

RECOMMENDATION ITU-R SA.1742

Technical and operational characteristics of interplanetary and deep-space systems operating in the space-to-Earth direction around 283 THz

(Question ITU-R 235/7)

(2006)

Scope

This Recommendation specifies technical parameters (frequencies, link, signal and data characteristics, antenna parameters, etc.) and operational characteristics of interplanetary and deep-space systems operating in the space-to-Earth direction around 283 THz, which could be used in sharing studies.

The ITU Radiocommunication Assembly,

considering

- a) that telecommunication links are being planned for use on some satellite systems for deep-space and interplanetary radiocommunications at frequencies in the region of 283 THz;
- b) that, using recent technological developments, astronomers are making a concerted effort to build telescopes and make observation in this segment of the spectrum;
- c) that this segment of the spectrum is also being used for other terrestrial and space services;
- d) that this segment of the spectrum is also being used for scientific and industrial purposes other than radiocommunication;
- e) that mechanisms of interference between satellites operating in deep-space and passive systems such as astronomy operating above 20 THz may differ from those in the radio-frequency portion of the spectrum,

recognizing

1 that No. 78 of Article 12 of the ITU Constitution states a function of the Radiocommunication Sector includes, "... carrying out studies without limit of frequency range and adopting Recommendations ...";

2 that, under Note 2 of No. 1005 in the Annex to the ITU Convention, Study Groups may consider "radiocommunication" to include electromagnetic spectrum above 3 000 GHz propagated through space without artificial guide in the course of their studies and in the creation of draft new Recommendations;

3 that use and sharing of this segment of the spectrum has not been thoroughly studied within the ITU-R,

recommends

1 that sharing studies considering space research satellites operating in the space-to-Earth direction around 283 THz in deep space should take into account the technical and operational parameters presented in Annexes 1 and 2.

Annex 1

1 Introduction

The increased pressure for use of the radio spectrum and the advancement of technology, there is more attention being given to the use of frequencies above 3 000 GHz for free space radiocommunications. Radiocommunication links have become a reality in the frequency bands above 3 000 GHz as a result of many recent technological developments in the field of optical fibre telecommunication especially in the area of lasers, modulation and receiver technology. Free space radiocommunication at frequencies above 3 000 GHz has the ability to support higher data rates with less mass than traditional radio-frequency systems as well as meet gain and directivity requirements of beams used for deep-space applications.

1.1 Frequency considerations

Currently, most of the interest in free space radiocommunication links above 3 000 GHz is focused around the frequencies 200, 283, 311 and 353 THz¹, whose corresponding wavelengths are approximately 1.5, 1.06, 0.965 and 0.850 μm . These frequencies are the same as those most widely used for telecommunications in optical fibres. For interplanetary and deep-space radiocommunication in the space-to-Earth direction, attention is being focused on use of Ytterbium-doped (Yb) fibre-optical amplifier at 1.06 μm in a Master-Oscillator-Power Amplifier (MOPA) configuration, Q-switched, Neodymium: Yttrium Aluminium Garnet (Nd:YAG) or Neodymium: Yttrium Vanadate (Nd:YVO₄) lasers operating around 283 THz (1.06 μm) though, depending on mission requirements, other frequencies are feasible. The use of Yb, Nd:YAG and Nd:YVO₄ are of primary use due to availability and reliability.

1.2 Generic mission parameters

Technical parameters suitable for interference analyses should be based on generic interplanetary missions to Mars and Jupiter. For the purposes of minimizing weight and power consumption, the links must support radiocommunication requirements beginning shortly after launch and continuing throughout the duration of the mission negating the need for additional radiocommunication systems. Therefore, link distances will vary from a few thousand km to several Astronomical Units (AU²). Distances from the Earth to Mars or Jupiter vary from 0.5 to 6.2 AU. A summary of the fundamental technical parameters of a 283 THz deep-space link operating in the space-to-Earth direction is provided in Table 1.

2 Link considerations

Deep-space links operating at 283 THz in the space-to-Earth direction may utilize Yb, Nd:YAG or Nd:YVO₄ lasers. The beam would be transmitted from a 30 cm telescope on board the spacecraft and received by a telescope on the Earth with an effective diameter of 4.2 m to 10 m.³

¹ 1 THz = 1 000 GHz.

² 1 AU \approx 149 597 870 km.

³ For the purposes of telecommunication through free space at around 283 THz, a telescope is effectively an antenna.

TABLE 1

**Technical parameters of two reference deep-space missions
operating at 283 THz in the space-to-Earth direction**

Parameter	Mars	Jupiter
Transmitter power	5 W (average)	
Transmitter aperture	30 cm	
Transmitter frequency (wavelength)	283 THz (1.06 μm)	
Modulation	Pulse position modulation (PPM) ($M = 64$ to 256) with concatenated coding	
Pointing accuracy	0.35 μrad	
Range	0.5 to 2.5 AU	4.2 to 6.2 AU
Data rate (during the day at Earth terminal) ⁽¹⁾	3 to 30 Mbit/s (4.2 m Earth terminal)	1.5 to 3 Mbit/s (10 m terminal)
Receiver aperture	4.2 to 10 m equivalent size	
Detector type	An array of Geiger-mode InGaAsP/InP APDs	
Required link margin	2 to 3 dB	

⁽¹⁾ Night-time data rates on Earth are about 30% (1.13 dB) higher.

