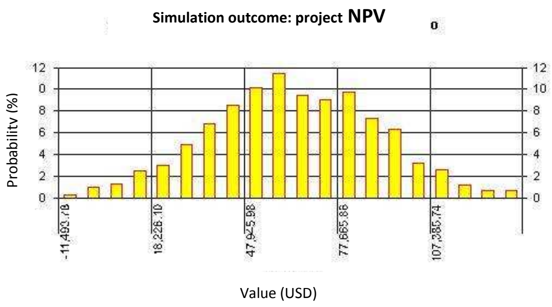
In terms of payout, the trust game has a similar structure to the prisoner's dilemma, as the reward for mutual cooperation is higher than that for mutual betrayal. The repeated trust game is potentially very stable, because it gives the maximum reward to players that have established a practice of trust and mutual cooperation. In spite of that, the problem is that the players may not be aware that it is in their interest to cooperate, or do not foresee that the negative reciprocity of the other player undermines reputation, cooperation and trust in the process.

The Monte Carlo simulation

The Monte Carlo simulation is a technique that combines statistical concepts (random sampling) with the capacity of computers to generate pseudo-random numbers and automate calculations.

The key to this kind of simulation is to create a mathematical model of the system, process or activity to be analysed, identifying those entry variables (model inputs) whose random behaviour determines the system's overall behaviour. Once those random inputs or variables have been identified, an experiment is carried out that consists in: (1) using the computer to generate random samples (tangible values) for those inputs; and (2) analysing how the system behaves towards the values generated. Repeating the experiment **n** times results in **n** observations on how the system behaves, which will be useful for understanding how it functions. Obviously, the higher the number of **n** experiments performed, the more accurate the analysis will be.

This technique stems from the work done by Stan Ulam and John von Neumann in the late 1940s at the Los Alamos laboratory, when they were researching the random movement of neutrons. Since then, the Monte Carlo simulation has been applied in countless fields as an alternative to exact mathematical models or even as the only means of solving complex problems.



The Monte Carlo simulation is present in all fields in which random or probabilistic behaviour plays a fundamental role. Indeed, the name Monte Carlo comes from Monaco's famous capital, home to countless casinos, where chance, probability and random behaviour are a lifestyle!

The latest versions of Excel programmed using Visual Basic for Applications can create authentic simulation applications for the end user. Various Excel add-ins available on the market have also been designed to perform Monte Carlo simulations; the best-known are @Risk, Crystall Ball, Insight.xla and SimTools.

The method for applying Monte Carlo simulations is simple. In the example below, the method is applied to project evaluation by analysing the behaviour of net cash flows.

1. First calculate the anticipated value of cash flows for every year by multiplying the probability of each cash flow and then adding up the products. For example, in year 1 (100 + 400 + 900 + 1 600) the sum is USD 3 000.

2. Next discount the cash flow for year 1 at a rate of 4% for one year, the cash flow for year 2 at a rate of 4% for two years, and lastly the cash flow for year 3 at a rate of 4% for three years. The result will be the current value from which the investment of USD 5 000 is deducted to obtain the current net value.

3. To obtain the standard deviation in cash flow for each year, relate each cash flow to the average or anticipated value for each year. Those deviations are squared and multiplied by the probability of occurrence. Repeat the process for every cash flow in year 1 and then add.

4. Apply the total standard deviation formula.

5. This standard deviation implies discounting the variance (the square of the standard deviation) for each year at an annual rate of 4%; the result is the square root of the sum of the current values of those variances to obtain the total standard total deviation.

6. On the basis of the net current value obtained, incorporate the area under the normal curve using the standard deviation obtained under point 5 above.

7. For example, the probability of obtaining a net negative current value is expressed as follows:

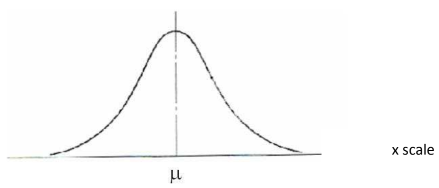
$$Z = \frac{0 - \text{net current value}}{\text{Standard deviation}}$$

Method for calculating z values

Before the method for calculating z values can be discussed, several basic concepts have to be defined. One is normal distribution, as explained by Aguirre, Betancourt and Mayo in their notes.[1]

Chief characteristics

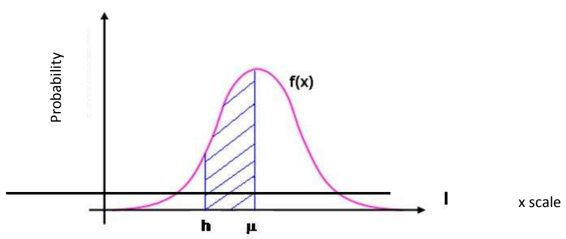
1. It is bell-shaped.



2. It is symmetrical with respect to the arithmetical average of the distribution, which is situated in the middle, i.e. the number of values is the same to the right and to the left of centre.

3. It extends from -∞ to + ∞, i.e. it never touches the x axis.

4. The total area of the curve comprises 100% of the values considered.



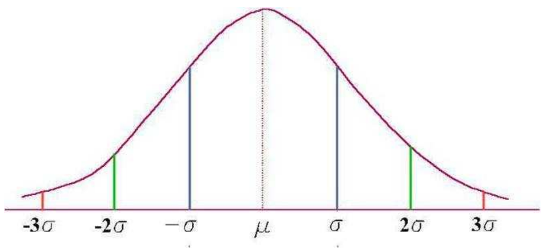
5. The area under the curve between two reference points on the x axis

[1] O. Aguirre, J. Betancourt, O. Mayo. *Estadística Básica Aplicada*, National Polytechnical Institute, Mexico.

corresponds to the probability that a variable with a normal distribution will have a value between the two.

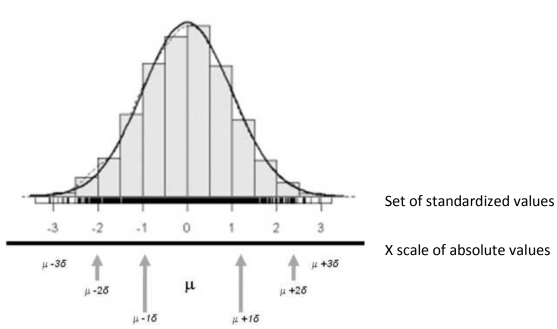
6. The probability will always have an interval value defined between the arithmetic average and the standard deviation.

7. The area under the curve is a determination of the number of standard deviations between the arithmetic average and a reference point to the right or left.



As indicated above, to make it easier to obtain the area under the standard curve the curve is standardized. It is transformed into a set of "z" values with respect to the x axis. If this were not done, on a scale of absolute values, integrated calculus would be required.

$$Z = \frac{x - \mu}{\sigma}$$



Whereby:

x = reference value

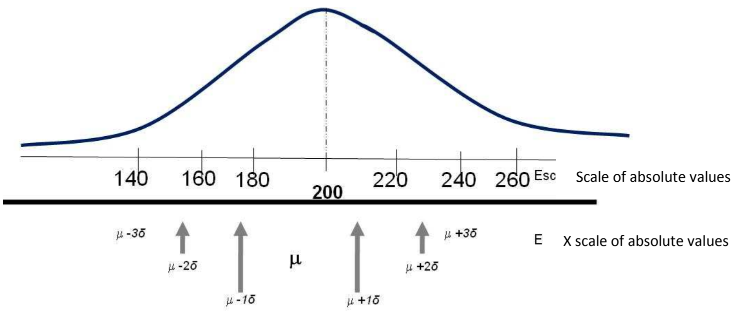
δ = the standard deviation of the entire population

μ = the arithmetic average of the entire population

Application to the telecommunication sector

Step-by-step example of consumption analysis using the Monte Carlo method

A market study has determined that average monthly consumption of short messages is 200, with a standard deviation of 20. On the basis of that data, the normal distribution with a scale of absolute values is illustrated as below.



1. The various values are standardized using the formula:

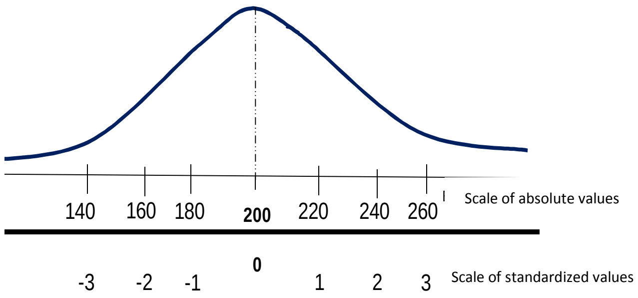
$$z = \frac{x - \mu}{\sigma}$$

2. The reference values are those noted in the scale of absolute values in the above figure, namely:

$x_1 = 140$
$x_2 = 160$
$x_3 = 180$
$x_4 = 200$
$x_5 = 220$
$x_6 = 240$
$x_7 = 260$

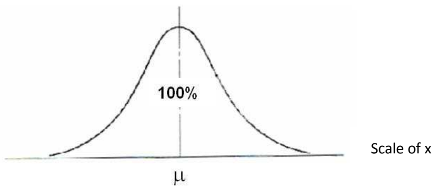
3. If the arithmetic average is μ = 200 and the standard deviation is σ = 20, then:

$$Z_1 = \frac{x_1 - \mu}{\delta} = \frac{140 - 200}{20} = \frac{-60}{20} = -3$$
$$Z_2 = \frac{x_2 - \mu}{\delta} = \frac{160 - 200}{20} = \frac{-40}{20} = -2$$
$$Z_3 = \frac{x_3 - \mu}{\delta} = \frac{180 - 200}{20} = \frac{-20}{20} = -1$$
$$Z_4 = \frac{x_4 - \mu}{\delta} = \frac{200 - 200}{20} = \frac{0}{20} = 0$$
$$Z_5 = \frac{x_5 - \mu}{\delta} = \frac{220 - 200}{20} = \frac{20}{20} = 1$$
$$Z_6 = \frac{x_6 - \mu}{\delta} = \frac{240 - 200}{20} = \frac{40}{20} = 2$$
$$Z_7 = \frac{x_7 - \mu}{\delta} = \frac{260 - 200}{20} = \frac{60}{20} = 3$$

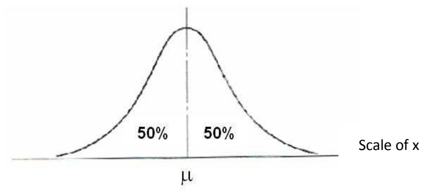


4. The transformed values are used to plot the corresponding standardized values under the scale of absolute values and to observe that the two are equivalent.

5. There is a logical relationship between the scale of absolute values and the z scale of standardized values, which show the distance in standard deviations of a specific "xi" reference value to the arithmetic average.

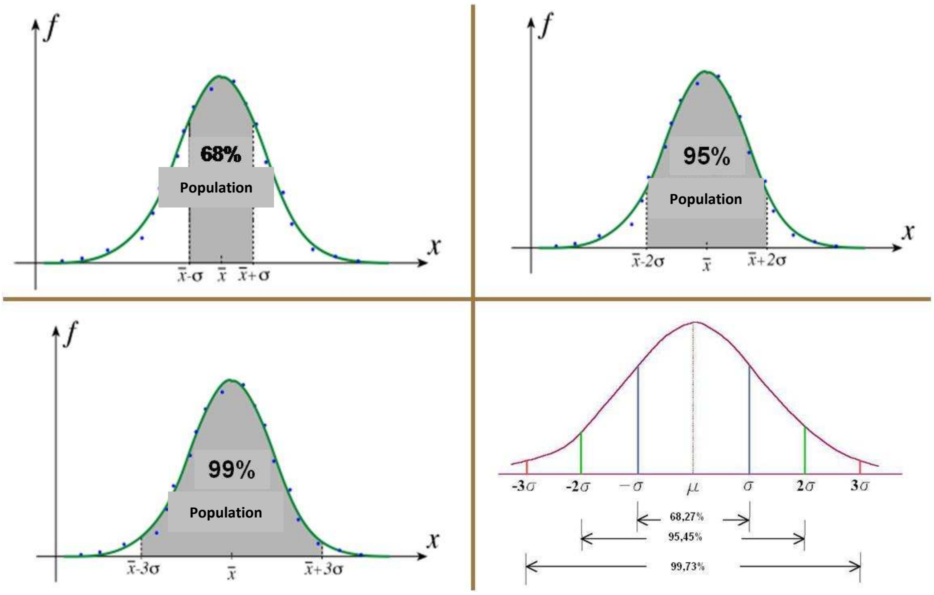


6. To obtain the area under the standardized normal curve, i.e. the probability, it must be remembered that the distribution encompasses 100% of the values, and that either side therefore contains exactly 50% of those values with regard to the centre (arithmetic average).

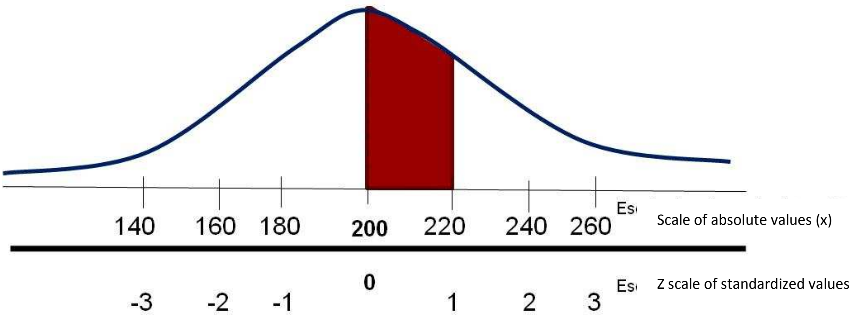


Application of probabilities

The same example can be used to determine the probability of there being a customer who consumes between 200 and 220 messages per month, considering that average monthly consumption the previous year rose to 200, with a standard deviation of 20.



In graphic terms, this means identifying the area under the curve highlighted in red.



When standardizing the values:

Como = Since   entonces = then

Como x_1 = 200 entonces: $$Z_1 = \frac{x_1 - \mu}{\delta} = \frac{200 - 200}{20} = \frac{-0}{20} = 0$$

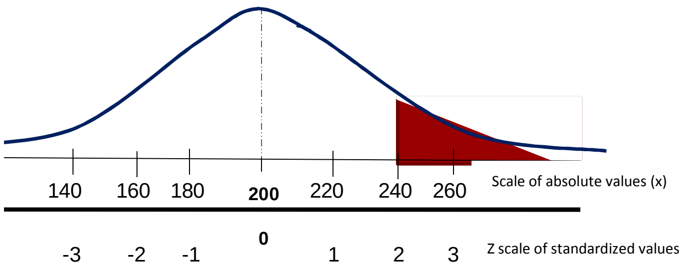
Como x_2 = 220 entonces: $$Z_2 = \frac{x_1 - \mu}{\delta} = \frac{220 - 200}{20} = \frac{20}{20} = 1$$

Then look for the area under the curve that lies between 0 and 1 of the Z scale of standardized values, or the shaded zone in the above figure.

($0 \leq Z \leq 1$) = area under the normal curve between 0 and 1 = 0.3413 or 34.13%

The probability of there being a customer who consumes between 200 and 220 messages per month is 34.13%.

Using the same example, what is the probability of there being a customer who consumes more than 240 messages per month?



$$Z_1 = \frac{x_1 - \mu}{\delta} = \frac{240 - 200}{20} = \frac{40}{20} = 2$$

Area under the curve

(2 ≤ Z) = area under the normal curve greater or equal to 2

0.5 – (0 ≤ Z ≤ 2)

The area situated between 0 and 2 is subtracted from half of the normal curve.

(0≤ Z ≤ 2) = 0.47725

The remainder is applied:

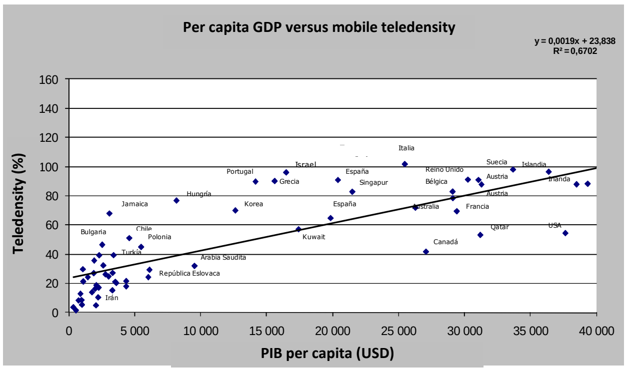
0.5 – (0 ≤ Z ≤ 2) = 0.5 – 0.47725 = 0.02275 or 2.275% or 2.3%

The probability of there being a customer who consumes more than 240 messages is 2.3%.

Regulatory application

Operators logically set their prices in the light of demand, peak times and, less often, equipment capacity, and the regulator must therefore ensure optimum service grounded in economic well-being. The above application allows the regulator to give operators guidance as to the probability that their demand projections are correct, i.e. it can demonstrate, using historical information, whether the size of the operator's service networks or platforms is in line with market behaviour, when the data behave normally. This method can be applied to any variable (voice, data or video consumption, etc.).

3. Analysis of time series



A sequence of values observed over time, i.e. ordered chronologically, in the broad sense is a time series. It is hard to imagine a branch of science in which there are no data that can be considered as a time series.

There are various reasons for analysing time series, chief among them to produce a forecast, control a process, simulate processes or generate new physical or biological theories.

Obviously, even though the future value of a time series cannot be predicted with absolute certainty, if its study is to be of interest the result cannot be completely random either. If there is any regularity in the series' behaviour over time, it can be used as a model and, ultimately, for forecasts. The search for regularity and patterns has always been one of the basic tasks of science, and symmetries are often discovered that underpin forecasts of how phenomena will behave, even before the reason for or cause of the regularity has been understood.

Therefore, if patterns of regularity can be found in different sections of a time series, those sections can be described using models based on distributions of probability. The ordered sequence of random variables X(t) and its associated distribution of probability is called the stochastic process. A stochastic process is therefore the mathematical model for a time series.

