



Recommendation ITU-R SF.675-4
(01/2012)

**Calculation of the maximum power density
(averaged over 4 kHz or 1 MHz)
of angle-modulated and digital carriers**

SF Series
**Frequency sharing and coordination between
fixed-satellite and fixed service systems**

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SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
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Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

Electronic Publication
Geneva, 2012

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RECOMMENDATION ITU-R SF.675-4

**Calculation of the maximum power density (averaged over 4 kHz or 1 MHz)
of angle-modulated and digital carriers**

(1990-1992-1993-1994-2012)

Scope

This Recommendation provides methods for calculating the maximum power density of various types of carriers averaged over 4 kHz or 1 MHz.

The ITU Radiocommunication Assembly,

considering

- a) that administrations are requested to prepare the information listed in Appendices 3 and 4 to the Radio Regulations (RR) for coordination and notification purposes;
- b) that one item of the information listed in RR Appendices 3 and 4 is the maximum power density per Hz at the input of the antenna;
- c) that RR Appendix 4 prescribes that the maximum power density per Hz of a carrier is averaged over the worst 4 kHz band for frequencies below 15 GHz and is averaged over the worst 1 MHz band for frequencies above 15 GHz;
- d) that generalized methods are necessary for the calculation of the maximum power density of an angle-modulated carrier;
- e) that tracking, telemetry and telecommand (TT&C) carriers generally have modulation characteristics, and hence spectral density characteristics, that differ from other types of carriers,

recommends

- 1** that the methods described in Annex 1 should be used for the calculation of the maximum power density, averaged over 4 kHz, of angle-modulated, digital or tracking, telemetry and telecommand (TT&C) carriers;
- 2** that the methods described in Annex 2 should be used for calculation of the maximum power density, averaged over 1 MHz, of digital or TT&C carriers;
- 3** that for the determination of the maximum spectral power density, with agreement of the other administrations concerned, the objectives of Resolution 703 (Rev.WRC-07) should be followed.

Annex 1

Calculation of the maximum power density (averaged over 4 kHz) of an angle-modulated carrier

Given below is the method of calculating the power level in the worst 4 kHz (W/4 kHz). The power density per Hz required by the RR is obtained by dividing this value by 4 000.

1 FM carrier

1.1 FM carrier modulated by a multi-channel telephony signal

The maximum power spectral density at full baseband loading is determined either by the residual carrier or by the peaks of the continuous spectrum, depending upon the nature of the modulation.

The power of the residual carrier is given by the expression:

$$P_t \cdot e^{-\psi_0} \quad \text{W} \quad (1)$$

where:

$$\psi_0 = \frac{m^2}{\varepsilon} \left[C_0 + C_2 \cdot \varepsilon + \frac{C_4}{3} (\varepsilon + \varepsilon^2 + \varepsilon^3) \right] \quad (2)$$

In equation (2), m is the multi-channel r.m.s. modulation index and the constants C_0 , C_2 and C_4 describe the prevailing pre-emphasis characteristic in the general expression for the pre-emphasis:

$$p(f/f_h) = C_0 + C_2(f/f_h)^2 + C_4(f/f_h)^4 \quad (3)$$

where f is the specific baseband frequency under consideration, to be given in the same units as f_h . In the range $\varepsilon \leq f/f_h \leq 1$, the pre-emphasis characteristic is well approximated by:

$$p(f/f_h) = 0.4 + 1.35(f/f_h)^2 + 0.75(f/f_h)^4 \quad (4)$$

Thus, for a system with pre-emphasis:

$$\psi_0 \approx \frac{m^2}{\varepsilon} (0.4 + 1.6 \varepsilon + 0.25 \varepsilon^2 + 0.25 \varepsilon^3) \quad (5)$$

