Managing the risk associated with bandwidth demand uncertainty

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Content

- Uncertain bandwidth requirements
  - Quantification
  - Risk management
- Implementation of capacity instalment process
- Estimating time to capacity expiry
- Optimal timing of capacity instalment
- Use of real options

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Bandwidth demand risk

- “*Every day I look at the decision: should we build or should we lease*”?
- Unknown demand for bandwidth
  - Uncertain future applications
  - Uncertain uptake of future applications
  - Uncertain customer base

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Bandwidth price risk

- Unknown price of bandwidth
  - Prices are on their way down
  - The rate of decline is unknown $\Rightarrow$ risk

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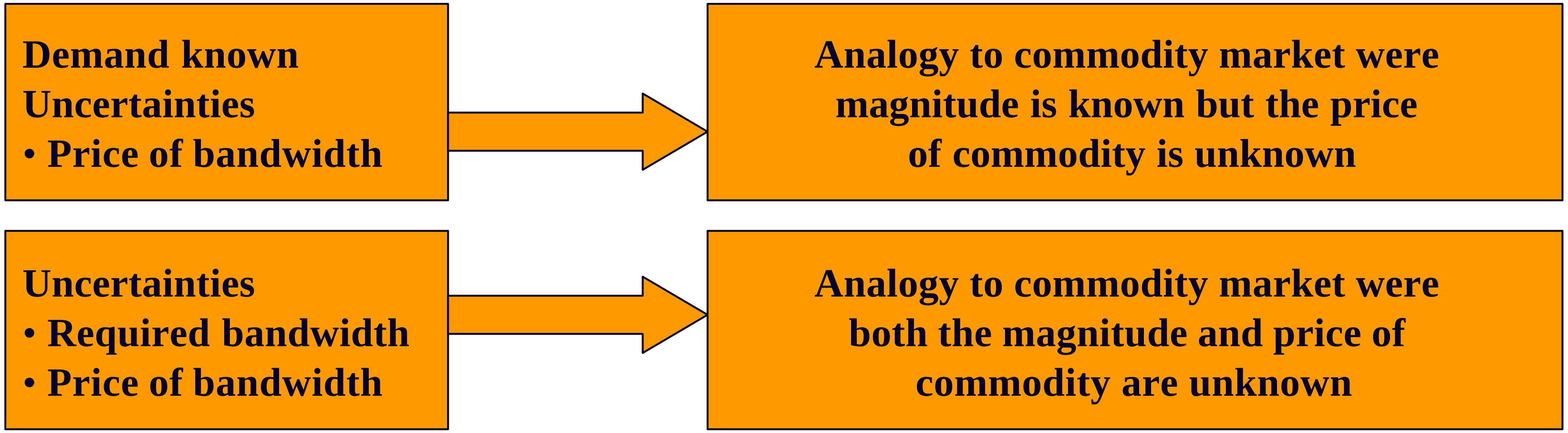
What uncertainties?

- Possible scenarios



- Require different risk management
- Most risk management procedures assume
  - Known demand (currency, commodity,…)
  - Uncertain price

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Aim of risk management

- Identify risk sources
- Quantify the risk caused
- Control the risk
  - Hedge
    - only some risks can be hedged (commodities markets,….)
  - More efficient decision making
    - operational caution
    - insurance
    - real options

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Bandwidth demand risk

- Required bandwidth can be acquired in different ways
  - Building networks
  - Install additional capacity, lit fibre
  - Lease
  - Enter derivative contracts (futures, swaps, options)
- Before deciding on action
  - Model bandwidth demand evolution
  - Model price evolution

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Modelling bandwidth demand

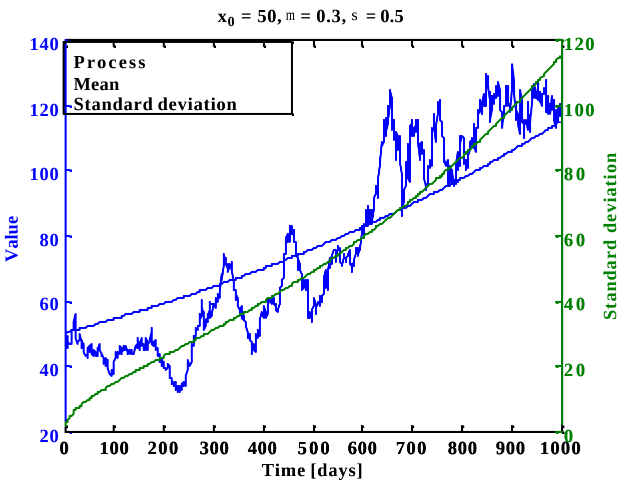
- Evidence for “exponential growth”
- But, rate of growth is uncertain
- Model as geometric Brownian motion

$$dD_t = mD_t dt + s D_t dW_t$$

$$dW_t = h_t \sqrt{dt}$$

$$h_t \in N(0,1)$$

- Therefore



gbmo (50,0.3,0.50,1/360,1000,1)

