

RECOMMENDATION ITU-R SA.1156^{*,**}**Methods of calculating low-orbit
satellite visibility statistics**

(1995)

The ITU Radiocommunication Assembly,

considering

- a) the increasing use of space stations in circular low-Earth orbit in the space research and other services;
- b) that because of the orbits used by these satellites, interference to or from these stations will be a time varying function;
- c) that, because of these variations, dynamic sharing models may need to be developed,

recommends

that the equations and graphs contained in Annexes 1 and 2 be considered for developing models of dynamic sharing situations involving low-Earth orbit satellites.

Annex 1**Analysis of low-orbit satellite visibility statistics****1 Introduction**

The increasing use of space stations in circular low-orbit in the space research service (and other services) necessitates the development of dynamic sharing models in which the potential interference from the space station can be treated as a time varying function. Even for the simplest of dynamic sharing models, at least six specific system parameters must be evaluated to define precisely the primary time dependent statistics of a low-orbit space station as seen from a location on the Earth's surface.

* This Recommendation should be brought to the attention of Radiocommunication Study Groups 8 and 9.

** Radiocommunication Study Group 7 made editorial amendments to this Recommendation in 2003 in accordance with Resolution ITU-R 44.

The time dependent statistics are:

- the longest time of passage of a space station through the main beam of a ground antenna (discussed in § 3 of this Annex);
- the long-term percentage of time that the space station spends in various areas of the orbit sphere as seen from the ground station.

The first statistic is important in that it defines the longest continuous duration of noise power into the ground receiving system from the space station. The second set of statistics, after convolution with transmit and receive antenna patterns, and range loss, can be used to develop interference-to-noise (I/N) relations as a function of time for the dynamic sharing model. In one sense then, I/N versus time relations can be treated in a method similar to the signal strength versus time relations derived from atmospheric propagation statistics. However, instead of a receiver experiencing change in the signal-to-noise ratio as a statistical function of time, it experiences a change in signal-to-noise-plus-interference ratio, as a statistical function of time, based upon the low-orbit space station model parameters.

The specific parameters which define the long-term visibility statistics of a space station in a low circular inclined orbit (see Note 1) as seen from a receiving system on the Earth's surface are:

- altitude of the space station H (km);
- inclination of the space station orbit i (degrees);
- latitude of the ground station La (degrees);
- pointing azimuth of the ground station antenna measured from North, Az (degrees);
- pointing elevation of the ground station antenna measured from the local horizontal plane, El (degrees);
- angular area of the region of interest δA .

NOTE 1 – This Annex only deals with circular satellite orbits in which the orbital period is not an even multiple of the Earth's rotational period.

The last parameter may take on several different physical interpretations depending upon the purpose of the analysis. For instance, it may be the angular area of the main beam of the ground station antenna or it may be taken as an angular area expressed by an azimuth "width" of δAz (degrees) and an elevation "height" expressed as δEl (degrees).

2 Bounding equation

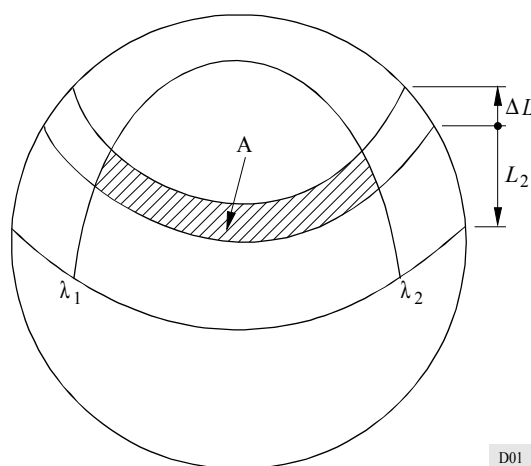
The bounding equation (derived in Annex II of Report 684, Geneva, 1982) is given below and may be used to determine the percentage of time that a low-orbit spacecraft will reside in certain regions visible to a ground station over long periods of time:

$$T(\%) = \frac{\delta \lambda}{2\pi^2} \left(\sin^{-1} \left[\frac{\sin(L + \Delta L)}{\sin i} \right] - \sin^{-1} \left[\frac{\sin L}{\sin i} \right] \right) \times 100 \quad (1)$$

where:

- $L, \Delta L$: latitude limits of the region on the orbital shell (see Fig. 1)
- $\delta \lambda$: longitudinal extent of the region on the orbital shell, between the longitude limits of λ_1 and λ_2 (as seen in Fig. 1)
- i : inclination of the satellite orbit (all angles in rad).

