# ITU / BDT- COE workshop Nairobi, Kenya, 7-11 October 2002 

## Network Planning

## Lecture NP-4. 1

## Service and applications matrix forecasting

## Service matrix forecasting:

## - Traffic matrix

The bases for effective network planning is the traffic data between each two nodes of the network.

Such traffic values are typically shown in an origindestination traffic matrix.

| to |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| from | 1 | $i$ | $j$ | $n$ | $S O$ |
| $l$ | $A(l 1)$ |  |  | $A(\ln )$ | $O(1)$ |
| $i$ |  | $A(i i)$ | $A(i j)$ |  | $O(i)$ |
| $j$ |  | $A(j i)$ | $A(j j)$ |  | $O(j)$ |
| $n$ | $A(n 1)$ |  |  | $A(n n)$ | $O(n)$ |
| $S T$ | $T(1)$ | $T(i)$ | $T(j)$ | $T(n)$ | $A(11)$ |

Here: $\quad A(i j) \quad$ is the traffic from i to $j$;
$A(j i) \quad$ is the traffic from $j$ to $i$;
A(ii) is the local traffic in $i$;
$O(i) \quad$ is the sum of all traffic originating in $i$;
$T(j) \quad$ is the sum of all traffic terminating in $j$.

Adding all the row-totals $O(i)$, i.e. the entries in column SO (sum originating traffic) give the total traffic $A$.

The same result is obtained by adding all the columntotals $T(\mathbf{j})$, i.e. the entries in the row $S T$ (sum terminating traffic).

In the ideal case service matrices are the result of point-to point measurement of traffic and further mathematical traffic predictions.

If complete data for a present (first) traffic matrix are not available be measurements they have to be created by other means.

The generation of such first traffic matrix is based on information about the subscribers and corresponding traffic per subscriber (also Calling rate).


Example:
$\mathbf{a}_{\text {orig }}=0.05$ Erlang $\quad$ for Voice in PSTN
$\mathbf{a}_{\text {orig }}=5 \mathrm{Kbit} / \mathrm{sec} \quad$ for VoIP

## - Estimation of total traffic

Taking into account that different categories of subscribers initiate different amounts of traffic, it may sometimes be possible to estimate a future traffic from:

$$
A(t)=N_{1}(t) \cdot \alpha_{1}+N_{2}(t) \cdot \alpha_{2}+--
$$

where $N_{l}(t), N_{2}(t)$, etc., are the forecasted number of subscribers of category 1,2 , etc., and $\alpha_{1}, \alpha_{2}$, etc., are the traffic per subscriber of category 1,2 , etc.

If it is not possible to separate the subscribers into categories with different traffic, the future traffic may simply be estimated as:

$$
A(t)=A(0) \frac{N(t)}{N(0)}
$$

where $N(t)$ and $N(0)$ are the number of subscribers at times $t$ and zero.

## - Distribution of point-to-point traffic

## Homogenous distribution

For estimation of the traffic from one exchange to another, various formulae may be applied. The main idea is to take into account the increase of subscribers in the two exchanges and to apply certain weight factors to these growths.

$$
A_{i j}(t)=A_{i j}(0) \frac{W_{i} G_{i}+W_{j} G_{j}}{W_{i}+W_{j}}
$$

where $w_{i}$ and $w_{j}$ are the weights and $G_{i}$ is the growth of subscribers in exchange $i$, and $G_{j}$ in exchange $j$.

$$
G_{i}=\frac{N_{i}(t)}{N_{i}(0)} \quad G_{j}=\frac{N_{j}(t)}{N_{j}(0)}
$$

Different methods exist for $w_{i}$ and $w_{j}$.

## Distribution according to the gravity model

The traffic between two exchanges can be expressed as:

$$
A_{i j}=K\left(d_{i j}\right) \cdot N_{i} \cdot N_{j}
$$

where $K\left(d_{i j}\right)=$ community of interest factor.

That value can be calculated from a known traffic matrix. It may be necessary to adjust the expression for $A_{i j}$ for pairs of exchanges with special relations to each other; for example, a big factory in one part of a country and the head office in another part.