$$D_t = D_0 \exp\left\{\left(m - \frac{1}{2}s^2\right)t + s W_t\right\}$$

$$E[D_t] = D_0 \exp(mt) \quad ; \quad \mathrm{var}[D_t] =$$

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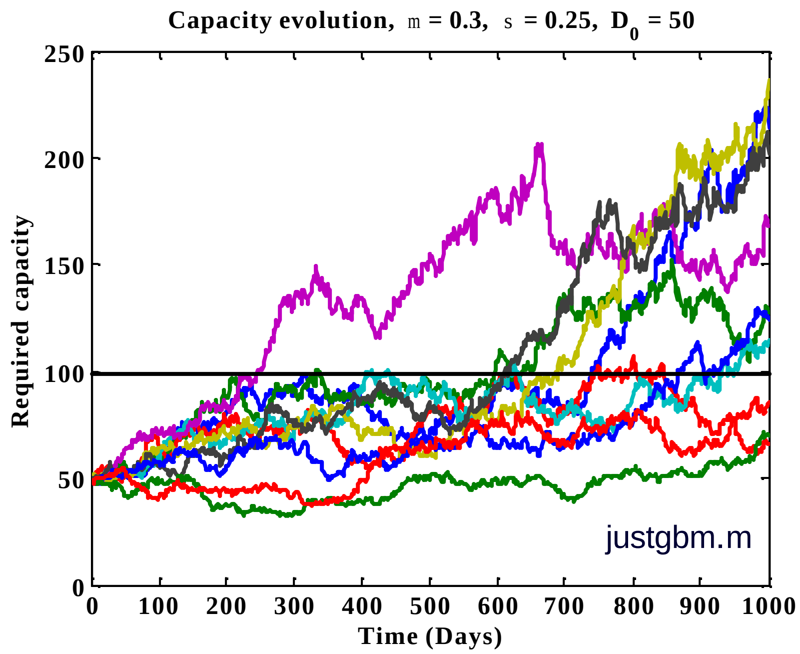
Possible demand scenarios

- Monte Carlo simulations – iterate a large number of scenarios each compatible with the assumptions



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Assumptions and questions

- Given
  - Present demand D_0
  - Presently available capacity C_0, $D_0 < C_0$
- Questions
  - What is the probability of exceeding the installed capacity within a given time?
  - What is the proper capacity instalment rate?
- Contrast the expected life of presently installed capacity with expectations about price evolution

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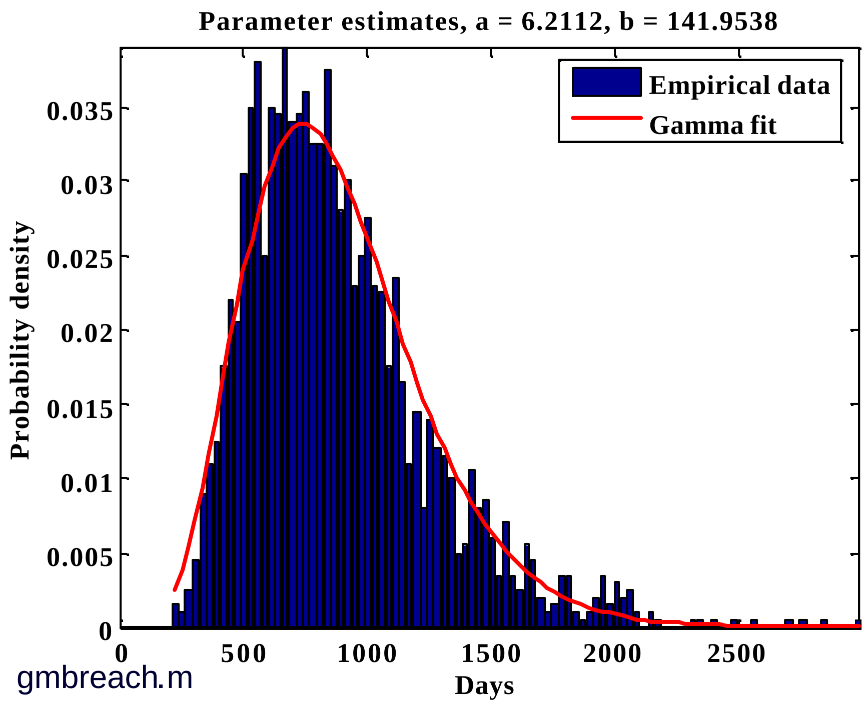
Probability of exceeding capacity

- Probability of exceeding installed capacity
- Probability density function
- Cumulative probability function



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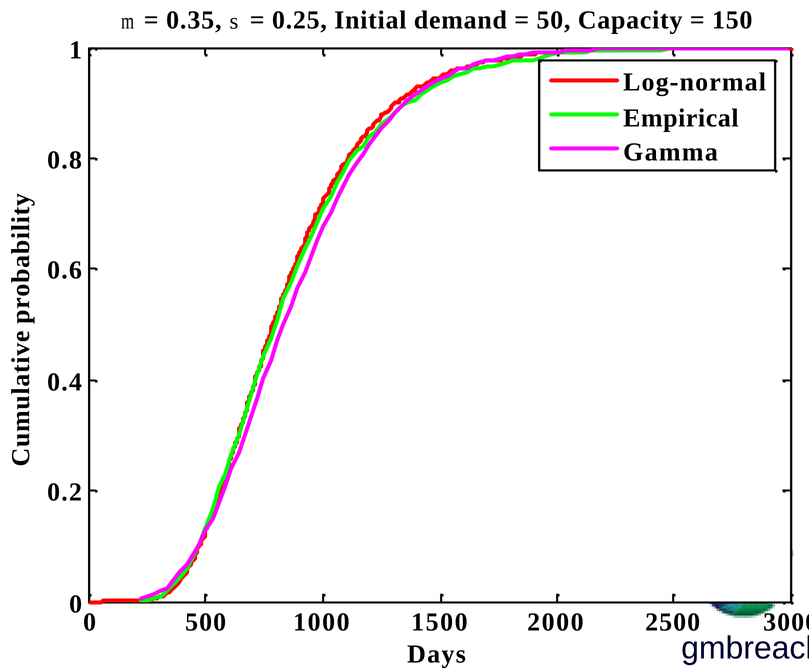