FIGURE 1



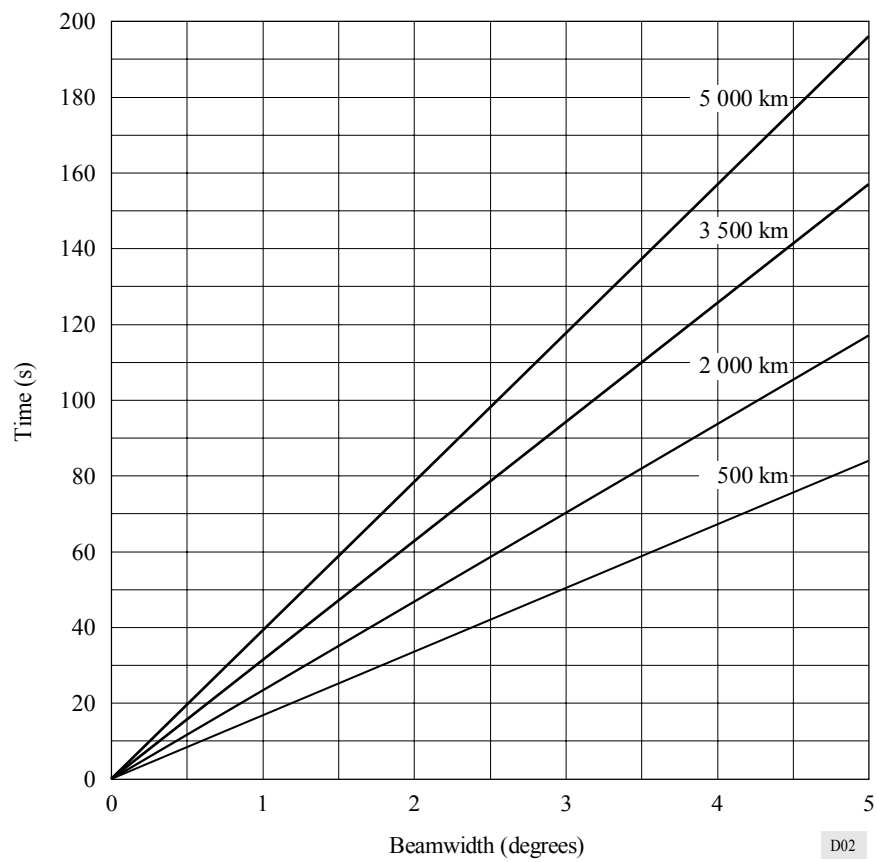
3 The maximum time a satellite spends in the beam of a ground station

This section provides worst case numerical data on one aspect of frequency sharing with low-orbit, inclined orbit satellites. Such sharing is influenced by the amount of time that an “unwanted” and potentially interfering satellite appears within the 3 dB beamwidth of a ground station. This parameter is evaluated for several orbit altitudes and for two “bounding” elevations of the receiving antenna. The numerical results developed in this paper represent an upper bound on the length of time a spacecraft at a given altitude will appear within the beam of a ground station.

The time a satellite spends in a ground station’s beam is a function of the beam’s width, the elevation of the beam and the altitude of the satellite. The worst case, i.e. when the satellite spends the maximum possible time in the beam, occurs when the ground station is located at the equator with a beam of elevation = 0° and the satellite is travelling east along an orbit with 0° inclination. The time the satellite spends in the beam depends upon the satellite’s velocity relative to the velocity of the beam as it rotates with the Earth, and upon the length of the intersection of the orbit with the beam.