2.1 Link performance

Like a deep-space system operating in the traditional radio-frequency spectrum, performance of a link operating at 283 THz is measured in terms of data rate and bit error rate (BER). Performance is calculated as a function of power, telescope quality, propagation considerations, noise and receiver sensitivity. Each of these parameters is function of additional variables.

2.1.1 Data rate

Unlike a deep-space system operating in the traditional radio-frequency spectrum, when all other parameters are held constant, the data rate is not exactly inversely proportional to the square of the propagation distance; however, it is a very close approximation for links operating from the vicinity of Mars and Jupiter and thus a good rule of thumb. Data rates from Mars will vary depending on many parameters including link range and geometry with the sun. Data rates from Mars will generally be around an order of magnitude higher than from Jupiter.

2.1.2 BER

Frames of data must have a BER of less than 10^{-6} after error correction in order to be retained. A link must retain 99% of data frames.

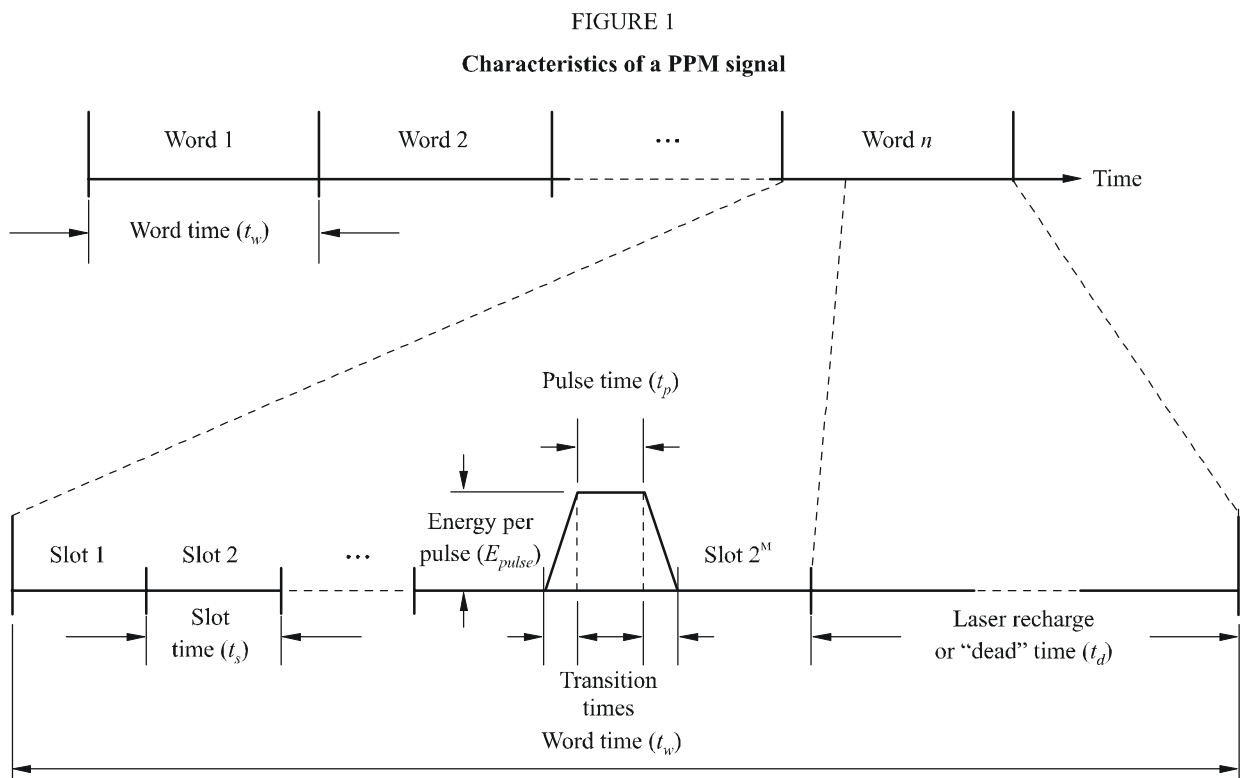
2.1.3 Margin requirement

The typical margin requirement of a deep-space or interplanetary link operating at 283 THz in the space-to-Earth direction is on the order of 2 to 3 dB. Conditions are dependent on factors which include *inter alia* weather, time of day and elevation angle.

2.2 Modulation

Deep-space and interplanetary links operating around 283 THz will utilize PPM. This modulation technique allows for direct detection (specifically photon counting) by the receiver rather than implementing coherent receivers. The PPM signal will be encoded with a concatenated code.

PPM uses a single pulse of energy within the time of a word. M bits of data may be transmitted with a single pulse of energy temporally located within 2^M time slots of a word. A portion of the total word time is used for recharging the laser and will never contain a pulse. This recharging or “dead” time often accounts for the majority of the word time at low data rates but becomes less of a factor at higher data rates. The temporal characteristics of a PPM signal and its relevant measures are illustrated in Fig. 1.



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2.3 Deliverable power

The Yb, Q-switched, Nd:YAG or Nd:YVO₄ transmitter will typically produce 5 W of average power. Peak power will vary with data rate but may be on the order of 30 to 40 dBW. The following procedure calculates the peak power, P_{peak} , of a PPM transmitter. The following parameters are required:

- M : modulation index
- P_{ave} : average transmitter power (W)
- t_d : dead time (s)
- t_p : transmitter pulse time (s)
- t_s : slot time (s).

