RECOMMENDATION ITU-R SM.1271-0*

EFFICIENT SPECTRUM UTILIZATION USING PROBABILISTIC METHODS

(1997)

Scope

This Recommendation serves as a basis for the performance criteria developed through the application of probabilistic methods to achieve more efficient spectrum utilization.

Keywords

Probability of interference, spectrum utilization, interference model, co-frequency, interference criteria

The ITU Radiocommunication Assembly,

considering

a) that communications technology has advanced rapidly during the past decade and the use of radiocommunication services by administrations has multiplied and placed new demands on the radio spectrum;

b) that frequency sharing is one of the important aspects of efficient frequency spectrum utilization;

c) that many guidelines and sharing criteria are based on the most unfavourable interference assumptions;

d) that more efficient spectrum utilization may depend on the acceptance of performance criteria developed through the application of probabilistic methods;

e) that the statistical characteristics of both the desired and interference signals would be necessary to evaluate spectrum sharing situations and spectrum utilization against performance standards;

recommends

1 that in order to achieve more efficient spectrum utilization, administrations should consider the use of the probability of interference and its impact on system performance.

2 that in order to assess fully the interference potential of introducing a new system into the environment, the probability of interference due to multiple interference sources should be considered using techniques such as those shown in Annex 1 and for calculation a probability of interference for base to mobile and mobile to base interference modes in a land mobile duplex system a methodology given in Annex 2 should be used.

NOTE 1 – Future examples will be included to explain how probabilistic methods be used to estimate characteristics of desired and interfering signals with the objective of increasing spectrum utilization.

ANNEX 1

(Example 1)

Calculation of the received voltage due to the radiation from multiple co-frequency sources

1 Introduction

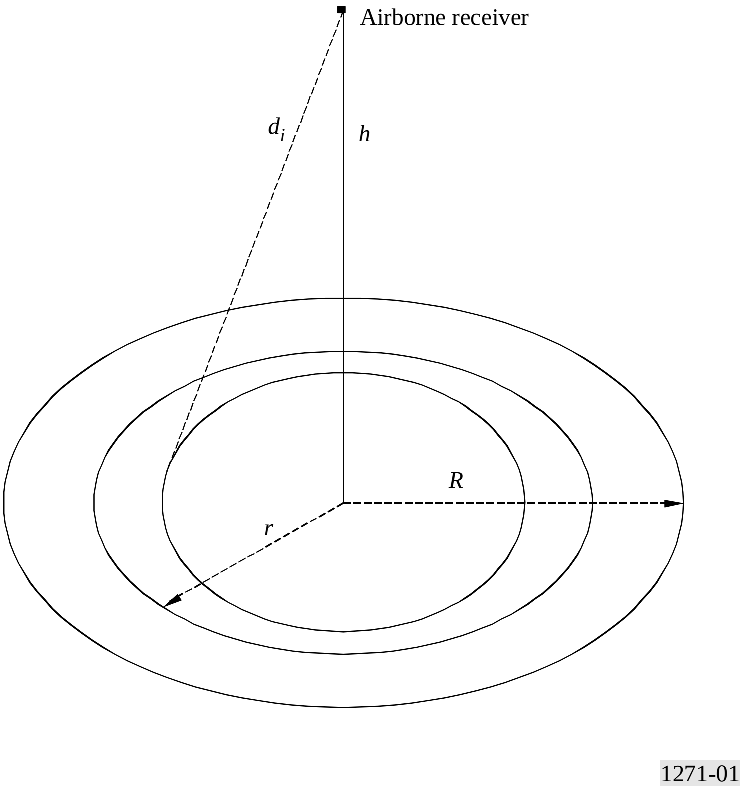
There are many frequency sharing situations where interference may occur. In some situations the number and location of possible interfering sources are not known (e.g. when the interference is from land-mobile radio transmitters). In these situations, interference can be estimated using probabilistic methods.

* Radiocommunication Study Group 1 made editorial amendments to this Recommendation in the years 2018 and 2019 in accordance with Resolution ITU R 1.