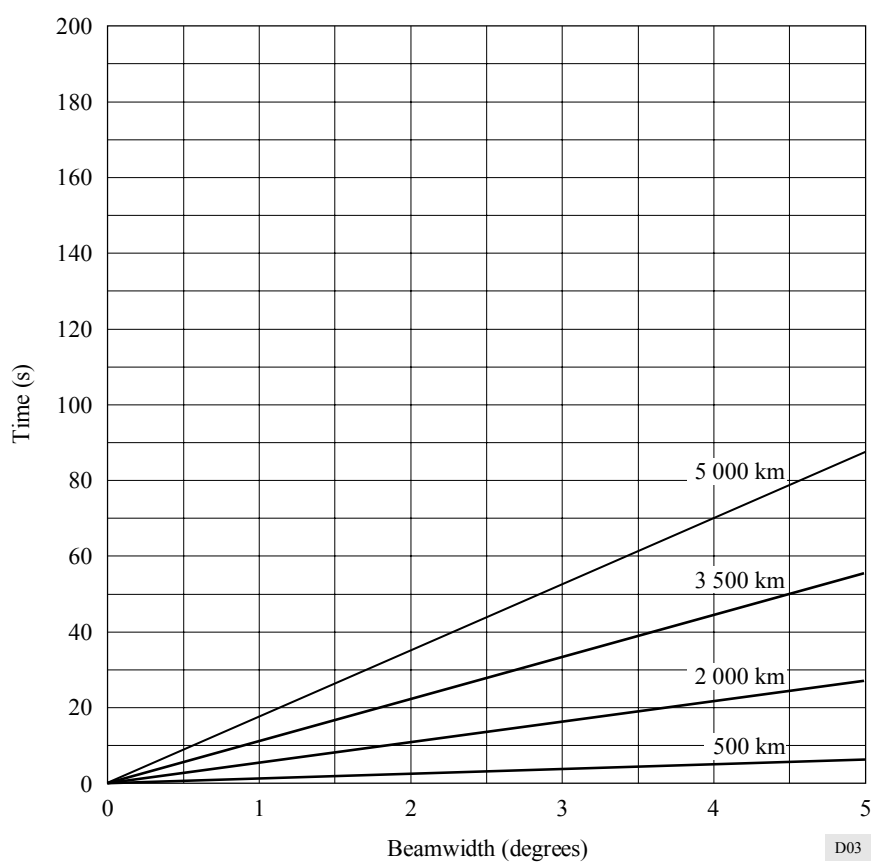
The maximum time that a spacecraft can spend in the main beam of an antenna is shown in Figs. 2 and 3 for antenna elevations of 0° and 90° respectively, and refers to a variety of orbital altitude and beamwidths.

FIGURE 2
Maximum time in beam plotted against beamwidth at elevation of 0°



D02

FIGURE 3
Maximum time in beam plotted against beamwidth at elevation of 90°



D03

Annex 2

A method of calculating the cumulative statistics for assessing interference between low-orbit satellites and terrestrial stations

1 Introduction

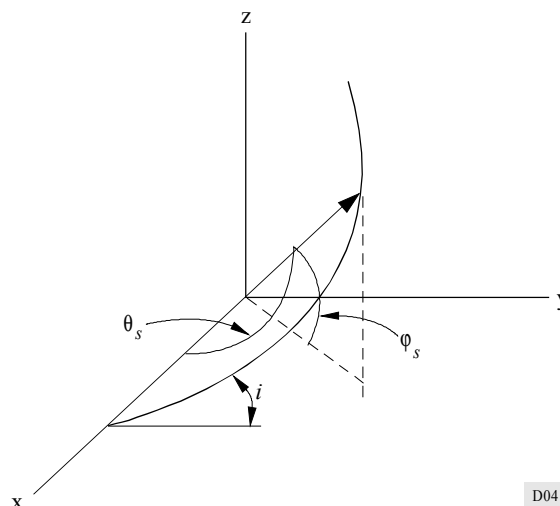
Annex 1 describes the factors affecting the visibility statistics, presents a bounding equation for determining the percentage of time that a low-orbiting satellite will occupy specified regions of the orbit shell that are visible to an earth station and contains summary charts giving the maximum duration a low-orbiting satellite spends in certain regions of the orbit shell as a function of several parameters. The purpose of this Annex is to:

- develop the probability density function (pdf) of a satellite occupying specific locations on the orbit shell;
- illustrate how the probability density function may be used to calculate the pdf of interference to low-orbiting satellites resulting from emissions from stations in the fixed service (FS); and
- to illustrate how to compute the pdf of the interference to FS systems assuming the e.i.r.p. of the low-orbiting satellite is shaped to produce a specific power flux-density profile on the surface of the Earth.

