

## RECOMMENDATION ITU-R SF.675-3\*

CALCULATION OF THE MAXIMUM POWER DENSITY (AVERAGED OVER 4 kHz)  
OF AN ANGLE-MODULATED CARRIER

(1990-1992-1993-1994)

The ITU Radiocommunication Assembly,

*considering*

- a) that administrations are requested to prepare the information listed in Appendix S4 to the Radio Regulations (RR) for coordination and notification purposes;
- b) that one item of the information listed in RR Appendix S4 is the maximum power density per Hz at the input of the antenna averaged over the worst 4 kHz band;
- c) that generalized methods are necessary for the calculation of the maximum power density of an angle-modulated carrier,

*recommends*

1. that the methods described in Annex 1 be used for the calculation of the maximum power density, averaged over 4 kHz, of an angle-modulated carrier;
2. that for the determination of the maximum spectral power density, with agreement of the other administrations concerned, administrations should make use of the most precise calculation method available to them.

## ANNEX 1

**Calculation of the maximum power density (averaged over 4 kHz)  
of an angle modulated carrier**

Given below is the method of calculating the power level in the worst 4 kHz (W/4 kHz). The power density per Hz required by the RR is obtained by dividing this value by 4 000.

## 1. FM carrier

### 1.1 FM carrier modulated by a multi-channel telephony signal

The maximum power spectral density at full baseband loading is determined either by the residual carrier or by the peaks of the continuous spectrum, depending upon the nature of the modulation.

The power of the residual carrier is given by the expression:

$$P_t \cdot e^{-\psi_0} \quad \text{W} \quad (1)$$

where:

$$\psi_0 = \frac{m^2}{\epsilon} \left[ C_0 + C_2 \cdot \epsilon + \frac{C_4}{3} (\epsilon + \epsilon^2 + \epsilon^3) \right] \quad (2)$$

\* Radiocommunication Study Group 4 and 9 made editorial amendments to this Recommendation in 2000 in accordance with Resolution ITU-R 44.

In equation (2),  $m$  is the multi-channel r.m.s. modulation index and the constants  $C_0$ ,  $C_2$  and  $C_4$  describe the prevailing pre-emphasis characteristic in the general expression for the pre-emphasis:

$$p(f/f_h) = C_0 + C_2(f/f_h)^2 + C_4(f/f_h)^4 \quad (3)$$

where  $f$  is the specific baseband frequency under consideration, to be given in the same units as  $f_h$ . In the range  $\varepsilon \leq f/f_h \leq 1$ , the ITU pre-emphasis characteristic (Recommendation ITU-R F.275) is well approximated by:

$$p(f/f_h) = 0.4 + 1.35(f/f_h)^2 + 0.75(f/f_h)^4 \quad (4)$$

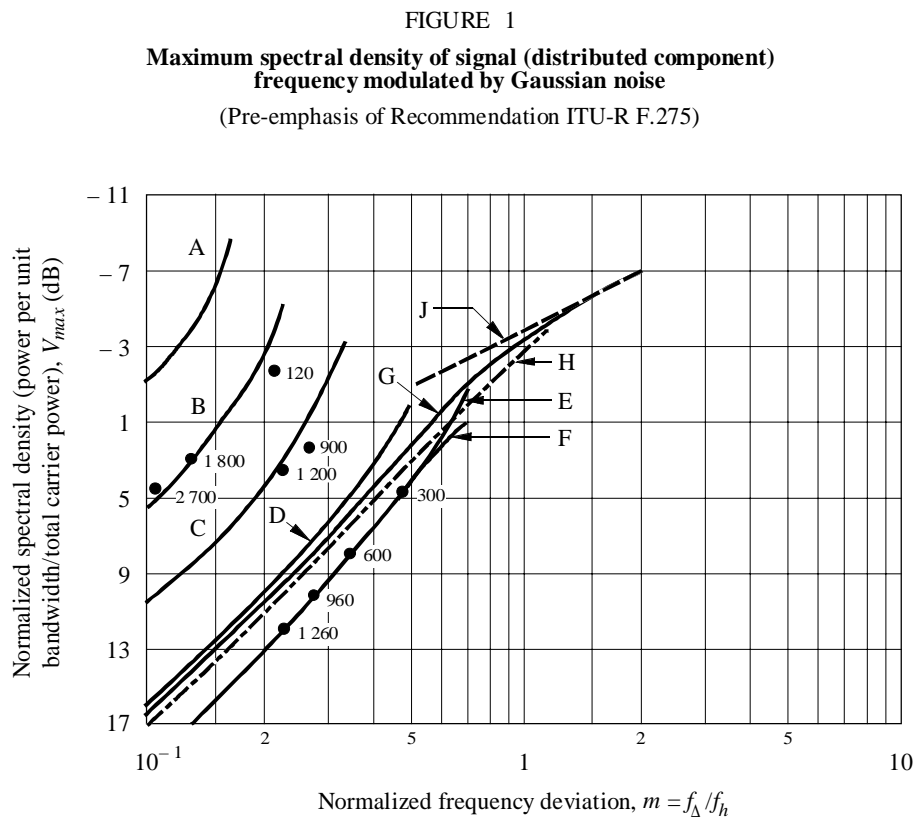
Thus, for a system with ITU pre-emphasis:

$$\psi_0 \approx \frac{m^2}{\varepsilon} (0.4 + 1.6 \varepsilon + 0.25 \varepsilon^2 + 0.25 \varepsilon^3) \quad (5)$$

where  $m = f_\Delta/f_h$ .

*Note 1* – Administrations should make available details of the spectrum shape and the value of the coefficients used in equations (2) and (3), for detailed coordination purposes.

The maximum power of the spectral density in the continuous part of the spectrum can be obtained approximately from Figs. 1 and 2.



Curves A:  $\psi_0 = 0.1$

B:  $\psi_0 = 0.2$

C:  $\psi_0 = 0.4$

D:  $\psi_0 = 1.0$

E:  $\psi_0 = 2.0$

F:  $\psi_0 = 4.0$

G:  $\psi_0 = \infty$

H: small-deviation approximation

J: large-deviation approximation

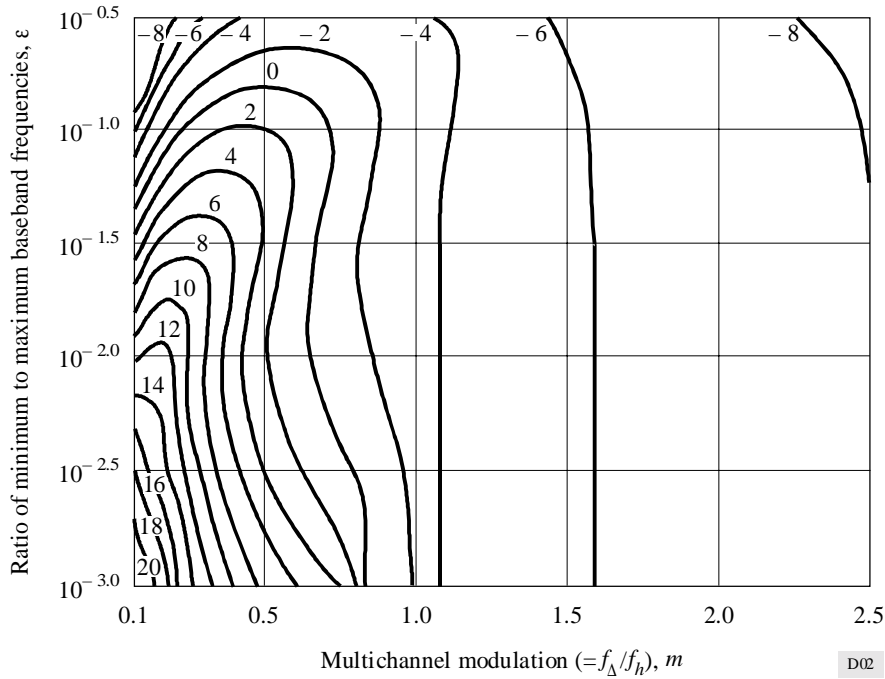
● Values for standard radio-relay systems (as labelled) are for the following base band limits:

120 channels 60-552 kHz

960 channels 60-4 028 kHz

1 260 channels 60-5 636 kHz

FIGURE 2  
 Equal contour of  $V_{max}$  (maximum spectral density) of the FDM-FM signal  
 (Pre-emphasis of Recommendation ITU-R F.275)



In the equations and the figures, the symbols have the following meanings:

$P_t$ : total power of carrier (W)

$f_{\Delta}$ : multi-channel r.m.s. deviation (Hz)

$$f_{\Delta} \begin{cases} f_d \times 10^{(-15+10 \log N_c)/20} & \text{for } N_c \geq 240 \\ f_d \times 10^{(-1+4 \log N_c)/20} & \text{for } 240 > N_c \geq 60 \\ f_d \times 10^{(2.6+2 \log N_c)/20} & \text{for } 60 > N_c \geq 12 \end{cases}$$

$$\text{or } \begin{cases} f_d \times 10^{(-15+10 \log N_c)/20} & \text{for } N_c \geq 240 \\ f_d \times 10^{(-1+4 \log N_c)/20} & \text{for } 12 \leq N_c < 240 \end{cases}$$

$f_d$ : r.m.s. test tone deviation (Hz)

$N_c$ : number of channels

$f_h$ : top frequency of the baseband (Hz)

$f_1$ : bottom frequency of the baseband (Hz)

$m$ : multi-channel modulation index ( $= f_{\Delta}/f_h$ )

$\epsilon = f_1 / f_h$

$V_{max} = W_{max} f_h / P_t$

$W_{max}$ : maximum spectral power per unit bandwidth (W/Hz).

For carriers for which  $1 < N \leq 12$ , the maximum power density per 4 kHz is approximated by the expression:

$$P_t \cos^2 \frac{m_b}{1.5} \quad \text{W/4 kHz} \quad \text{for } m_b < 1 \quad (6)$$

where:

$P_t$ : total power of the carrier (W)

$m_b$ : peak modulation index (rad) due to a 0 dBm test tone in the highest frequency baseband channel.

### 1.2 FM carrier modulated by a multi-channel telephony signal and an energy dispersal signal of a triangular waveform with fixed amplitude

Triangular wave dispersal systems are normally designed to ensure that the maximum power spectral density per 4 kHz centred on the carrier frequency is maintained within 3 dB of the fully loaded value.

The power spectral density centred on the carrier frequency is given by:

$$\frac{P_t}{\Delta F} \times 4000 \quad \text{W/4 kHz} \quad (7)$$

where:

$P_t$ : total power of the carrier (W)

$\Delta F$ : peak-to-peak frequency deviation due to the energy dispersal signal (Hz).

*Note 1* – Equation (7) assumes the use of a perfectly linear triangular dispersal waveform.

### 1.3 FM carrier modulated by a television video signal

– For the case where an energy dispersal signal of a triangular waveform is superimposed on the video signal, the maximum power density per 4 kHz in the worst case is given by:

$$\frac{P_t}{\Delta F} \times 4000 \quad \text{W/4 kHz} \quad (8)$$

where:

$P_t$ : total power of the carrier (W)

$\Delta F$ : peak-to-peak frequency deviation due to the energy dispersal signal (Hz).

*Note 1* – Equation (8) assumes the use of a perfectly linear triangular dispersal waveform. Negligible error results from this assumption for current FM-TV transmissions.