One important concept in this field is that of stationary processes. For example, telephone use by a group of customers over a number of years will tend to rise, even though there may be fluctuations. In simplified fashion, it is possible to indicate that a series is stationary when it is in statistical equilibrium, in the sense that its properties do not vary over time, and no trends can therefore exist.

A process is non-stationary if its properties vary over time, as in the case of climate. In view of the above, it is important for the regulator to analyse the information provided by the operator, to evaluate data trends using statistical techniques and to correct as required, using regression and autoregressive models. There are tools for this, such as models for measuring the effect of inflation, exchange rates, monetary policies and other factors affecting operators. It is important to know the historical information to forecast market behaviour. The data can prompt a business to expand and justify an investment. In some cases an operator can gauge future capacity. This tool is an additional element for assessing such events, protecting the market from possibly over-inflating and hence preventing overpricing.

Classic time series models

The first thing to do when analysing a time series is to draw up a chart indicating changes in the variable over time, as illustrated in Figure 3.1 below.

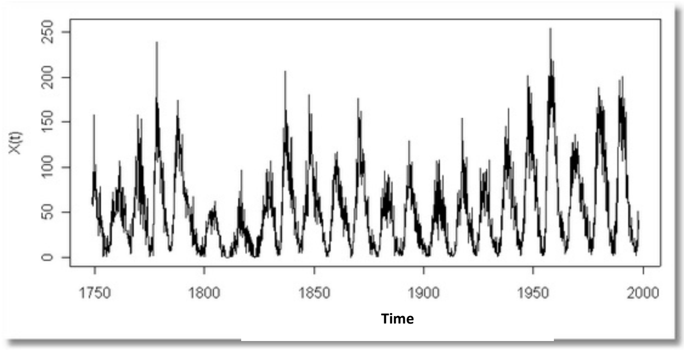


Figure 3.1 Representation of a time series

The next step is to determine whether the sequence of values is completely random, or whether some pattern can be detected over time, for only in the latter case can the analysis continue.

The traditional method for studying time series is basically to break the series down into various parts: trend, seasonal or periodic variation, and other irregular fluctuations.

- **Trend.** This is the general direction of the variable during the period of observation, i.e. long-term change in the series average.

- **Seasonality**. These are periodic fluctuations in the variable over relatively short periods of time.

- **Other irregular fluctuations**. Once the trend and the cyclical variations have been extracted from the series, a series of residual values will remain that may or may not be totally random. The analysis has to start over, because the point now is to determine whether this time sequence of residual values is or is not to be considered as purely random. Figure 3.2 below gives an example of a time series reflecting the various components mentioned.

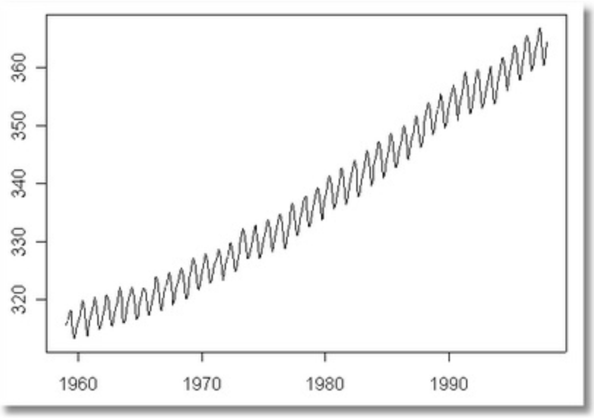


Figure 3.2 Time series with a trend

Analysing trend

The graphic representation of the time series will already give an idea of whether there is a trend, but the trend will not always be expressed as clearly as in Figure 3.2. It is less noticeable, for example, in Figure 3.3.

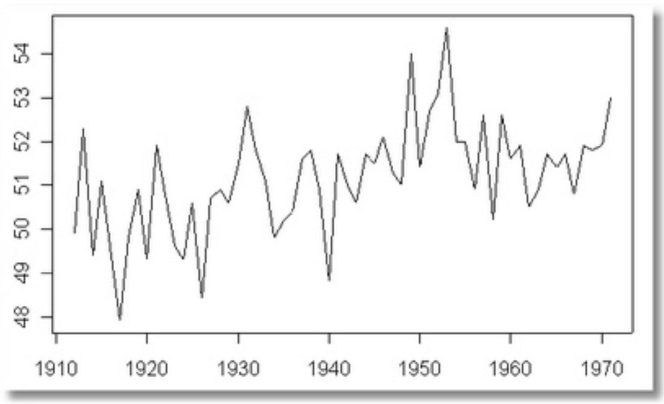


Figure 3.3 Time series with no marked trend

The means most often used to detect and eliminate the trend from a series are based on the application of data filters. A filter is nothing more than a mathematical function which, when applied to the values of a series, produces a new series with specific features. Such filters include moving averages.

A moving average is calculated, for each point, as an average for the same number of values on either side of that point. A three-point moving average is therefore calculated as follows:

$$m(x_t) = \frac{x_{t-1} + x_t + x_{t+1}}{3}$$

A four-point moving average is calculated as follows:

$$m(x_t) = \frac{(x_{t-2}/2) + x_{t-1} + x_t + x_{t+1} + (x_{t+2}/2)}{4}$$

When the number of points in the moving average is even, the values at either end are divided in two.

There are other means of extracting the trend, such as polynomial adjustments and exponential smoothing. One kind of filter that is particularly useful in eliminating trend applies differences to the series until it is stationary. A first-order difference is obtained by subtracting two contiguous values:

$$\nabla x_{t+1} = x_{t+1} - x_t$$

If the series is redifferentiated by subtracting the new contiguous values, the outcome is a new, smoother series.

$$\nabla^2 x_{t+2} = \nabla x_{t+2} - \nabla x_{t+1}$$

When a classic breakdown process implying moving averages is applied to the data in Figure 3.3, the following series is obtained:

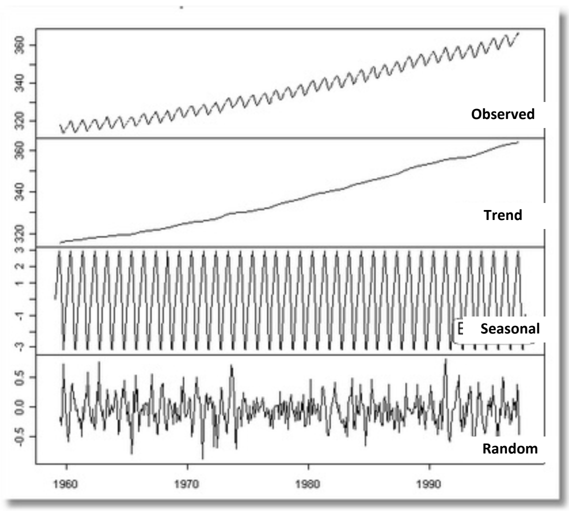


Figure 3.4 Breakdown of a time series

The autocorrelation function is of great interest in the analysis of time series and is used to analyse seasonality. It measures the correlation between the values in the series separated by a lapse of time k.

The formula for the simple correlation coefficient, given N pairs of y, x observations, is:

$$r = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sqrt{\sum (y_i - \bar{y})^2 \sum (x_i - \bar{x})^2}}$$

In the same way, where there is a time sequence of N observations x1...xN, N-1 contiguous pairs of observations can be formed (x1, x2), (x2, x3), ... (xN-1, xN) and their correlation coefficient calculated. This is called the first-order autocorrelation coefficient and written r1.

By analogy, pairs can be formed with points separated by a distance of 2, i.e. (x1, x3), (x2, x4), etc., and the new, second-order autocorrelation coefficient calculated. Generally speaking, whenever pairs with points separated by distance k are formed, the k-order autocorrelation coefficient can be calculated.

As with the simple linear correlation coefficient, a standard error, and hence a confidence interval, can be calculated for the autocorrelation coefficient.

The autocorrelation function is the set of autocorrelation coefficients rk from 1 to a maximum that cannot exceed half of the values observed. It is hugely important for studying series seasonality, because if seasonality exists, the values separated by intervals equivalent to the season should be in some way correlated. In other words, the autocorrelation coefficient for an interval equal to the seasonal period should be significantly different from 0.

The partial autocorrelation function is similar to the autocorrelation function. In the k-order partial autocorrelation coefficient, the correlation between pairs of values separated by distance k is calculated, but the effect of the correlation produced by previous intervals in k is eliminated.

Figure 3.5 shows a typical partial autocorrelation function in which the confidence intervals are marked to help detect significant values and whose position on the x axis indicates the probable presence of a seasonal factor for this interval.

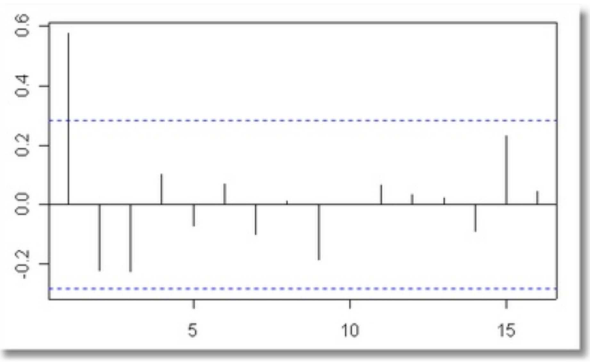


Figure 3.5 Partial autocorrelation function

Causal models

As discussed in section I, causal models link a dependent variable to one or several explanatory variables, for example the number of sales (dependent variable) and the population (explanatory variable). It is also possible to relate the amount sold to price, income or other variables.

These are known as econometric models. "Econometrics" literally means economic measurement and is an amalgam of economic, mathematical and statistical theory. According to Gujarati, the method can be summed up in the points below.

Method

1. Specification of the mathematical model for the theory
2. Specification of the econometrical model for the theory
3. Statement of the theory or hypothesis
4. Data collection
5. Estimation of the parameters of the econometric model
6. Hypothesis testing
7. Forecast or prediction
8. Use of the model for control or policy

For example, the statement of the theory or the hypothesis can be explained by Keynes's theory of consumption. Keynes's fundamental psychological law of consumption states that men and women will generally and on average be willing to consume more as their income rises, but not at the same rate as the rise in income.

Keynes postulated the marginal propensity to consume, i.e. the rate of change in consumption generated by an additional unit of income.

The idea is to demonstrate that the model must have a theoretical foundation, be logical. A model cannot be generated with one occurrence. It must be maintained, it has to be valid and demonstrable.

- The specification for the mathematical model of consumption is as follows:

Keynesian function:

$$C = \beta 1 + \beta 2X \qquad 0 < \beta 2 < 1$$

Whereby:

C= Consumption

X= Income

ß1 and ß2 = the model's parameters, respectively intercept and residual coefficient. ß2 is the marginal propensity to consume.

This is the deterministic mathematical model. The resulting equation is:

C = ß1 + ß2X

In this equation, the sign to the left of the equal sign is known as the dependent variable and the signs to the right as independent or explanatory variables. In the Keynesian function of consumption, consumption is the dependent variable and income the explanatory variable. However, the mathematical model is limited for econometrics because of the exact or deterministic relationship between consumption and income. This is why an econometric model of consumption must be specified.

- Specification of the econometric model of consumption

Econometric model:

C = ß1 + ß2X + u

u = is the disturbance or error term, it is a random or stochastic variable. This implies that u has clearly defined probabilistic properties. In our branch, we work with variables that are affected by human behaviour, which is why they are not exact.

For example, if we analyse the consumption/income relationship in a sample of 500 families and plot the result on a chart, we would not expect all the observations to be exactly on the line, because spending is also affected by other variables, not just income, such as size of the family, age and religion.

The econometric model, technically known as the linear regression model, can be plotted as follows:

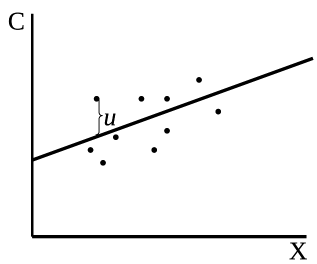


Figure 3.6 Simple linear regression model

- Data collection

To estimate our econometric model we need to obtain numerical values for the betas, and for that we need data. Gujarati's example of the method, shown below, illustrates this.

PERSONAL CONSUMPTION AND GDP		
YEAR	C	X
1980	2447	3776
1981	2476	3843
1982	2503	3760
1983	2619	3906
1984	2746	4184
1985	2865	4279
1986	2969	4404
1987	3052	4539
1988	3162	4718
1989	3162	4718
1990	3223	4838
1991	3260	4877

- This information should allow us to obtain beta values. The calculation is explained in the section on least squares and their application in Excel using regression analysis.

- The estimated function is thus:

$$\hat{C} = -231.8 + 0.7194\ X$$

The cap on the C indicates that it is an estimated value.

The coefficient of residual B2, or the marginal propensity to consume, was about 0.72, suggesting that for the period under consideration, an increase in one dollar would lead on average to an increase of 72 cents in spending. This is an average because the consumption/income relationship is inexact.

- Hypothesis testing

According to the positivists, a theory or hypothesis that cannot be verified using empirical evidence is inadmissible in scientific research.

- Projection or forecast

If the model selected confirms the hypothesis or the theory being considered, it can be used to forecast the value of dependent variable C based on the future value of the explanatory variable. For example:

If real GDP is expected to amount to USD 6 billion in the coming year in a given country, what is the forecast for consumer spending?

$$\hat{C} = -231.8 + 0.7196\ (6)$$

= 4084.6

In economic terms, it can be estimated that consumer spending will be approximately USD 4.085 billion in the coming year.

This type of analysis is very useful in the regulatory field and for telecommunication operators in general, because the analysis of minute consumption series can be used to estimate future traffic on the networks and hence provide reliable statistical inputs for the estimation of, for example, investment and interconnection costs.

Least squares

To establish the algebraic derivation of the least squares regression model, it must be indicated that for an estimated value there exists an error i-th (*ei*) between the coordinates "x" and "y". This is expressed:

yi = α + βxi + ei thus error i-th is: ei = yi - α - βxi

The point is to minimize the error towards zero (0), which means all the errors must be added in such a way that they express the absolute average deviation of the estimated value, expressed as:

yi = α + βxi as an estimated datum, and y = + x as a function

The observations remaining above the line have a positive residual, those remaining below it have a negative residual. If the line is geometrically equidistant from the observations above and below it, then the sum of the positive residuals (ei) is equal to the sum of the negative residuals (ei).

∑ ei (+) = ∑ ei(-) or as: ∑ ei (+) + ∑ ei (-) = 0

In order for the positive residuals not to cancel out the negative residuals, a bit of well-known mathematical sleight-of-hand is applied: each residual is squared, so that the negative values become positive. The absolute value of the residuals can then be computed:

∑ ei^2 = ∑ (yi- - xi) ^2 where the cap (^) on the α and the β indicate "to be estimated".

Infinitesimal calculus is used to minimize the value of the function of the errors obtained. If the first derivative of the function equalized to zero, for the partial derivative to give the function's critical values:

∑ ei^2 and therefore

d∑ ei^2 = (2) (-1) ∑ (yi – α – xβi)

$$\frac{\quad}{d\alpha} \qquad \frac{\quad}{(2)\ (-1)}$$

$\sum ei\hat{}2 \quad (2)\ (-1) \sum (yi - \ - xi)$

Applying the chain rule, the equation is made to equal zero (0) with the result:

$\sum (yi- \ - xi) = 0$

$\sum yi - n\alpha - \beta nX = 0$ (where $\sum Zi = nZ$)

If X is the arithmetic average:

$$\frac{n\beta - n\alpha - \beta nX = 0}{n}$$

$y - \alpha - \beta X = 0$

Where $Y-\beta x$ is less than or equal to alpha (α)

The same procedure is now used to obtain beta (β):

$\frac{\sum ei2 - =}{d\beta}$ $(2)\ (-xi) \sum(yi-\alpha-xi) = 0$

$\sum xiyi- \ \alpha\sum xi-\beta\sum xi\hat{}2 = 0$

$\sum xiyi= \alpha\sum xi+\beta\sum xi\hat{}2$ (if $\sum zi = nZ$)

$\sum xiyi= \alpha nX+\beta\sum xi2$ replacing α gives

$\sum xiyi= (Y-\beta X)nX+\sum xi\hat{}2$

$\sum xiyi= nXY-n\beta X\hat{}2+\sum xi2$

$\sum xiyi- nXY=\sum xi2-\beta Xn\hat{}2$

$\sum xiyi- nYX= (\sum xi\hat{}2-nX\hat{}2)$

$\beta = \frac{\sum xiyi - nXY}{\sum xi\hat{}2-nX\hat{}2}$

$\alpha = Y - \beta x$

This gives α and β of the regression equation.

$Y = \alpha + \beta X$ where α and β are the parameters to be estimated

Things become more difficult when the forecast is for the medium or long term (several years into the future). Here the history of what has happened so far is just one of the items of information, since normally the environment and the conditions will change. The assumptions therefore have to be explicitly stated, not fall back on ongoing predictions or rely only on laws of the past. As a result, forecasts are required that incorporate aspects other than past information, that give precedence to a broad vision of the future, that consider possible alternatives. In other words, it is advisable to go from forecasting a single future to simulating alternative futures, in answer to questions such as, "What would happen if ...?"

It is no easy task, however, to obtain alternative forecasts based on how the environmental factors shaping them are expected to evolve, nor does this solve the problem of decision-making.

Making a decision requires, inevitably, considering risks that no forecast can eliminate. This modest but decisive contribution can provide elements for gauging this uncertain future.

Simple linear regression method

The simple linear regression method, or least squares, is used to determine the equation for a line. It allows the regulator to plot a trend line for a variable of interest - such as consumption, messages, traffic, relative market share - so that it can forecast or project the variable's future behaviour.

Say, for example, that an operator or the regulator needs to forecast the consumption of mobile voice minutes for the period t+1. To calculate that figure, it needs information on historic consumption comprising at least 60 observations. If that information is not available it will have to fall back on whatever information is and use it on the basis of the R^2 result obtained. Excel is used to calculate this, as described below.