This example describes a method that clearly demonstrates the concept of calculating interference levels due to multiple sources using probabilistic methods. Consider, for the sake of simplicity, the case of an airborne receiver flying over an urban area. In this particular case the effects of the earth curvature can be neglected. The airborne antenna will be assumed to be a half-wave dipole with a cosine pattern. The N interfering sources are assumed to be uniformly distributed over the area which is assumed to be circular of radius R as shown in Fig. 1.

FIGURE 1

Airborne receiver over an urban area with uniformly distributed interference sources



The objective will be to derive a simple expression for the probability density function of the received r.m.s. voltage at the receiving antenna terminals. This example is based on several simplifying assumptions as follows:

– The number of the interfering transmitters, N, is large.

– The interfering transmitters are uniformly distributed, in geographical terms, over a circular area of radius R.

– All interfering transmitters have equal amount of radiated power.

– The receiver, which is subjected to interference from all the mobiles, is airborne at an altitude h, directly above the centre of the circular area that contains all the interfering sources.

– The propagation law from any interferer to the airborne receiver is approximated by the free-space propagation rule.

2 The method

Considering the airborne receiver now flying over the urban area, the voltage at the receiving antenna terminals can be written as follows:

$$V = \sum_{i=1}^{N} C_A \; A(d_i) \, K_i \; \mathrm{e}^{j\varphi_i} \tag{1}$$

C_A is the relationship between the field strength at an antenna and the voltage across its terminals when matched to a 50 Ω load:

$$C_A = \frac{\lambda}{2\pi}\sqrt{\frac{50}{73}} \tag{2}$$

$A(d_i)$ is the complex antenna factor which is related to the interferer-to-antenna distance d_i as follows:

$$A\left(d_i\right) = a\left(d_i\right) \mathrm{e}^{j\alpha(d_i)} \tag{3}$$

K_i and φ_i are the amplitude and phase of the field at the receiving antenna due to interferer i and are defined as follows:

$$K_i = E_L \left(\frac{d_L}{d_i}\right) K_s \tag{4}$$

where E_L is the specified field strength limit (mV/m) at a distance d_L (m) from a single interfering source. K_s is the interferer radiation pattern factor, accounting for the reduction of the mean level compared to the maximum level. d_i is the interferer-to-antenna distance in (m).

$$\varphi_i = \varphi(0) + \frac{2\pi\, d_i}{\lambda} \tag{5}$$

where λ is the wavelength and $\varphi(0)$ is the initial phase of the signal as it leaves source i.

Equation (1) can therefore be rewritten as follows:

$$V = \sum_{i=1}^{N} v_i\ \mathrm{e}^{j\psi_i} \tag{6}$$

where:

$$V_i = C_A\ E_L\, K_S\, d_L\, \frac{a(d_i)}{d_i} \tag{7}$$

$$\psi_i = \varphi(0) + \frac{2\pi\, d_i}{\lambda} + \alpha\left(d_i\right) \tag{8}$$

where ψ_i is uniformly distributed between 0 and 2π.

It can be easily shown that $|v|$ follows a Rayleigh distribution such that:

$$p\left(|v|\right) = \frac{|v|}{\xi^2}\ \exp\left[-\frac{|v|^2}{2\xi^2}\right] \tag{9}$$

where $\xi^2 = (N/2)$. $E(v_i^2)$ is the expected value of v_i^2. The r.m.s. voltage is $v_{r.m.s.} = \sqrt{2}\xi$.

From the variables already stated, the r.m.s. voltage can be calculated as follows:

$$v_{r.m.s.} = \sqrt{N}\ C_A\ \ E_L\ K_S\ \ d_L\ \sqrt{E\left[\left(\frac{a(d_i)}{d_i}\right)^2\right]} \tag{10}$$

The antenna/distance factor:

$$\sqrt{E\left[\left(\frac{a(d_i)}{d_i}\right)^2\right]}$$

is due to the combined effect of the receiving antenna pattern and the spread of the interfering sources over a given area and may be calculated by considering the probability distribution of the interfering sources. Assuming a uniform distribution for purposes of simplicity, referring to Fig. 1, the probability density function of the location *r* of the interfering sources is given by:

$$p(r) = \frac{(2\pi r)}{(\pi R^2)} = 2\frac{r}{R^2} \tag{11}$$

Since the earth curvature is negligible, the interferer-to-receiving antenna distance is given by:

$$d_i = \sqrt{r^2 + h^2} \tag{12}$$

Also since we assumed a cosine law for the antenna:

$$a\left(d_i\right) = h / d_i \tag{13}$$

Thus the antenna factor is given by:

$$a(d_i) \,/\, d_i = h \,/\, d_i^2 = h \,/\, \left(r^2 + h^2\right) \tag{14}$$

of which the expected value is:

$$E\left[\left(\frac{a(d_i)}{d_i}\right)^2\right] = \int_0^R \left(\frac{h}{r^2 + h^2}\right)^2 \frac{2r}{R^2}\,\mathrm{d}r = \frac{1}{R^2 + h^2} \tag{15}$$