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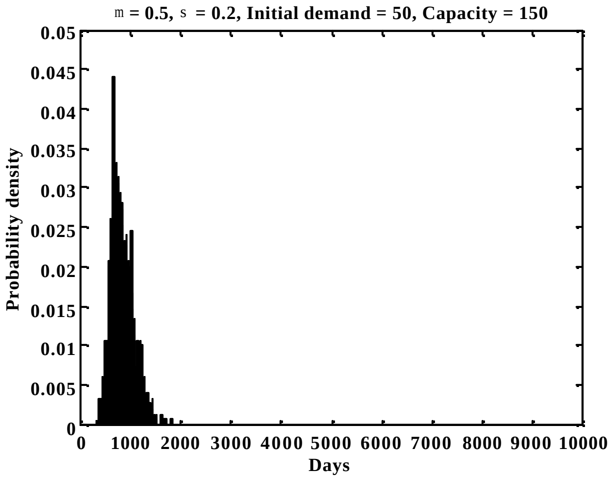
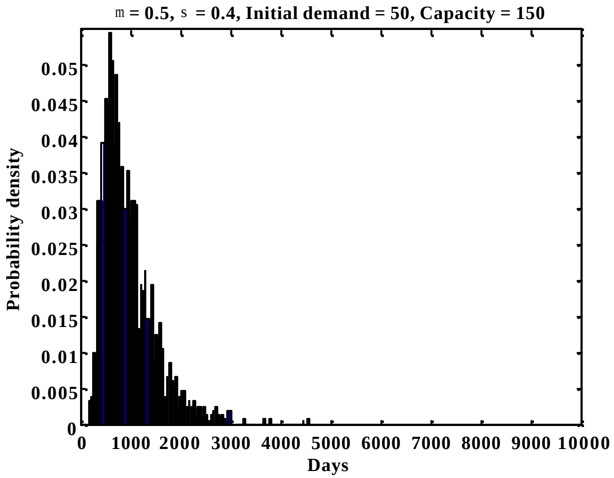
Impact of uncertainty

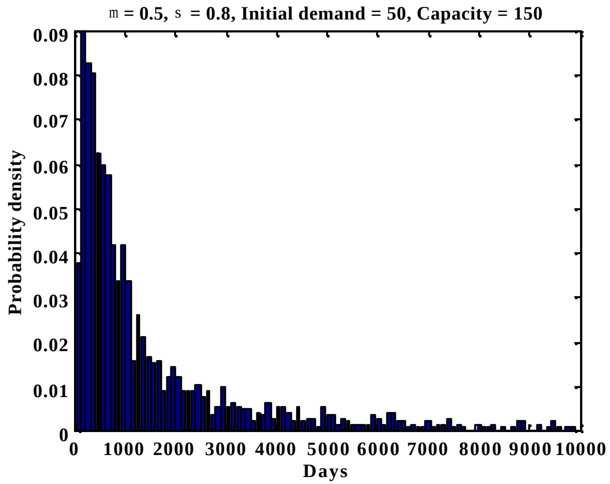
All scenarios have the same mean capacity life

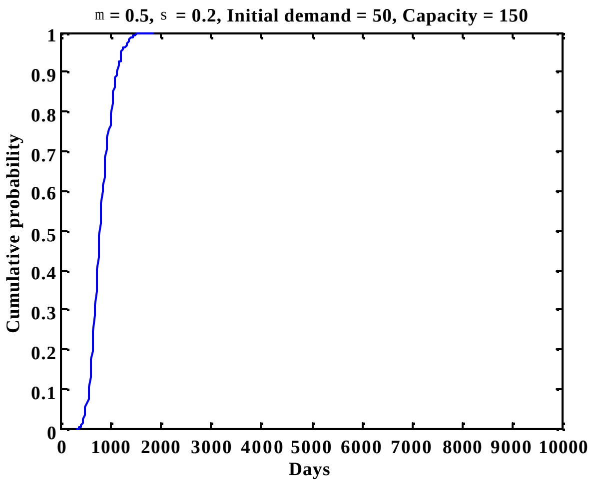
$$t = \frac{1}{m} \log\left(\frac{C_0}{D_0}\right)$$

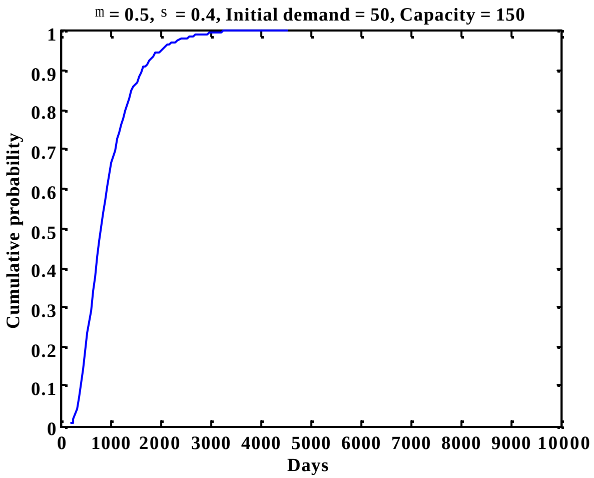
$m = 0.5$, $s = 0.2$ $m = 0.5$, $s = 0.4$ $m = 0.5$, $s = 0.8$