Step 1: Calculate the word time, t_w , by:

$$t_w = M \cdot t_s + t_d \quad \text{s} \quad (1)$$

Step 2: Calculate the energy per word, E_{word} , by:

$$E_{word} = P_{ave} \cdot t_w \quad \text{J} \quad (2)$$

NOTE 1 – As only one pulse occurs during each word time, the energy per pulse is equivalent to the energy per word (i.e. $E_{pulse} = E_{word}$).

Step 3: Calculate the peak transmitter power, P_{peak} , by:

$$P_{peak} = \frac{E_{pulse}}{t_p} \quad \text{W} \quad (3)$$

2.4 Received signal

The general method for calculating the signal level at 283 THz received by the earth station is the same as that used with traditional radio-frequency systems.

$$P_S = P_t + G_t + G_r + L_t + L_r + L_p + L_a + L_s \quad \text{dBW} \quad (4)$$

where:

P_S : receiver signal power

P_t : average laser output power (typical value is 4.7 to 7.0 dBW)

G_t : transmitter antenna gain (typical value is 119 dB) is discussed in detail in § 2.6.2

G_r : receiving antenna gain (typical value is 129 to 149 dB) is discussed in detail in § 2.6.3

L_t : transmitter losses

L_r : receiver losses

L_p : pointing losses

L_a : atmospheric losses along the space-to-ground link

L_s : free space loss.

2.5 Link losses

There are five primary sources of link losses:

- internal transmitter losses, L_t , that include the effects of absorption, scattering and reflection losses in the optical train of the transmitter;
- internal receiver losses, L_r , that include the effects of absorption, scattering and reflection losses in the optical train of the receiver;
- pointing losses, L_p , that include the effects of antenna or spacecraft jitter and mispointing of the transmitting antenna;
- atmospheric losses, L_a , that include the effects of atmospheric scatter and turbulence;
- free space loss, L_s , that is due to the physical separation between the transmitter and receiver.

Values of each source of loss will vary with hardware design, hardware age, mission requirements and the phase of the mission. Suggested values of losses to be used in generic interference analyses are provided in Table 2. Atmospheric propagation in this region of the spectrum has been addressed in detail by Radiocommunication Study Group 3 through Recommendations ITU-R P.1621 and ITU-R P.1622.

TABLE 2

**Technical parameters of two reference deep-space missions
operating at 283 THz in the space-to-Earth direction**

Mechanism of loss	Typical value
Transmitter losses, L_t	0.63 (= -2 dB)
Receiver losses, L_r	0.63 (= -2 dB)
Pointing losses, L_p	0.63 (= -2 dB)
Atmospheric losses, L_a	0.89 (= -0.5 dB) at 90° 0.56 (= -2.5 dB) at 30°

Free space loss, L_s , is calculated at 283 THz in the same manner as with traditional radio-frequency systems:

$$L_s = \left(\frac{\lambda}{4\pi R} \right)^2 = \left(\frac{c}{4\pi f R} \right)^2 \quad (5a)$$

which, at 283 THz, reduces to:

$$L_s = \frac{7.169 \times 10^{-15}}{R^2} \quad (5b)$$

where:

R : distance between the transmitter and receiver (m).

2.6 Transmit/receive telescope parameters

Deep-space and interplanetary radiocommunication links operating at around 283 THz will utilize telescopes as transmitting and receiving antennas. The typical parameters of the transmitting and receiving telescopes will differ greatly from each other. These differences will effect each telescope's respective gain pattern. The transmitter and receiver antenna patterns are also different since the transmitter optics are usually fed by a Gaussian distributed beam while the receiver optics have a planar detector. For an envelope of the antenna gain patterns of transmitting and receiving antennae operating around 283 THz, refer to Annex 2.

2.6.1 Diameter

For the purposes of interference analyses, the diameter of the transmitting antenna should be assumed to be 30 cm. The transmitting aperture will either be unobstructed or have a 3 cm obscuration.

The effective diameter of the receiving antenna may vary between 1 and 10 m, but for most applications will be at least 4.2 m. For the purposes of interference analyses, antennas of 1, 4.2 and 10 m should be considered. The primary receiving aperture will have a secondary obscuration with a diameter of no more than 20% of the diameter of the primary aperture.

2.6.2 Transmitting gain pattern

The transmitter utilizes a telescope that is fed by a laser. Such lasers normally operate only in the lowest cavity mode, TEM₀₀, which results in a beam that has a Gaussian distribution of energy with a maximum intensity along its axis of transmission. The beam pattern is tailored such that as the intensity of the beam falls off in amplitude with angular separation from the axis of transmission, no more than a few percent of the beam power is wasted. Two points of reference are the angles at

which the beam amplitude falls off to either 37% or 13% of the amplitude on axis. These points are called the $1/e$ and $1/e^2$ points respectively and are referred to frequently in the characterization of emitted laser energy patterns.

The full-angle beamwidth at the $1/e^2$ point is approximated by:

$$\theta_{1/e^2} = \frac{4\lambda}{\pi D} \quad \text{rad} \quad (6)$$

where:

- θ_{1/e^2} : beamwidth (rad)
- λ : wavelength (m)
- D : aperture diameter (m).

In the case of a 283 THz Gaussian beam transmitted from a 30 cm aperture, the beamwidth at the $1/e^2$ point is approximately 4.5×10^{-6} rad.

