RECOMMENDATION ITU-R S.736-3*

ESTIMATION OF POLARIZATION DISCRIMINATION IN CALCULATIONS OF INTERFERENCE BETWEEN GEOSTATIONARY-SATELLITE NETWORKS IN THE FIXED-SATELLITE SERVICE

(Question ITU-R 42/4)

(1992-1994-1995-1997)

The ITU Radiocommunication Assembly,

considering

a) that more than one geostationary-satellite network in the fixed-satellite service (FSS) operates in the same frequency band;

b) that interference between networks in the FSS contributes to noise in each network;

c) that it is necessary to protect a network in the FSS from interference by other such networks;

d) that the detailed estimation of mutual interference between satellite networks, due to increased orbit occupancy requires more accurate values of polarization discrimination, resulting from the use of different or identical polarizations by wanted and interfering systems;

e) that the use for actual coordination requirements of the values of polarization isolation factors in Appendix 29 to the Radio Regulations, would not provide a precise estimation of polarization discrimination in the calculation of actual interference margins,

recommends

1 that, to estimate the polarization discrimination between two satellite networks, the method described in Annex 1 should be used.

ANNEX 1

Estimation of polarization discrimination

1 Definition of the polarization of a wave

Polarization is defined as a vector of the electric field wave which is located in a plane orthogonal to the direction of the wave propagation.

Generally, this vector describes an ellipse. Two particular cases arise, firstly circular polarization where the two axes of the ellipse are equal, secondly linear polarization where one of the axes is zero.

If the radiated wave is linearly polarized, two orthogonal polarization planes exist, each polarization vector keeping a fixed direction. The polarization plane, for a linearly polarized wave, is the plane containing the direction of the wave propagation and the polarization vector.

If the radiated wave is circularly polarized, right- and left-hand rotations exist.

2 Definition of polarization angle and of relative alignment angle

The polarization angle ε is the angle between the vertical plane including the propagation direction (pointing of the earth station towards the satellite) and the polarization plane of the linearly polarized wave transmitted by the satellite or by the earth station pointed towards the satellite.

* This Recommendation should be brought to the attention of the Radiocommunication Working Party 10-11S.

The relative alignment angle β is, in linear polarization, the angle between:

– the planes of polarization of the wanted and interfering signals ($\varepsilon_1 - \varepsilon_2$) (see Appendix 2);

– or the plane of polarization of the received signal and the plane of polarization of the receiving antenna (see Appendix 1).

In the co-polarized case, the angle β is given by:

$$\beta = |\varepsilon_1 - \varepsilon_2| + \delta$$

with:

δ: tolerances.

For calculation of the angles β and ε, see Appendices 1 and 2.

3 Definition of polarization decoupling ratio and polarization discrimination

The ratio of polarization decoupling $D_p(\varphi)$ of an earth station or a satellite antenna is the ratio of the field component in the wanted polarization to the field component in the orthogonal polarization. φ is the angle between the directions of wanted and interfering signals.

The polarization discrimination Y (factor of polarization/isolation) of a receive antenna is the ratio of the received power of the two waves of different direction and polarization.

It should be noted that, if the interfering or the wanted network (or both of them) operates a set of transponders on one polarization, and a set of co-frequency transponders on the orthogonal polarization, then it is not valid to include the full polarization discrimination in calculations of interference between the two networks. The degree of such discrimination will depend on the extent of the overlap of the pass-bands of transponders in one network with those of the other network. In the worst case, i.e., when the transponders of the two networks are exactly aligned in frequency and bandwidth, no inter-network polarization discrimination should be included in interference calculations between the two networks.

4 Calculation of polarization discrimination, *Y*, in linear polarization

4.1 Calculation of polarization discrimination, Y_d, in the down-link

The purpose of this calculation is to determine, in the case of a wanted receiving earth station, the discrimination with respect to an interfering wave. Earth-station radiation patterns have been established for co- and cross-polarization planes using experimental data.

The polarization angles are calculated for wanted and interfering signals using the coordinates of the two pointing directions of wanted and interfering satellite antennas and the coordinates of the reference earth station.

The derived value of discrimination Y_d takes into account the co-polarized wave coming from the interfering satellite, received by the earth-station receiver (co-polarized $A_{//}(\varphi)$ and cross-polarized $A_+(\varphi)$ patterns).

The cross-polarized wave coming from the interfering satellite intercepted by the co-polarized pattern of the station is also taken into account. However, the additional isolation afforded by the ratio of crossed polarization transmit-to-crossed polarization receive may be neglected.

$$Y_d = -10 \log \left(\cos^2 \beta + \sin^2 \beta \cdot 10^{-D_p(\varphi_b)/10} + \sin^2 \beta \cdot 10^{-D_{psat}/10} \right) \quad \text{dB} \qquad (1)$$

where:

φ_b: topocentric separation between satellites

$D_p(\varphi_b)$: polarization decoupling of the wanted earth station:

$$D_p(\varphi_b) = A_{//}(\varphi_b) - A_+(\varphi_b) \quad \text{dB}$$

D_{psat}: polarization decoupling (dB) of the interfering satellite in the coverage area where the wanted station is located.

4.2 Calculation of the polarization discrimination, Y_u, in the up-link

The purpose of this calculation is to determine in a similar way to the preceding section, for a receive antenna of the wanted satellite, the discrimination Y_u with respect to an interfering wave.

$$Y_u = -10 \log \left(\cos^2 \beta + \sin^2 \beta \cdot 10^{-D_p(\psi_b)/10} + \sin^2 \beta \cdot 10^{-D_{pst}/10}\right) \quad \text{dB} \qquad (2)$$

where:

ψ_b: angle between the main radiation direction and the direction of interfering earth station

$D_p(\psi_b)$: polarization decoupling of the wanted satellite:

$$D_p(\psi_b) = S_{//}(\psi_b) - S_{+}(\psi_b) \quad \text{dB}$$

$S_{//}$ and S_{+}: co-polarized and cross-polarized patterns of the wanted satellite antenna

D_{pst}: polarization decoupling (dB) of the interfering earth station.