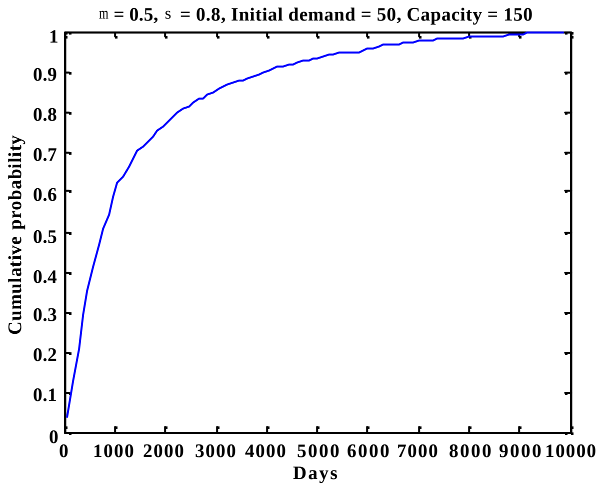












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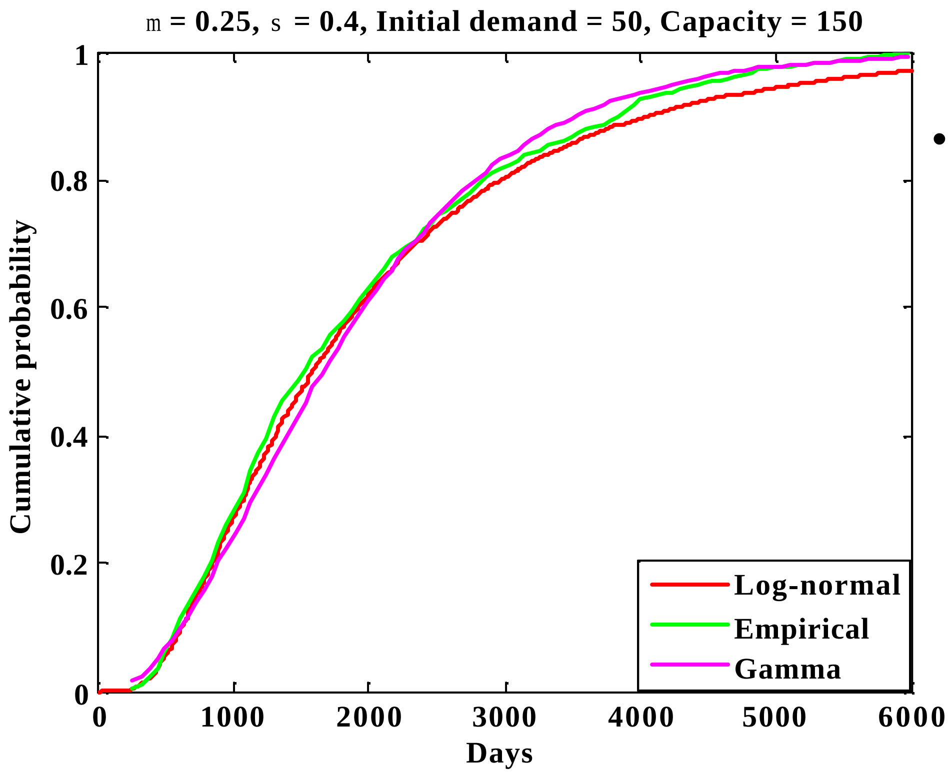
Justification for log-normal modelling

- The empirical cumulative probability is well approximated by the log-normal distribution

$$CP(D_t \le C, t) = \frac{1}{2}\left(1 + erf\left(\frac{\log(C/D_0) - \left(m - s^2/2\right)t}{s\sqrt{2t}}\right)\right)$$



- Deviations are explained by demand exceeding installed capacity and then go down below installed capacity again

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Criteria for delaying instalment

- There are benefits in delaying the acquirement of additional capacity
  - Cost
  - Efficient usage
- Risks
  - QoS reduction
  - Loss of customers
- Quantifying the criteria requires assumptions on the price evolution

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Bandwidth demand risk

- Bandwidth evolution is a stochastic process $D(t)$
- Match installed capacity optimally to demand
- Upgrade sequence

$$\Omega = \{t_1, \Delta C_1, \ldots, t_k, \Delta C_k; \ldots\}$$

- The process $C(t)$ should stochastically dominate $D(t)$

$$\Pr(C(t) \geq D(t)) \to 1$$

- Approach
  - Dynamic programming, simulation, real options

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Controlled instalments

- Probability to exceed installed capacity

$$CP\left(D_t \le C_0, t\right) = \frac{1}{2}\left(1 + erf\left(\frac{\log\left(C_0 / D_0\right) - \left(m - s^2/2\right)t}{s\sqrt{2t}}\right)\right)$$

$$D_t \to D_{t+1} = D_t + dD_t \qquad [GBM, uncontrolled]$$

$$C_T \to C_{T+1} = C_T + dC_T \qquad [controlled\ \ instalment]$$

- The instalment process will depend on
  - Expected growth
  - Expected volatility
  - Required QoS