## Fixed percentage of internal traffic

## Interest factor or destination factor method.

Percentage of outgoing/incoming long-distance, national, international traffic.

## - Equalization of the traffic matrix

## Kruithof method

Kruithof's method enables us to estimate the future individual traffic values, $A(i, j)$ in a traffic matrix. The values, at present, are assumed to be known and so is the future row and column sums.

The procedure is to adjust the individual $A(i, j)$ so as to agree with the new row and column sums, i.e.

$$
A(i, j) \text { is changed to } A\left(i, j \frac{S_{I}}{S_{o}}\right.
$$

where $S_{o}$ is the present sum and $S_{l}$ the new sum for the individual row or column.

If we start with adjusting the $A(i, j)$, with regard to the new row sums, $S_{j}$, these sums will agree, but the column sums will not. Next step is then to adjust the found values of $A(i, j)$ to agree with the column sums. This makes the row sums disagree, so the next step is to adjust the new $A(i, j)$ to agree with the row sums. The procedure is continued until sufficient accuracy for both column and row sums has been reached. The iteration is rather fast and gives in general a satisfying result after about three corrections.

ANNEX:

## Example of the use of Kruithof's Double Factor Method

Given: The present traffic interests $A_{i j}(0)$

| i | j | 1 | 2 |
| :---: | :---: | :---: | :---: |
| sum |  |  |  |
| 1 | 10 | 20 | 30 |
| 2 | 30 | 40 | 70 |
| sum | 40 | 60 | 100 |

Forecast of the future total originating and terminating traffic per exchange: $A_{i}(t)$ and $A_{j}(t)$ :

| i | j | 1 | 2 | sum |
| :--- | :---: | :---: | :---: | :---: |
| 1 |  |  |  | 45 |
| 2 |  | $?$ |  | 105 |
| sum | 50 | 100 |  | 150 |

Problem: Estimate the traffic values $A(i, j / t)$ with Kruithof's method.

Solution: Iteration 1: Row multiplication.
$A_{i}$ is distributed as given by present traffic interest.

| i | j | 1 | 2 |
| :---: | :---: | :---: | ---: |
| 1 | 15 | 30 | 45 |
| 2 | 45 | 60 | 105 |
| su | 60 | 90 | 150 |
| m |  |  |  |

$$
A_{i j}(I)=\frac{A_{i j}(0)}{A_{i} \cdot(0)} A_{i}(t)
$$

After row multiplication, the sums of columns differ from the forecast. Next iteration will be column multiplication.
Iteration 2:
Column multiplication.
$A_{j}(2)$ is distributed as given by iteration 1.

| i | j | 1 | 2 |
| :---: | :---: | :---: | :---: |
| sum |  |  |  |
| 1 |  | 12.5 | 33.33 |
| 2 | 37.53 |  |  |
| 2 | 66.67 | 104.17 |  |
| sum | 50 | 100 | 150 |

$$
A_{i j}(2)=\frac{A_{i j}(1)}{A_{j}(0)} A_{i j}(t)
$$

After column multiplication, the sums of rows differ from the forecasted values. Next iteration will be row multiplication.

Iteration 3: Row multiplication.
$A_{i}$ is distributed as given by iteration 2 .

| i | j | 1 | 2 |
| :---: | :---: | :---: | :---: |
| sum |  |  |  |
| 1 | 12.27 | 32.73 | 45 |
| 2 | 37.80 | 67.20 | 105 |
| sum | 50.07 | 99.93 | 150 |

Iteration 4:
Column multiplication.
$A_{j}$ is distributed as given by iteration 3.

| i | j | 1 | 2 |
| :---: | :---: | :---: | ---: |
| sum |  |  |  |
| 1 | 12.25 | 32.75 | 45 |
| 2 | 37.75 | 67.25 | 105 |
| sum | 50 | 100 | 150 |

After 4 iterations, the sums of rows and columns are equal to the forecasted values.

We can now write: $A_{j j}(t)=A_{i j}(4)$