5 Calculation of polarization discrimination, *Y*, in the case of two polarizations: one circular, the other linear

In the case of an interfering wave in linear polarization (the linear polarization vector can be derived from two circular polarization vectors, right- and left-hand rotation), the discrimination obtained at the receive wanted antenna operating in circular polarization can be expressed in a way similar to the above:

$$Y = -10 \log \frac{1}{2} \left(1 + 10^{-D_p(\varphi)/10}\right) \quad \text{dB} \qquad (3)$$

where:

$D_p(\varphi)$: polarization decoupling of the receive antenna (dB).

Similarly, in the case of an interfering wave in circular polarization (the circular polarization vector can be composed of two orthogonal linear polarization vectors), the discrimination obtained at the receive wanted antenna operating in linear polarization is described by the same formula.

6 Application to actual cases

From the above relationships, the calculation of the polarization discrimination in various cases permits estimation of the improvement obtained in interference calculations. In particular the use of a polarization orthogonal to the one of the interference, may be considered. These relationships do not take into account the effects of propagation conditions on the signal polarization plane; these are included in Appendix 1.

7 Orientation of polarization planes

It is possible to orient the polarization plane of a receive (linearly polarized) satellite or earth-station antenna for two different optimum conditions, one to minimize clear-sky interference from an orthogonally polarized signal and the other to minimize the effects of depolarization due to rain. The means of calculating the optimum alignments in the two cases are described in Appendix 1.

8 Depolarization due to rain (see Appendix 3)

After atmospheric propagation, a wave in a given polarization (linear or circular) does not keep this polarization, because of the Faraday effect by the ionosphere and the cross-polarization effect by the troposphere. The area concerned is the radioelectric area from about some tens of megahertz to 50 GHz. Cross-polarization is the fact that part of the energy transmitted in a polarization is found in an orthogonal polarization after propagation.

The troposphere (non-ionized atmosphere) goes from the ground to about 15 km altitude and may create a cross-polarization of waves in the following cases:

– clear air effect (reflection phenomenon): the effect is small on oblique links;

– ice crystals effect: this effect becomes evident on satellite links and is due to the refraction of waves in altitude on the ice crystals, partially changed into liquid water (isotherm 0 °C). This cross-polarization can be very constricting because there is no attenuation;

– precipitation effect: the drops of rain can create, because of their non-spherical shape, a wave in the orthogonal polarization.

The effects of rain and ice crystals are dependent on the climate. The effects of drops of rain are studied for the satellite links in Appendix 3.

APPENDIX 1

TO ANNEX 1

Optimization of polarization orientation

1 Simple model of a satellite-to-Earth link

Depolarization effects on satellite-to-Earth (or Earth-to-satellite) links have been extensively analysed and mathematical procedures which take into account all known contributions to depolarization have been formulated. These procedures are, however, extremely complex and cumbersome. A simplified model, suitable for use in a first-order analysis of interference has therefore been devised which uses the concept of equivalent gain of a partial link (i.e. either the Earth-to-space or the space-to-Earth connection).

The equivalent gain (as a power ratio) for one partial link can be represented by the following approximation:

$$G = G_1 \cdot \cos^2 \beta + G_2 \cdot \sin^2 \beta$$

$$G_1 = G_{tp} \cdot G_{rp} \cdot A + G_{tc} \cdot G_{rc} \cdot A + G_{tp} \cdot G_{rc} \cdot A \cdot X + G_{tc} \cdot G_{rp} \cdot A \cdot X$$

$$G_2 = \left(\sqrt{G_{tc} \cdot G_{rc} \cdot A} + \sqrt{G_{tc} \cdot G_{rp} \cdot A}\right)^2 + G_{tp} \cdot G_{rp} \cdot A \cdot X + G_{tc} \cdot G_{rc} \cdot A \cdot X \qquad (4)$$

where:

β: relative alignment angle, for linear polarization, between the received signal polarization plane and the plane of polarization of the receive antenna

G_{tp}: co-polar gain characteristic of the transmit antenna expressed as a power ratio (Recommendations ITU-R S.465, ITU-R S.580 for earth stations and Recommendation ITU-R S.672 for satellites)

G_{tc}: cross-polar gain characteristic of the transmit antenna expressed as a power ratio (Recommendation ITU-R S.731 for earth stations)

G_{rp}: co-polar gain characteristic of the receive antenna expressed as a power ratio (Recommendations ITU-R S.465, ITU-R S.580 and ITU-R S.672)

G_{rc}: cross-polar gain characteristics of the receive antenna expressed as a power ratio (Recommendation ITU-R S.731 for earth stations)

A: rain fade as a power ratio ≤ 1

X: rain depolarization as a power ratio $<< 1$.

The following sections give derivations for the angle, β, for the two scenarios:

– polarization aligned to minimize effects of rain fade, and

– polarization aligned to minimize interference.

Using the equivalent gain concept, the wanted carrier power, C, or the single-entry interfering power, I, on each partial link is simply given by:

$$C \text{ (or } I) = P_T - L_{FS} - L_{CA} + 10 \log G \qquad \text{dBW} \tag{5}$$

where:

P_T: wanted (interfering) transmitting antenna power (dBW)

L_{FS}: free-space loss on the wanted (interfering) link (dB)

L_{CA}: clear-air absorption on the wanted (interfering) link (dB)

G: equivalent gain on the wanted (interfering) link (dBi).

2 Polarization orientation to minimize interference due to rain depolarization

As illustrated in Fig. 1, the apparent polarization plane at the Earth's surface is a function of the geographical coordinates of the boresight, the test point under consideration, and the satellite. The following formula for this polarization angle ε can be used:

$$\tan \varepsilon = \frac{\sin \psi_b \cdot \cos \psi_p \cdot \sin(\lambda_p - \lambda_s) - \cos \psi_b \cdot \sin \psi_p \cdot \sin(\lambda_b - \lambda_s)}{\sin \psi_b \cdot \sin \psi_p + \cos \psi_b \cdot \cos \psi_p \cdot \sin(\lambda_b - \lambda_s) \cdot \sin(\lambda_p - \lambda_s)} \tag{6}$$

where:

ψ: latitude

λ: longitude

b: boresight

p: test point

s: satellite.

In the above, it is assumed that the polarization plane is optimized for minimum rain fade at the boresight, i.e. aligning with either the local horizontal or the local vertical. In functional form, equation (6) may be rewritten:

$$\varepsilon = f(\psi_b, \lambda_b, \psi_p, \lambda_p, \lambda_s)$$

To determine the interference component at test point, p, the difference in polarization angles is required.