For the transmitting terminal, the following equations can be used to calculate the far field radiation pattern of a laser with a Gaussian amplitude plane wave feeding a telescope. Use of these equations makes the following basic assumptions:

- the laser source is characterized as single mode Gaussian emission;
- the antenna gain patterns are measured in the far field;
- the aperture is circular.

The gain pattern of a transmitting telescope of radius, a , fed with a Gaussian amplitude plane wave having a waist radius of ω , where ω is the distance from the central axis of the optical system to the $1/e^2$ intensity point, and having a central obscuration of radius, b , is given by equation (7). The term, G_0 , is the upper limit on antenna gain which is obtained for a uniformly illuminated unobscured circular aperture. The second term, $g_t(\alpha, \gamma, X)$, is a gain efficiency term which accounts for obscuration, truncation, off-axis intensity, and defocusing effects.

$$G_t(\alpha, \gamma, X) = G_0 \cdot g_t(\alpha, \gamma, X) \quad \text{dBi} \quad (7)$$

where:

$$G_0 = \frac{4\pi A}{\lambda^2} = \left(\frac{2\pi a}{\lambda}\right)^2 \quad \text{dBi} \quad (8)$$

$$g_t(\alpha, \gamma, X) = 2\alpha^2 \left| \int_{\gamma^2}^1 J_0(X\sqrt{u}) e^{-\alpha^2 u} du \right|^2 \quad (9)$$

$$\gamma = \frac{b}{a} \quad (10)$$

- A : area of the telescope aperture (m^2)
- a : radius of the primary aperture (m)
- b : radius of the obscuration (m)
- J_0 : Bessel function of the first kind of order zero
- α : the ratio, α/ω , of the radius of the transmitter aperture, a , to the radius of the Gaussian feed beam waist, ω , at the $1/e^2$ point
- u : the variable of integration

$$X: \left(\frac{2\pi}{\lambda} \right) \cdot a \cdot \sin(\theta)$$

θ : angle off the optical axis (rad).

For the on-axis, $X = 0$ and the gain efficiency term in equation (9) becomes:

$$g_t(\alpha, \gamma, 0) = \left[\frac{2}{\alpha^2} \left[e^{-\alpha^2} - e^{-\gamma^2 \alpha^2} \right]^2 \right] \quad (11)$$

Then the on-axis maximum main beam gain in (7) becomes:

$$G_t(\alpha, \gamma, 0) = \frac{4\pi A}{\lambda^2} \left[\frac{2}{\alpha^2} \left[e^{-\alpha^2} - e^{-\gamma^2 \alpha^2} \right]^2 \right] \quad \text{dBi} \quad (12)$$

Any obscuration b will reduce the main beam gain, fill in the nulls and increase the side lobes.

2.6.3 Receiving gain pattern

The size of the field of view is related to the physical size of the detector and the focal length of the telescope. It may be determined by the equation:

$$\varphi = \frac{d}{F} \quad (13)$$

where:

- φ : field of view (rad)
- d : diameter of the detector (typically 10^{-4} to 10^{-3}) (m)
- F : focal length of the telescope (m).

The pattern of a receiving antenna is typically matched to the detector. The detector is isolated from unwanted energy with the use of field stops and exposed only to the portion of the main beam within φ rad of the axis of the main beam. Therefore, unwanted energy received in the side lobes of the receiving antenna pattern does not arrive at the detector and may be neglected in the course of interference analyses.

Assuming the receiving aperture is in the far-field of the transmitting antenna, the received energy is normally treated as a plane wave. The receiving system may use a common or separate aperture from the transmitting system. The beamwidth of the receiving aperture is also typically measured in terms of its $1/e^2$ point.

The maximum, on-axis, gain of a receiving antenna, G_R , is given by:

$$G_R = 10 \log \left(\frac{4\pi A}{\lambda^2} \right) + 10 \log(1 - \gamma^2) + \delta \quad \text{dBi} \quad (14)$$

where:

- A : area of the receiving aperture (m^2)
- λ : wavelength of the incoming signal (m)

and

$$\gamma = \frac{b}{a} \quad (15)$$

where:

- a : radius of the primary mirror (m)
- b : radius of the secondary mirror (m).

The gain calculated in equation (14) represents the quantity of energy incident on the detector. The term G_R assumes that the receiving antenna is located in the far-field of the transmitter, and the aperture and the detector are round. The first term of equation (14) is the classic antenna gain realized by an ideal unobscured antenna of area A . The second term accounts for losses due to the obscuration introduced by the secondary mirror of a Cassegrain system. In the case of systems without secondary mirrors, the value of b in equation (15) becomes zero and the second term of equation (14) may be neglected.

The third term, δ , of equation (14) accounts for losses (dB) due to spillover of the signal energy beyond the edge of the detector. For direct detection systems such as PPM, δ reduces as the ratio of the detector size to focal length of the telescope increases. For most practical values, δ will be no more than -0.5 dB.

2.7 Pointing and tracking

The narrow beamwidth and long range of a deep-space or interplanetary link operating at around 283 THz impose strict pointing and tracking requirements on a system. Typical pointing requirements require a pointing accuracy of approximately 1/10th of a beamwidth. For the reference system outlined in Table 1, this equates to 0.35 microradian (μrad) and a pointing loss of no more than 2 dB.