The above expression is suitable for computing the combined radiation from many sources over a relatively small area, such as small towns where earth curvature can be neglected. For large cities where the earth curvature cannot be neglected, a relatively more complex formula can be obtained.

3 Conclusion

In this example, we can see that in order to estimate the effect of multiple sources of interference which have unknown locations, it is necessary to use probabilistic methods under the assumptions given in the Introduction. The two random variables, namely the received voltage and the antenna factor, have to be considered in their expected value form. Equations (10) and (15) are example formulae for computing the interference voltage from the summation of a relatively large number of interfering sources.

ANNEX 2

(Example 2)

A methodology to calculate a probability of interference for base station to mobile station and mobile station to base station interference modes in a land mobile duplex system

1 Introduction

In land mobile systems, the reuse distance between co-channel base stations is traditionally determined according to acceptable level of interference at the victim mobile station. This is a "deterministic" methodology in which probability of interference is not considered. Hence, the reuse distance calculation is based on worst case scenarios. Clearly, such systems do not frequently operate in such scenarios, hence leading to inefficient use of spectrum.

The methodology presented herein is based on a "probabilistic" approach. It defines a probability of interference that is dependent on distance between co-channel base stations. Reuse distance can hence be determined according to an acceptable level of interference probability, which is a function of specific system parameters, and also to the off-channel rejection (OCR) which is a measure of the capability of a receiver to reject interference. This parameter was defined in Recommendation ITU-R SM. 337.

The probability of interference is defined as the ratio of the area within the coverage area that would suffer interference to the total coverage area. In this example, the distribution of mobiles inside both the victim and the interfering coverage areas is assumed to be uniform. One would think that this probability would decrease as the distance of separation between base stations increases. The analysis in this paper shows that this is not always the case. For certain values of OCR, the probability of interference will actually increase and then decrease as distance increases. Using the probability curves, it is possible to select a smaller distance of separation at an acceptable probability of interference, hence increasing the utilization of the spectrum. This method of calculating the probability of interference is independent of the modulation technique used. It can also be applied to both analog and digital radio systems of different channel widths.

2 Interference criteria calculation

This example provides a methodology to determine the probability of interference in a land mobile duplex system. The base to mobile interference and the mobile to base interference are dealt with separately. First, it is necessary to define a criterion to determine if there is harmful interference or not.

In the interference criterion, interference is assumed to occur if the interference power level P_i is higher than the desired power level P_r by a certain protection ratio, ε (dB) (i.e. if $P_i - P_r > \varepsilon$). By using the 4th power law, it is possible to express the power levels in terms of distances so that the interference criterion becomes $d_2 < k\, d_1$ where the parameters are defined as follows:

d_1: distance in km, between the desired transmitter and the desired receiver;

d_2: distance in km, between the interfering transmitter and the desired receiver;

k: interference criterion factor and is given by:

$$k = 10^{\dfrac{\varepsilon \; - \; 20 \log\left(\dfrac{h_{DTx}}{h_{ITx}}\right) \; - \; 10 \log\left(\dfrac{G_{DTx}}{G_{ITx}}\right) \; - \; P_{t_{DTx}} \; + \; P_{t_{ITx}} \; - \; OCR}{40}}$$

where:

h: antenna height (m)

G: isotropic antenna gain (dB)

P_t: transmitted power (dBW)

OCR: off-channel rejection (dB)