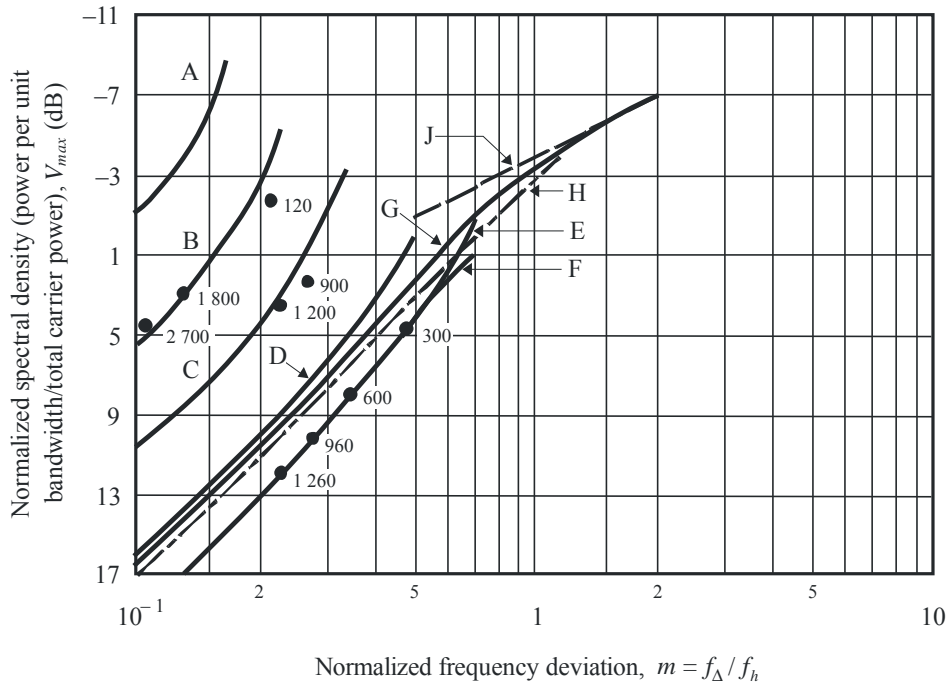
Where $m = f_\Delta/f_h$.

NOTE 1 – Administrations should make available details of the spectrum shape and the value of the coefficients used in equations (2) and (3), for detailed coordination purposes.

The maximum power of the spectral density in the continuous part of the spectrum can be obtained approximately from Figs 1 and 2.

FIGURE 1

Maximum spectral density of signal (distributed component)
frequency modulated by Gaussian noise

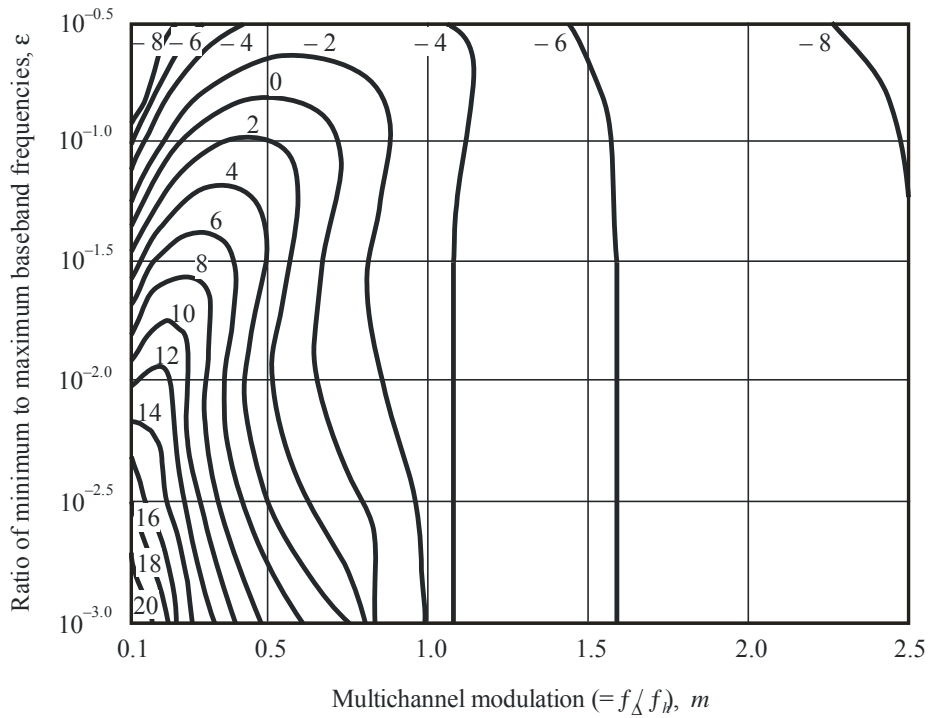


- Curves A: $\psi_0 = 0.1$
 B: $\psi_0 = 0.2$
 C: $\psi_0 = 0.4$
 D: $\psi_0 = 1.0$
 E: $\psi_0 = 2.0$
 F: $\psi_0 = 4.0$
 G: $\psi_0 = \infty$
 H: small-deviation approximation
 J: large-deviation approximation