Thus, if the wanted and interfering signals are "co-polar", the angle, β, in equation (4) may be expressed as:

$$\beta = |\varepsilon_1 - \varepsilon_2| + \delta \tag{7}$$

where:

$\varepsilon_1 = f(\psi_{b_1}, \lambda_{b_1}, \psi_p, \lambda_p, \lambda_{s_1})$

$\varepsilon_2 = f(\psi_{b_2}, \lambda_{b_2}, \psi_p, \lambda_p, \lambda_{s_2})$

b_1: boresight of wanted satellite s_1

b_2: boresight of interfering satellite s_2

δ: allowance for mis-alignment of earth-station antenna and rotational tolerances of satellite beams.

If the wanted and interfering signals are "cross-polar" the angle β_χ in the worst case, is:

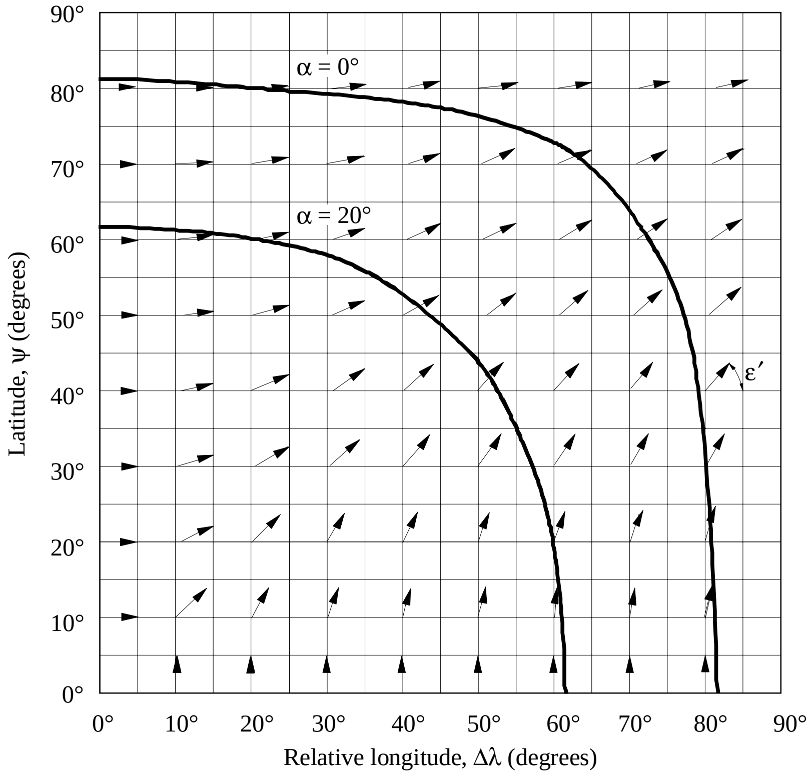
$$\beta_\chi = \frac{\pi}{2} - |\varepsilon_1 - \varepsilon_2| - \delta \tag{8}$$

Under some circumstances the distinction between co-polar and cross-polar may be academic. Thus for the purposes of this analysis the following definition will be assumed:

– if both wanted signals are aligned to the respective local horizontals or both are aligned to the respective local verticals, they will be considered co-polar.

FIGURE 1

Variation of received angle of polarization on the Earth



α: angle of elevation 0736-01

3 Polarization orientation to minimize clear-sky interference

Minimum interference occurs when the polarization planes at the satellite orbit are orthogonal, i.e. when the polarization planes are either in the equatorial plane or in the plane of the Earth's North-South axis. The following formula for the angle of polarization, ε' , can be used when the polarization vector of the transmitted wave is parallel to the equatorial plane:

$$\tan \varepsilon' = \frac{\sin(\lambda_p - \lambda_s)}{\mathrm{tg}\,\psi} \sqrt{1 + \left(\frac{\alpha' \sin \xi}{1 - \alpha' \cos \xi}\right)^2} \tag{9}$$

$$\cos \xi = \cos(\lambda_p - \lambda_s) \cdot \cos \psi_p$$

α': radius of earth divided by radius of orbit $\sim$ 0.151.

Thus, in functional form:

$$\varepsilon' = g\left(\psi_p, \lambda_p, \lambda_s\right)$$

When the wanted and interfering signals are "co-polar", i.e. when both have polarization planes parallel to the equatorial plane or when both are perpendicular to the equatorial plane, the relative angle, β, between wanted and interfering polarization planes is given by:

$$\beta = \left|\varepsilon_1' - \varepsilon_2'\right| + \delta \tag{10}$$

where:

$\varepsilon_1' = g\left(\psi_p, \lambda_p, \lambda_{s_1}\right)$

$\varepsilon_2' = g\left(\psi_p, \lambda_p, \lambda_{s_2}\right)$

s_1 : wanted satellite

s_2 : interfering satellite.

Similarly when the two signals are cross-polar, the relative angle β_χ is given by:

$$\beta_\chi = \frac{\pi}{2} - \left|\varepsilon_1' - \varepsilon_2'\right| - \delta \tag{11}$$

The variation of ε′ is given in Fig. 1.

APPENDIX 2

TO ANNEX 1

Calculation of polarization angle and relative alignment angle

1 Calculation of polarization angle ε of a wave in linear polarization

1.1 Definition of orthonormal coordinate systems (see Fig. 2)

The orthonormal coordinate systems used are the following:

– *the base Earth-centre coordinate system* $\boldsymbol{R_g}$: $(\vec{X}_g, \vec{Y}_g, \vec{Z}_g)$: $\vec{Z}_g$ is directed to the North and $\vec{X}_g$ in the equatorial plane is determined by the direction $\overrightarrow{GS}$, G is the Earth centre and the origin of the system and S the satellite;

– *the coordinate system* $\boldsymbol{R_s}$ *connected with the satellite S*: $(\vec{X}_s, \vec{Y}_s, \vec{Z}_s)$: $\vec{Z}_s = -\vec{X}_g$ and $\vec{Y}_s$ is directed to the East in the equatorial plane;