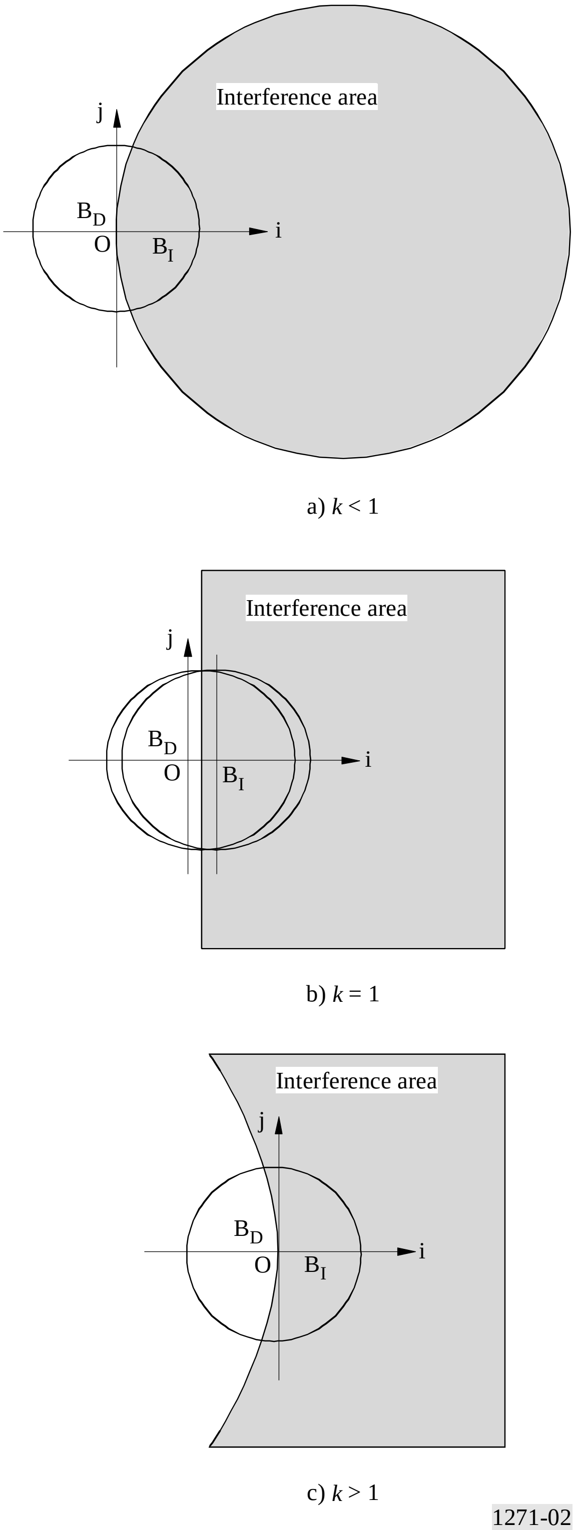
DTx: desired transmitter

ITx: interfering transmitter.

Depending on the values of these parameters, k can assume one of three possible values: $0 < k < 1$, $k = 1$, $k > 1$. The interference areas for the three values of k are shown in Figs 2a), 2b) and 2c). Note that the shaded area corresponds to the area where interference exists and $OCR = 1$ corresponds to an OCR value of 18 dB. Analysis results in § 6 show that small values of OCR correspond to high probability of interference and large values of OCR correspond to low probability of interference for the same distance of separation.