Step-by-step estimates using Excel

Using the Windows XP or Windows 7 operating system, click on the Data tab and **right click** on Data Tools. A dialog box will open; select **Customize Quick Access Toolbar...**

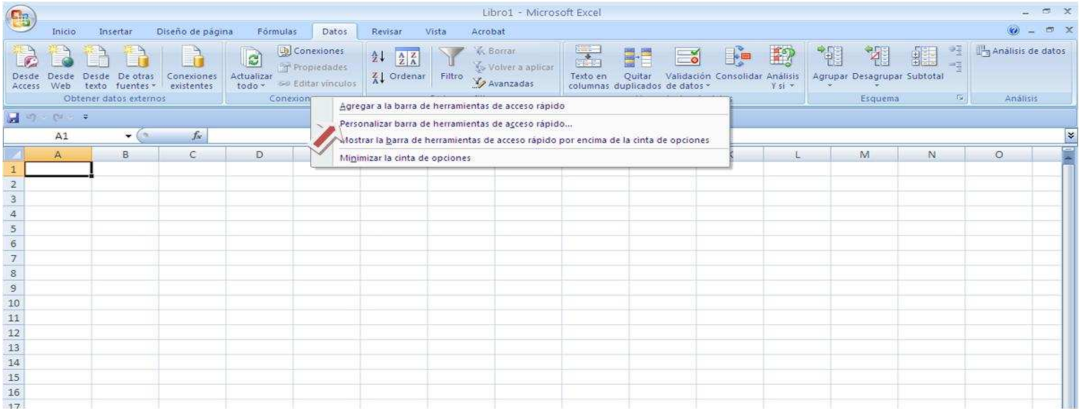


Table 3.1 Excel Data Tool function

Another window will open. Select **Add-Ins** and then **Analysis Toolpak.**

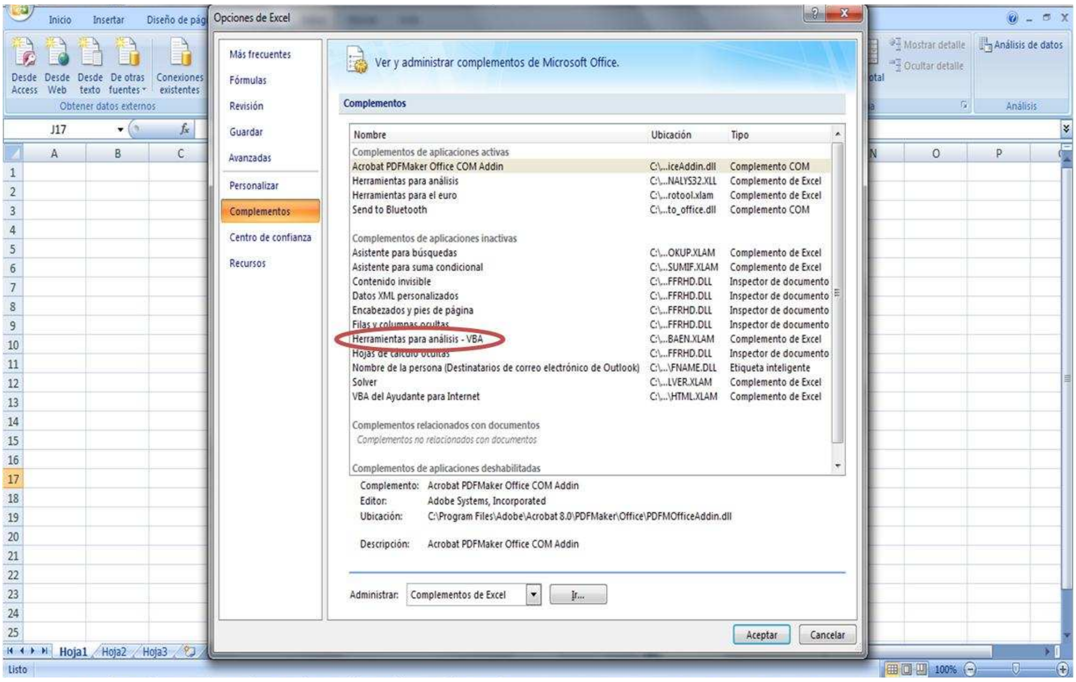


Figure 3.2 Excel Add-Ins function

Click on Go, and when another dialog box appears, tick **Analysis Toolpak.**

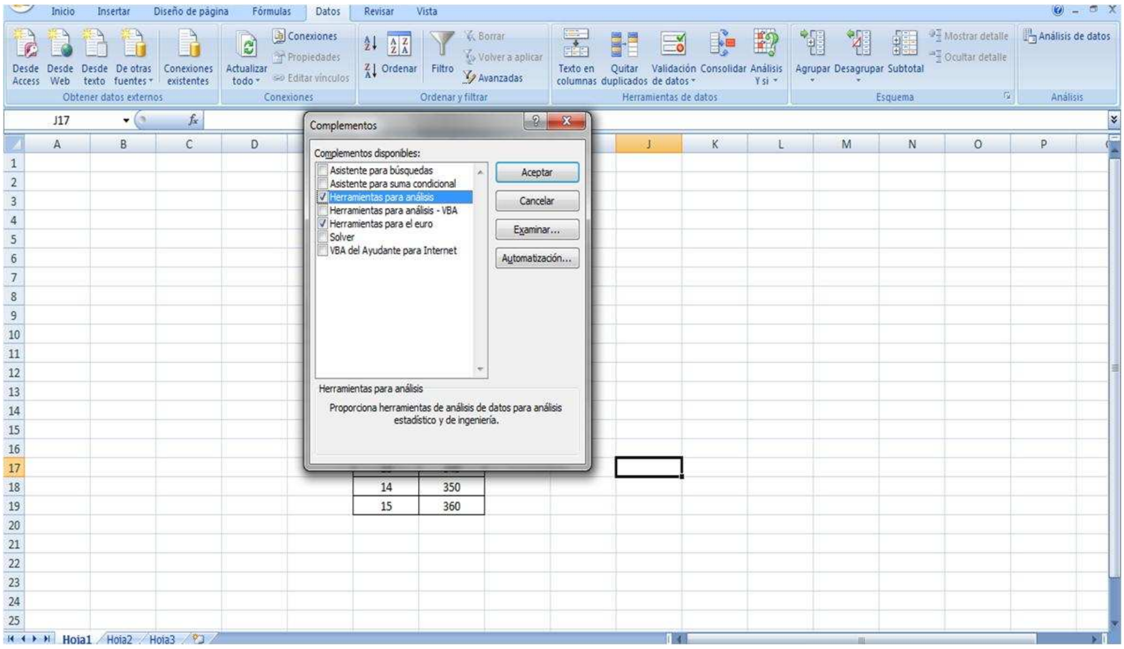


Figure 3.3 Excel Analysis Toolpak function

Click on **OK**. The **Data Analysis** tab will appear in the upper right-hand corner.

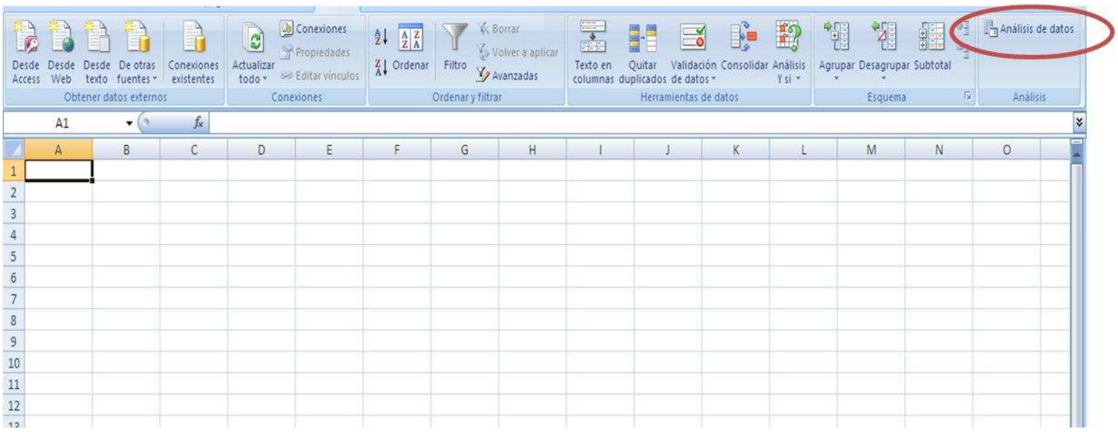


Figure 3.4 Excel Data Analysis function

To run the regression, open an Excel workbook, give it a name and make sure you have the data for the exploratory (x) and the dependent variables (y). In this specific case, the exploratory variable is the years and the dependent variable is average consumption of voice minutes.

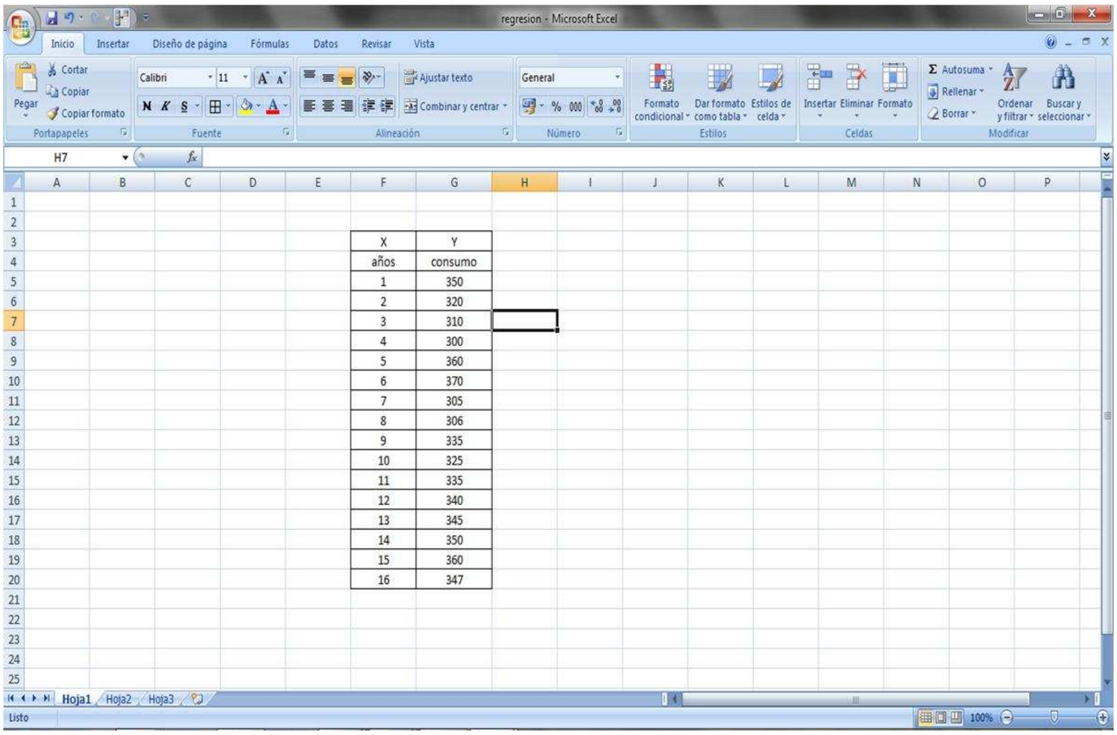


Figure 3.5 Series of historical data for calculating simple regression

Click on **Data Analysis**. Select **Regression** in the dialog box.

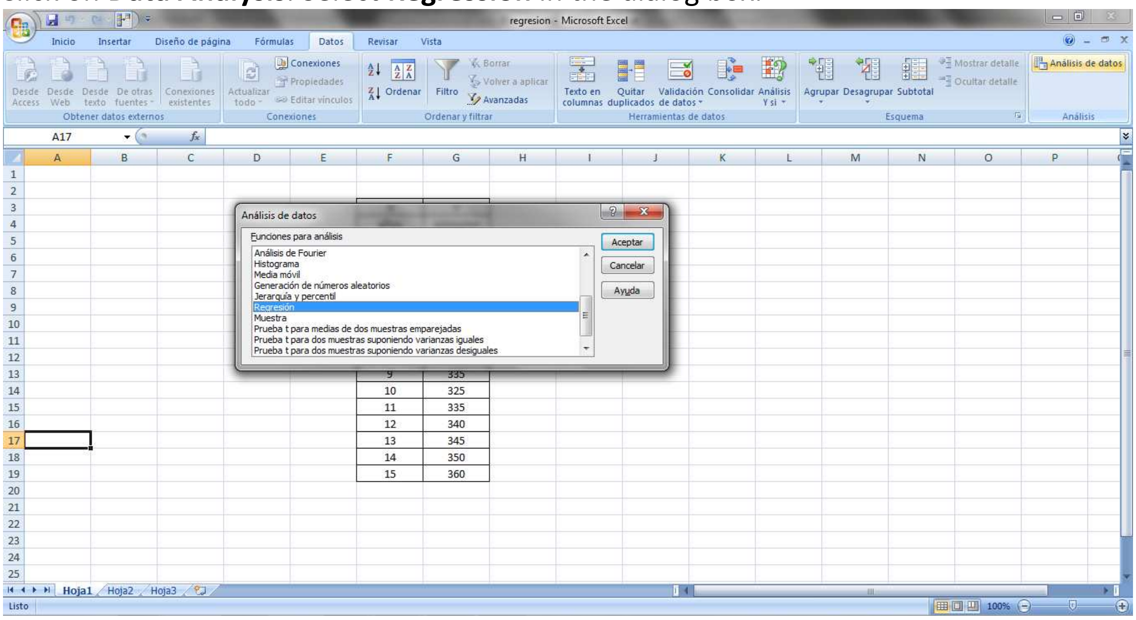


Figure 3.6 Excel Regression function

A new window will open for entering the data, in this case consumption as variable Y, which is dependent, and X, which is exploratory. Enter the data. In this case, the exercise involves a series of 15 data.

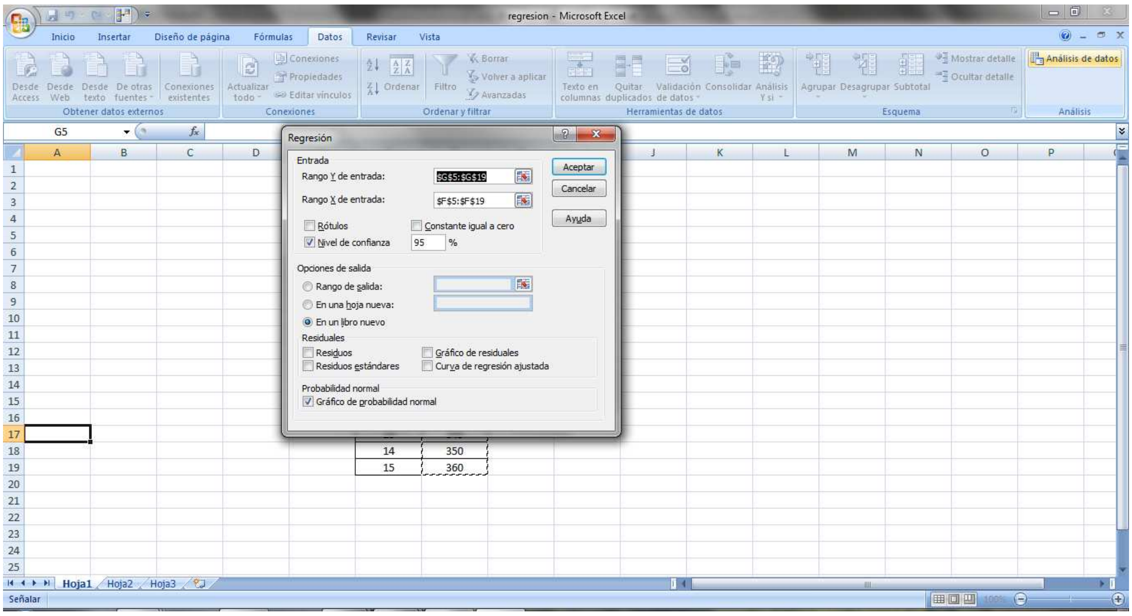


Figure 3.7 Parameters for calculating regression

In the output options, tick **New Workbook**, make sure the option **Normal probability plot** has been ticked, click on **OK**, and you get the following information:

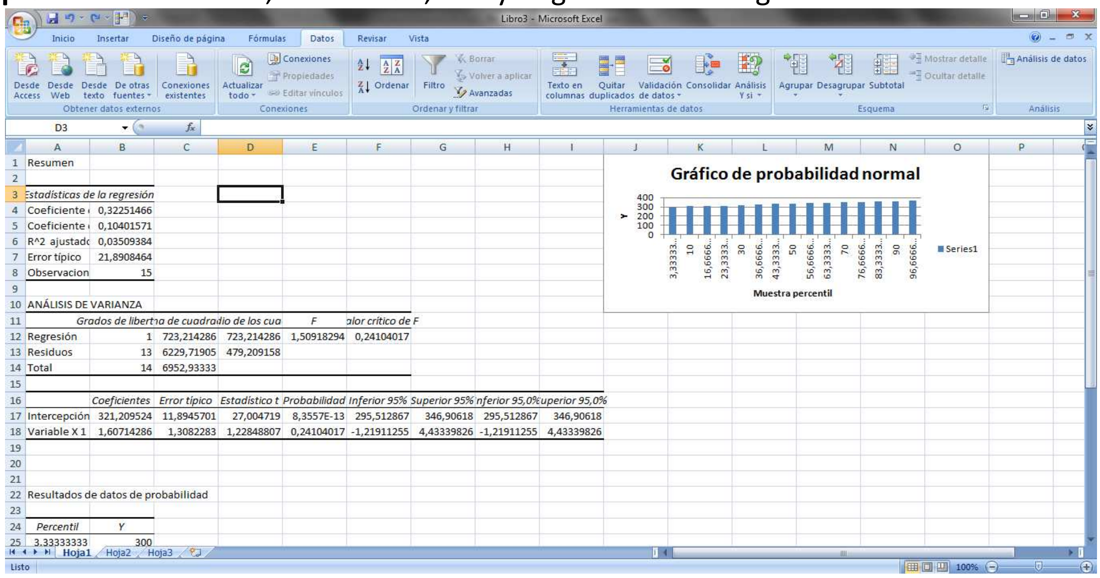


Figure 3.8 Regression calculation outcome

The results are the linear regression coefficients. The coefficient for residual B2 is in square B18, and the intercept in square B17. The resulting equation is:

Consumption = 321,209524+1,60714286X, where X= 16

Consumption = 346,9248 = 347 minutes, meaning that consumption for period 16 is 347 minutes.