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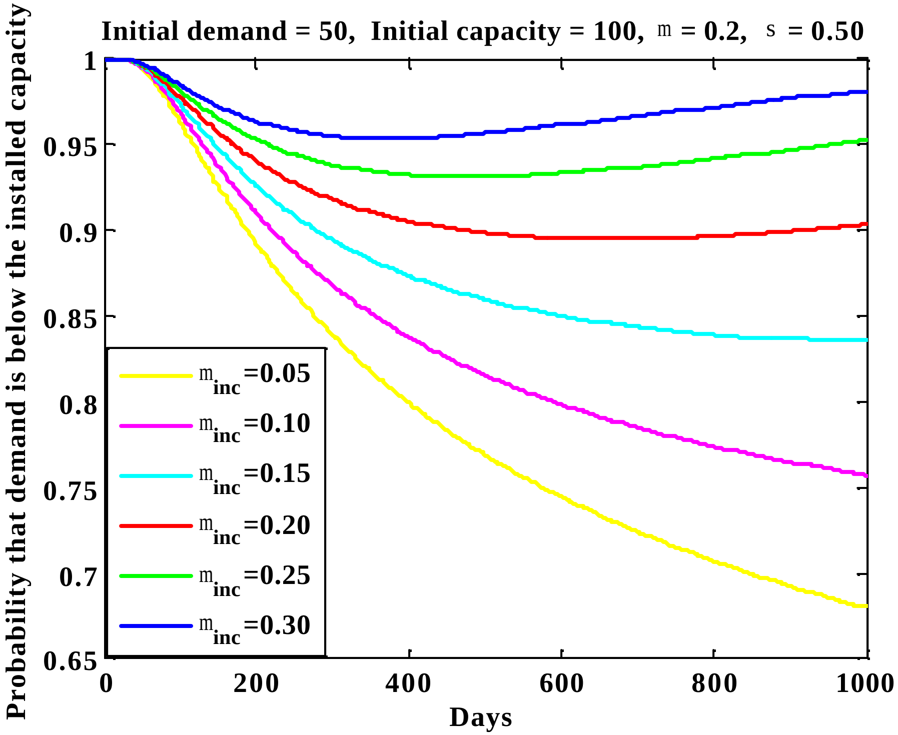
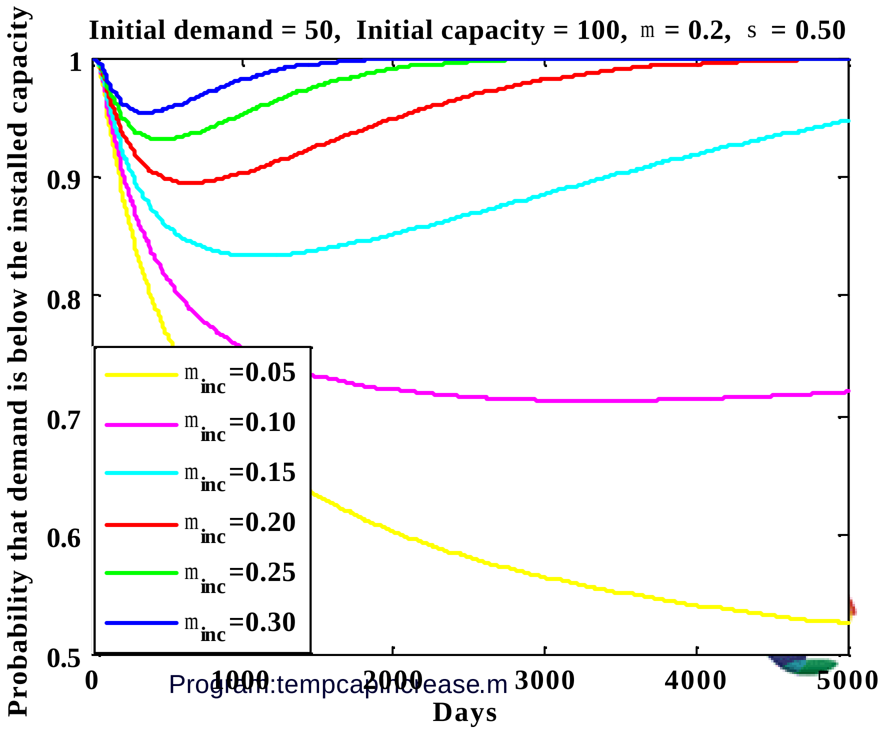
Instalment strategy

- Probability that demand does not exceed installed capacity for different instalment strategies





Program:tempcapincrease.m

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Time to capacity exhaustion

- Assume additional capacity is installed at the average rate m_i

$$D_t = D_0 \exp\left(\left(m - \frac{s^2}{2}\right)t + sW(t)t\right) = C_0 \exp(m_{in} t)$$

- Time to exhaustion

$$t = \frac{\log\left(\frac{C_0}{D_0}\right)}{m - m_i - \frac{s^2}{2} + sW(t)} \underset{E(W)=0}{\Longrightarrow} E(t) = \frac{\log\left(\frac{C_0}{D_0}\right)}{m - m_i - \frac{s^2}{2}}$$

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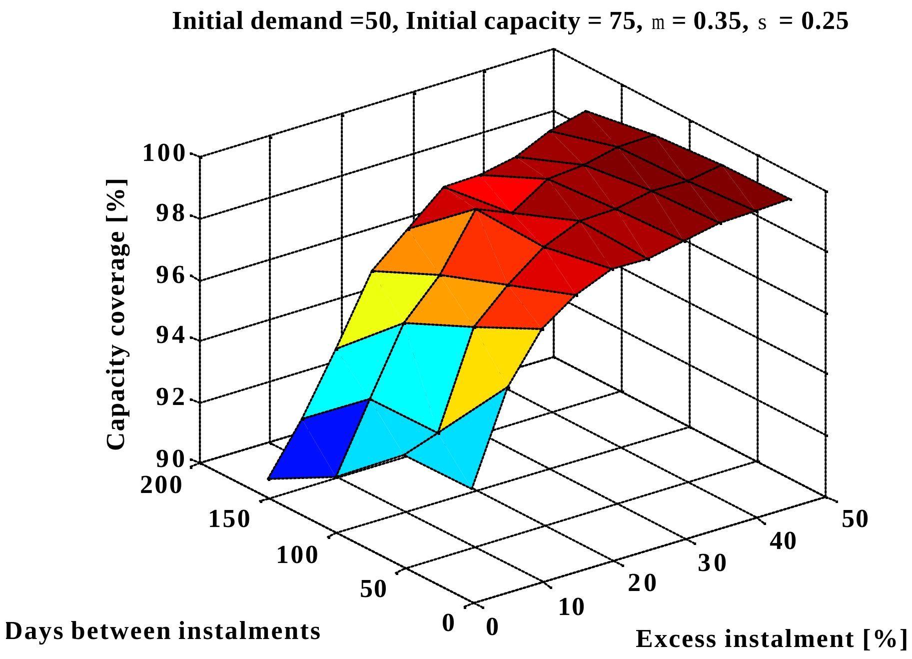
Different instalment strategies