The $1/e^2$ contour footprint of a link 2 and 6 AU long is presented in Figs. 2 and 3 respectively. The Figures assume the spacecraft was transmitting along the Earth's equatorial plane and is pointed at the NASA experimental optical downlink facility in Wrightwood, California, United States of America. The inner solid contour in each Figure represents the footprint assuming perfect pointing. The outer, dashed contour assumes the range of the contour with a random 0.35 μrad pointing error. The distances of 2 and 6 AU equate to typical distances to Earth from Mars and Jupiter respectively.

FIGURE 2

Footprint an 8 μrad beam from 2 AU

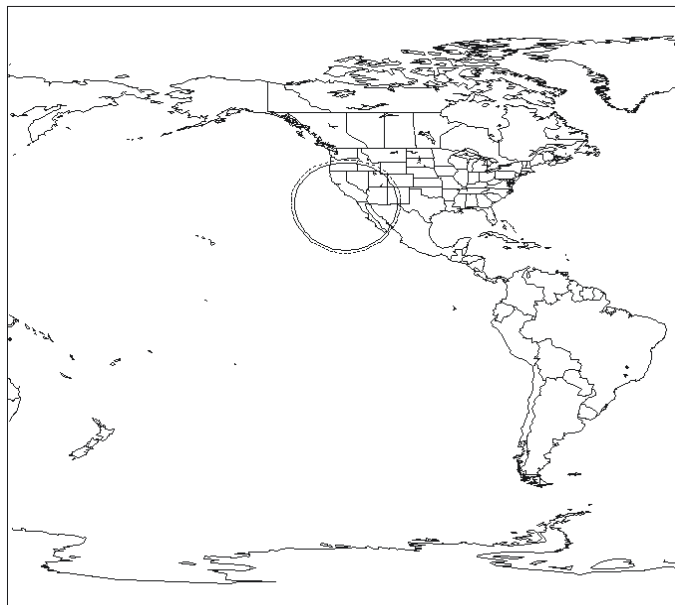
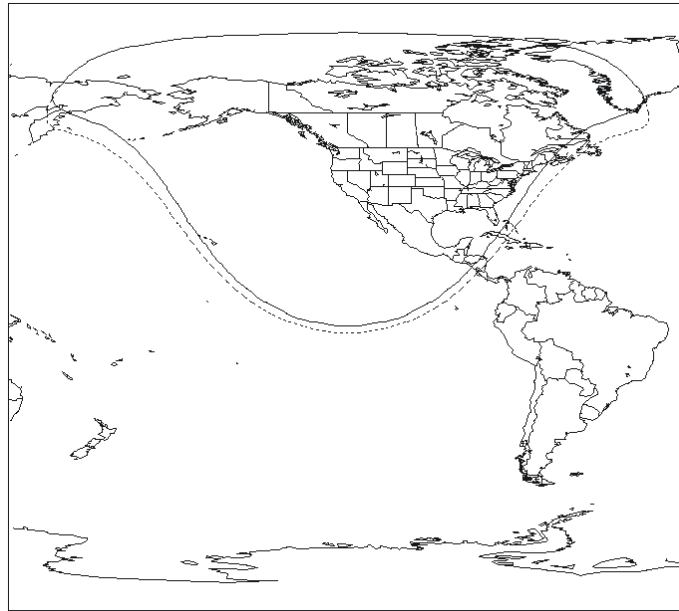


FIGURE 3

Footprint an 8 μ rad beam from 6 AU

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3 Signal-to-noise ratio (SNR)

The performance of deep-space and interplanetary telecommunication links operating at around 283 THz depends directly on achieving a high SNR at the receiver. The higher the SNR, the lower the BER. In general:

$$SNR = \frac{P_s}{N_t} \quad (16)$$

where:

P_s : received signal power as given by equation (4)

N_t : noise power from all sources.

Noise comes from two independent sources, detector noise and background signal. Background signal, discussed in § 3.1, is due to extraneous energy from the sky reaching the detector. Detector noise, discussed in § 3.2, is due to the inherent noise within the detector.

The basic equations describing the performance of a laser optical crosslink or a link through the atmosphere can be simplified by the following basic assumptions:

- Optical transmitting and receiving antennas have no central obstructions.
- Transmitted waveforms are Gaussian and are truncated at the $1/e^2$ points.
- Received waves are plane waves.
- Airy disks are truncated at the first null of the airy disk pattern.

3.1 Background signal

During daytime operations or when an illuminated object appears in the field of view of the detector, the background signal will be the dominant source of noise. The power of the background signal, P_{back} , incident on the detector may originate from diffuse energy from the sky, planets or stars. Sky noise is greater during the daylight hours and varies with local meteorological conditions

but is always present at some finite level. Noise from planets or stars will be a factor when they are within the field of view of the receiver. The noise from each source may be summed to calculate the total background signal.

The following procedure should be used to calculate background signal levels at the detector of the earth station. The following parameters are required:

- D : aperture diameter of the receiver (m)
- γ : obscuration ratio of the earth station telescope
- ϕ : field of view of the detector (rad)
- B : bandwidth of the receiver (μm).