2 Probability density function of the position of a low-orbiting satellite on the orbit shell

The position, i.e. the latitude and longitude of an orbiting satellite on the orbit shell relative to a fixed point on Earth, is a function of two independent parameters: the position of the satellite in its orbit plane; and the longitude of the observation point on Earth relative to the orbit plane. The geometry used for this analysis is shown in Fig. 4. It is assumed that the satellite is in a circular orbit at an altitude, h , the inclination of the orbit plane is i , and the period of rotation of the satellite and of the Earth are not directly related.

FIGURE 4
Simplified geometric model of an Earth-orbiting satellite



The coordinate system shown in Fig. 4 is a right-handed, geocentric system with the x-y plane corresponding to the equatorial plane, and the x-axis pointing in an arbitrary direction in space (usually the First Point of Aries).

For simplicity, assume that the intersection of the orbit plane and the equatorial plane is the x-axis. The latitude φ_s of the position of the satellite in space is given by:

$$\sin \varphi_s = \sin \theta_s \sin i \quad (2)$$

where θ_s is the central angle between the x-axis and the position vector of the satellite. For satellites in circular orbits, θ_s is a linear function of time t , i.e. $\theta_s = 2\pi t / \tau$, where τ is the period of the orbit. Equation (2) relates the latitude of the satellite as a function of the central angle θ_s and the orbit inclination angle i .

If the central angle of the position vector of a satellite in a circular orbit is sampled at random times, the angle θ_s will be found to be uniformly distributed between 0 and 2π radians. Stated as a probability density function $p(\theta_s)$:

$$p(\theta_s) = \frac{1}{2\pi} \quad (3)$$

The probability density function of the latitude of the position vector of the satellite may be found using a straightforward transformation technique from probability theory. It may be shown for a random variable x with pdf $p(x)$ that undergoes the transformation $y = g(x)$, that the pdf $p(y)$ of the random variable y is given by:

$$p(y) = \frac{p(x_1)}{|g'(x_1)|} + \dots + \frac{p(x_n)}{|g'(x_n)|} \quad (4)$$

where:

$$g'(x) = \frac{dg(x)}{dx}$$

and $x_1, \dots, x_n$ are the real roots of $y = g(x)$.

Applying the procedure described above to equations (2) and (3) yields the probability density function of the latitude of the position vector of the satellite in its orbital plane:

$$p(\varphi_s) = \frac{1}{\pi} \frac{\cos \varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (5)$$

Equation (5) represents the function that would be obtained if the latitude of the satellite were randomly sampled a large number of times. Inspection of equation (5) shows that the expression is defined only for real values of $|\varphi_s| \leq i$ as expected. It may also be shown that:

$$\int_{-i}^i p(\varphi_s) d\varphi_s = 1 \quad (6)$$

also as expected.

For the satellite to appear at a specific longitude λ_s on the orbit shell relative to the reference point on the surface of the Earth, the orbit plane must intersect the orbit shell at that longitude. The probability of this occurring is uniformly distributed over 2π radians, i.e.:

$$p(\lambda_s) = \frac{1}{2\pi} \quad (7)$$

Finally, since it has been assumed that the period of the satellite and the rotation of the Earth are not directly related, the probability density function of the satellite position is the joint probability of two independent events which is given by the product of the individual probability density functions:

$$p(\varphi_s, \lambda_s) = \frac{1}{2\pi^2} \frac{\cos \varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (8)$$

The probability $P(\Delta\varphi, \Delta\lambda)$ of the satellite occupying the region on the orbit shell bounded by latitude φ_s , $\varphi_s + \Delta\varphi_s$ and longitude $\Delta\lambda_s$ is given by:

$$P(\Delta\varphi, \Delta\lambda) = \frac{1}{2\pi^2} \int_0^{\Delta\lambda_s} \int_{\varphi_s}^{\varphi_s + \Delta\varphi_s} \frac{\cos \varphi_s d\lambda_s d\varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (9)$$