- Installing capacity at different
  - Rates
  - Time intervals



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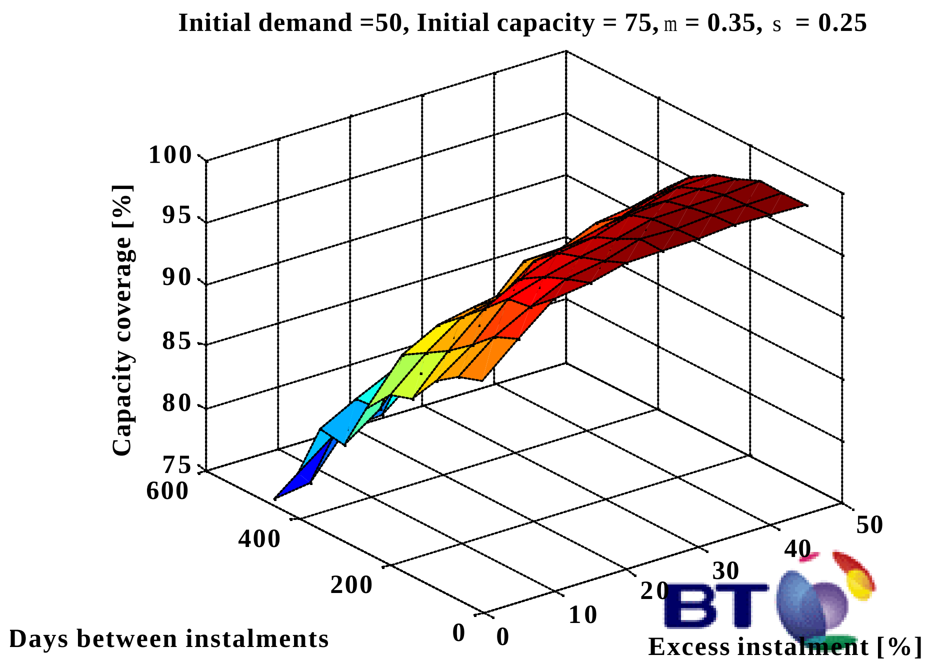

capacityplot([0:0.05:0.45],[1:50:500],50,75,0.35,0.25,1/360,1000,1000,1.5)

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Simple model for price of bandwidth

- Price of bandwidth has been going down
  - [P] = $/year/mile/megabit
- The real uncertainty is regarding the rate of decline in price

$$S(t+1) = aS(t) + h_{t+1}$$

$$h_t \in N(0, s_h), \quad E_t\{h_{t+1}\} = 0, \quad \mathrm{var}(h_t) = E\{h_t^2\} = s_h^2$$

$$S(t+k) = a^k S(t) + \sum_{i=1}^{k} a^{k-i} h_{t+i}$$

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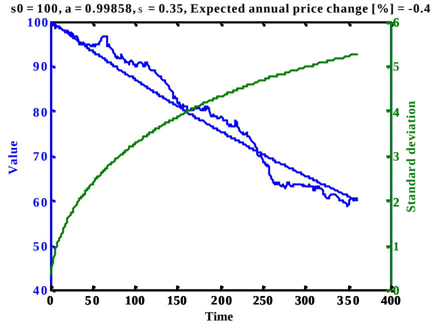
Simple model for price of bandwidth



- Price expectations and variance

$$E_t\{S(t+k)\} = a^k S(t)$$

$$s_S^2(k) = \mathrm{var}(S(t+k)) = s_h^2 \sum_{n=0}^{k-1} a^{2n} = s_h^2 \left(\frac{1-a^{2k}}{1-a^2} \right)$$

- Therefore - even if the future expected spot price *decreases* its variance *increases* as long as $a < 1$

$$E\{S(t+k)\} > E\{S(t+k+1)\} > \ldots > E\{S(t+\infty)\}$$

$$s_S^2(k) < s_S^2(k+1) < \ldots < s_S^2(\infty)$$

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When to install additional capacity

- Take into account the “damage” of capacity exhaustion
- Develop analogies to efficient frontier in portfolio management
- Optimal decisions
  - » Attitudes to risk
  - » Utility function
  - » etc

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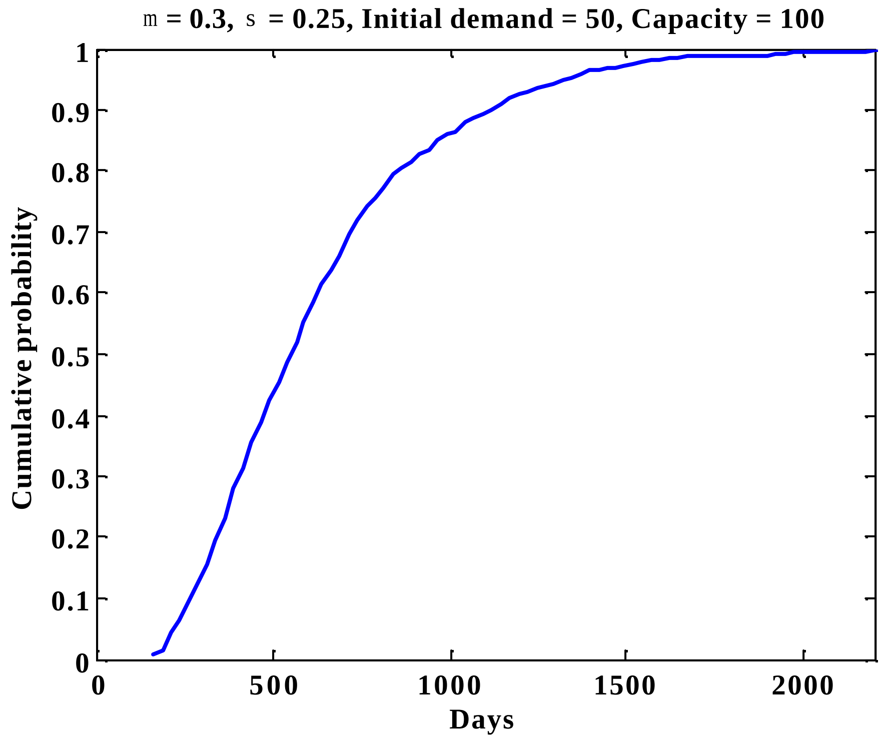
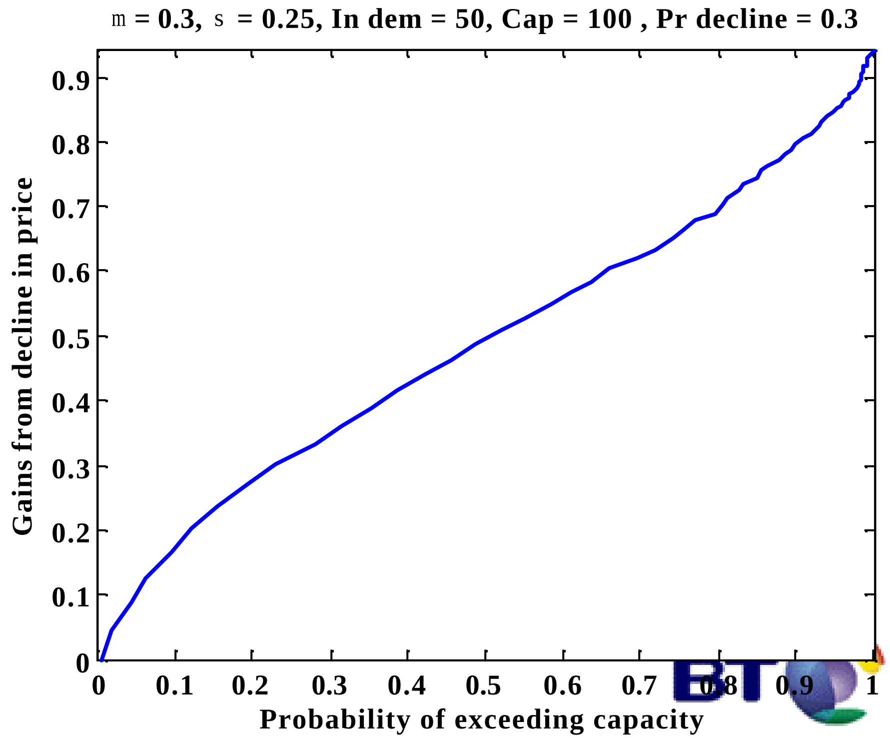
When to install additional capacity?