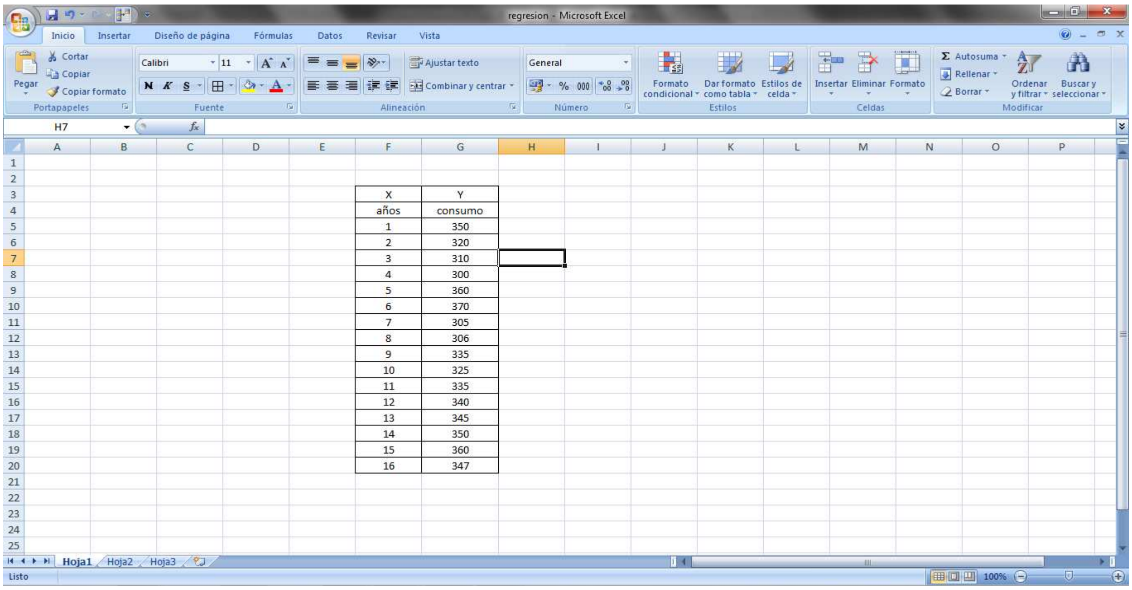


Figure 3.9 Estimate for period t+1

4. Practical application I

Evaluation of universal access/service projects

The various tools described above can be used by the regulator to demonstrate that universal access/service projects can be developed in a manner that makes them profitable to the project implementer.

One of the main barriers to the implementation of plans and projects aimed at developing the services associated with universal access/service is the associated risk. In most cases, the service is to be developed in remote areas that are hard to reach and have low population densities. Those factors signal slow or zero return on investment.

Even when the project is driven by resources administered by the regulatory entity and provided by the various market players, the invitations to tender for project implementation attract few bidders because of the uncertainty surrounding the potential outcome of the project's execution.

The project's potential implementers do not want to invest more than the amount estimated for the project's development, meaning that bearing the cost of the project's financial evaluation is not considered an option when it comes to deciding whether or not to participate in bidding processes of this kind.

The fact of incorporating the project feasibility study into the tender specifications, which shows that on certain premises established as fundamental for the project's proper development and for achievement of the objectives of the project, not only makes it attractive for the operators as a business proposal, it also enables the regulator to start with parameters that will allow it to evaluate the project ex post. On the basis of the estimates in the feasibility study, targets and achievement of objectives can be measured and the efficient allocation and use of resources proven.

In the following case, the tools are applied to a rural public telephony project.

Application to a rural telephony project

It is well-known that in various countries in Latin America rural municipalities do not have access to telephone services. In this section we analyse a project to develop rural telephony. Our analysis is not linked to a specific country or place, and the data and references used should be considered fictitious.

The analysis method developed in this section is real and applicable to any universal service/access project, so long as the main indicators are adjusted for each specific case.

The inhabitants of the rural community of Tierra Blanca do not have access to public telephone services. Communicating with the rest of the country is costly in terms of time, resources and travel. In some cases, additional costs are incurred because a timely communication cannot be established.

As in the majority of universal access/service projects, it is assumed that it is not profitable, from the economic and financial points of view, to supply this kind of service, given the high investment costs, the low population density and the operating costs.

From the market point of view, there is an unmet need that stands in addition to the regulator's goals of universal access/service.

From the technical point of view, there are two alternative solutions: physical links or radio links. In the case of physical links, the communication is established via a copper pair that physically connects the telephone to a telephone exchange. In rural telephony projects specifically, links of this kind are installed in communities situated next to a road or whose population density is equal to or greater than one subscriber every 2 kilometres.

Radio links, for their part, establish the communication through space, using a frequency selected in the light of the number of conversations that will have to be carried simultaneously, distance, the geographical conditions and transmission quality.

An additional element to consider in this case is the possibility to apply the solution in conjunction with single-channel equipment or gateways. In the first instance, two communities can be linked to a telephone channel, while in the second, seven or eight channels can be connected to the same exchange at a lower marginal cost. The single-channel solution is recommended when the aim is to connect 11 or fewer communities in low-density or far-flung areas.

To determine the relevant costs, the prices of all the resources the project will use must be identified. It is important to bear in mind, in this case, the difference between private costs and social costs. The former are assessed on the basis of market prices, the latter on the basis of social or shadow prices.

Revenues are estimated by quantifying the largest number of telephone calls originated by the project to install public telephones in rural areas; they constitute the project's private revenues.

The revenues generated by the project are received by the company to which it was awarded. In addition, and depending on the business model that company decides to use, other stakeholders may receive a share of the revenues corresponding to their participation in the value chain: the signal carrier, the telephone administration dealer, even the person delivering the messages, in the case of communities in which only one public telephone is installed for the entire population.

In terms of social benefits, one of the main beneficiaries will be the community, which obtains direct user benefits thanks to the increase in the well-being of families. That benefit is two-way, in that it is received by both the person originating the call and the person receiving it.

There is also a whole series of collateral benefits that are hard to quantify and that are in practice considered to be intangible, such as: geographical, cultural, economic and social integration, income redistribution, reduced travel times, etc.

From the point of view of cost identification, it must be remembered that the costs will be all disbursements the company will have to make to provide the service.

Very generally speaking, there should at least be a difference between investment, operations and maintenance.

To determine the time horizon for evaluation, the reference is the project's economic useful life, which is never longer than the technical useful life. It is standard procedure in telephony projects to work on the basis of ten years, without considering residual value.

Tierra Blanca has 25 communities with an average population of 600 inhabitants per community. The communities are over 5 kilometres from the municipal administrative centres. Most of the inhabitants have to go into the hills or walk as much as several kilometres to call their relatives. With a telephone in the community, people would not have to take risks, especially at night in an emergency.

The project also benefits the communities from the socio-economic point of view, as they are located in overwhelmingly agricultural and livestock-raising areas. It will in particular help them do business with other sectors and towns. It must also be borne in mind that, for a variety of reasons, people from the community have emigrated to the towns and abroad, and it is indispensable that they have a means of communication.

The project's implementation means the inhabitants of these communities will have a means of communication when a natural event occurs (landslide, flood, earthquake, etc.). It is also indispensable from the point of view of protecting citizens from the crimes that can be perpetrated in the community.

The project entails the provision and installation of turnkey telecommunication systems and comprises, at a minimum, the provision, factory testing, fitting, packaging and installation of the systems. The initial investment is estimated at around USD 2.5 million. The operating and maintenance costs are estimated at an annual equivalent of 5% of total investment in equipment. The associated administrative costs amount to 2% of the operating and maintenance costs.

The table below sums up the project's minimum investment components.

Terminal equipment
Transmission subsystems (carrier means)
Switching subsystems (telephone exchange)
Interconnection subsystem (rural public network telephone exchange)
Power subsystem
Protection subsystem
Network administration and management subsystem (HW & SW)

Figure 4.1 Investment components

The cost of acquiring land and buildings is borne by the regulatory body via an independent bidding process, and is therefore not taken into account in the project cash flow.

This particular case is analysed using the average demand method. This implies the use of demand for similar communications, which assumes that distance, duration and timing are similar and homogeneous.

To calculate the benefits, the number of calls placed after project implementation and the price charged for those calls have to be determined. For outgoing calls, the estimation is based on the population of each community. In this specific case, each community is estimated to have a population of 600. This means that the following function has to be used to determine actual monthly traffic per terminal.

AMT = 2.24* Inhab

Where:

AMT is actual monthly traffic.
2.24 is the average duration, in minutes, of phone calls in the beneficiary communities, according to the regulator's data.
Inhab is the number of inhabitants in the community.

The number of incoming calls is estimated as 30% of all outgoing traffic, according to the information provided by the regulator.

The estimate by type of call is obtained from a study of caller behaviour in rural communities, which resulted in the following distribution:

Type of call	% of calls
Local	48%
National trunk call	52%
For the sake of simplicity, international calls are not taken into account.	

Figure 4.2 Types of calls

In the same way, average prices are used to determine revenues. In this case, the average price per minute of local call is assumed to be 30 cents, and per minute of national trunk call, 40 cents.

Part of the benefits are calculated as being the savings accruing to the inhabitants of the communities in not having to travel to make calls. To quantify this social benefit, it is estimated as corresponding to 80% of the cost of outgoing calls. The tax rate is 30% and the cut-off ratio[2] is 18% annually.

On the basis of these assumptions, effective cash flow is put together by identifying, determining and calculating revenues and expenditure for each year of the project, and the result obtained, as observed in the table below.

Regulatory body Evaluation rural telephony project Year 1 - Year 10 (dollars)											
	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Revenues from calls		826 608	892 737	964 156	1 041 289	1 124 592	1 214 559	1 311 724	1 416 662	1 529 994	1 652 394
Revenues from access		413 132	446 182	481 877	520 427	562 061	607 026	655 588	708 035	764 678	825 852
Savings on travel costs		661 287	714 190	771 325	833 031	899 673	971 647	1 049 379	1 133 329	1 223 996	1 321 915
Total revenues		**1 239 740**	**892 737**	**964 156**	**1 041 289**	**1 124 592**	**1 214 559**	**1 311 724**	**1 416 662**	**1 529 994**	**1 652 394**
Expenditure											
Operation and maintenance		125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000
Administrative		2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500
Interconnection		206 393	222 905	240 737	259 996	280 796	303 259	327 520	353 722	382 019	412 581
Depreciation		250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000
Total spending		**583 893**	**600 405**	**618 237**	**637 496**	**658 296**	**680 759**	**705 020**	**731 222**	**759 519**	**790 081**
Initial invesment	**2 500 000**										
Income tax		196 754	87 700	103 776	121 138	139 889	160 140	182 011	205 632	231 142	258 694
Adjustment for depreciation		250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000
Net effective cash flow	**(2 500 000)**	**709 093**	**454 633**	**492 143**	**532 655**	**576 407**	**623 660**	**674 692**	**729 808**	**789 332**	**853 619**

Figure 4.3 The project's net effective cash flow

Once the net effective cash flows have been established for each year, the financial indicators are calculated. Excel has the formulae needed for each calculation.

The first step is to discount the current value cash flows. In the **Formulas** tab, select **Insert function**. In the window that opens, select the category **Financials**, then click on **NPW**, as shown in the next screen shot.

[2] This is the minimum acceptable rate of return, used as a reference to measure the profitability of the project being evaluated.

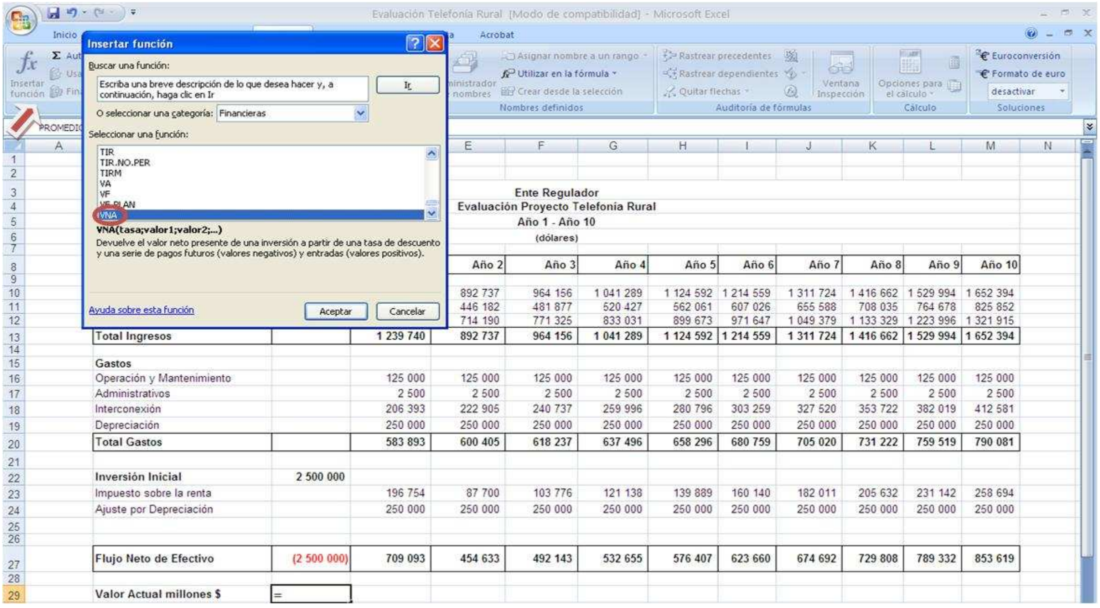


Figure 4.4 Excel NPW function

Click on **OK**. A window opens in which to provide the parameters for the calculation. **Rate** is the percentage used as the cut-off rate (in this case, 18%), **Value 1** refers to the cash flows from Year 1 to Year 10.

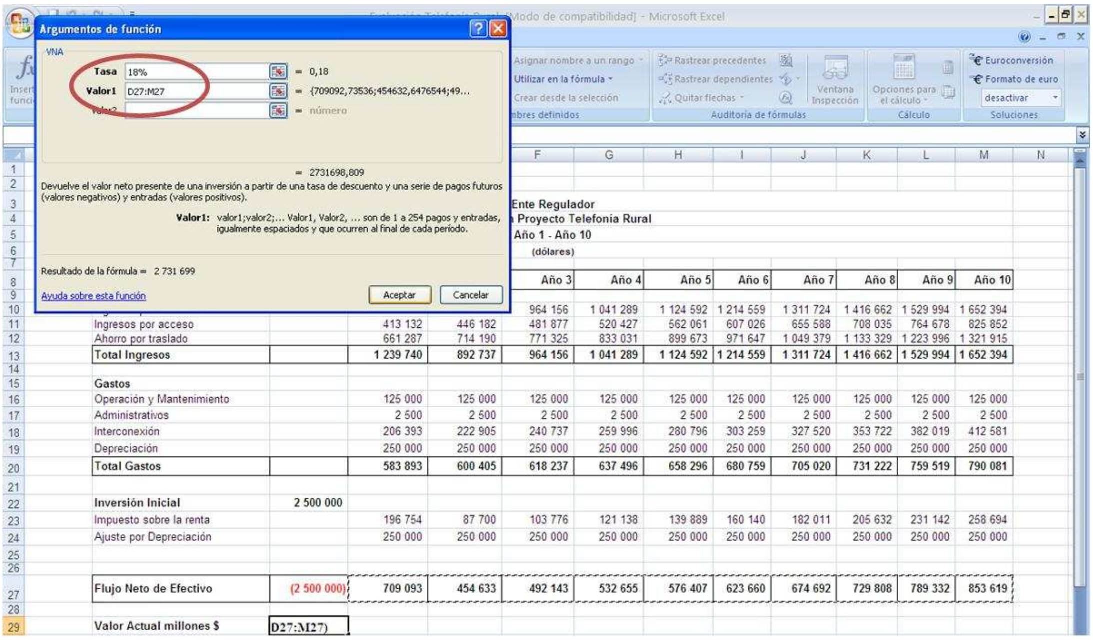


Figure 4.5 Parameters for calculating NPW

Once the data have been entered, click on **OK**. Excel generates the calculation in the cell selected.

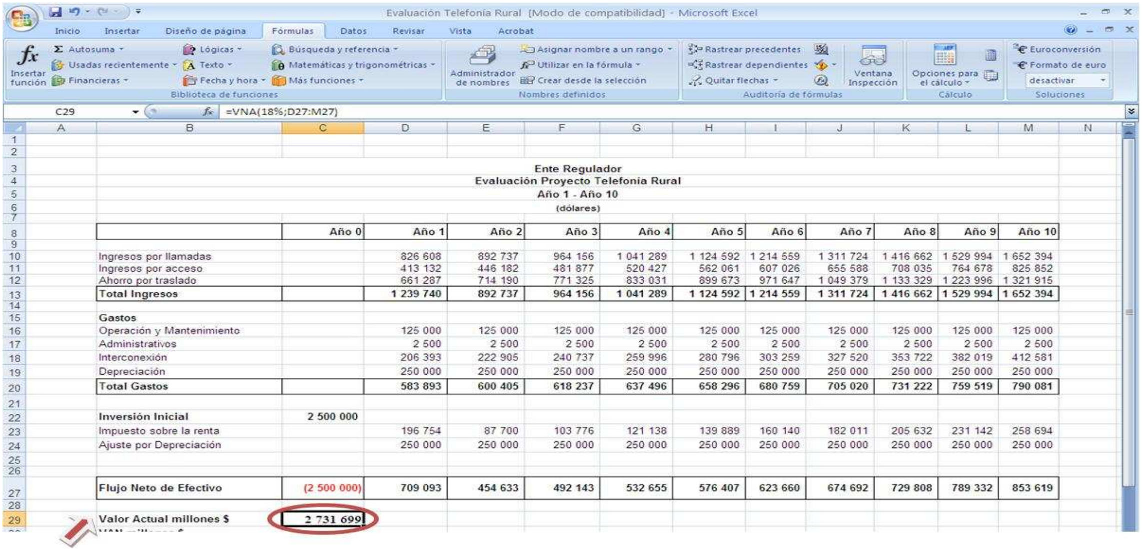


Figure 4.6 Excel calculation of NPW

The result obtained shows the total value of the cash flows generated by the project during the period being analysed, discounted for current value at an annual rate of 18%. Using this datum, the net present worth (NPW) that the project will produce can be calculated by subtracting the initial investment from the current value of the cash flows.