– *the coordinate system* $\boldsymbol{R_a}$ *connected with the antenna A of satellite S*: $(\vec{X}_a, \vec{Y}_a, \vec{Z}_a)$: $\vec{Z}_a$ is directed to boresight Pv and $\vec{Y}_a$ is directed to the East in the equatorial plane;

and the system $\boldsymbol{R_a(\gamma)}$ *linked with the polarization vector of the antenna A of satellite S*: $(\vec{X}_a(\gamma), \vec{Y}_a(\gamma), \vec{Z}_a)$: the transformation from coordinate system $\boldsymbol{R_a}$ into coordinate system $\boldsymbol{R_a(\gamma)}$ is made by a rotation $(-\gamma)$ around $\vec{Z}_a$. That axis $\vec{Y}_a(\gamma)$ is located in the plane of symmetry of the reflector of antenna A and defines the direction of the linear main polarization of antenna A for a propagation direction $\vec{Z}_a$ i.e. towards the boresight Pv of the antenna A of satellite S;

– *the coordinate system $\boldsymbol{R_p}$ connected with earth station P and satellite S on which is connected the earth station:* $(\vec{X}_p, \vec{Y}_p, \vec{Z}_p)$: $\vec{Z}_p$ is directed to satellite S and $\vec{X}_p$ is directed to the left of an observer who, placed at station P, faces at satellite S: direction $\vec{X}_p$ is $\vec{Vert}(P) \wedge \vec{Z}_p$;

$\vec{Vert}(P) = \vec{GP}$ gives the direction of the local vertical on the Earth at earth station P

$\vec{Vert}(P) = \left[R_t \cos\psi_p \cos(\lambda_p - \lambda_s);\ R_t \cos\psi_p \sin(\lambda_p - \lambda_s);\ R_t \sin\psi_p\right]$ in system $\boldsymbol{R_g}$

hence:

$$\vec{X}_p = (\vec{Vert}(P) \wedge \vec{Z}_p) \Big/ \left\|\vec{Vert}(P) \wedge \vec{Z}_p\right\| = (\vec{GP} \wedge \vec{PS}) \Big/ \left\|\vec{GP} \wedge \vec{PS}\right\|$$

$$\vec{Y}_p = \vec{Z}_p \wedge \vec{X}_p$$

$$\vec{Z}_p = \vec{PS} \Big/ \left\|\vec{PS}\right\| \text{ with } \vec{PS} = h\,\vec{X}_g - \vec{GP}$$

Parameters and notations are as follows:

index 1: wanted satellite S_1 and earth station P_1

index 2: interfering satellite S_2 and earth station P_2

index d: down-link

index m: up-link

Pv: boresight of the antenna A of satellite S

P: any point on the Earth

R_t: radius of the Earth: 6 378 km

h: radius of the geostationary-satellite orbit (GSO): 42 164 km

k: GSO radius divided by radius of the Earth
$= 6.62 = h / R_t$

ψ_p: latitude of earth station P

λ_p: longitude of earth station P

ψ_{pv}: latitude of boresight Pv

λ_{pv}: longitude of boresight Pv

λ_s: longitude of satellite S

θ_p: geocentric angle $S\hat{G}P$: angle at the centre of the Earth between the sub-satellite point and earth station P

γ: inclination angle of the ellipse of the satellite antenna coverage pattern: angle between the equatorial plane and the polarization vector (or direction of the ellipse major axis), in the plane perpendicular to the main radio axis, and is chosen to be positive for the trigonometric direction from the equator towards the ellipse major axis

δ: allowance for mis-alignment of earth station antenna and rotational tolerance of satellite beams

$(\psi, \theta, (\varphi - \gamma))$: Euler angles appearing in the transformation matrix M from system $\boldsymbol{R_s}$ to system $\boldsymbol{R_a}(\gamma)$

^: cross product

$\|\vec{Z}\|$: magnitude of vector $\vec{Z}$.

1.1.1 Vectors

1.1.1.1 The direction of the local vertical of an earth station at any point P is expressed in the base earth-centre coordinate system $\boldsymbol{R_g}$ by:

$$\vec{Vert}(P) = \left[R_t \cos\psi_p \cos(\lambda_p - \lambda_s); R_t \cos\psi_p \sin(\lambda_p - \lambda_s);\ R_t \sin\psi_p\right]$$

1.1.1.2 The unit vectors coordinates of the orthonormal system $\boldsymbol{R_p}$ of earth station P are expressed in system $\boldsymbol{R_g}$ as follows:

$$\vec{Z_p} = \left[h - R_t \cos\psi_p \cos\left(\lambda_p - \lambda_s\right);\ -R_t \cos\psi_p \sin\left(\lambda_p - \lambda_s\right); -R_t \sin\psi_p\right] \Big/ \left\| norm\,\vec{Z_p} \right\|$$

with:

$$\left\| norm\,\vec{Z_p} \right\| = \left(h^2 + R_t^2 - 2h\ R_t \cos\Theta_p\right)^{1/2}$$

$$\vec{X_p} = \left[0;\ R_t \sin\psi_p;\ -R_t \cos\psi_p \sin\left(\lambda_p - \lambda_s\right)\right] \Big/ \left\| norm\,\vec{X_p} \right\|$$

with:

$$\left\| norm\,\vec{X_p} \right\| = \left| R_t \sin\Theta_p \right|$$

$$\vec{Y_p} = \left[R_t^2;\ R_t \left(k - \cos\Theta_p\right) \cos\psi_p \sin\left(\lambda_p - \lambda_s\right);\ R_t^2 \left(k - \cos\psi_p\right) \sin\psi_p\right] \Big/ \left\| norm\,\vec{Y_p} \right\|$$

with:

$$\left\| norm\,\vec{Y_p} \right\| = R_t^2 \left|\sin\Theta_p\right| \left(\sin^2\Theta_p + (k - \cos\Theta_p)^2\right)^{1/2}$$

with:

$$\cos\Theta_p = \cos\psi_p \cos\left(\lambda_p - \lambda_s\right)$$

1.1.1.3 Direction satellite-station $\overrightarrow{SP}$ is represented by vector $-\vec{Z_p}$ and its coordinates are expressed as follows:

– in base earth-centre coordinate system $\boldsymbol{R_g}$:

$$\left(-\vec{Z_p}\right) = \left[X_g;\ Y_g;\ Z_g\right]$$

$$\left(-\vec{Z_p}\right) = \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} -\left(h - R_t \cos\psi_p \cos(\lambda_p - \lambda_s)\right) \\ R_t \cos\psi_p \sin(\lambda_p - \lambda_s) \\ R_t \sin\psi_p \end{bmatrix} \Big/ \left(h^2 + R_t^2 - 2h\ R_t \cos\Theta_p\right)^{1/2}$$

– in satellite system $\boldsymbol{R_s}$:

$$\left(-\vec{Z_p}\right) = \left[X_s;\ Y_s;\ Z_s\right] = \left[Z_g;\ Y_g;\ -X_g\right]$$

$$\left(-\vec{Z_p}\right) = \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} Z_g \\ Y_g \\ -X_g \end{bmatrix} = T\,(3 \times 3) \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \ldots \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix}$$

with $T\,(3 \times 3)$ the transformation matrix between systems $\boldsymbol{R_g}$ and $\boldsymbol{R_s}$

– in antenna system $\boldsymbol{R_a}(\gamma)$:

$$\left(-\vec{Z_p}\right) = \left[X_a(\gamma);\ Y_a(\gamma);\ Z_a\right]$$

$$\left(-\vec{Z_p}\right) = \begin{bmatrix} X_a(\gamma) \\ Y_a(\gamma) \\ Z_a \end{bmatrix} = {}^tM\,(3 \times 3) \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

with $M\,(3 \times 3)$ the Euler transformation matrix between systems $\boldsymbol{R_a}(\gamma)$ and $\boldsymbol{R_s}$.

1.2 Particular case: the earth station is at the boresight: P = Pv

1.2.1 Definition of main polarization of a linearly polarized wave transmitted from a satellite antenna A

In this particular case, the planes of the antenna of earth station Pv ($\vec{X}_{pv}$, $\vec{Y}_{pv}$) and of the satellite antenna A ($\vec{X}_a$, $\vec{Y}_a$) are parallel, polarization vector $\vec{V}_{pol}$ is parallel to the plane of GSO and is directed towards the East, i.e. normal to $\vec{Z}_a$ and $\vec{Z}_g$ therefore:

$$\vec{Z}_a = -\vec{Z}_{pv} = \left(h\,\vec{X}_g + \vec{Vert}(Pv)\right) / \left\|h\,\vec{X}_g + \vec{Vert}(Pv)\right\|$$

with:

$$\left\|-h\,\vec{X}_g + \vec{Vert}(Pv)\right\| = \left(h^2 + R_t^2 - 2h\,R_t\ \cos\Theta_{pv}\right)^{1/2}$$

$$\vec{V}_{pol} = \vec{Z}_a \wedge \vec{Z}_g = \left(1 / \left\|\vec{Z}_a \wedge \vec{Z}_g\right\|\right) \cdot \left[R_t\ \cos\psi_{pv} - \sin(\lambda_{pv} - \lambda_s);\ h - R_t\ \cos\Theta_{pv};\ 0\right]$$

in system $\boldsymbol{R_g}$

with:

$$\cos\Theta_{pv} = \cos\left(\lambda_{pv} - \lambda_s\right)\ \cos\psi_{pv}$$

1.2.2 Approximate definition of polarization angle in the system $\boldsymbol{R_{pv}}$ of a receiving earth station at boresight Pv: determination of polarization angle of a linearly polarized wave transmitted from a satellite antenna A

The approximate polarization angle is defined in system $\boldsymbol{R_{pv}}$ by:

$$\begin{aligned}\varepsilon &= \text{arc tan}\left[\left(\vec{V}_{pol} \cdot \vec{Y}_{pv}\right) / \left(\vec{V}_{pol} \cdot \vec{X}_{pv}\right)\right] \\ &= \text{arc tan}\left[\left(\sin(\lambda_{pv} - \lambda_s) / \tan\psi_{pv}\right)\left(1 + \sin^2\Theta_{pv} / (k - \cos\Theta_{pv})^2\right)^{1/2}\right]\end{aligned} \qquad (12a)$$

Vectors $\vec{V}_{pol}$, $\vec{X}_{pv}$ and $\vec{Y}_{pv}$ must be expressed in the same system.

Taking into account the angle γ characterizing the polarization of the transmitting satellite antenna, the value of the polarization angle may be expressed as:

$$\varepsilon(\gamma) = \varepsilon + \gamma \qquad (12b)$$

1.3 General case: the earth station is distinct from the boresight: P ≠ Pv

1.3.1 Definition of main polarization and cross-polarization of a linearly polarized wave transmitted from a satellite antenna A (see Fig. 2a)

The electric field $\vec{E}(P)$ is defined in the antenna system $\boldsymbol{R_a}$ of satellite S. The most used definition is the third definition of Arthur C. Ludwig for a wave transmitted by an antenna. The main polarization and cross-polarization components $E_p(P)$ and $E_c(P)$ at any point P are expressed as follows:

$$E_p(P) = \vec{E}(P) \cdot \left(\vec{e}^{\,=}\right) : \text{main or co-polar polarization}$$

$$E_c(P) = \vec{E}(P) \cdot \left(\vec{e}^{\,+}\right) : \text{cross-polarization}$$

1.3.1.1 The definition used is the third definition of Arthur C. Ludwig: The expressions of vectors $(\vec{e}^{\,=})$ and $(\vec{e}^{\,+})$ of linear co- and cross-polarization, in any point P, for a wave transmitted from the antenna A of satellite S are the following in the case of a linearly polarized field parallel to $\vec{Y_a}$:

$$\left(\vec{e}^{\,=}\right) = \sin\varphi_a \cdot \vec{e}_{\theta a} + \cos\varphi_a \cdot \vec{e}_{\varphi a} \text{ in } \boldsymbol{R_a}\text{: polarization vector}$$

$$\left(\vec{e}^{\,+}\right) = \cos\varphi_a \cdot \vec{e}_{\theta a} - \sin\varphi_a \cdot \vec{e}_{\varphi a} \qquad \text{: cross-polarization vector}$$

with:

– $(\vec{e}_{\theta a}$ and $\vec{e}_{\varphi a})$: usual unit vectors of spherical coordinates in antenna system $\boldsymbol{R_a}$:

$$\left(\vec{e}_{\theta a}\right) = \left[\cos\theta_a\ \cos\varphi_a;\ \cos\theta_a\ \sin\varphi_a;\ -\sin\theta_a\right] \text{ in } \boldsymbol{R_a}$$

$$\left(\vec{e}_{\varphi a}\right) = \left[-\sin\varphi_a;\ \cos\varphi_a;\ 0\right] \text{ in } \boldsymbol{R_a}$$

– (θ_a and φ_a): angles for determining direction $\overrightarrow{SP}$ or vector $-\vec{Z_p}$ (satellite S – any earth station P) in antenna system $\boldsymbol{R_a}$:

$$\left[\theta_a;\ \varphi_a\right] = \left[\text{arc}\cos Z_a;\ \text{arc}\tan\left(Y_a / X_a\right)\right] \text{ in } \boldsymbol{R_a}$$

X_a, Y_a and Z_a are the components of vector $-\vec{Z}_p$ in antenna system $\boldsymbol{R_a}$

therefore:

$$\begin{matrix}\left(-\vec{e}^{\,=}\right) = \\ \text{in } \boldsymbol{R_a}\end{matrix} \begin{bmatrix} \cos\varphi_a\ \sin\varphi_a\ (\cos\theta_a - 1) \\ \sin^2\varphi_a\ \cos\theta_a + \cos^2\varphi_a \\ -\sin\varphi_a\ \sin\theta_a \end{bmatrix} = \begin{bmatrix} \cos\varphi_a\ \sin\varphi_a\ \cos\theta_a - \sin\varphi_a\ \cos\varphi_a \\ \sin\varphi_a\ \sin\varphi_a\ \cos\theta_a + \cos\varphi_a\ \cos\varphi_a \\ -\ \sin\varphi_a\ \sin\theta_a \end{bmatrix}$$

1.3.1.2 In the case of an inclination γ between the equatorial plane and the ellipse major axis of the antenna radiation pattern of satellite S in the plane perpendicular to radio axis (axis $\vec{Z_a}$ or direction $\overrightarrow{SPv}$), the definition of main polarization vector stays similar but must therefore get its direction in comparison with the direction of linearly polarized field parallel to $\vec{Y}_{\boldsymbol{a}}(\gamma)$ of antenna system $\boldsymbol{R_a}(\gamma)$ and not as $\vec{Y_a}$ of antenna system $\boldsymbol{R_a}$.

The definition of new polarization vector $\left(\vec{e}^{\,=}(\gamma)\right)$ is deducted from a rotation $(-\gamma)$ around $\vec{Z_a}$:

$$\left(\vec{e}^{\,=}(\gamma)\right)_{Ra} = \sin(\varphi_a + \gamma) \cdot \vec{e}_{\theta a} + \cos(\varphi_a + \gamma) \cdot \vec{e}_{\varphi a}$$

$$\left(\vec{e}^{\,=}(\gamma)\right)_{Ra} = M_{za}(-\gamma) \cdot \left(\vec{e}^{\,=}(\gamma)\right)_{Ra(\gamma)}$$

with:

– $(\vec{e}_{\theta a}$ and $\vec{e}_{\varphi a})$: usual unit vectors of spherical coordinates which are allowed to be also expressed in antenna system $\boldsymbol{R_a}(\gamma)$:

$$\vec{e}_{\theta a} = \left[\cos\theta_a\ \cos(\varphi_a + \gamma);\ \cos\theta_a\ \sin(\varphi_a + \gamma);\ -\sin\theta_a\right] \text{ in } \boldsymbol{R_a}(\gamma)$$

$$\vec{e}_{\varphi a} = \left[-\sin(\varphi_a + \gamma);\ \cos(\varphi_a + \gamma);\ 0\right] \text{ in } \boldsymbol{R_a}(\gamma)$$

– $(\theta_a$ and $\varphi_a)$: angles for determining direction $\overrightarrow{SP}$ (satellite S – any earth station P) in antenna system $\boldsymbol{R_a}$, therefore:

$$\underset{\text{in } \boldsymbol{R_a}}{\left(\vec{e}^{\,=}(\gamma)\right)} = \begin{bmatrix} \cos\theta_a \ \cos\varphi_a \ \sin(\varphi_a+\gamma) \ - \ \sin\varphi_a \ \cos(\varphi_a+\gamma) \\ \cos\theta_a \ \sin\varphi_a \ \sin(\varphi_a+\gamma) \ + \ \cos\varphi_a \ \cos(\varphi_a+\gamma) \\ -\sin\theta_a \ \sin(\varphi_a+\gamma) \end{bmatrix}$$

and

$$\underset{\text{in } \boldsymbol{R_a}(\gamma)}{\left(\vec{e}^{\,=}(\gamma)\right)} = \begin{bmatrix} \cos\theta_a \ \cos(\varphi_a+\gamma) \ \sin(\varphi_a+\gamma) \ - \ \sin(\varphi_a+\gamma) \ \cos(\varphi_a+\gamma) \\ \cos\theta_a \ \sin(\varphi_a+\gamma) \ \sin(\varphi_a+\gamma) \ + \ \cos(\varphi_a+\gamma) \ \cos(\varphi_a+\gamma) \\ -\sin\theta_a \ \sin(\varphi_a+\gamma) \end{bmatrix}$$

– Rotation matrix $M_{za}(-\gamma)$ with rotation $-\gamma$ around axis $\vec{Z}_a$ of system $\boldsymbol{R_a}$ (direction $\overrightarrow{SPv}$) which allows to go from system $\boldsymbol{R_a}(\gamma)$ to system $\boldsymbol{R_a}$ and is expressed as:

$$M_{za}(-\gamma)\ (3\times 3) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} = M_{za}(-\gamma)$$

This rotation matrix $M_{za}(-\gamma)$ expresses the change of the direction of the polarization vector of the antenna of satellite S characterized by angle γ, inclination angle between the equatorial plane and the ellipse major axis (elliptic radiation pattern) in the plane perpendicular to the main radio axis, which is chosen to be positive for the trigonometric direction from the equator towards the ellipse major axis.