- Values for standard radio-relay systems (as labelled) are for the following base band limits:
 120 channels 60-552 kHz
 960 channels 60-4 028 kHz
 1260 channels 60-5 636 kHz

FIGURE 2

Equal contour of V_{max} (maximum spectral density) of the FDM-FM signal



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In the equations and the figures, the symbols have the following meanings:

P_t : total power of carrier (W)

f_Δ : multi-channel r.m.s. deviation (Hz)

$$f_\Delta \begin{cases} f_d \times 10^{(-15+10 \log N_c)/20} & \text{for } N_c \geq 240 \\ f_d \times 10^{(-1+4 \log N_c)/20} & \text{for } 240 > N_c \geq 60 \\ f_d \times 10^{(2.6+2 \log N_c)/20} & \text{for } 60 > N_c \geq 12 \end{cases}$$

$$\text{or } \begin{cases} f_d \times 10^{(-15+10 \log N_c)/20} & \text{for } N_c \geq 240 \\ f_d \times 10^{(-1+4 \log N_c)/20} & \text{for } 12 \leq N_c < 240 \end{cases}$$

f_d : r.m.s. test tone deviation (Hz)

N_c : number of channels

f_h : top frequency of the baseband (Hz)

f_1 : bottom frequency of the baseband (Hz)

m : multi-channel modulation index ($= f_\Delta / f_h$)

$$\varepsilon = f_1 / f_h$$

$$V_{max} = W_{max} f_h / P_t$$

W_{max} : maximum spectral power per unit bandwidth (W/Hz).

For carriers for which $1 < N \leq 12$, the maximum power density per 4 kHz is approximated by the expression:

$$P_t \cos^2 \frac{m_b}{1.5} \quad \text{W/4 kHz} \quad \text{for } m_b < 1 \quad (6)$$

where:

- P_t : total power of the carrier (W)
 m_b : peak modulation index (rad) due to a 0 dBm test tone in the highest frequency baseband channel.

1.2 FM carrier modulated by a multi-channel telephony signal and an energy dispersal signal of a triangular waveform with fixed amplitude

Triangular wave dispersal systems are normally designed to ensure that the maximum power spectral density per 4 kHz centred on the carrier frequency is maintained within 3 dB of the fully loaded value.

The power spectral density centred on the carrier frequency is given by:

$$\frac{P_t}{\Delta F} \times 4000 \quad \text{W/4 kHz} \quad (7)$$

where:

- P_t : total power of the carrier (W)
 ΔF : peak-to-peak frequency deviation due to the energy dispersal signal (Hz).

NOTE 1 – Equation (7) assumes the use of a perfectly linear triangular dispersal waveform.

1.3 FM carrier modulated by a television video signal

- For the case where an energy dispersal signal of a triangular waveform is superimposed on the video signal, the maximum power density per 4 kHz in the worst case is given by:

$$\frac{P_t}{\Delta F} \times 4000 \quad \text{W/4 kHz} \quad (8)$$

where:

- P_t : total power of the carrier (W)
 ΔF : peak-to-peak frequency deviation due to the energy dispersal signal (Hz).

NOTE 1 – Equation (8) assumes the use of a perfectly linear triangular dispersal waveform. Negligible error results from this assumption for current FM-TV transmissions.

- For the case where there is no modulation by video and energy dispersal signals, the maximum power density per 4 kHz in the worst-case is given by:

$$P_t \quad \text{W/4 kHz}$$

2 Phase modulated (PM) carrier modulated by a multi-channel telephony signal

When a PM carrier is modulated by a multi-channel telephony signal, the maximum power density is found at the centre frequency of the carrier. This is true if the top baseband frequency is much larger than the bottom baseband frequency. An expression for the maximum power density assuming this condition is given as follows:

– for $\beta\sigma_a \geq 2$:
$$\frac{P_t}{(\beta\sigma_a) f_h} \sqrt{\frac{3}{2\pi}} \times 4000 \quad \text{W/4 kHz} \quad (9)$$

– for $\beta\sigma_a < 2$, the maximum power density per 4 kHz is the sum of following two terms:

– continuous spectrum: $P_t \times S(0) \times 4000 \quad \text{W/4 kHz} \quad (10)$

$S(0)$ can be found from Fig. 3 which gives values for the ratio of the total carrier power-to-power density in a bandwidth of f_h (Hz).