Step 1: Calculate the area of the receiver, A_{rec} , by:

$$A_{rec} = (1 - \gamma^2) \left(\frac{\pi D^2}{4} \right) \quad \text{m}^2 \quad (17)$$

Step 2: Calculate the solid angle of the receiving detector field of view, ϕ' , by:

$$\phi' = 2\pi \left(1 - \cos\left(\frac{\phi}{2}\right) \right) \quad \text{Sr} \quad (18a)$$

which for small angles is approximated by:

$$\phi'_{det} = \frac{\pi\phi^2}{4} \quad \text{Sr} \quad (18b)$$

Step 3: Determine the radiance value of the sky, H_{sky} , at 283 THz. If measured values are not available, select from the values in Table 3.

TABLE 3
Radiance at 283 THz for arbitrary sky
brightness conditions, H_{sky} ($\text{W}/\text{m}^2/\mu\text{m}/\text{Sr}$)

Source	Radiance
Bright daylight sun	54.45
Normal daytime	25.32
Cloudy daytime	17.99
Night	1.000×10^{-5}

Step 4: Calculate the power at the detector from the sky, P_{sky} , by:

$$P_{sky} = H_{sky} \cdot A_{rec} \cdot \phi' \cdot B \quad \text{W} \quad (19)$$

Step 5: Determine the irradiance value, N_{star} , at 283 THz of the star, if any, that appears in the earth station's field of view. If measured values are not available, select from the values in Table 4. If there is no star in the field of view, let $P_{star} = 0$ and proceed to Step 7.

TABLE 4

Irradiance at 283 THz for selected stars, N_{star} (W/m²/μm)

Source	Irradiance	Source	Irradiance
Achernar	1.94462×10^{-9}	Capella	1.67642×10^{-8}
Aldebaran	2.87647×10^{-8}	Pollux	1.61359×10^{-8}
Altair	2.68864×10^{-9}	Procyon	1.22510×10^{-8}
Arcturus	3.22719×10^{-8}	Rigel	4.76926×10^{-9}
Betelgeuse	3.99278×10^{-8}	Rigil Kent	1.67642×10^{-8}
Canopus	2.09429×10^{-8}	Sirius	2.09013×10^{-8}

Step 6: Calculate the power at the detector from the star, P_{star} , by:

$$P_{star} = N_{star} \cdot A_{rec} \cdot B \quad \text{W} \quad (20)$$

Step 7: Determine the diameter, D_{planet} , bond albedo, χ_{planet} , and the power spectral density, Ψ_{planet} , of the planet in the field of view of the receiver from the values in Table 5. If there is no planet in the field of view of the receiver, let $P_{planet} = 0$ and proceed to Step 10.

TABLE 5

Diameters, albedo and spectral density of the planets

Planet	Diameter (m)	Albedo of planet	Spectral density incident on planet (W/μm)
Mercury	4 866 070	0.119	8.336×10^{16}
Venus	12 108 756	0.75	1.469×10^{17}
Mars	6 778 400	0.25	1.043×10^{16}
Jupiter	142 989 171	0.343	3.950×10^{17}
Saturn	120 582 610	0.342	8.228×10^{16}
Uranus	51 204 220	0.3	3.891×10^{15}
Neptune	49 508 383	0.29	1.373×10^{15}
Pluto	2 308 404	0.145	1.803×10^{12}

Step 8: Calculate the angle subtended by the planet as seen from the receiver, θ_{planet} , by:

$$\theta_{planet} = \frac{D_{planet}}{R_{planet}} \quad \text{rad} \quad (21)$$

where:

R_{planet} : distance from Earth to planet (m).

Step 9: If $\theta_{planet} < \phi$, calculate the power at the detector from the planet, P_{planet} , by:

$$P_{planet} = \left(\frac{\Psi_{planet} \cdot \chi_{planet}}{R_{planet}^2} \right) \cdot A_{rec} \cdot B \quad \text{W} \quad (22a)$$

otherwise,

$$P_{planet} = \left(\frac{\Psi_{planet} \cdot \chi_{planet}}{R_{planet}^2} \right) \cdot A_{rec} \cdot B \cdot \left(\frac{\phi'}{\theta'_{planet}} \right) \quad \text{W} \quad (22b)$$

where θ'_{planet} is the solid angle of the planet subtending the detector given by:

$$\theta'_{planet} = 2\pi \left(1 - \cos \left(\frac{\theta_{planet}}{2} \right) \right) \approx \frac{\pi \theta_{planet}^2}{4} \quad \text{S/r} \quad (23)$$

Step 10: Calculate the total power of the background signal, P_{back} , incident on the detector by:

$$P_{back} = P_{sky} + P_{star} + P_{planet} \quad \text{W} \quad (24)$$

3.2 Detector noise

Most planned applications for deep-space and interplanetary radiocommunications will use a direct detection receiver with an avalanche photo-diode (APD). APD detectors will normally operate in one of two noise-limited detection regions. Detectors receiving high input power levels will generally be limited by photon shot noise. However, detectors receiving low input power levels, such as those associated with deep-space and interplanetary links, will be detector noise limited. The SNR for the commonly used APD followed by a next-stage amplifier in a direct detection system is developed below.

Calculate the excess noise factor, N_E , by:

$$N_E = Gk + \left(2 - \frac{1}{G} \right) (1-k) \quad (25)$$

where:

- G : gain
- k : electron/hole ionization rate.