– For the case where there is no modulation by video and energy dispersal signals, the maximum power density per 4 kHz in the worst-case is given by:

$$P_t \quad \text{W/4 kHz}$$

## 2. Phase modulated (PM) carrier modulated by a multi-channel telephony signal

When a PM carrier is modulated by a multi-channel telephony signal, the maximum power density is found at the centre frequency of the carrier. This is true if the top baseband frequency is much larger than the bottom baseband frequency. An expression for the maximum power density assuming this condition is given as follows:

$$\text{– for } \beta\sigma_a \geq 2: \quad \frac{P_t}{(\beta\sigma_a) f_h} \sqrt{\frac{3}{2\pi}} \times 4000 \quad \text{W/4 kHz} \quad (9)$$

– for  $\beta\sigma_a < 2$ , the maximum power density per 4 kHz is the sum of following two terms:

– continuous spectrum:  $P_t \times S(0) \times 4000$  W/4 kHz (10)

$S(0)$  can be found from Fig. 3 which gives values for the ratio of the total carrier power-to-power density in a bandwidth of  $f_h$  (Hz).

– residual carrier:  $P_t \exp \{ - (\beta\sigma_a)^2 \}$  W (11)

where:

$P_t$ : total power of the carrier (W)

$\beta\sigma_a$ : multi-channel phase deviation (rad)

$\beta$ : r.m.s. test tone phase deviation (rad)

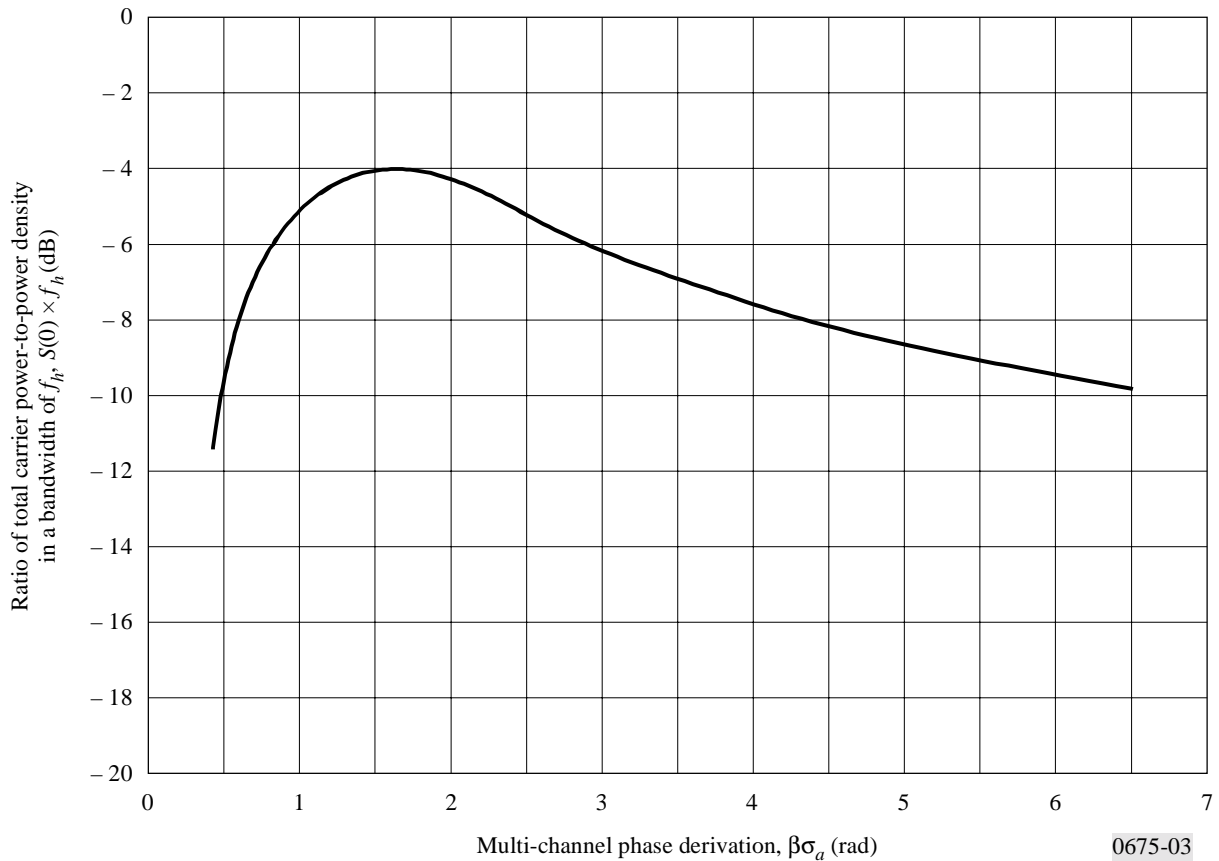
$\sigma_a$ : loading factor of the multi-channel telephony signal.