Carrying out the integration yields:

$$P(\Delta\varphi, \Delta\lambda) = \frac{\Delta\lambda_s}{2\pi^2} \left(\sin^{-1} \left[\frac{\sin(\varphi_s + \Delta\varphi_s)}{\sin i} \right] - \sin^{-1} \left[\frac{\sin \varphi_s}{\sin i} \right] \right) \quad (10)$$

3 Probability density function of interference to low-orbiting satellites caused by emissions from FS systems

The pdf of interference to low-orbiting satellites caused by emissions from FS systems is a function of the geometry and the pdf of the satellite position. If the interference can be expressed as a function of the coordinates (latitude and relative longitude) of the visible orbit shell, i.e. $I(\varphi_s, \lambda_s)$, then the pdf of the interference to the low-orbiting satellite $p(I)$ is given by Recommendation ITU-R S.699:

$$p(I) dI = \iint_S p(\varphi_s, \lambda_s) d\varphi_s d\lambda_s \quad (11)$$

where S indicates that the integration is to be performed over the segment of the surface of the orbit sphere that contributes a level of interference between the values of I and $I + dI$.

The function $I(\varphi_s, \lambda_s)$ is a complex function of a number of parameters that include the location of the FS station, the transmitter power spectral density, the directional characteristics of the transmitting antenna gain, the azimuth and elevation angle of the transmitting antenna, the altitude and orbit inclination angle of the satellite, the range to the satellite, the gain of the satellite receiving antenna in the direction of the interference and the operating frequency. An integral involving a function of this complexity is most readily handled through the use of numerical techniques.

The steps used in the numerical procedure are:

Step 1: define φ_s and λ_s as independent variables over the surface of the visible orbit sphere.

Step 2: define an array $I(n)$ corresponding to the range of interest (maximum to minimum value of interference ($I(\varphi_s, \lambda_s)$) where n corresponds to the number of desired increments (e.g. 0.25 dB increments) (this array will be used to store the differential pdf).

Step 3: evaluate $I(\varphi_s, \lambda_s)$ at specific values of φ and λ (this value will be used to point to a specific element n_0 in the array $I(n)$).

Step 4: calculate $p(\varphi_s, \lambda_s) d\varphi_s d\lambda_s$ and add to the value stored in $I(n_0)$.

Step 5: increment φ and λ over the surface of the visible orbit sphere.

Step 6: repeat steps 3 to 5.

It is noted that evaluating equation (11) numerically results in the transformation of the integral to a summation.

The geometrical parameters required to evaluate $I(\varphi_s, \lambda_s)$ are obtained by using a geocentric coordinate system similar to the one shown in Fig. 4. The main difference is that the coordinate system rotates at the same rate and direction as the Earth. The x-y plane is the equatorial plane and the z-axis is the rotational axis of the Earth. The position of the FS station is assumed, for simplicity, to lie in the x-z plane. The scale of the coordinate system is normalized to the radius of the Earth. Therefore, any distances computed in this coordinate system must be multiplied by the radius of the Earth (6 378 km) to obtain the correct value. The normalized components of the station position vector $\mathbf{P}$ are given by:

$$\mathbf{P} = \begin{bmatrix} \cos \varphi_p \\ 0 \\ \sin \varphi_p \end{bmatrix} \quad (12)$$

where φ_p is the latitude of the FS station.

The direction the FS transmitting antenna is pointed, is given by a unit vector that lies in the plane of the local horizontal and is offset from the direction of North by a specified azimuth angle θ_{az} . The components of the antenna pointing vector $\mathbf{U}_A$ are given by:

$$\mathbf{U}_A = \begin{bmatrix} \cos \varphi_r \cos \lambda_r \\ \cos \varphi_r \sin \lambda_r \\ \sin \varphi_r \end{bmatrix} \quad (13)$$

where:

$$\varphi_r = \sin^{-1} (\cos \varphi_p \cos \theta_{az}) \quad (14a)$$

$$\lambda_r = \cos^{-1} \left[\frac{-\sin \varphi_p \cos \theta_{az}}{\sqrt{1 - \cos^2 \varphi_p \cos^2 \theta_{az}}} \right] \quad (14b)$$