– residual carrier: $P_t \exp \{ -(\beta\sigma_a)^2 \} \quad \text{W} \quad (11)$

where:

P_t : total power of the carrier (W)

$\beta\sigma_a$: multi-channel phase deviation (rad)

β : r.m.s. test tone phase deviation (rad)

σ_a : loading factor of the multi-channel telephony signal.

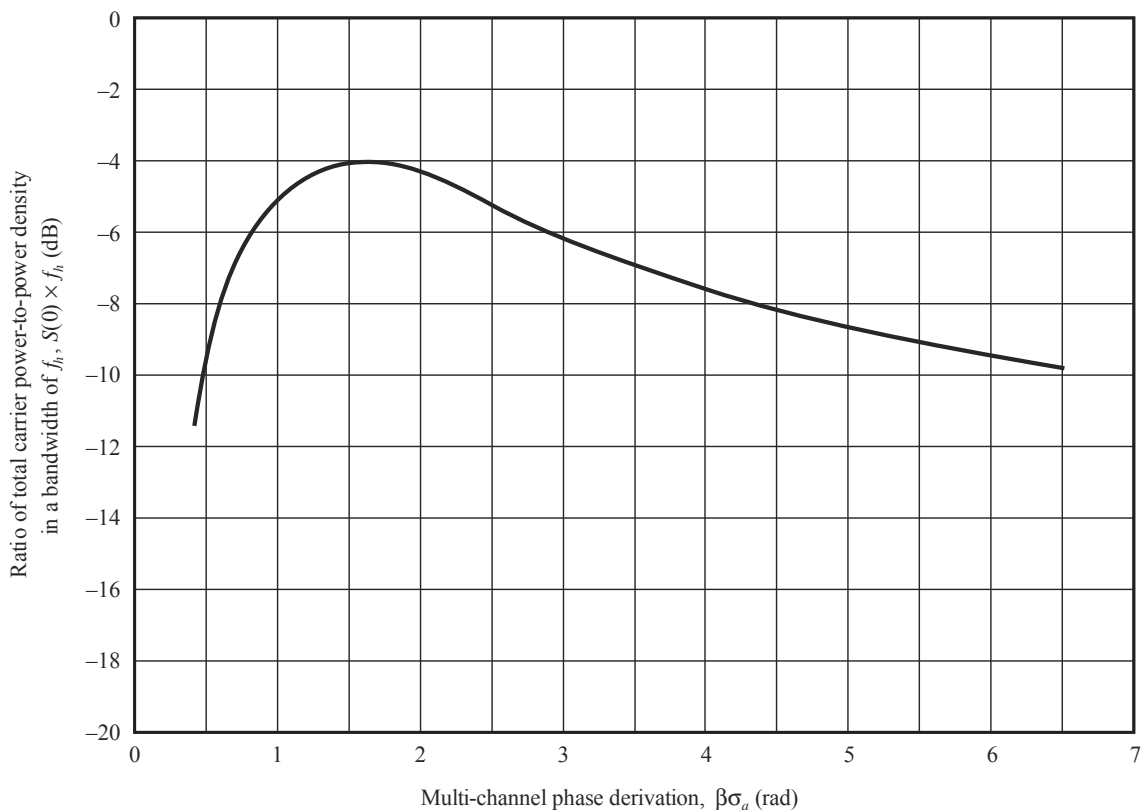
$$\sigma_a = \begin{cases} 10^{(-15+10 \log N)/20} & \text{for } N \geq 240 \\ 10^{(-1+4 \log N)/20} & \text{for } N < 240 \end{cases}$$

N : channel number

f_h : top baseband frequency (Hz).

FIGURE 3

Power density at the centre frequency of continuous power spectrum of PM carrier in a bandwidth of f_h



3 Digital carrier with necessary bandwidth greater than 4 kHz

The maximum power density per Hz of a digital carrier is given by:

$$P_o = P_t/B \quad (12)$$

where:

- P_t : total power of the carrier (W)
- B : necessary bandwidth of the digital emission (Hz).