$$\sigma_a = \begin{cases} 10^{(-15+10 \log N)/20} & \text{for } N \geq 240 \\ 10^{(-1+4 \log N)/20} & \text{for } N < 240 \end{cases}$$

$N$ : channel number

$f_h$ : top baseband frequency (Hz).

FIGURE 3  
Power density at the centre frequency of continuous power spectrum  
of PM carrier in a bandwidth of  $f_h$



### 3. PSK carrier

The maximum power density per 4 kHz of a PSK carrier modulated by a digital energy dispersal signal of a PN (pseudo-noise) sequence is given by (see Note 1):

$$P_t \times (4\,000 / B) \quad \text{W/4 kHz} \quad (12)$$

when the repetition cycle of the PN sequence is longer than 250  $\mu\text{s}$ ; and by:

$$P_t \cdot \frac{L + 1}{L^2} \left\{ \left( \frac{4\,000}{1 / L t} \right) + 1 \right\} \quad \text{W/4 kHz} \quad (13)$$

when the repetition cycle of the PN sequence is equal to or less than 250  $\mu\text{s}$ ,

where:

$P_t$ : total power of the carrier (W)

$B$ : symbol rate (symbol/s)

$L$ : length of the PN sequence (symbol)

$t$ : symbol duration (s)

$$\left( \frac{4\,000}{1 / L t} \right) = \text{integer part } \frac{4\,000}{1 / L t}$$

The expressions given above apply to the case of PN sequence modulation of a PSK carrier and would be applicable to cases where the PSK message signal is continuously overlaid by a PN scrambling sequence. Large errors could result from the application of these expressions to systems, such as TDMA, in which the preamble portion of the signal, added parity bits, and the like, are not scrambled. Furthermore, in multi-phase systems the spectral uniformity expected from PN energy dispersal may be destroyed by the common operation of differential encoding.

It should be noted that the above treatment does not give any guidance on the assumptions which should be made to cover the case of PSK systems without energy dispersal, under conditions in which bit patterns could repeat in such a manner as to concentrate the power in a relatively small number of spectral lines.

For the remaining radio-relay systems shown in Fig. 1, the maximum power spectral densities are valid for any of the baseband limits specified in Recommendation ITU-R F.380, since their  $\beta$  values are relatively constant and match those used in Fig. 1.

*Note 1* – The equations (12) and (13) can be used in calculating the maximum power density per 40 kHz of a PSK carrier modulated by a digital energy dispersal signal of a PN sequence, by simply replacing the number 4 000 by the number 40 000. Equations (12) and (13) would then be, respectively, applicable to a PN sequence longer than 25  $\mu\text{s}$  and to a PN sequence equal to or less than 25  $\mu\text{s}$ . If the symbol rate,  $B$ , is less than 40 000, the maximum power density per 40 kHz is the total power of the PSK carrier.

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