FIGURE 2

Interference areas for the three values of *k*

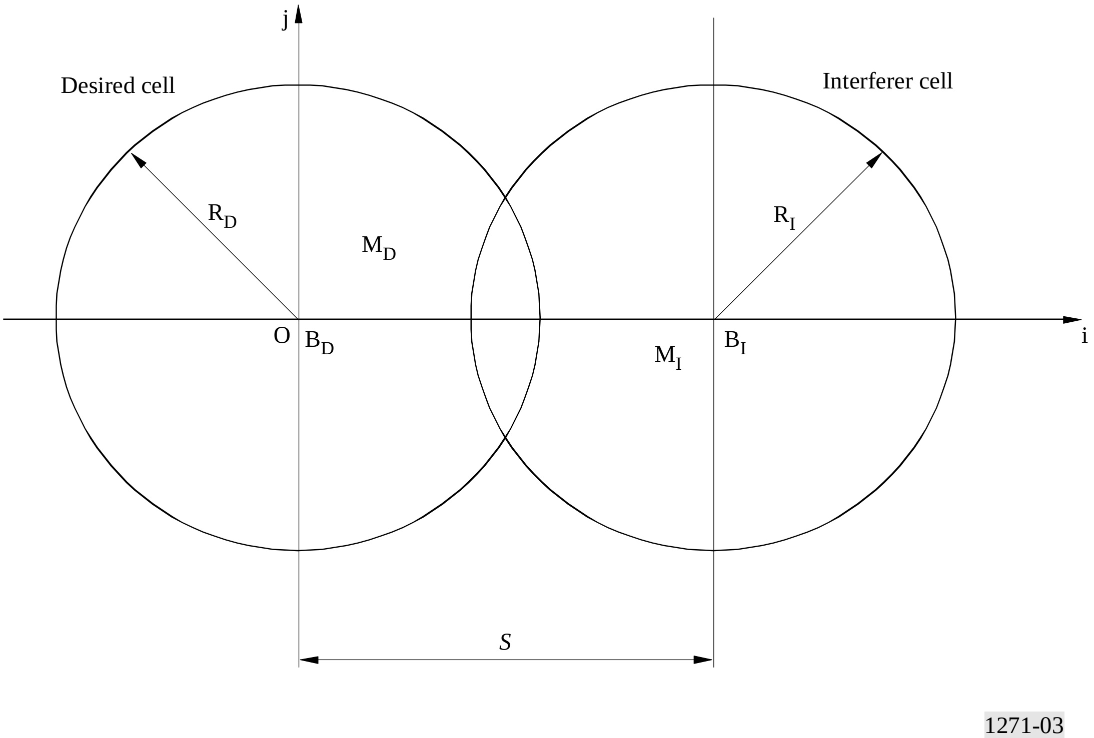


3 Probabilistic model

Figure 3 shows the coverage areas – or cells – of two base stations separated some *S* km from each other.

FIGURE 3

Desired and interfering base stations



Within both the desired and interferer cells are mobile stations. Here the desired base station is referred to as B_D, the interfering base station as B_I and in the same way, the mobiles station as M_D and M_I. The constant R defines the radius of the coverage areas. The base station B_D is at the origin and the base station B_I is some *S* km away along the i-axis. In both models d_1 is defined to be the distance between the desired Rx and the desired Tx. Similarly, d_2 is defined to be the distance between the desired Rx and the interfering Tx. All distance parameters are in kilometres.

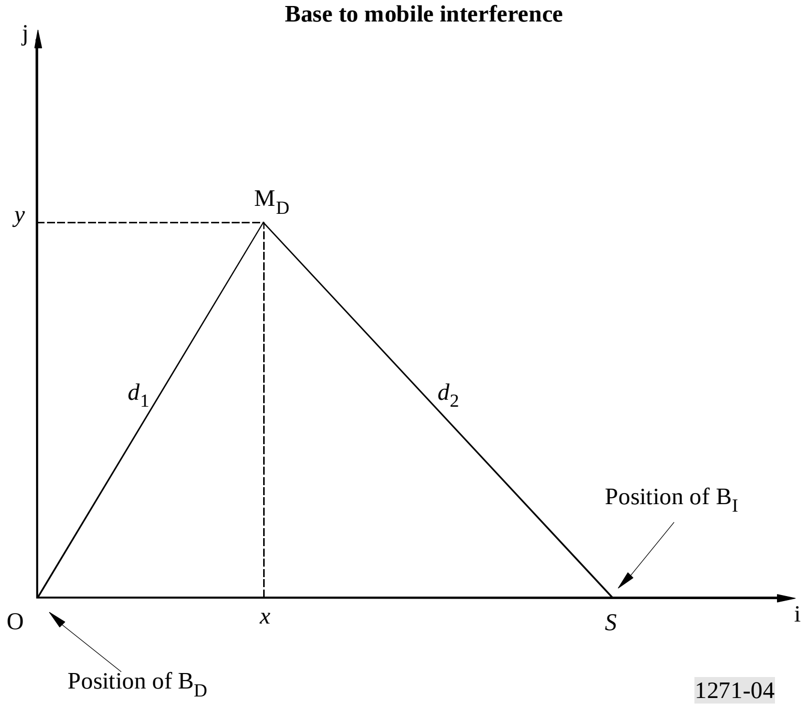
4 Base to mobile interference

This type of interference occurs when a desired mobile M_D is interfered with by an interfering base station B_I.