The maximum power density per 4 kHz in the worst case is given by multiplying the result of equation (12) by 4×10^3 as:

$$P_{4\text{ kHz}} = P_o * 4 \times 10^3 \text{ (W/4 kHz)} \quad (13)$$

4 Digital carrier with necessary bandwidth less than 4 kHz

For those cases with multiple identical carriers with a bandwidth less than 4 kHz where it is known that any 4 kHz will not be completely filled with such carriers, the following equation should be applied:

$$P_{4\text{ kHz}} = (P_t * N) \text{ (W/4 kHz)} \quad (14)$$

where:

- P_t : total power of a single carrier (W)
- N : maximum number of carriers, or portions of carriers, with a bandwidth less than 4 kHz to occupy any given 4 kHz band.

Equation (14) can be generalized for the case where it is known that there will be multiple different types of carriers with bandwidths less than 4 kHz occupying any given 4 kHz band by summing the power of the various individual carrier types to occupy the 4 kHz band and assuming this total power is the power per 4 kHz.

5 TT&C carrier

When narrow-band TT&C carriers are involved, in accordance with RR No. 1.23, in frequency bands below 15 GHz, care must be taken in assessing the maximum power per 4 kHz for such carriers. This is due to the fact that such carriers can have multiple distinct and significant spectral components. As such, it is important to consider the actual spectral shape of such TT&C carriers when selecting the worst 4 kHz bandwidth in order to assess the maximum power density.

Annex 2

Calculation of the maximum power density (averaged over 1 MHz) of a digital or TT&C carrier

Given below is the method of calculating the power level in the worst 1 MHz (W/MHz).

1 Digital carrier with necessary bandwidth greater than 1 MHz

The maximum power density per Hz of a digital carrier is given by:

$$P_o = P_t / B \quad (\text{W/Hz}) \quad (15)$$

where:

P_t : total power of the carrier (W)

B : necessary bandwidth of the digital emission (Hz).

The maximum power density per MHz in the worst case is given by multiplying the result of equation (15) by 1×10^6 as:

$$P_{1 \text{ MHz}} = P_o * 1 \times 10^6 \quad (\text{W/MHz}) \quad (16)$$

2 Digital carrier with necessary bandwidth less than 1 MHz

Note that for digital carriers with a necessary bandwidth less than 1 MHz, a determination must be made as to the maximum number of carriers, and portions of carriers, that will be operating in any 1 MHz band. Where this maximum number of carriers is not known, an assumption should be made that the 1 MHz reference bandwidth will be filled with multiple identical carriers. Under this assumption equations (15) and (16) provide the estimate of the maximum power density per MHz in the worst case. For those cases with carriers with a necessary bandwidth less than 1 MHz where it is known that any 1 MHz will not be completely filled with multiple identical carriers, the following equation for calculating the maximum power density per 1 MHz should be applied:

$$P_{1 \text{ MHz}} = (P_t * N) \quad (\text{W/MHz}) \quad (17)$$

where:

P_t : total power of a single carrier (W)

N : maximum number of carriers, including portions of carriers, with a necessary bandwidth less than 1 MHz to occupy any given 1 MHz band.

Equation (17) can be generalized for the case where it is known that there will be multiple different types of carriers with necessary bandwidths less than 1 MHz occupying any given 1 MHz band by summing the power of the various individual carrier types to occupy the 1 MHz band and assuming this total power is the power per 1 MHz.

3 TT&C carrier

When, in accordance with RR No. 1.23, narrow-band TT&C carriers are involved, the calculation of maximum power density per MHz can lead to an overestimation of the potential interference power. This is due to the fact that simply scaling the power of a single TT&C carrier up to 1 MHz by applying a ratio of the bandwidths would result in a power level greater than the TT&C carrier itself when a single TT&C carrier with a necessary bandwidth less than 1 MHz is used in a given frequency assignment. In such cases, the interference potential caused by a TT&C carrier would be

more accurately assessed if recognition were given to the fact that, in most cases, only a single TT&C carrier will be transmitted in any given 1 MHz band.

As such, the maximum power density per 1 MHz for a TT&C carrier having a necessary bandwidth less than 1 MHz is given by:

$$P_t \quad (\text{W/MHz}) \quad (18)$$

where:

P_t : total power of the TT&C carrier (W).

For the case of a TT&C carrier having a necessary bandwidth greater than 1 MHz, but not more than 1.5 MHz, the maximum power density per 1 MHz is given by:

$$P_t \times (1 \times 10^6/B) \quad (\text{W/MHz}) \quad (19)$$

where:

P_t : total power of the TT&C carrier (W)

B : necessary bandwidth of the TT&C emission which is greater than 1 MHz (Hz).