The values of φ_s and λ_s defining the limits of the visible surface of the orbit sphere may be readily determined. The limits for φ_s are given by:

$$\varphi_{max} = \varphi_p + \varphi_{lim}, \quad \varphi_{max} \leq i, \text{ otherwise } \varphi_{max} = i \quad (15a)$$

$$\varphi_{min} = \varphi_p - \varphi_{lim}, \quad \varphi_{min} \leq i \quad (15b)$$

where:

$$\varphi_{lim} = \cos^{-1}(1/\beta)$$

$$\beta = 1 + h/r_e$$

h : altitude of the satellite

r_e : radius of the Earth.

If $\varphi_{min} < i$, then the low-orbiting satellite is not visible at the FS station.

For an arbitrary value of φ_s between the limits φ_{min} and φ_{max} , the limiting values of the relative longitude λ_{min} and λ_{max} on the visible segment of the orbit sphere are given by:

$$\lambda_{max} = -\lambda_{min} = \cos^{-1} \left[\frac{\cos \varphi_{lim} - \sin \varphi_p \sin \varphi_s}{\cos \varphi_p \cos \varphi_s} \right] \quad (16)$$

Given values for φ_s and λ_s between the limits derived above, the range to the satellite and the angle between the direction the FS station antenna is pointed and the direction to the satellite is most easily obtained using vector analysis. Specifically, the vector to the satellite $\mathbf{R}$ is given by:

$$\mathbf{R} = \mathbf{S} - \mathbf{P} \quad (17)$$

where $\mathbf{P}$ is the position vector of the FS station as given by equation (12), and $\mathbf{S}$ is the position vector of the sample point of the location of the satellite on the orbit sphere given by:

$$\mathbf{S} = \beta \begin{bmatrix} \cos \varphi_s \cos \lambda_s \\ \cos \varphi_s \sin \lambda_s \\ \sin \varphi_s \end{bmatrix} \quad (18)$$

The normalized range to the satellite $|\mathbf{R}|$ is given by the square root of the sum of the squares of the components of the range vector given in equation (17). The off-axis angle to the satellite is obtained using the scalar product of the antenna pointing vector $\mathbf{U}_A$ (whose normalized components are given by equation (13)) and the range vector $\mathbf{R}$. The off-axis angle $\varphi_{off-axis}$ is given by:

$$\varphi_{off-axis} = \cos^{-1} \left[\frac{\mathbf{R} \cdot \mathbf{U}_A}{|\mathbf{R}|} \right] \quad (19)$$

Recommendation ITU-R S.699 sets forth the reference radiation pattern to be used for FS station transmitting antenna with $D/\lambda_f < 100$ and for antennas with $D/\lambda_f > 100$, where D is the diameter of the antenna and λ_f is the wavelength at the operating frequency. The reference radiation pattern to be used for the receiving system on the low-orbiting satellite will be assumed to be isotropic. Using the above assumptions:

$$I(\varphi_s, \lambda_s) = \frac{P_T G_T(\varphi_{off-axis}) G_R \lambda_f^2}{(4\pi R_s)^2} \quad (20)$$

where:

P_T : transmitter power (or power spectral density)

$G_T(\varphi_{off-axis})$: transmitting antenna gain in the direction of the sample point (φ_s, λ_s)

G_R : receiving antenna gain at the sample point in the direction of the FS station

λ_f : wavelength of the operating frequency

R_s : range (in the same dimensions as λ_f) from the FS station to the sample point (i.e. $|\mathbf{R}| r_e$).