- Delaying capacity instalment
  - Provides monetary benefits
  - Incurs risk





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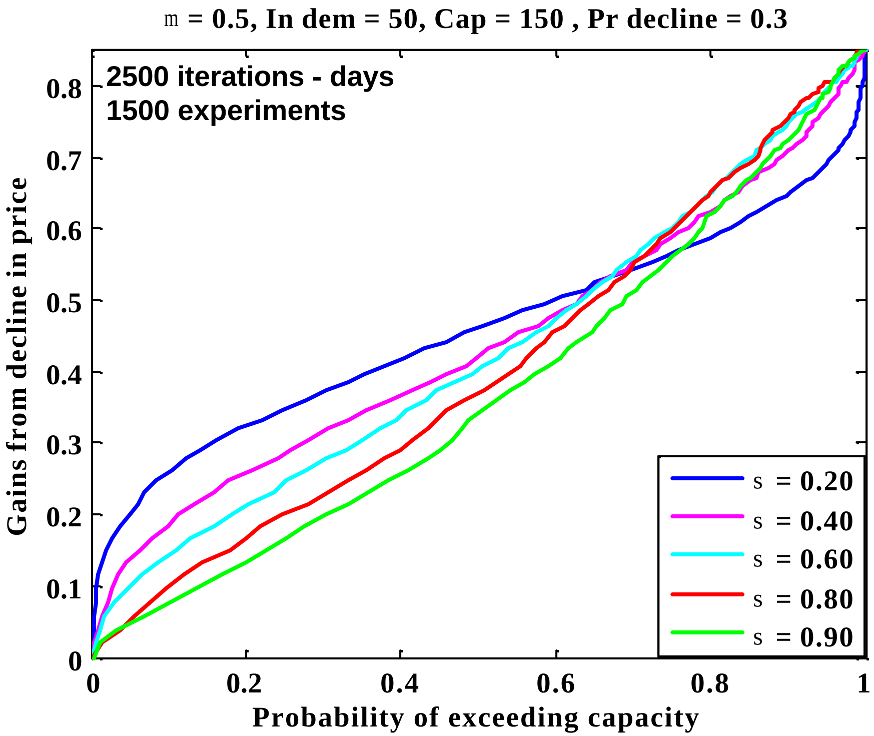
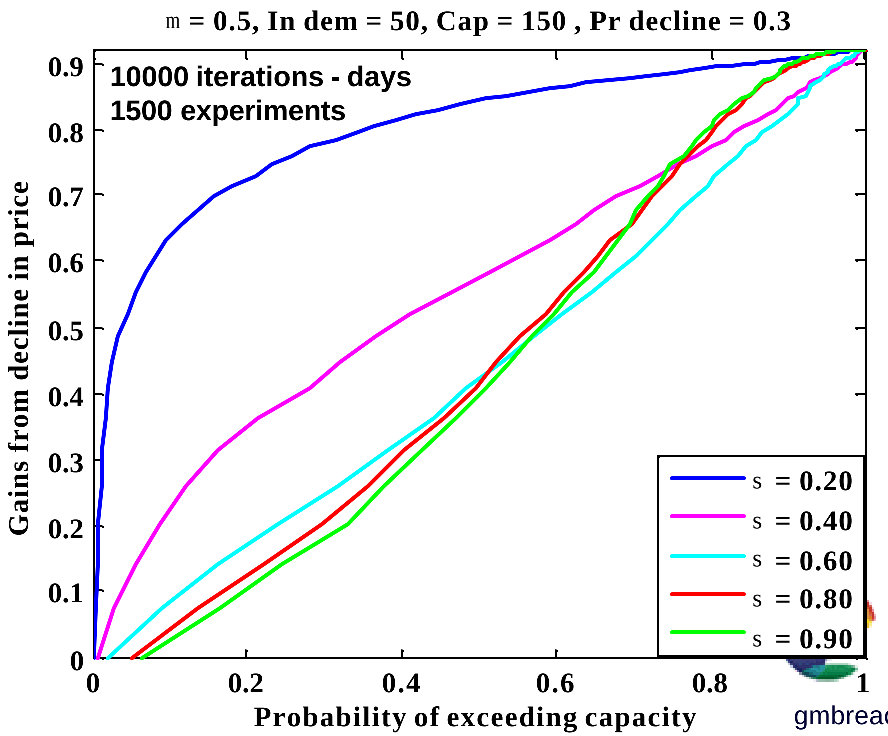
When to install additional capacity?