To calculate the internal rate of return, the calculation process is that same as for NPW, but NPW is replaced by IRR.

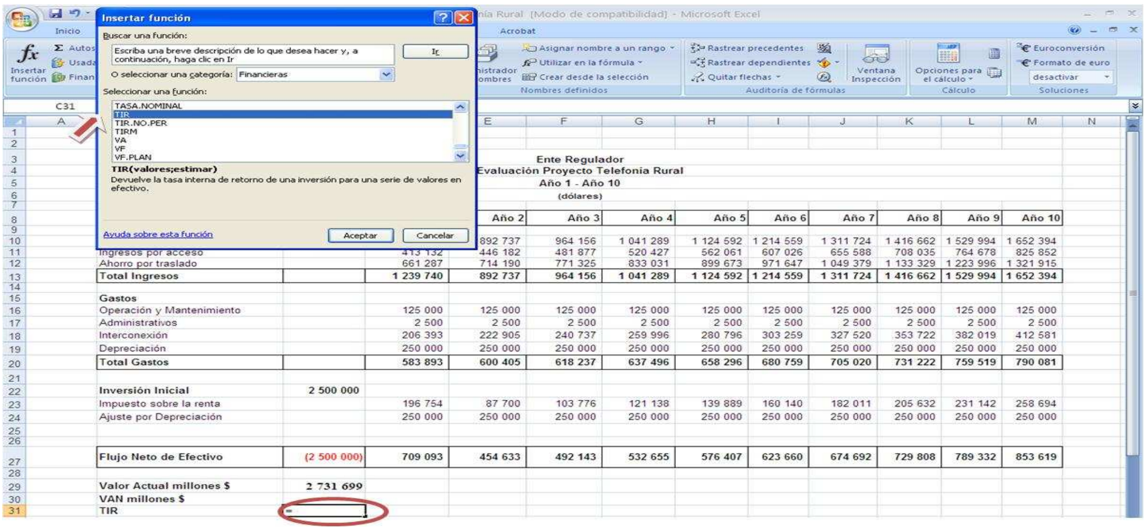
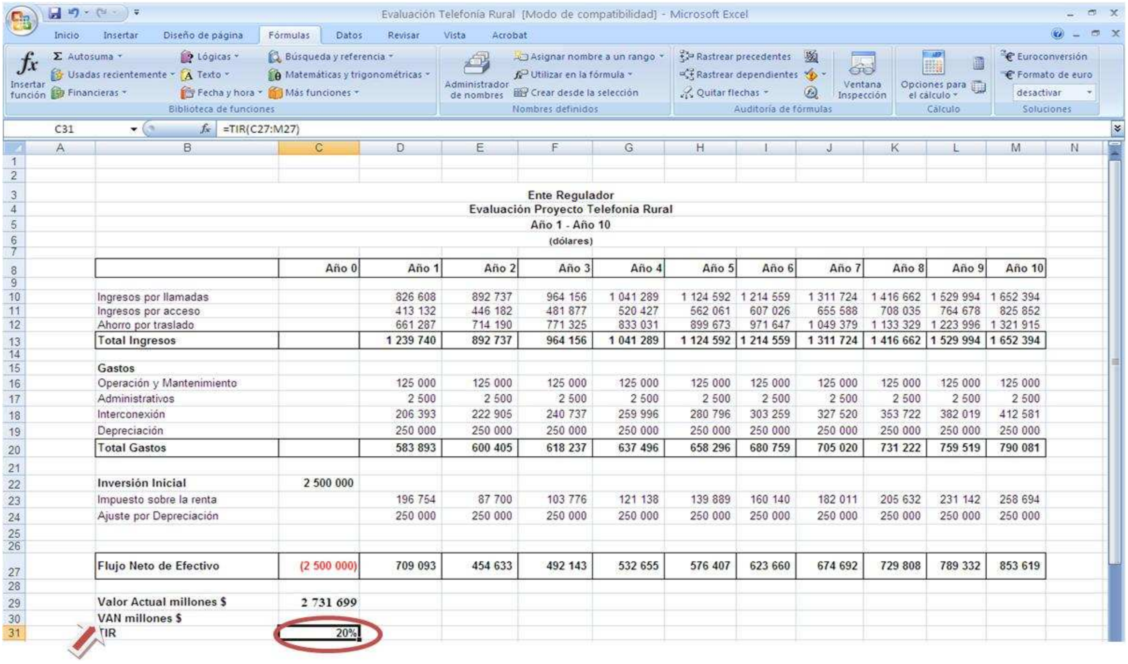


Figure 4.7 Excel IRR function

Click on **OK**. A window opens that asks for the **Values**, which means the cash flows from Year 0 to Year 10. Once the data have been entered, click on **OK** and Excel generates the calculation in the cell selected.



=TIR(C27:M27)

Ente Regulador
Evaluación Proyecto Telefonia Rural
Año 1 - Año 10
(dólares)

	Año 0	Año 1	Año 2	Año 3	Año 4	Año 5	Año 6	Año 7	Año 8	Año 9	Año 10
Ingresos por llamadas		826 608	892 737	964 156	1 041 289	1 124 592	1 214 559	1 311 724	1 416 662	1 529 994	1 652 394
Ingresos por acceso		413 132	446 182	481 877	520 427	562 061	607 026	655 588	708 035	764 678	825 852
Ahorro por traslado		661 287	714 190	771 325	833 031	899 673	971 647	1 049 379	1 133 329	1 223 996	1 321 915
Total Ingresos		1 239 740	892 737	964 156	1 041 289	1 124 592	1 214 559	1 311 724	1 416 662	1 529 994	1 652 394
Gastos											
Operación y Mantenimiento		125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000
Administrativos		2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500	2 500
Interconexión		206 393	222 905	240 737	259 996	280 796	303 259	327 520	353 722	382 019	412 581
Depreciación		250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000
Total Gastos		583 893	600 405	618 237	637 496	658 296	680 759	705 020	731 222	759 519	790 081
Inversión Inicial	2 500 000										
Impuesto sobre la renta		196 754	87 700	103 776	121 138	139 889	160 140	182 011	205 632	231 142	258 694
Ajuste por Depreciación		250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000
Flujo Neto de Efectivo	(2 500 000)	709 093	454 633	492 143	532 655	576 407	623 660	674 692	729 808	789 332	853 619
Valor Actual millones $	2 731 699										
VAN millones $											
TIR	20%										

Figure 4.8 Excel calculation of IRR

In short, the project evaluation shows that, based on the above assumptions, the project generates an NPW of USD 231 699, with an IRR of 20% and a desirability index that indicates that the return on the initial investment will be 1.09 during the period under analysis.

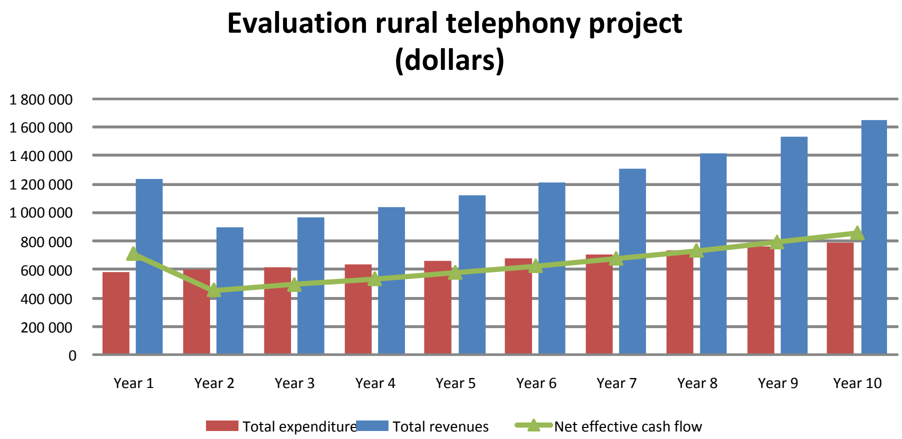


Figure 4.1 Net effective cash flow

The results obtained from the evaluation of net effective cash flow in the conditions given are used to measure the risk of implementing the project under the conditions indicated in the feasibility study.

If the statistical analysis of probabilities is applied, in this specific case there is a 33% probability that the project will generate an additional cost of USD 325 827. There is also a 33% probability that it will generate extraordinary earnings in the order of USD 790 000, as demonstrated in the comparative chart below.

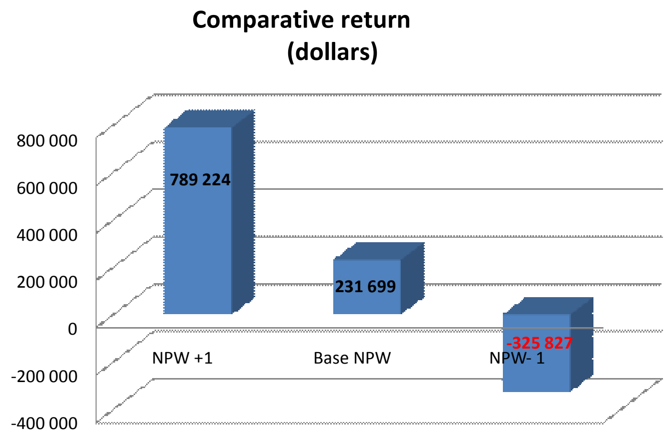


Figure 4.2 Comparative return by scenario

Thanks to the outcome of this simple exercise, the regulator can invite potential investors to bid for the project and inform them that a feasibility study has shown that once the project is implemented, extraordinary earnings will amount to approximately USD 231 699 over 10 years, with a minimum acceptable rate of return of 18%, and that during that period, according to the assumptions used, the project cash flow is positive. The project is therefore self-sustaining, and has a 33% probability of incurring losses amounting to approximately USD 325 000.

5. Model for analysing consumer surplus

The consumer surplus

Mankiw[3], in his discussion of consumers, producers and market efficiency, says that buyers always want to pay less and sellers always want to charge more. But is there a "right price" from the point of view of society as a whole?

The author establishes, from the positive point of view (what is), that prices and quantities are determined by market forces, so that the market allocates scarce resources. The question that immediately springs to mind is whether such allocations are desirable and rule-based (what should be). It is known that the prices of goods and services fluctuate so as to ensure that supply is equal to demand. But is the quantity produced and consumed of good x too little, too much or simply right in this equilibrium?

The economics of well-being studies how the allocation of resources affects economic well-being. It is therefore important to analyse the benefits to consumers and sellers of market participation, and therefore to study the consumer surplus.

Mankiw explains that an individual's willingness to pay is the maximum amount that a buyer will pay for a good. Thus, if an individual is interested in acquiring good "x" and is willing to pay USD 100, but manages to buy at USD 80, then he has received a surplus of USD 20. This is the consumer surplus, i.e. "the price the buyer is willing to pay minus the amount actually paid". The consumer surplus measures the benefit to buyers of participating in the market. Buyers who value good "x" at a price under USD 80 receive no surplus. However, if there exists another buyer who is willing to pay, say, USD 90, then the market surplus is USD 30, because the first buyer's surplus of USD 20 is added to the second buyer's surplus of USD 10.

The demand curve reflects the willingness of buyers to pay. It can also be used to measure consumer surplus. The area corresponding to the consumer surplus is shown below.

[3] N. Gregory Mankiw, *Principles of Economics*, Harvard University, McGrawHill,1998, p. 129.

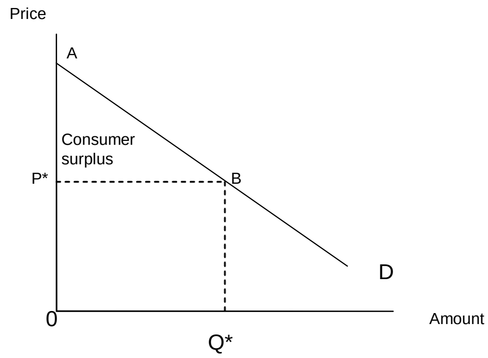


Figure 5.1 Consumer surplus

Suppose there are four telephone customers: the first is willing to pay between 5 000 and 6 000 monetary units (customer A), the second between 4 000 and 5 000 (customer B), the third between 3 000 and 4 000 (customer C) and the fourth less than 3 000 (customer D).

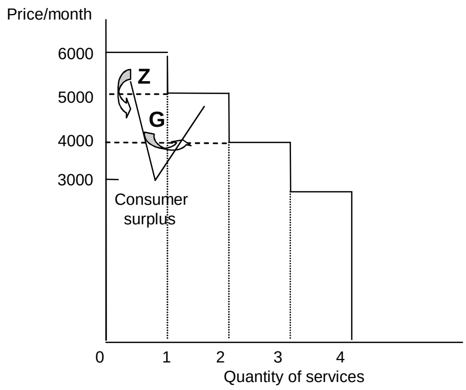


Figure 5.2 Buyer willingness to pay

The above figure shows the relationship between the height of the demand curve and buyer willingness to pay. For any given quantity, the price indicated by the demand curve shows the willingness to pay of the marginal buyer, who would be the first to quit the market if the price were too high.

If the quantity of services is four, for example, the demand curve is 3 000 monetary units high, which is the price of buyer D, who accepts a monthly subscription of 3 000 monetary units or less. If the quantity is three services, the demand curve is 4 000 monetary units high, which is the price of customer C, who is willing to pay between 3 000 and 4 000 monetary units and who is now the marginal buyer.

Since the demand curve shows the buyers' willingness to pay, it can also be used to measure consumer surplus. In the figure, at a price of 5 000 or somewhat higher, demand falls to 1. Note that the area situated above the price and below the demand curve is equal to 1 000 monetary units (Z area). That amount is the consumer surplus. If the price is 4 000, then the consumer surplus is the sum of area Z plus area G, adding 2 000 monetary units.

The consumer surplus increases as the price falls, because buyers always want to pay less for the good they acquire, and a price reduction therefore increases their well-being.

The purpose of developing the consumer surplus concept is to obtain rule-based views of how desirable market outcomes are and to determine the economic well-being of most markets using consumer surplus.

This would be seen as follows on a curve with a negative slope:

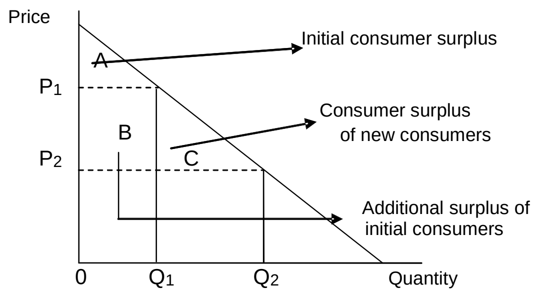


Figure 5.3 Determining consumer surplus as prices change

Measuring consumer surplus is a key component of cost-benefit analysis.[4] For example, in order to decide whether or not to construct a new electricity-generating plant, the value to consumers of the electricity produced by the plant has to be ascertained. The value of the electricity generated is not equal to the price of the electricity multiplied by the amount generated by the new plant. The total value to be weighed against the costs of the plant include the consumer surplus accruing to the users of the electricity if the new plant is constructed.

The producer surplus

The analysis of the other side of the market concerns producers (operators in the case at hand) and is very similar to the analysis of consumers.

Suppose there are four suppliers - A, B, C, D - and each has a cost for its service, cost being understood as opportunity cost, which comprises both actual spending and the value they attribute to their time. Suppose the cost values are those in the table below:

[4] Karl E. Case and Ray C. Fair, *The Principles of Microeconomics*, Pearson Prentice Hall, 2007.

The costs of four possible sellers (operators)	
Seller	**Cost (USD)**
A	900
B	800
C	600
D	500

When the four sellers of service "X" send in their offers, the price can start very high but fall rapidly as they compete for the job. If seller **D** offers USD 600 (or less), it is the only bidder left. It is happy to do the job at that price, since its cost is USD 500. **A**, **B** and **C** are not willing to do the job for less than it costs them. Ultimately, the job will go to the seller that can do it at the lowest cost.

What benefit will seller **D** obtain from being awarded the job? Since it is willing to do it for USD 500, but will earn USD 600, it is said to receive a producer surplus of USD 100. The producer surplus is the amount that the seller receives less the cost of production. It measures the benefit to the sellers of their participation in a market.

If the price was USD 800, then **B**, **C** and **D** would be willing to do the job and the producer surplus would be USD 300 for **D** and USD 200 for **C**. The total producer surplus for the market is USD 500.

In the same way that the consumer surplus is closely linked to the demand curve, so the producer surplus is closely linked to the supply curve.

The concept is that the area situated under the price and above the supply curve measures the producer surplus in a market. The logic is simple: the height of the supply curve measures the costs of the sellers and the difference between the price and the production cost is the producer surplus of each seller. Therefore, the total area is the sum of the producer surplus of all the sellers.

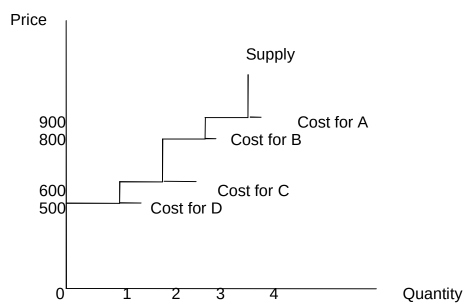


Figure 5.4 Producer/operator surplus

The way in which the producer surplus increases as the price goes up can be seen in the following figure.