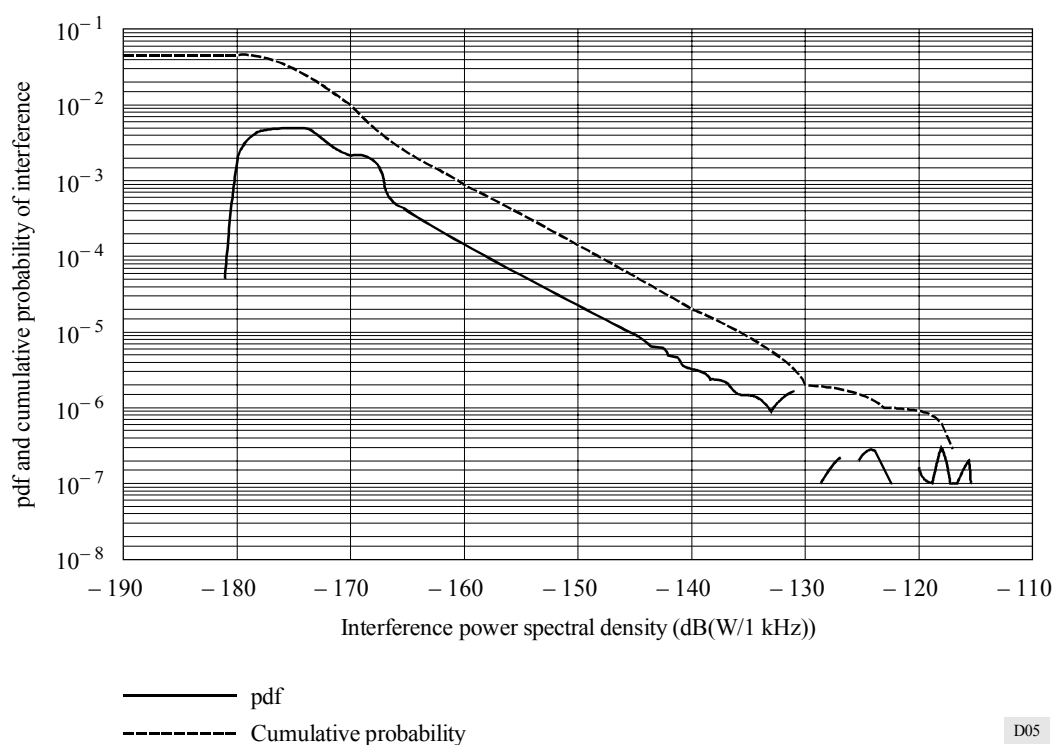
Using the procedure described earlier in this section, the probability density function of the interference to a low-orbiting satellite caused by emissions of a FS station is obtained using equations (11) and (20).

An example case has been evaluated to illustrate the results that are obtainable using the analytical procedure described above. For this case it has been assumed that:

- the FS station is located at 38° N latitude;
- the antenna gain is 50 dBi;
- the azimuth angle of the antenna is 90°;
- the operating frequency is 2 050 MHz;
- the transmitter power spectral density at the input to the antenna is 0 dB(W/1 kHz);
- the satellite is in a circular orbit at an altitude of 800 km;
- the inclination of the orbit plane is 90°; and
- the satellite uses an isotropic receiving antenna with a gain of 0 dBi.

The results of the analysis are shown in Fig. 5. The solid curve is the probability density function of interference received by a low-orbiting satellite. The dashed curve gives the cumulative probability that the interference exceeds a specific value. For example, the solid curve shows that the pdf of the interference being on the order of -150 dB(W/1 kHz) is about 2×10^{-5} . Similarly, the dashed curve indicates that the probability of interference exceeding -170 dB(W/1 kHz) is about 1×10^{-2} or 1%.

FIGURE 5
Interference from the example troposcatter station



4 Probability density function of interference to FS systems caused by emissions from low-orbiting satellites

The approach used to compute the pdf of interference to FS stations caused by emissions from low-orbiting satellites is a minor extension of the approach described in the previous section. In this case, the incident interference at the FS station is assumed to conform to values of power flux-density (pfd) that are specified as a function of the elevation angle at the FS station. The steps of the procedure described in § 3 are used. The computation of $I(\varphi_s, \lambda_s)$ becomes:

$$I(\varphi_s, \lambda_s) = \rho(\delta) G_T(\varphi_{off\ axis}) \frac{\lambda_f^2}{4\pi} \quad (21)$$

where $\rho(\delta)$ is the spectral power flux-density, δ is the elevation angle and the other parameters are as defined previously. Equation (19) is used to compute the off-axis angle and the spectral power flux-density is given by:

$$\rho(\delta) = \begin{cases} -154 & \text{dB(W/(m}^2 \cdot 4 \text{ kHz))} & \text{for } 0^\circ \leq \delta < 5^\circ \\ -154 + 0.5(\delta - 5) & \text{dB(W/(m}^2 \cdot 4 \text{ kHz))} & \text{for } 5^\circ \leq \delta < 25^\circ \\ -144 & \text{dB(W/(m}^2 \cdot 4 \text{ kHz))} & \text{for } 25^\circ \leq \delta < 90^\circ \end{cases} \quad (22)$$