- The impact of volatility on expected benefits





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Cost benefit analysis

- Delaying capacity instalment is not only a question of making monetary savings
- What are the implications for deterioration in QoS on customers?
- The expected gains from delaying investment

$$g(t) = p(0)(1 - \exp(-kt))$$

- We assume the following loss as a function of time

$$c(t) = aq(t - t_c)t \;\; ; \;\; q(x) = \begin{cases} 1 \; ; \; if \;\; x > 0 \\ 0 \; ; \; if \;\; x \leq 0 \end{cases}$$

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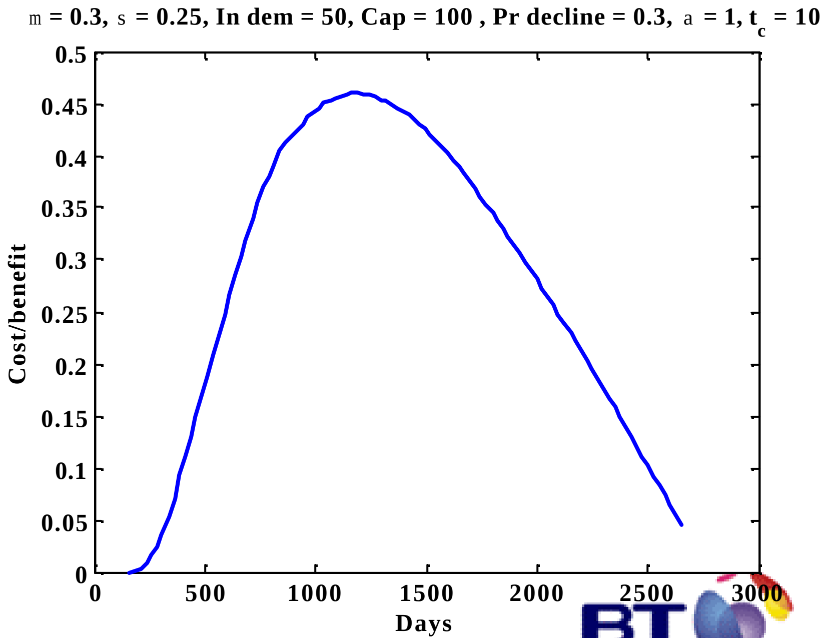
Cost benefit analysis

- Benefits from delaying capacity investment
- Disadvantage from exceeding installed capacity – resulting service deterioration
- “Optimal” instalment time depends on the assumptions made about
  - price decline
  - losses from QoS depreciation



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Hedge against price risk

- Hedge ratio,
  - h=(size of futures contract/size of exposure)
- Consider short hedge

$$\Pi = S - hF \quad \Rightarrow \quad \Delta\Pi = \Delta S - h\Delta F$$

$$\sigma_h^2 = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_H$$

$$\mathrm{var}(S) = \sigma_S^2$$

$$\mathrm{var}(F) = \sigma_F^2$$

$$\rho = \frac{\mathrm{cov}(S, F)}{\sigma_S \sigma_F}$$

$$\frac{\partial \sigma_h^2}{\partial h} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F = 0 \quad \Rightarrow \quad h = \rho\frac{\sigma_S}{\sigma_F}$$

- This assumes known demand but uncertain price

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Hedge against price and demand risk

- Demand for and price of future capacity are unknown
- Then, the correlation between demand and price matters
- Put together a portfolio

$$\Pi = DS - hF \;\Rightarrow\; s_{\Pi}^{2} = \mathrm{var}(DS) + h\,\mathrm{var}(F) - 2h\,\mathrm{cov}(DS, F)$$

- Optimal hedge ratio

$$h = \frac{E(D)\mathrm{cov}(S, F) + E(S)\mathrm{cov}(D, F) + E\big((D - E(D))(S - E(S))(F - E(F))\big)}{\mathrm{var}(F)}$$

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Real options approach

- The starting point is that demand follows

$$dD_t = mD_t dt + sD_t dW_t$$

- If $F = F(D,C,P,t)$ is value of investment which depends
  - Demand, $D(t)$
  - Instalment strategy, $C(t)$
  - Price evolution, $P(t)$
- The conditions $F = F(D,C,P,t)$ has to satisfy can be derived from Ito's Lemma

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Real options approach

- Differential equation for value of investment

$$\frac{\partial F}{\partial t} - \frac{1}{2} s^{2} D^{2} \frac{\partial^{2} F}{\partial D^{2}} - (m - ks) \frac{\partial F}{\partial D} + rF - \min(D, C(t)) LP(t) = 0$$

- With k, market price of risk, r risk free interest rate, $C(t)$ presently installed capacity
- Market price of risk captures the tradeoffs between risk and return for investments in capacity. The expected return on investment is

$$\overline{R} = r + ks$$

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Summary

- Stochastic models for bandwidth demand and price evolution are considered
- In spite of falling bandwidth prices there is still a considerable risk exposure
- Stochastic modelling of bandwidth demand and price evolution allow
  - Operator risk exposure to be quantified
  - Bandwidth instalment strategies to be formulated
- After making assumptions on the cost of running out of bandwidth the optimal timing of capacity instalment is decided
- We consider real options approach where value is controlled by price evolution and instalment strategy

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References