To find the probability of interference, it is necessary to determine the overlapping area between the coverage and the interfering area. This interfering area is delimited by the boundary of the coverage area on one side and the curve defined by $d_2 = k\, d_1$ on the other. Depending on the value of *k*, this curve can be a circle enclosing the interference region ($k < 1$), a straight line ($k = 1$) or a circle enclosing the interference-free region ($k > 1$). The interference circles, which are different from the coverage areas, have a centre and radius that vary with *k* and *S*. Once these areas are determined, the probability of interference can be obtained by dividing the interference area by the coverage area.

FIGURE 4

Base to mobile interference

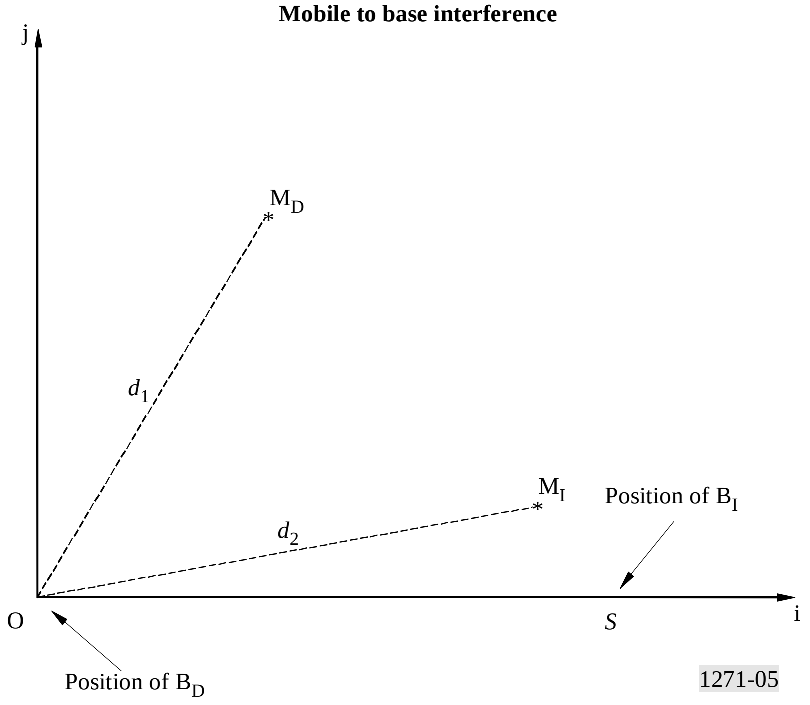


5 Mobile to base interference

This type of interference occurs when a desired base station B_D is interfered with by an interfering mobile M_I.

FIGURE 5

Mobile to base interference



For mobile to base interference to occur, the desired mobile must be r km from B_D and the interfering mobile must be less than $k\ r$ km from B_D. The probability of interference at the base station receiver is the product of the probabilities of these two events.