The SNR may then be calculated by:

$$SNR = \frac{G^2 R_D^2 P_S^2}{2eG^2 B(N_E) (R_D P_S + i_B) + 2ei_S + 4N_A B_F \left(\frac{k_B T}{R_L} \right)} \quad (26)$$

where:

- e : electron charge (1.6×10^{-19} coulomb)
- P_S : mean received signal power (W)
- R_D : APD responsivity

- k_B : Boltzmann's constant (1.38×10^{-23} J/K)
 T : temperature (K)
 i_S : detector surface dark current (A)
 i_B : detector bulk dark current (A)
 R_L : resistance of the transimpedance amplifier (Ω)
 N_A : amplifier noise figure
 B_F : receiver bandwidth (Hz).

4 List of variables

- a : radius of the telescope mirror (m)
 A : area of the telescope aperture (m^2)
 A_{rec} : area of the receiving telescope aperture (m^2)
 α : the ratio, α/ω
 b : radius of the secondary mirror (m)
 B : filter bandwidth (μm)
 B_F : filter bandwidth (Hz)
 c : speed of light (m/s)
 χ_{planet} : bond albedo of a planet
 d : diameter of the detector (m)
 D : diameter of the aperture (m)
 D_{planet} : diameter of the planet (m)
 δ : losses due to energy spilling over the edge of the detector (dB)
 e : electron charge (coulomb)
 E_{pulse} : energy per pulse (J)
 f : optical frequency (Hz)
 F : focal length of the telescope (m)
 φ : field of view of the detector (rad)
 φ' : solid angle field of view of the detector (srad)
 G_0 : maximum on-axis antenna gain (dBi)
 G_r : receiving antenna gain (dBi)
 g_i : gain efficiency
 G_t : transmitter antenna gain (dBi)
 γ : obscuration ratio
 H_{sky} : radiance of the sky ($W/m^2/\mu m/Sr$)
 i_b : bulk dark current in the detector (A)
 i_s : surface dark current in the detector (A)
 k : electron/hole ionization rate
 k_B : Boltzmann's constant
 L_t : transmitter losses

L_r :	receiver losses
L_p :	pointing losses
L_a :	atmospheric losses along the space-to-ground link
L_s :	free space loss
λ :	wavelength (m)
M :	modulation index
N_A :	noise figure of amplifier
N_E :	excess noise factor
N_{star} :	irradiance of a star ($\text{W}/\text{m}^2/\mu\text{m}$)
N_t :	noise power from all sources
P_{ave} :	average transmitter power (W)
P_{back} :	total received background power (W)
P_{peak} :	peak power (W)
P_{planet} :	received power from an illuminated planet (W)
P_s :	received signal power (W)
P_{sky} :	received power from the sky (W)
P_{star} :	received power from a star (W)
P_t :	average laser output power (W)
θ :	angle off the optical axis (rad)
θ_{planet} :	angle subtended by a planet (rad)
θ_{1/e^2} :	angular width of the beam at the $1/e^2$ point (rad)
θ'_{planet} :	solid angle subtended by a planet (rad)
R :	distance between transmitter and receiver (m)
R_D :	responsivity of the APD
R_L :	resistance of the transimpedance amplifier
R_{planet} :	distance between planet and receiver (m)
t_d :	transmitter dead time (s)
t_p :	transmitter pulse time (s)
t_s :	transmitter slot time (s)
t_w :	transmitter word time (s)
T :	temperature (K)
ω :	waist size of the beam at the $1/e^2$ point
Ψ_{planet} :	power spectral density of a moon or planet ($\text{W}/\mu\text{m}$).

Annex 2

1 Antenna pattern for unobscured, circular transmitting apertures

The envelope of the antenna gain pattern of an unobscured, circular transmitting aperture operating at around 283 THz is approximated by:

$$\begin{aligned}
 G_t(\varphi) &= G_{max} - 0.9 - (4.5 \times 10^{-4}) \left(\frac{D}{\lambda} \cdot \varphi \right)^{2.5} & \text{dBi} & \quad \text{for } 0 \leq \varphi \leq \varphi_m \\
 G_t(\varphi) &= G_1 & \text{dBi} & \quad \text{for } \varphi_m < \varphi \leq \varphi_r \\
 G_t(\varphi) &= G_{max} + 35 - 30 \log \left(\frac{D}{\lambda} \right) - 30 \log(\varphi) & \text{dBi} & \quad \text{for } \varphi_r < \varphi \leq \varphi_1 \\
 G_t(\varphi) &= -10 & \text{dBi} & \quad \text{for } \varphi_1 < \varphi \leq 180^\circ
 \end{aligned}$$

where:

$$G_{max} = 20 \log \left(\frac{\pi D}{\lambda} \right): \text{ gain of the mainbeam (dBi)}$$

$$G_1 = G_{max} - 25.8: \text{ gain of the first sidelobe (dBi)}$$

D : aperture diameter (m)

λ : wavelength (m)

φ : off-axis angle (degrees)

$$\varphi_r = 5.83 \left(\frac{180\lambda}{\pi^2 D} \right): \text{ angle of first sidelobe (degrees)}$$

$$\varphi_m = 0.75\varphi_r: \text{ angle where gain of mainlobe equals gain of first sidelobe (degrees)}$$

φ_1 : field of view limited by optical stops (degrees).