Vector $\vec{Z}_a = (-\vec{Z}_{pv})$ of system $\boldsymbol{R_a}$ is expressed in the base system $\boldsymbol{R_g}$ as follows:

$$\vec{Z}_a = \left[\ -\left(h - R_t \ \cos\psi_{pv} \ \cos(\lambda_{pv}-\lambda_s)\right);\ R_t \ \cos\psi_{pv} \ \sin(\lambda_{pv}-\lambda_s);\ R_t \ \sin\psi_{pv}\right] \Big/ \left(h^2 + R_t^2 - 2h \ R_t \ \cos\psi_{pv}\right)^{1/2}$$

with:

X_a, Y_a and Z_a: components of vector $(-\vec{Z}_p)$ in antenna system $\boldsymbol{R_a}$

X_g, Y_g and Z_g: components of vector $(-\vec{Z}_p)$ in earth base system $\boldsymbol{R_g}$

$X_s = Z_g$, $Y_s = Y_g$ and $Z_s = -X_g$: components of vector $(-\vec{Z}_p)$ in satellite system $\boldsymbol{R_s}$

1.3.1.3 The Euler transformation matrix M (3×3) from system $\boldsymbol{R_s}$ into system $\boldsymbol{R_a}(\gamma)$ takes into account the Euler angles (ψ, θ and $(\varphi - \gamma)$) defined as follows:

– rotation ψ around $\vec{Z}_s$: transformation from system $\boldsymbol{R_s}$ into intermediate system $\boldsymbol{R_1}$: rotation matrix $M_{zs}(\psi)$

$(\vec{X}_1,\ \vec{Y}_1,\ \vec{Z}_1) = (\vec{X}_1,\ \vec{Y}_1,\ \vec{Z}_s)$ with: $(\vec{X}_s,\ \vec{Y}_s,\ \vec{Z}_s) = M_{zs}(\psi) \cdot (\vec{X}_1,\ \vec{Y}_1,\ \vec{Z}_1)$ and $\vec{Z}_1 = \vec{Z}_s$

– rotation θ around $\vec{X}_1$: transformation from system $\boldsymbol{R_1}$ into intermediate system $\boldsymbol{R_2}$: rotation matrix $M_{x1}(\theta)$

$(\vec{X}_2,\ \vec{Y}_2,\ \vec{Z}_2) = (\vec{X}_1,\ \vec{Y}_1,\ \vec{Z}_a)$ with: $(\vec{X}_1,\ \vec{Y}_1,\ \vec{Z}_1) = M_{x1}(\theta) \cdot (\vec{X}_2,\ \vec{Y}_2,\ \vec{Z}_2)$ and $\vec{X}_2 = \vec{X}_1$ and $\vec{Z}_2 = \vec{Z}_a$

– rotation $(\varphi - \gamma)$ around $\vec{Z}_2$: transformation from system $\boldsymbol{R_2}$ into system $\boldsymbol{R_a}(\gamma)$: rotation matrix $M_{za}(\varphi - \gamma)$

$(\vec{X}_a(\gamma),\ \vec{Y}_a(\gamma),\ \vec{Z}_a)$ with: $(\vec{X}_2,\ \vec{Y}_2,\ \vec{Z}_2) = M_{za}(\varphi-\gamma) \cdot \vec{X}_a(\gamma), (\vec{Y}_a(\gamma),\ \vec{Z}_a)$ and $\vec{Z}_a(\gamma) = \vec{Z}_a$

with:

$$\theta = -\text{arc}\cos\left(Z_{pvs} \ / \ Z_{pva}\right)$$

$$\psi = -\text{sign}\left(\psi_{pv}\right) \cdot \left|\text{arc}\cos\left[Y_{pvs} \ / \ \left(\sin\theta \cdot Z_{pva}\right)\right]\right| \text{ and}$$

$$\tan\varphi = -\tan\psi \ \cos\theta$$

X_{pva}, Y_{pva} and Z_{pva}: coordinates of boresight Pv in system $\boldsymbol{R_a}$

X_{pvs}, Y_{pvs} and Z_{pvs}: coordinates of boresight Pv in system $\boldsymbol{R_s}$

with:

$$Z_{pvs} = h - R_t \cos \psi_{pv} \cos \left(\lambda_{pv} - \lambda_s\right)$$

$$Z_{pva} = \left[h^2 + R_t^2 - 2h\ R_t \cos \psi_{pv} \cos \left(\lambda_{pv} - \lambda_s\right)\right]^{1/2} \text{ and}$$

$$Y_{pvs} = R_t \sin \left(\lambda_{pv} - \lambda_s\right) \cos \psi_{pv}$$

Euler transformation matrix M (3 × 3) is defined as follows:

$$M\,(3 \times 3) = \begin{bmatrix} \cos\psi\cos(\varphi-\gamma) - \sin\psi\cos\theta\sin(\varphi-\gamma) & -\cos\psi\sin(\varphi-\gamma) - \sin\psi\cos\theta\cos(\varphi-\gamma) & \sin\theta\sin\psi \\ \sin\psi\cos(\varphi-\gamma) + \cos\psi\cos\theta\sin(\varphi-\gamma) & -\sin\psi\sin(\varphi-\gamma) - \cos\psi\cos\theta\cos(\varphi-\gamma) & -\sin\theta\cos\psi \\ \sin\theta\sin(\varphi-\gamma) & \sin\theta\cos(\varphi-\gamma) & \cos\theta \end{bmatrix}$$

because $M\,(3 \times 3) = M_{zs}(\psi)\,(3 \times 3) \cdot M_{x1}(\theta)\,(3 \times 3) \cdot M_{za}(\varphi)\,(3 \times 3) \cdot M_{za}(-\gamma)\,(3 \times 3)$

$= M_{zs}(\psi)\,(3 \times 3) \cdot M_{x1}(\theta)\,(3 \times 3) \cdot M_{za}(\varphi - \gamma)\,(3 \times 3)$

$$M\,(3 \times 3) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M\,(3 \times 3) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos(\varphi-\gamma) & -\sin(\varphi-\gamma) & 0 \\ \sin(\varphi-\gamma) & \cos(\varphi-\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

because

$$\vec{V}_{Rs} = M\,(3 \times 3) \cdot \vec{V}_{Ra(\gamma)} \text{ and } \vec{V}_{Ra} = M_{za}\,(-\gamma)\,(3 \times 3) \cdot \vec{V}_{Ra(\gamma)}$$

with vector $\vec{V}$ expressed in satellite and antenna systems $\boldsymbol{R_s}$, $\boldsymbol{R_a}$ and $\boldsymbol{R_a}(\gamma)$.