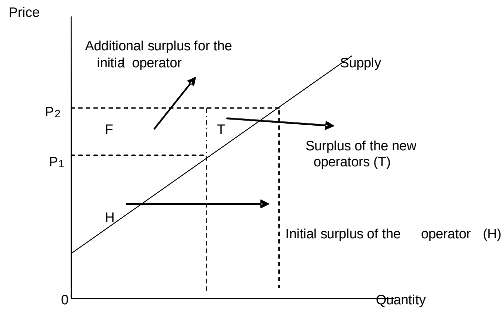


Figure 5.5 Determining the operator surplus when prices change

Sellers always want to charge the highest price for the goods they sell. But how does their well-being increase when the price goes up? The concept of producer surplus answers that question.

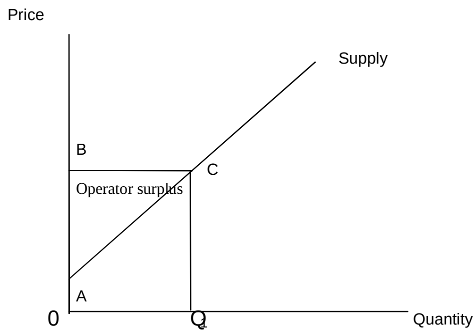


Figure 5.6 Operator surplus

The producer or operator surplus is the area situated under the price and above the supply curve. The producer surplus is the triangle formed by **ABC**.

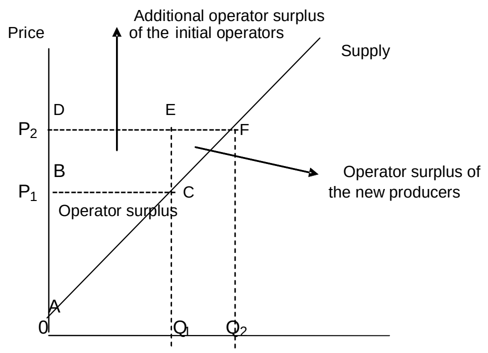


Figure 5.7 Operator surplus when prices change

The above figure shows what happens when the price goes up from **P1** to **P2**. Now the producer surplus is equal to the area **ADF**. That increase in producer surplus has two parts. First, the sellers that were already selling quantity **Q1** of the good at the lower **P1** price enjoy greater well-being because they now receive more for what they sell. The increase in producer surplus of the existing sellers is equal to the area of the rectangle **BCED**.

Second, new sellers have entered the market because they are now willing to produce the good at the higher price, therefore increasing the supply from **Q1** to **Q2**. The producer surplus for these new sellers is the area of the triangle **CEF**.

Producer and consumer surplus are measures of economic well-being. They are similar, and must therefore be analysed together.

Consumer surplus and producer surplus[5] are added as follows:

Total surplus = value for the buyers – amount paid by the buyers + amount received by the sellers – sellers' costs

The amount paid by the buyers is equal to that received by the sellers, so the equation can be expressed as follows[6]**:**

Total surplus = value for the buyers – sellers' costs

The total market surplus is the total value for the buyers of the goods, expressed by their willingness to pay, less the costs to the sellers of producing those goods.

If resources are allocated in such a way as to maximize the total surplus, the market is said to be efficient. If they are inefficiently allocated, some of the earnings derived from the exchange between the buyers and sellers are not being obtained. For example, resources are not efficiently allocated if the sellers are not producing a good at the lowest possible cost, or if the good is not being consumed by the buyers who place the highest value on it.

[5] See Richard A. Bilas, *Teoria microéconomica*, and Case and Fair, *op. cit.*
[6] *Ibid*.

The evaluation of market equilibrium

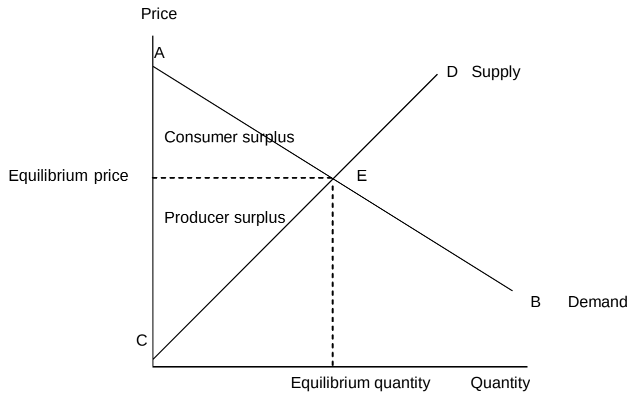


Figure 5.8 Market equilibrium

The above figure shows the consumer surplus and the producer or operator surplus when the market reaches supply/demand equilibrium. The total area between the supply and demand curves up to the equilibrium point represents the total market surplus.

It goes without saying that when a market is in equilibrium, the price is that determined by the buyers and sellers in the market. Buyers that place a higher value on the good than the price (segment AE on the demand curve) decide to buy it, while those who place a lower value on it (segment EB) do not. Likewise, sellers whose costs are lower than price (segment CE on the supply curve) decide to produce and sell the good, whereas those whose costs are higher than price (segment ED) will neither produce nor sell it.

This analysis leads to three conclusions:

1. Free markets allocate the supply of goods to the buyers who place higher value on them, represented by their willingness to pay.

2. Free markets allocate the demand for goods to the sellers that can produce them at the lowest cost.

3. Free markets produce the quantity of goods that maximize the sum of the consumer and producer surplus.

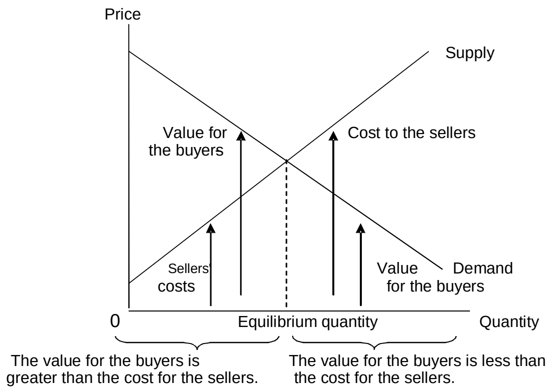


Figure 5.9 Market value

The above figure shows that for quantities that are less than the equilibrium level, the value for the buyers is higher than the cost for the sellers in a telecommunication or other market if there is none of the manoeuvring typical of an imperfect market. In this region, an increase in quantity raises the total surplus until the quantity reaches equilibrium. Beyond the equilibrium quantity, the value for the buyers is less than the cost for the sellers. Therefore, producing a quantity that is above the equilibrium would reduce the total surplus.

The three conclusions mentioned above therefore indicate that the supply/demand equilibrium maximizes the sum of the consumer and producer surplus. In other words, equilibrium reflects an efficient allocation of resources.

It is important to mention that transaction taxes reduce consumer and producer surplus. Some of the losses take the form of transfers of funds from consumers and producers to the beneficiaries of government spending. At the same time, however, there is also a loss of efficiency, as the volume of exchanges is reduced.[7]

Taxes hinder exchanges because of the effect they have on prices; they operate quantitative restrictions on exchange. At the same time, production quotas include a transfer of consumer surplus to the producers; the rationing of demand causes a transfer from the producers to the consumers.

In addition, both restrictions bring about a loss in efficiency due to the reduction in the volume of exchanges. There will generally be a subsequent loss in efficiency as a result of the non-ideal allocations of the rights to sell or buy.[8]

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[7] Jack Hirshleifer, *Microeconomía teoría y aplicaciones*, 3rd ed.
[8] See Case and Fair, *op. cit.*

Methodology and application

In this section we analyse the application of a linear econometric model and its methodology to fixed telephony, in order to provide a hypothetical demonstration of how consumer surplus is generated at both the economic and the social levels.

Calculating consumer surplus in a telecommunication company makes it possible to measure the degree of competitivity delivered by the producer (operator) to the consumers expressed as real savings in monetary terms. Economic theory clearly identifies the elements making up a price, namely variable costs, fixed costs, psychological factors and indirect factors such as marketing and image. All of these are combined with the modern strategy of price and cost, as Porter clearly indicates.

The following pages present the basic construction of two econometric models, the integration analysis and details of the models. They also present the guidelines and methodology for the empirical verification of the surpluses calculated using econometrics and the mathematical tools of integration.

6. Practical application II

Model for traditional fixed telephony

Conceptual definition

- Apply multiple linear regression analysis using ordinary least squares to obtain an equation for average consumption of minutes, which is influenced by the price of the minute and a proxy variable for disposable income, defined as real per capita GDP.

- Submit the model to various hypothesis tests, in order to detect problems in the behaviour of the variables and residuals.

- The resulting equation represents an estimator of how consumption will behave over time in response to variations in price and GDP. This temporal analysis is used to estimate the future value of consumption, subject to those variables. This is an important element in determining prices.

- Current consumer surplus is estimated by integration analysis of the equation obtained for the regression analysis with respect to the price variable. Since some prices may fluctuate little over time, the minimum is the real current price and the maximum the price in the period base year, assuming that normal behaviour is real price stability over time.

- The value determined using the combined regression and integration analyses represents the consumer surplus and also the amount that the operator is contributing to the economy, or the social contribution it no longer receives from each customer.

- The model is explained by adjusted R^2 (coefficient of determination).

- Verify the slope of the supply/demand curve.

- Conduct a survey to check whether the customer is receiving a very high or very low psychological price, so as to evaluate consumer and producer surplus ex ante.

- Very inelastic prices indicate that the operator can increase them without losing economic benefits, i.e. total revenues will always rise. As a result, consumer surplus will rise and vice versa.

- If the price in dollars for one minute of service were to rise from USD 1.72 to USD 3.25, consumption would drop by 16.9% (from 966.10 to 802.25 on the quantity axis).This indicates even more clearly that consumer surplus can be translated into a benefit for society, because the operator would not lose if a higher price were applied, it would simply reduce the quantity sold. This situation is explained in the figure below and in the calculations that follow.

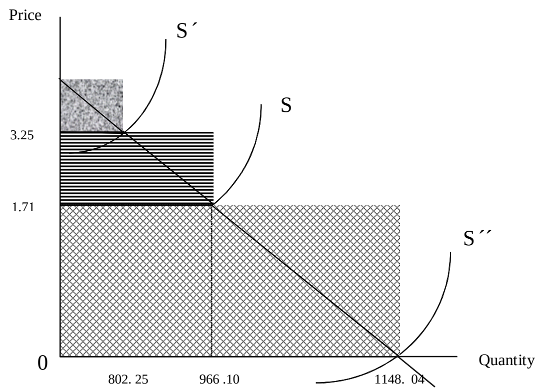


Figure 6.1 Consumer surplus for traditional fixed telephony

The demand curve is inelastic, has a negative slope and is the outcome of a normal service. The various supply curves, for their part, intersect the demand curve at the projected price, because of possible surplus demand during the period in which concessions for universal service are defined. Supply is thus represented by the level of price, for the sole purpose of representing the fixed telephone consumer surplus.

Zone **A** above the price of USD 3.25 is the consumer surplus if the operator had considered an increase. In that case, consumption would have been approximately 20% less than at present, but this would not affect the operator because demand is inelastic, meaning the increase in price is proportionately greater than the change in the amount consumed. This situation is evidence of the huge benefit to society as a whole.

Zone **B** shows the additional benefit received by the consumers when the real price is lower because prices are constantly falling in real terms.

Regression analysis methodology used to obtain the consumer surplus for traditional telephony

The purpose of applying the regression analysis model is to estimate the consumption curve, which is the best estimator of demand in the specific case of the surplus minutes component of traditional telephony. In principle, the first step is to examine the behaviour of the consumption variable (C, average consumption per customer of surplus minutes), which is a dependent variable, in response to price fluctuations (P, price of the surplus minutes) and GDP (real per capita GDP as the proxy variable for disposable income), which are independent variables.

The variables are observed over an 8-year time period on a quarterly basis, for a total of 32 observations.

Initial assumptions

- $E(u_i)= 0$ indicates that the mathematical expectancy of errors is zero.
- $Cov(u_i,u_j)=0$ indicates that the explanatory variables should be mutually independent; there should be no autocorrelation of errors.
- $Var(u_i)=\sigma^2$ for each i indicates that variance is unique for all errors; there is homoscedasticity.
- $Cov(u_i,x_n)$ $Cov(u_i,x_{n+x})=0$ indicates that the terms of error are not associated with the model's explanatory variables.
- The hypothesis can be tested using the test statistics χ^2, F, and t-statistics and their corresponding probabilities of occurrence according to the values tabulated.
- The level of significance is 5%.
- The analysis uses the multiple linear regression technique, estimating ordinary least squares (LS in the econometric package E-VIEWS).

Anticipated relationships between the variables:

△C/△P: the relationship between price per surplus minute and consumption of surplus minutes is theoretically positive, given that a higher price is expected to shrink consumption.

△C/△GDP: the relationship between real GDP and consumption of surplus minutes is theoretically positive, given that GDP is a good estimator as a proxy variable for disposable income, it being hoped that changes in the production of the economy's end goods and services reflect greater dynamism in the consumer surplus for telephone minutes.

The model as outlined above is intended to demonstrate, by estimating demand, the effect of prices and income as variables that have had a greater impact on the consumption of minutes over the past eight years.

Testing the model

First model

The initial equation to be estimated is the following: (eq1)

$$Consumption = \beta_0 +(\beta_1)*Price +(\beta_2)*GDP+(\beta_3)*Formationk + u_i$$

In this first model, the variable Formationk has been added. This is the gross formation of capital in real terms.

Results of the first model

Estimation Command:
LS AVECONS C PRICE GDP FORMATIONK

Estimation Equation:
AVECONS = C(1) + C(2)*PRICE + C(3)*GDP + C(4)*FORMATIONK

Substituted Coefficients:
AVECONS = 1196.491-112.863*PRICE+0.0077*GDP-0.0007*FORMATIONK

Behaviour of the coefficient sign

> **△C/△P:** the coefficient for this relationship is negative, indicating an inverse relationship between the variables and confirming the anticipated theoretical outcome, that the price of minutes has an inverse impact on consumption.
>
> **△C/△GDP:** the regression coefficient obtained for GDP is positive, confirming the anticipated theoretical outcome, that the increase in national production has a positive effect on the consumption of minutes.

Dependent Variable: AVECONS
Method: Least Squares
Date: 12/02/08 Time: 07:30
Sample: 2000:1 2008:4
Included observations: 32

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1196.492	347.9662	3.438528	0.0018
PRICE	-112.8636	33.67329	-3.351726	0.0023
GDP	0.007754	0.003130	2.477308	0.0195
FORMATIONK	-0.000728	0.001051	-0.692790	0.4941

R-squared	0.780899	Mean dependent var	1546.844
Adjusted R-squared	0.757424	S.D. dependent var	119.3388
S.E. of regression	58.77679	Akaike info criterion	11.10184
Sum squared resid	96731.92	Schwarz criterion	11.28506
Log likelihood	-173.6294	F-statistic	33.26494
Durbin-Watson stat	1.301046	Prob(F-statistic)	0.000000

Hypothesis testing

If probability is 5% of the significance level, no problems are observed in the coefficients of the variables GDP and PRICE, whereas in the case of the variable FORMATIONK, the null hypothesis that its regression coefficient is equal to 0 (t-Statistic=-0.692790, Prob=0.494) is not rejected. This is an indication that the variable FORMATIONK is statistically significant in the model.

In addition, the F-statistic of the regression (33.26494) exceeds the tabulated value; given the probability of 0.00000, the null hypothesis that all coefficients are simultaneously equal to 0 can be rejected.

In line with the probability values obtained, the variable for consumption of surplus minutes is influenced by the macro variables studied. However, redundant variables must be tested for the FORMATIONK variable, in order to verify that the latter is truly significant in the model.

The null hypothesis that the FORMATIONK variable is redundant is tested using the droptest for redundant variables, in order to determine which is the best model. In accordance with the specifications and outcomes of high probabilities, the variable is eliminated from the model.

Redundant variables: FORMATIONK

F-statistic	0.479958	Probability	0.494148
Log likelihood ratio	0.543875	Probability	0.460831

Second model

The equation to be estimated is as follows: (eq2)

$$\text{Consumption} = \beta_0 + (\beta_1)*\text{Price} + (\beta_2)*\text{GDP} + u_i$$

Results of the second model

Estimation Command:
LS AVECONS C PRICE GDP

Estimation Equation:
AVECONS = C(1) + C(2)*PRICE + C(3)*GDP

Substituted Coefficients:
AVECONS = 1148.04466 - 106.3968559*PRICE + 0.007599918794*GDP

Behaviour of the coefficient sign

> **△C/△P:** the coefficient for this relationship is negative, indicating an inverse relationship between the variables and confirming the anticipated theoretical outcome, that the price of minutes has an inverse effect on consumption.

△C/△GDP: the regression coefficient obtained for GDP is positive, confirming the theoretical anticipated outcome, that the increase in national production has a positive impact on the consumption of minutes.

For the purpose of interpreting the coefficients, the model was estimated using natural logarithms (growth).

Estimation Command:
=====================
LS LAVECONS C LPRICE LGDP

Estimation Equation:
=====================
LAVECONS = C(1) + C(2)*LPRICE + C(3)*LGDP

Substituted Coefficients:
=====================
LAVECONS = 2.568663048 - 0.**163553626***LPRICE + 0.4322605762*LGDP

These estimated coefficients indicate the percentage variation in consumption of minutes generated by a 1% change in the independent variables. For example, a 1% increase in price generates a 0.16% decrease in consumption, and a 1% increase in GDP generates a 0.43% increase in consumption of minutes (estimated marginal propensity to consume).

Dependent Variable: AVECONS
Method: Least Squares
Date: 12/03/02 Time: 05:42
Sample: 2000:1 2008:4
Included observations: 32

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1148.045	337.7968	3.398625	0.0020
PRICE	-106.3969	32.06226	-3.318445	0.0024
GDP	0.007600	0.003094	2.456408	0.0203

R-squared	0.777143	Mean dependent var	1546.844
Adjusted R -squared	0.761774	S.D. dependent var	119.3388
S.E. of regression	58.2474 0	Akaike info criterion	11.05634
Sum squared resid	98390.04	Schwarz criterion	11.19375
Log likelihood	-173.9014	F-statistic	50.56417
Durbin -Watson stat	1.376708	Prob(F -statistic)	0.000000

Hypothesis testing

If probability is 5% of the significance level, no problems are observed in the coefficients of the variables GDP and PRICE. In addition, the F-statistic of 50.56417 exceeds the tabulated value and has a probability of 0.00000. The null hypothesis that all coefficients are simultaneously equal to 0 can be rejected.