2 Antenna pattern for obscured, circular transmitting apertures

The envelope of the antenna gain pattern of an obscured, circular transmitting aperture operating at around 283 THz is approximated by:

$$\begin{aligned}
 G_t(\varphi) &= G_{max} - 0.9 + 32 \log(1 - \gamma^2) - \left(4 \times 10^{-4} + \frac{\gamma}{2000} \right) \left(\frac{D}{\lambda} \cdot \varphi \right)^{2.5} & \text{dBi} & \quad \text{for } 0 \leq \varphi \leq \varphi_m \\
 G_t(\varphi) &= G_1 & \text{dBi} & \quad \text{for } \varphi_m < \varphi \leq \varphi_r \\
 G_t(\varphi) &= G_{max} + 40 + 15\gamma - 30 \log \left(\frac{D}{\lambda} \right) - 30 \log(\varphi) & \text{dBi} & \quad \text{for } \varphi_r < \varphi \leq \varphi_1 \\
 G_t(\varphi) &= -10 & \text{dBi} & \quad \text{for } \varphi_1 < \varphi \leq 180^\circ
 \end{aligned}$$

where:

$$G_{max} = 20 \log \left(\frac{\pi D}{\lambda} \right): \text{ gain of the mainbeam (dBi)}$$

$$G_1 = G_{max} + 2.17 + 15\gamma - 30 \log (5.77 - 2.9\gamma^2): \text{ gain of the first sidelobe (dBi)}$$

D : aperture diameter (m)

λ : wavelength (m)

φ : off-axis angle (degrees)

$$\varphi_r = (5.77 - 2.9\gamma^2) \left(\frac{180\lambda}{\pi^2 D} \right): \text{ angle of first sidelobe (degrees)}$$

$$\varphi_m = (0.71 - 0.5\gamma) \cdot \varphi_r: \text{ angle where gain of mainlobe equals gain of first sidelobe (degrees)}$$

φ_1 : field of view limited by optical stops

$$\gamma = \frac{b}{a}: \text{ obscuration ratio}$$

a : radius of the primary aperture (m)

b : radius of the secondary aperture (m).

3 Antenna pattern for unobscured, circular receiving apertures

The envelope of the antenna gain pattern of an unobscured, circular receiving aperture operating at around 283 THz is approximated by:

$$G_t(\varphi) = G_{max} - (6 \times 10^{-4}) \left(\frac{D}{\lambda} \cdot \varphi \right)^{2.5} \quad \text{dBi} \quad \text{for } 0 \leq \varphi \leq \varphi_m$$

$$G_t(\varphi) = G_1 \quad \text{dBi} \quad \text{for } \varphi_m < \varphi \leq \varphi_r$$

$$G_t(\varphi) = G_{max} + 42 - 30 \log \left(\frac{D}{\lambda} \right) - 30 \log (\varphi) \quad \text{dBi} \quad \text{for } \varphi_r < \varphi \leq \varphi_1$$

$$G_t(\varphi) = -10 \quad \text{dBi} \quad \text{for } \varphi_1 < \varphi \leq 180^\circ$$

where:

$$G_{max} = 20 \log \left(\frac{\pi D}{\lambda} \right): \text{ gain of the mainbeam (dBi)}$$

$$G_1 = G_{max} - 17.5: \text{ gain of the first sidelobe (dBi)}$$

D : aperture diameter (m)

λ : wavelength (m)

φ : off-axis angle (degrees)

$$\varphi_r = 5.14 \left(\frac{180\lambda}{\pi^2 D} \right): \text{ angle of first sidelobe (degrees)}$$

- $\varphi_m = 0.65\varphi_r$: angle where gain of mainlobe equals gain of first sidelobe (degrees)
 φ_1 : field of view limited by optical stops (degrees).

4 Antenna pattern for obscured, circular receiving apertures

The envelope of the antenna gain pattern of an obscured, circular receiving aperture operating at around 283 THz is approximated by:

$$G_t(\varphi) = G_{max} + 20 \log(1 - \gamma^2) - \left(6 \times 10^{-4} + \frac{\gamma}{3000}\right) \left(\frac{D}{\lambda} \cdot \varphi\right)^{2.5} \quad \text{dBi} \quad \text{for } 0 \leq \varphi \leq \varphi_m$$

$$G_t(\varphi) = G_1 \quad \text{dBi} \quad \text{for } \varphi_m < \varphi \leq \varphi_r$$

$$G_t(\varphi) = G_{max} + 44 + 8\gamma - 30 \log\left(\frac{D}{\lambda}\right) - 30 \log(\varphi) \quad \text{dBi} \quad \text{for } \varphi_r < \varphi \leq \varphi_1$$

$$G_t(\varphi) = -10 \quad \text{dBi} \quad \text{for } \varphi_1 < \varphi \leq 180^\circ$$

where:

$$G_{max} = 20 \log\left(\frac{\pi D}{\lambda}\right): \quad \text{gain of the mainbeam (dBi)}$$

$$G_1 = G_{max} - 15.15 + 8\gamma: \quad \text{gain of the first sidelobe (dBi)}$$

D : aperture diameter (m)

λ : wavelength (m)

φ : off-axis angle (degrees)

$$\varphi_r = 5.14 \left(\frac{180\lambda}{\pi^2 D}\right): \quad \text{angle of first sidelobe (degrees)}$$

$$\varphi_m = (0.62 - 0.3\gamma) \cdot \varphi_r: \quad \text{angle where gain of mainlobe equals gain of first sidelobe (degrees)}$$

φ_1 : field of view limited by optical stops

$$\gamma = \frac{b}{a}: \quad \text{obscuration ratio}$$

a : radius of the primary aperture (m)

b : radius of the secondary aperture (m).