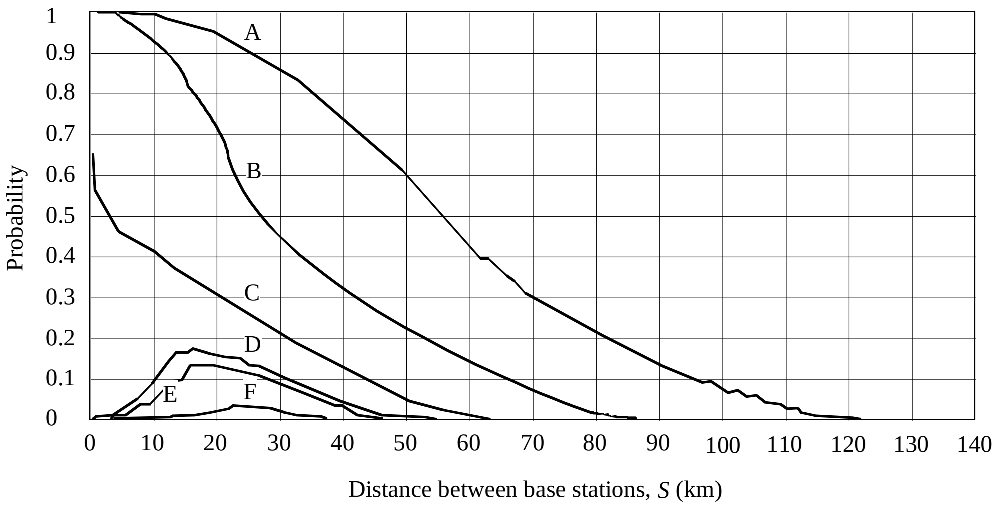
6 Results

Based on this methodology, the base to mobile and mobile to base interference is calculated for various values of OCR and S. The different values of OCR take into consideration that different land mobile systems (both analog and digital with different channel widths) may interfere with each other.

The probability curves for base to mobile and mobile to base interference are plotted in Figs. 6 and 7. As an example, the values: $R_D = R_I = 32$ km, $\varepsilon = 18$ dB have been assumed. Also, the significance of the OCR is described in Table 1.

FIGURE 6

Probability of interference from base to mobile



Curves	OCR (dB)
A	0.0
B	8.5
C	17.8
D	26.4
E	29.0
F	42.9

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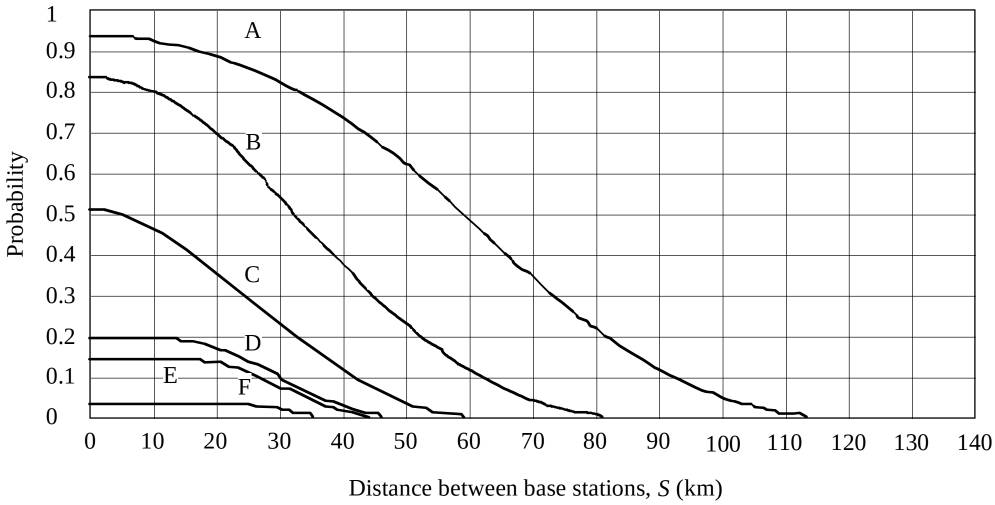
With these curves it is possible to determine the distance of separation between two land mobile duplex systems when the OCR and an acceptable probability of interference are known. For example, given an OCR value of 8.5 dB and an acceptable probability of interference of 0.05, the curves give us a distance between transmitters of 73 km for the base to mobile interference and 68 km for the mobile to base interference. Hence, 73 km should be chosen, as it will guarantee minimum requirements for both types of interference.

FIGURE 7

Probability of interference from mobile to base



Curves	OCR (dB)
A	0.0
B	8.5
C	17.8
D	26.4
E	29.0
F	42.9

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TABLE 1

Curve	OCR (dB)	Δf (kHz)	Significance
A	0.0	0.00	One system type interfering with a similar system
B	8.5	6.25	Two 12.5 kHz systems interfering each other
C	17.8	12.50	Two 25 kHz systems interfering each other
D	26.4	12.50	A 25 kHz system interfering into a 12.5 kHz one
E	29.0	12.50	A 12.5 kHz system interfering into a 25 kHz one
F	42.9	12.50	Two 12.5 kHz systems interfering each other

7 Conclusion

This example has presented a methodology to calculate the probability of interference for land mobile duplex systems. By selecting an acceptable level of probability of interference, the distance of separation between two such systems can be determined. In this way, the minimum distance of separation required can be determined, hence increasing spectrum utilization.