Other important statistics, such as R^2 and adjusted R^2, combined with the F-statistic, are evidence of an adequate goodness-of-fit level. In the case of the Durbin Watson statistic, the value calculated is between the maximum and the minimum (L_i 1.244 ,

L_S1.650), and the evidence for the presence of a positive first-order serial correlation is therefore inconclusive.

The Ramsey second-order RESET test is then performed, to test that the model has the right specifications.

The probabilities obtained lead to the conclusion that the null hypothesis is not rejected, in other words, that the model is correctly specified.

Ramsey RESET Test:

F-statistic	0.001605	Probability	0.968323
Log likelihood ratio	0.001835	Probability	0.965834
R-squared	0.777156	Mean dependent var	1546.844
Adjusted R-squared	0.753280	S.D. dependent var	119.3388
S.E. of regression	59.27671	Akaike info criterion	11.11878
Sum squared resid	98384.40	Schwarz criterion	11.30200
Log likelihood	-173.9005	F-statistic	32.54945
Durbin-Watson stat	1.375820	Prob(F-statistic)	0.000000

Other tests

Normality

A limiting factor is the paucity of data, which affects the reliability of the estimators and test statistics used. However, economically and statistically accepted decision-making criteria are used.

Four aspects are analysed with regard to residual normality:

1. the histogram, which is a graphic representation of the distribution of residuals;

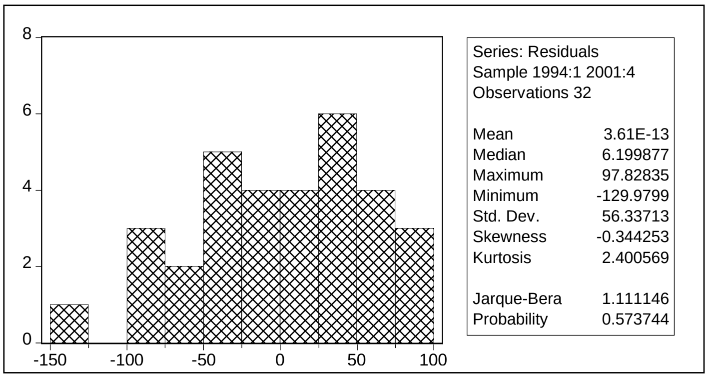
2. the skewness coefficient, which should be close to zero (-0.344);

3. kurtosis, which should have a value close to 3 (2.40);

4. the Jarque-Bera test, which is the statistic used to test that the null hypothesis has a normal distribution of residuals.

The value obtained for that statistic (1.11) is less than the tabulated value (5.99), which corresponds to a distribution of χ^2 with two degrees of freedom on a significance level of 5%.

In accordance with the above, and with the outcomes of the probability test, there is insufficient statistical evidence to reject Ho, and hence the null hypothesis is not rejected. (The residuals are normally distributed).



Multicollinearity

Multicollinearity is the linear relationship between observations of explanatory variables. There are three possible situations.

1. **Perfect multicollinearity:** this occurs when there is an exact linear relationship between several or all regressors in the model.

2. **Orthogonality:** this occurs when there is no linear relationship between several or all regressors in the model.

3. **Imperfect multicollinearity:** this is the existence of a strong linear relationship between the model's regressors. It is betrayed by the following symptoms:

    - large variations in the estimators;

    - estimator instability in the face of small sample variations (the high variation in the estimators implies that the estimators are unstable);

    - difficulty in interpreting the coefficients and hence their estimation. In the case of imperfect multicollinearity, it cannot be assumed that the remaining variables will stay constant if there are high linear correlations between them. This is why the parameters lose significance and their estimation.

The following tests can be used to detect the presence of imperfect multicollinearity.

- The matrix of correlations between the regressors and the matrix determinant is calculated, as this indicates the degree of linear correlation between each pair of regressors and among the regressors as a whole. If one or several of those coefficients is high (near + -1), the existence of multicollinearity is indicated, as a consequence of the relationship between more than two variables.

- The individual and combined significance of the regressors is analysed. In the case of imperfect multicollinearity, there is a strong likelihood that the regressors will be individually insignificant, but have a high combined explanatory capacity.

In the case of the model analysed, there are adequate levels of individual significance for the regressors and a combined explanatory capacity of 0.777143 measured using R^2 and of 0.761774 using adjusted R^2, which are the coefficients of determination.

These coefficients of determination are significant because it is important to have a measure of goodness-of-fit for deciding if the linear adjustment is sufficient or whether other models are needed.

The coefficient of determination is defined as follows:

$$R^2 = \frac{scE}{scG} = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \qquad (1)$$

In short, the coefficient of determination measures the ratio of total variability of the dependent variable (Y) to its mean, which is explained by the regression model.

Heteroscedasticity

Heteroscedasticity signifies that the variance in disturbances is not constant for all observations and supposes that one of the hypotheses on which the model of basic linear regression is based has been violated. The assumption is that the data used are heterogeneous because they come from distributions of probability with different variance.

The main consequences of the failure to meet the hypothesis of homoscedasticity of results is:

- an error in the calculation of the estimator for the matrix of variances and covariances of the least squares estimators;
- a loss of efficiency in the least squares estimator.

Detection

Heteroscedasticity is detected using the White test.

- Ho: $\delta_i^2 = \delta^2$ for every i, i.e. the model is homoscedastic
- H_1: Ho is not verified

White Heteroskedasticity Test:

F-statistic	1.603290	Probability	0.202188
Obs*R-squared	6.141924	Probability	0.188797

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/03/02 Time: 06:01
Sample: 2000:1 2008:4
Included observations: 32

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-100552.3	164399.9	-0.611632	0.5459
PRICE	-11975.42	12817.70	-0.934288	0.3584
PRICE^2	1672.712	2506.128	0.667449	0.5102
GDP	2.976563	3.729349	0.798145	0.4318
GDP^2	-1.80E-05	2.11E-05	-0.849897	0.4029

R-squared	0.191935	Mean dependent var	3074.689
Adjusted R-squared	0.072222	S.D. dependent var	3696.984
S.E. of regression	3560.981	Akaike info criterion	19.33606
Sum squared resid	3.42E+08	Schwarz criterion	19.56508
Log likelihood	-304.3770	F-statistic	1.603290
Durbin-Watson stat	2.891528	Prob(F-statistic)	0.202188

The above test shows that there is insufficient statistical evidence to reject Ho; in other words, the model is homoscedastic.

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>. |**. |</td><td>. |**. |</td><td>1</td><td>0.273</td><td>0.273</td><td>2.6135</td><td>0.106</td></tr>
<tr><td>.**| . |</td><td>.**| . |</td><td>2</td><td>-0.210</td><td>-0.307</td><td>4.2130</td><td>0.122</td></tr>
<tr><td>. *| . |</td><td>. |* . |</td><td>3</td><td>-0.081</td><td>0.093</td><td>4.4576</td><td>0.216</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>4</td><td>0.047</td><td>-0.023</td><td>4.5431</td><td>0.337</td></tr>
<tr><td>.**| . |</td><td>***| . |</td><td>5</td><td>-0.255</td><td>-0.330</td><td>7.1690</td><td>0.208</td></tr>
<tr><td>***| . |</td><td>.**| . |</td><td>6</td><td>-0.413</td><td>-0.257</td><td>14.309</td><td>0.026</td></tr>
<tr><td>. *| . |</td><td>. | . |</td><td>7</td><td>-0.102</td><td>-0.042</td><td>14.758</td><td>0.039</td></tr>
<tr><td>. |* . |</td><td>. | . |</td><td>8</td><td>0.130</td><td>-0.041</td><td>15.526</td><td>0.050</td></tr>
<tr><td>. *| . |</td><td>.**| . |</td><td>9</td><td>-0.123</td><td>-0.290</td><td>16.241</td><td>0.062</td></tr>
<tr><td>. *| . |</td><td>. *| . |</td><td>10</td><td>-0.184</td><td>-0.173</td><td>17.912</td><td>0.056</td></tr>
<tr><td>. |* . |</td><td>. *| . |</td><td>11</td><td>0.109</td><td>-0.107</td><td>18.528</td><td>0.070</td></tr>
<tr><td>. |*** |</td><td>. |* . |</td><td>12</td><td>0.413</td><td>0.185</td><td>27.818</td><td>0.006</td></tr>
<tr><td>. |**. |</td><td>. |* . |</td><td>13</td><td>0.235</td><td>0.082</td><td>30.981</td><td>0.003</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>14</td><td>-0.027</td><td>-0.036</td><td>31.024</td><td>0.006</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>15</td><td>0.035</td><td>-0.011</td><td>31.104</td><td>0.009</td></tr>
<tr><td>. | . |</td><td>. *| . |</td><td>16</td><td>0.040</td><td>-0.076</td><td>31.214</td><td>0.013</td></tr>
</table>

Test for second-order autocorrelation

To conduct the test for autocorrelation of a higher order, the F-statistic is used, for which the null hypothesis that there is no autocorrelation is tested. For an F-statistic lower than the value tabulated, or a level of probability greater than 5% of the level of significance, the conclusion is that there is insufficient statistical evidence to reject Ho; in other words, the model does not present a serial correlation.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.909258	Probability	0.071747
Obs*R-squared	5.673398	Probability	0.058619

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 12/03/08 Time: 10:39
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	186.2896	356.4343	0.522648	0.6055
PRICE	-14.39991	32.89040	-0.437815	0.6650
GDP	-0.001744	0.003264	-0.534433	0.5974
RESID(-1)	0.403423	0.202045	1.996698	0.0560
RESID(-2)	-0.320016	0.198241	-1.614275	0.1181

R-squared	0.177294	Mean dependent var	3.61E-13
Adjusted R-squared	0.055411	S.D. dependent var	56.33713
S.E. of regression	54.75403	Akaike info criterion	10.98618
Sum squared resid	80946.11	Schwarz criterion	11.21520
Log likelihood	-170.7789	F-statistic	1.454629
Durbin-Watson stat	1.954432	Prob(F-statistic)	0.243472

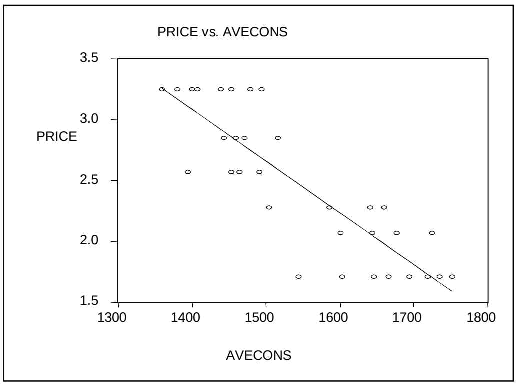
On the basis of the statistical analysis and the tests conducted of the behaviour of the variables, the residuals and the model in general, the coefficients can finally be validated to interpret the economic meaning of the model and the economic hypotheses it implies.

Analysis of consumer surplus

The equation obtained using the regression analysis serves to estimate a level of variance in the consumer surplus that is in line with changes in price. To calculate the value of this surplus, the integration method is used to determine the area under the curve, obtained by linear procedure, with respect to the independent variable of price per minute.

Equation:

AVECONS = 1148.04466 - 106.3968559*PRICE + 0.007599918794*GDP



Mean consumer surplus, in the case of traditional fixed telephony, is obtained by the equation's simple integration with regard to the independent variable of price per surplus minute. The integration limits are the current price (in real terms), which is the minimum, and the price that should apply, which is the maximum. The price in real terms should be at least equal to the real price at the start of the series in 1994 (USD 3.25 per minute).

Calculating the integral

$$\int_{1.71}^{3.25} (1148.04466 - 106.3968559 * PRICE + 0.007599918794 * GDP)\, dPRICE =$$

GDP is assumed to be constant for the integration analysis. The last recorded value is used, that of the fourth quarter of the last year (93254.2562665228).

$$\int_{1.71}^{3.25} (1148.04466–106.3968559*PRICE+0.007599918794*93254.2563)dPRICE =$$

$$\int_{1.71}^{3.25} (1856.7694 – 106.3969 * PRICE)\ dPRICE =$$

$$\left\{ 1856.7694 * PRICE \quad – 106.3969 * \frac{(PRICE)^2}{2} \right\}_{1.71}^{3.25}$$

(1856.7694 * (3.25) – 106.3969 * (3.25)2 / 2) – (1856.7694 * (1.71) – 106.3969 * (1.71)2/ 2) = 5472.59 – 3019.51 = 2453.08 Quarterly

The conclusion of this analysis is more simply expressed as the quantification of economic well-being, the regulators' own work, as it can be seen that the consumer surplus obtained via prices lower than equilibrium prices represents USD 2 453.08 per quarter (integration outcome). If prices increase, the surplus accrues to the producer / operator. This is a unique reference for the regulators, who can thus find the market equilibrium for those services for which prices can be regulated.

The application of the survey also helps verify the outcomes obtained by the econometric model. Key variables for the study are considered to be the price of the service (in order to link consumer surplus), coverage, problems and price information.

The operators can use the tools explained here to detect both the operator and the consumer surplus, and thus to improve their price, cost and project evaluation systems. This is an additional means of more accurately determining price elasticity and demand curves.

The applied theory of consumer surplus is verified thanks to the survey, which shows that, in general, customers see the price as fair and recognize telecommunications as a vehicle for competitivity.

The risks implicit in recognition and measurement have to be considered in the light of the changing economic environment. Changes in the global and hence the national economy could modify the analysis outcomes. The same applies to investment decisions and price policies influenced by political considerations.

The operationalization strategy should be applied early in the tariff policy and the company's investment decisions, for the purposes of ranking and thereby obtaining maximum economic and social benefits.

The application contributes to national economic life, demonstrating how an operator can generate social economic benefits for the country when applying corporate practices such as those mentioned in this paper. The regulator in particular can measure the benefit of the company's macroeconomic function by calculating the consumer surplus.

Appendix

Technical glossary

Variable: any individual characteristic or experimental situation that can vary from one person to another, or from one environment to another; for example, the price or sales revenues of a service are variables.

Data: numbers or measurements obtained as a result of observations; one example is the frequency of telephone use by class.

Parameter: any measureable characteristic in a population, such as the proportion of customers that consume above the basic income of the mobile telephone service. The Greek letters μ and σ are usually used to name or identify parameters.

Qualitative variables: these are expressed verbally as categories or attributes. Examples are a person's sex, colour, political affiliation, nationality, motivation, academic field or profession.

Quantitative variables: these vary in terms of quantity and are recorded or expressed in numerical form. Examples are age, academic average, test scores, crime rate and temperature.

Independent variables: these are characteristics controlled by the researcher and assumed to have an impact on other variables.

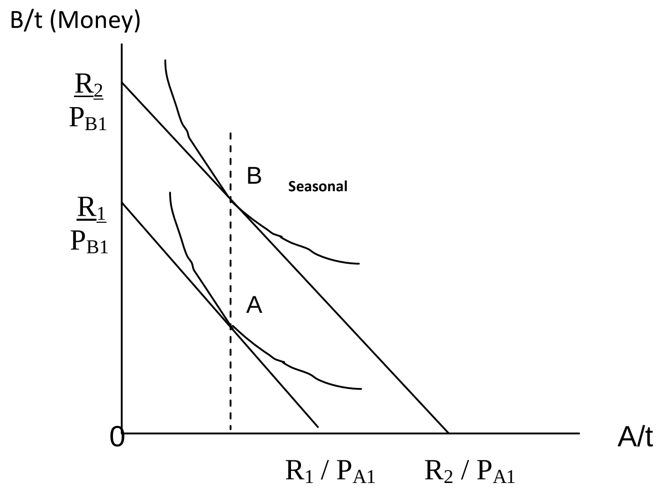
Dependent variables: these are characteristics or aspects that change as a result of the control exercised by the researcher on other variables.

Constant variable: this refers to a characteristic that does not change in a group or population, in particular during the time during which the research is conducted.

Technical note on the study of consumer surplus

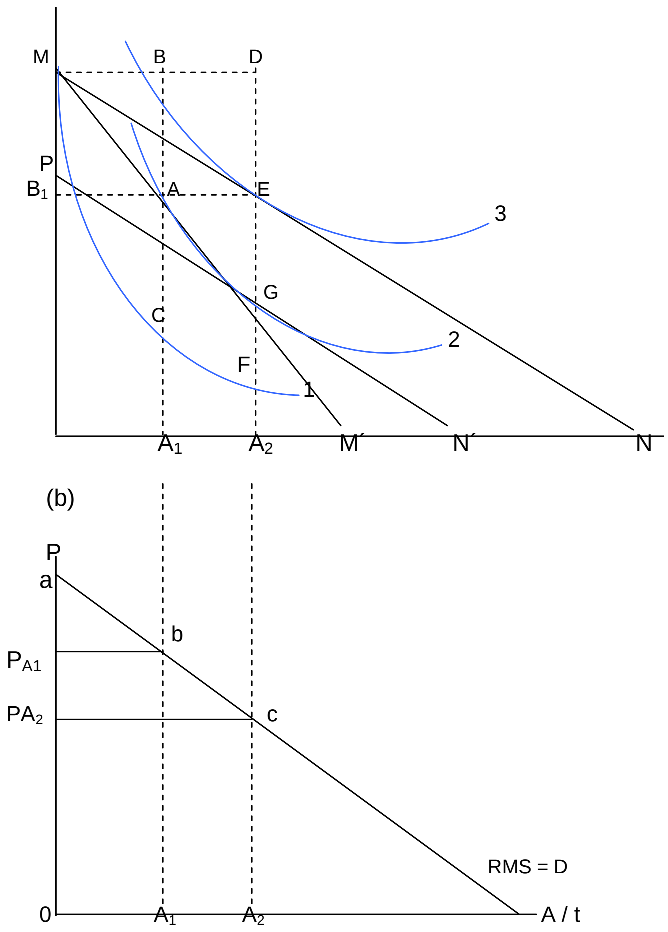
Consumer surplus can also be studied by means of indifference curves.[9]

Suppose there are two products, A and B, and that B is money. Further suppose that the marginal utility of money is constant. On that assumption, the indifference curves will run parallel to each other, i.e. all the indifference curves will have the same slope no matter what the amount of A.