The elevation angle is obtained using the scalar product of the range vector $\mathbf{R}$ and the FS station position vector $\mathbf{P}$. Noting that $\cos(90 - \delta) = \sin \delta$, then:

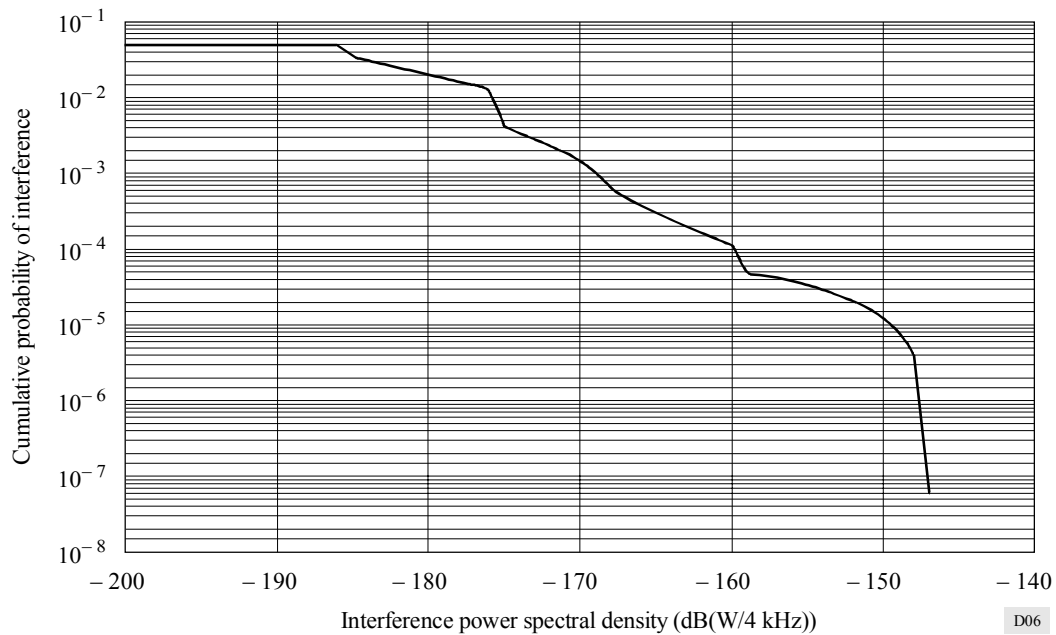
$$\delta = \sin^{-1} \left[\frac{\mathbf{R} \cdot \mathbf{P}}{|\mathbf{R}|} \right] \quad (23)$$

An example has been evaluated for this interference case. Here it has been assumed that:

- the FS station is located at 38° N latitude;
- the receiving antenna gain is 35 dBi;
- the azimuth angle of the antenna is 90°;
- the operating frequency is 2 250 MHz;
- the incident spectral power flux-density at the FS station is given by equation (22);
- the satellite is in a circular orbit at an altitude of 800 km; and
- the inclination of the orbit plane is 90°.

The results of the analysis are shown in Fig. 6. The solid curve is the cumulative probability that the interference exceeds a particular value. The Figure shows that the probability of the interference exceeding -167 dB(W/4 kHz) is on the order of 4×10^{-4} .

FIGURE 6
Interference to example radio-relay station



5 Summary

This Recommendation has summarized the development of the probability density function (pdf) of a satellite occupying specific locations on the orbit shell; illustrated how the probability density function may be used to calculate the statistical characteristics of interference to low-orbiting satellites resulting from emissions from stations in the FS; and illustrated the computation of the pdf of the interference to FS systems assuming the pdf of the emissions of the low-orbiting satellites conform to a specific power flux-density profile.