In A and B, the slopes of the two indifference curves are equal. For this reason, RMS_{BA} in A is equal to RMS_{BA} in B, and in both A and B UMa_A / P_A = UMa_B / P_B. Since P_A and P_B are equal in position A and B, and since UMa_B is constant, UMa_A must also be equal in B and A. This only holds true if the quantity of A does not change. This means that only the quantity of B can change when income rises from R_1 to R_2. Thus, RMS_{BA} on point A and RMS_{BA} on point B are equal and only the amount of B has changed, as income varies. (The marginal utilities of A and B are assumed to be independent.)

[9] Richard A. Bilas, *Microeconomic Theory: A Graphical Analysis,* McGraw-Hill, 1985, pp. 135-147.



In this figure, the indifference curves have been drawn so that they run parallel to each other. B represents money. Good A has to be valued in units of money. By definition, the price of one unit of money has to be 1. Since the slope of an indifference curve at any point is given by RMS_{BA} or a price relationship at that point, it follows that when $P_B = 1$, $RMS_{BA} = P_A$, no matter what the value of A.

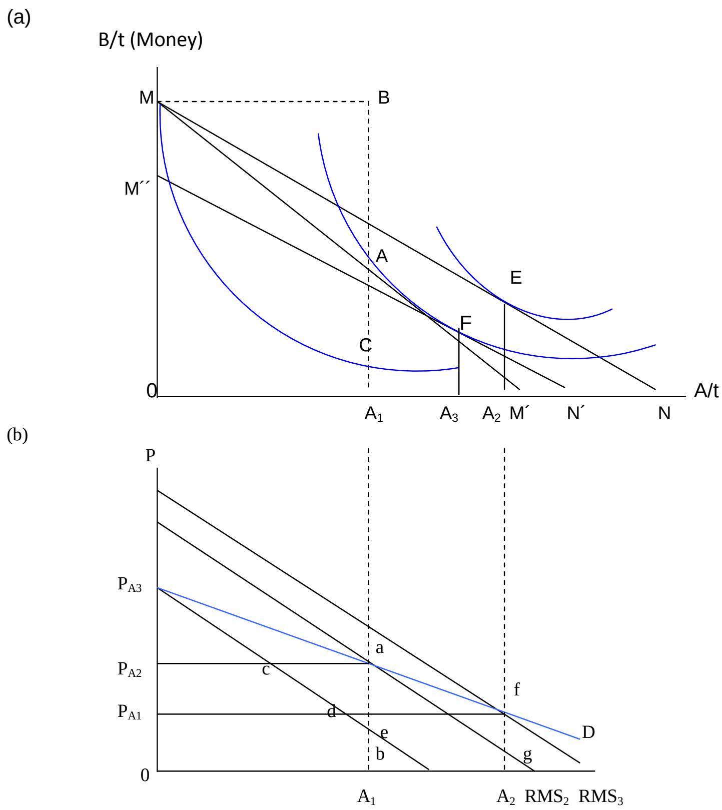
Since for a specific quantity of A, $RMS_{BA} = P_A$ (the price of A) on any indifference curve, the system of indifference curves is reduced to only one RMS curve. This becomes the demand curve for A. According to the previous figure, the demand curve has to be elastic with regard to the price equal to 1, given that BA = DE is the amount spent on A in the two equilibrium states indicated by points A and E on the previous figure.

If the consumer is at point A when income is M, and the price structure is represented by the budget restriction line MM´, the consumer buys A_1, spending AB on A and OB_1 on other goods. The price of A is OM / OM´ (slope of indifference curve 2 at point A, where $RMS_{BA} = P_A$), which is P_{A1}.

Since the slopes of the indifference curves in A and C are equal, to be equal to the slope of the budget restriction line MM´, RMS for all indifference curves on A_1 is equal to P_{A1}. For a lower price for A, indicated by the budget restriction line MN, the person will go to E, where OM / ON = P_{A2}.

If the consumers are in equilibrium at A, they will pay a maximum of up to BC for A_1 before forgoing the good, because ceding BC will not leave them worse off than if they had deprived themselves of A. Indeed, the consumers are equally satisfied at C and M (both points on indifference curve 1). It makes no matter to them, therefore, whether they are at C or M, and they would be willing to pay BC before going without A. Clearly, therefore, AC is the consumer surplus at A_1.

In figure (a), AC is equal to the area P_{A1}ab in figure (b). If price falls to P_{A2}, the consumer surplus will become EF in figure (a), which is the same as area P_{A2}ac in figure (b). The area P_{A1} P_{A2} bc of the figure represents the gain in consumer surplus due to the fall in price.

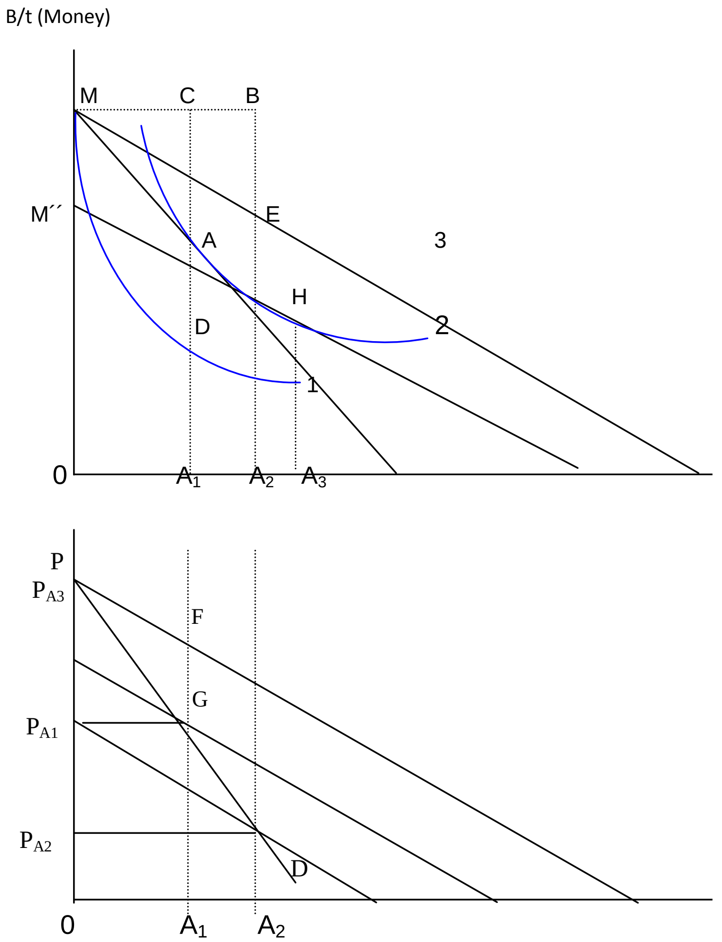


Now suppose that RMS_{BA} increases when B rises to a certain value of A. Here the price elasticity of A with regard to income is positive, since a move outside the line of budget restriction, parallel to itself, will increase the demand for A. This is the case of a normal good. Because of this, the system of indifference curves cannot be reduced to a single RMS curve; the slopes of the indifference curves become steeper, for a specific value of A, as they rise to ever greater levels of satisfaction.

As in the previous case, B represents money. The lines RMS_1, RMS_2 and RMS_3 are the RMS lines for indifference curves 1, 2 and 3 respectively.

If MM´ is the budget restriction line, the consumer will demand A_1 and the price will be P_{A2}, which is the slope of indifference curve 2 at point A, given that at that point $RMS_{BA} = P_A / P_B$ y $P_B = 1$. At the same time, P_{A1} on line RMS_3 is the slope of indifference curve 3 at point E, and the quantity demanded at that price is less than A_2. When those points are connected on figure (b), the curve represents consumer demand for A. What happens then is that consumer surplus is not equal to the triangular area under the demand curve.

The surplus in A_1 is AC. BC (the maximum quantity that the consumer would be willing to pay before going without the good) corresponds to $OP_{A3}b_{A1}$, which is the area under the RMS_1 curve, up to A_1.



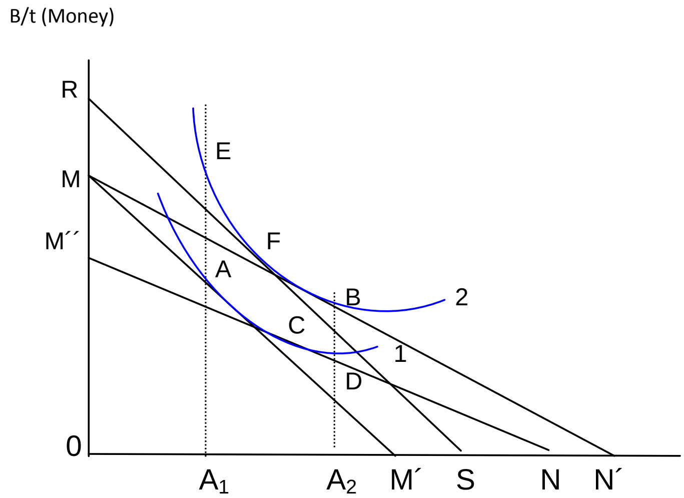
The curve RMS_1 indicates the maximum price per unit that the consumer would pay for A before going without the good; the area under the curve is simply the maximum price of the first unit multiplied by 1, plus the maximum price of the second unit multiplied by 1 and so on for the first A_1 units. AB corresponds to $OP_{A2}aA_1$. If those two areas are subtracted, the lower figure has $P_{A2}P_{A3}c$ – cab. That difference is obviously less than the triangle under the demand curve, $P_{A2}P_{A3}a$. In conclusion, it would have to be said that the triangular area under the demand curve is the consumer surplus, which would be an overstatement in the case of a normal good.

If an inferior good is chosen, the consumer surplus in the simple case in which UMa_B is constant is less than the true value of consumer surplus.

In this example, RMS_{BA} falls when B increases for a specific value of A. Demand elasticity for A with regard to income is negative, because a concomitant movement outwards (M´´N´ is parallel to MN) in respect of the budget restriction line would lower the quantity of A demanded. A is therefore an inferior good.

Here again, the system of indifference curves cannot be reduced to a single RMS line, because the slopes of the curves ease off for a specific value of A as ever higher levels of satisfaction are reached.

If the budget restriction line is MM´, the consumer will be in equilibrium at point A, with demand being A_1 and price P_{A1}. The price is the slope of indifference curve 2 at point A. Furthermore, when the price of A falls, the consumer is in equilibrium at E on indifference curve 3 and demands the quantity A_2 at price P_{A2}, which is the slope of indifference curve 3 at E.



For quantity A_1, the consumer surplus in the upper part of the figure is AD. CD, which is the maximum amount that the consumer would spend before going without the good, corresponds to the area $OP_{A3}FA_1$, which is the area under the RMS_1 line to A_1.

AC, which is the amount spent at A_1, corresponds to the area $OP_{A1}GA_1$. If those two areas are subtracted from each other, the outcome is the consumer surplus for quantity A_1. The outcome of that subtraction is $P_{A1}P_{A3}FG$. This is greater than $P_{A1}P_{A3}G$, which is the triangular area under the demand curve.

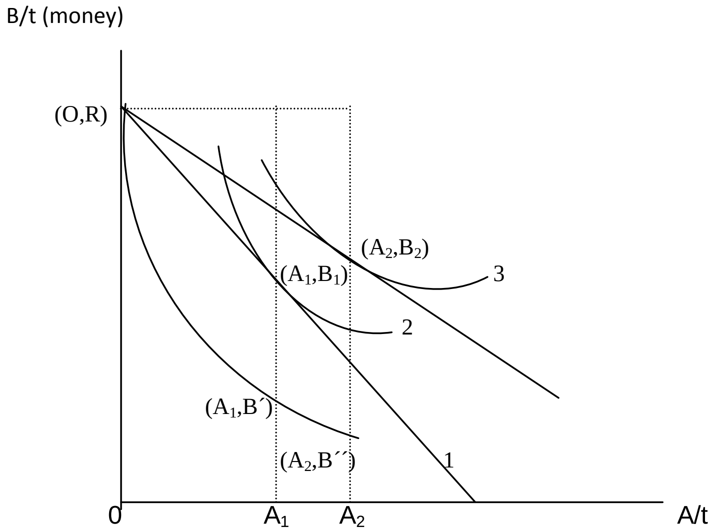
In the case of an inferior good, the consumer surplus is a value less than the true surplus.

Since consumer surplus measures the "extra" value that the consumer would receive if he bought an amount of the merchandise, the evaluator might wish to measure the variation in surplus when the price of the merchandise in question changes.

According to Hicks, there are four other methods of measuring consumer surplus in addition to the above. To study the above, the definition of consumer surplus must first be slightly modified. Consumer surplus is defined as the surplus brought about by a variation in price. It is the additional amount of money that the consumer might be obliged to pay in order to consume the new quantity and stay in the same situation of well-being as before the price changed.

Taking the budget line MM´ in the previous figure, the consumer is in equilibrium at point A on indifference curve 1 and consumes A_1 units of A. If the price of A falls to the point that the budget line becomes MN´, then equilibrium moves to point B on indifference curve 2. The consumer could be asked to pay BC and would be no worse off than before the price change. Therefore, BC is a measure of consumer surplus. BD (the compensating change in price) is another measure of consumer surplus.

If the consumer is at point A, how much money will be needed to reach indifference curve 2? There are two measures. AE can be used to reach indifference curve 2, the indifference curve it would reach with the fall in P_A indicated. MR would also allow the consumer to reach indifference curve 2, but at point F, not point E. The previous measure is equal to the triangular area under the demand curve. The two compensatory payments are less than the triangular area, while the two equal payments or variations are greater than the triangular area.



Consumer surplus can also be analysed using integral calculus. If a specific function of utility has an indifference curve that crosses through points (O,R), (A_1, B´) and (A_2, B´´), as shown in the following formula:

If the equation for this utility function is: U = Φ(A) + KB, where K is a constant.

Then:

$\delta \bar{U}/\delta A = \Phi'(A) = \Phi A$ and

$\delta \bar{U}/\delta B = K$ and

$d\bar{U} = \Phi A \, dA + K \, dB = 0$

Therefore:
$-dB / dA = \Phi A / K = RMS_{BA} = f(A) > 0$

As a rule, if U = ʉ (A,B), then B = ʉ$^{-1}$ (A, U)

Where U is constant, and therefore:

dB / dA = ʉ$^{-1\prime}(A,\bar{U}) = f(A) < 0$ o $-dB / dA = f(A) > 0$

At equilibrium and with $P_B = 1$, the outcome is f (A) = P_A, which is a demand function.

Therefore:

$$C = CB' - CB_1 = \int_0^{A1} f(A)dA - A_1P_{A1} \quad \text{and}$$

$$\Delta C = (DB'' - DB_2) - (CB' - CB_1)$$

$$\Delta C = - \left[\int_0^{A_1} f(A)\,dA - A_1P_{A1} \right]$$

$$= \int_{A_1}^{A_2} f(A)\,dA - (A_2P_{A2} - A_1P_{A1})$$

On all the above, see Jack Hirshleifer, *op. cit.,* pp. 244-246.

Bibliography

Baca Urbina, Gabriel, *Evaluación de Proyectos,* Editorial McGraw Hill, Mexico, 1987.

Bilas, Richard A. *Teoría Microeconómica*, Alianza Editorial, Madrid, 1974.

Case and Fair, *Principios de microeconomía*, Editorial Prentice – may Hispanoamericana, S.A., 1998.

Esser, Klaus, *Competitividad Sistémica*, Berlin, 1994.

González, Yolanda, Carrascal, Ursicino and Rodríguez, Beatriz, *Análisis econométrico con Eviews*, Editorial RA-MA, Madrid, 1997.

Maddala, G.S., *Introducción a la Econometría*, Mexico, 1988.

Gujarati, Damodar, *Econometría*, Santafé de Bogotá, Colombia, 1997.

Habermas, Jürgen, et al, *Competitividad internacional de las empresas y políticas requeridas*, Berlin, 1994.

Hirshleifer, Jack, *Microeconomía teoría y aplicaciones*, third edition, Prentice, 1998.

Jáuregui Gómez, Alejandro, *Determinantes de la competitividad*, 1996.

Martín, Guillermina, Labeaga, José María and Mochón, Francisco, *Introducción a la Econometría*, Spain, 1997.

Mercado Ramírez, Ernesto, *Productividad base de la competitividad*, 1998.

Mankiw, N. Gregory, *Principios de economía*, McGrawHill.

Pelayo, Carmen Maria, Internet document.

Porter, Michael, *Ventaja competitiva de las naciones*, Harvard University, 1989.

Porter, Michael, *Estrategia Competitiva,* CECSA, 1992.

Sapag Chain, Nassir, *Proyectos de Inversión. Formulación y Evaluación*, first edition, Editorial Pearson Educación, Mexico, 2007.

Thompson, Arthur, *Administración Estratégica*, McGrawHill, 2001.

Bibliography consulted

Dornbusch-Fisher, *Macroeconomía*, sicth edition, Barcelona, 1994.

Hamilton, Lawrence C., *Statistic with stata 5,* Duxbury Press, California, 1988.

Jonston, Jack and Dinardo, John, *Econometric Methods,* McGrawHill, Singapore, 1997.

Laffont-Tirol, *Competition in telecomunicacations,* first edition, London, 1999.

Leftwich, Richard, *Microeconomía*, first edition, Mexico, 1972.

Spiegel, Murray R., *Teoría y problemas de estadística*, McGrawHill, Mexico DF, 1974.

Tamales, Ramon, *Un Nuevo Orden Mundial,* Esapasa-Calpe, Madrid, 1991.

Raventós Vorst, Pedro, *Privatización y Regulación en Telecomunicaciones,* San José, 1997 edition.

Runyon/Haber, *Estadísticas para las ciencias sociales,* Mexico, 1992.

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