The rotation $(-\gamma)$ due to the inclination angle γ of the elliptic coverage of the beam of satellite S is therefore taken into account Euler transformation matrix M (3 × 3) which allows to go from antenna system $\boldsymbol{R_a}(\gamma)$ into satellite system $\boldsymbol{R_s}$.

1.3.2 Precise definition of polarization angle in the system $\boldsymbol{R_p}$ of a receiving earth station at any point P: determination of the polarization angle of a linearly polarized wave transmitted from a satellite antenna A

Therefore in the case of linear polarization parallel to $\vec{Y}_a$, the components of main polarization vector $\left(\vec{e}^{\,=}\right)$ are expressed in system $\boldsymbol{R_a}$ by:

$$\left(\vec{e}^{\,=}\right) = \left[\cos\varphi_a \sin\varphi_a\ (\cos\theta_a - 1);\ \sin^2\varphi_a \cos\theta_a + \cos^2\varphi_a;\ -\sin\varphi_a \sin\theta_a\right]$$

The polarization angle is defined in system $\boldsymbol{R_p}$ by:

$$\varepsilon = \text{arc tan}\left(\left[\left(\vec{e}^{\,=}\right) \cdot \vec{Y}_p\right] / \left[\left(\vec{e}^{\,=}\right) \cdot \vec{X}_p\right]\right) \tag{13a}$$

Vectors $\left(\vec{e}^{\,=}\right)$, $\vec{X}_p$ and $\vec{Y}_p$ must be expressed in the same coordinate system.

Taking into account the angle, γ, characterizing the polarization vector $\left(\vec{e}^{\,=}(\gamma)\right)$ of the transmitting satellite antenna, the value of the polarization angle may be expressed as:

$$\varepsilon(\gamma) = \text{arc tan}\left(\left[\left(\vec{e}^{\,=}(\gamma)\right) \cdot \vec{Y}_p\right] \;/\; \left[\left(\vec{e}^{\,=}(\gamma)\right) \cdot \vec{X}_p\right]\right) \tag{13b}$$

Vectors $\left(\vec{e}^{\,=}(\gamma)\right)$, $\vec{X}_p$ and $\vec{Y}_p$ must be expressed in the same coordinate system.

2 Calculation of relative alignment angle β for the linear polarization

2.1 Down-link case: reception by a wanted earth station P_1 (see Fig. 3)

Calculation, for an earth station P_1, of the alignment angle between the signals coming from a wanted satellite S_1 and an interfering satellite S_2

2.1.1 Determination of linear polarization vector of a wave transmitted off-axis from the antenna A_1 of satellite S_1 to wanted earth station P_1:

General case: the earth station is distinct from the boresight: $P_1 \neq Pv_1$

Earth station P_1 is pointed to satellite S_1 (characterized by its boresight Pv_1 and its inclination angle γ_1 of its elliptic transmitting beam)

Hypothesis of calculation: the on-axis receive linear polarization vector of wanted earth station P_1 is matched to the off-axis transmit linear polarization vector $\left[\vec{e}_{1\downarrow}^{\,=}(\gamma_1)\right]$ of the wave transmitted by the antenna A_1 of wanted satellite S_1.

2.1.1.1 Calculation of $\vec{X}_{p1}$, $\vec{Y}_{p1}$, $\vec{Z}_{p1}$ in base earth centre coordinate system $\boldsymbol{R_{g1}}$:

The coordinates of the unit vectors of the orthonormal base of system $\boldsymbol{R_{p1}}$ of earth station P_1 are expressed in system $\boldsymbol{R_{g1}}$ as follows:

$$\vec{Z}_{p1} = \left[h - R_t \cos\psi_{p1} \cos\left(\lambda_{p1} - \lambda_{s1}\right);\; -R_t \cos\psi_{p1} \sin\left(\lambda_{p1} - \lambda_{s1}\right);\; -R_t \sin\psi_{p1}\right] \Big/ \left\|norm\,\vec{Z}_{p1}\right\|$$

with:

$$\left\|norm\,\vec{Z}_{p1}\right\| = \left(h^2 + R_t^2 - 2h\;R_t \cos\Theta_{p1}\right)^{1/2}$$

$$\vec{X}_{p1} = \left[0;\; R_t \sin\psi_{p1};\; -R_t \cos\psi_{p1} \sin(\lambda_{p1} - \lambda_{s1})\right] \Big/ \left\|norm\,\vec{X}_{p1}\right\|$$

with:

$$\left\|norm\,\vec{X}_{p1}\right\| = \left|R_t \sin\Theta_{p1}\right|$$

$$\vec{Y}_{p1} = \left[R_t^2;\; R_t\left(k - \cos\Theta_{p1}\right) \cos\psi_{p1} \sin\left(\lambda_{p1} - \lambda_{s1}\right);\; R_t^2\left(k - \cos\psi_{p1}\right) \sin\psi_{p1}\right] \Big/ \left\|norm\,\vec{Y}_{p1}\right\|$$

with:

$$\left\|norm\,\vec{Y}_{p1}\right\| = R_t^2\left|\sin\Theta_{p1}\right|\left(\sin^2\Theta_{p1} + (k - \cos\Theta_{p1})^2\right)^{1/2}$$

and with:

$$\cos\Theta_{p1} = \cos\psi_{p1} \cos\left(\lambda_{p1} - \lambda_{s1}\right)$$

2.1.1.2 Calculation of $\overrightarrow{S_1P_1}$ in base earth-centre coordinate system $\boldsymbol{R_{g1}}$:

Direction satellite-earth station $\overrightarrow{S_1P_1}$ is represented by vector $\left(-\vec{Z}_{p1}\right)$ and its coordinates are expressed as follows:

– in base earth station system $\boldsymbol{R_{g1}}$:

$$\left(-\vec{Z}_{p1}\right) = \left[X_{g1};\ Y_{g1};\ Z_{g1}\right]$$

$$= \left[-(h - R_t \cos \psi_{p1} \cos (\lambda_{p1} - \lambda_{s1}));\ R_t \cos \psi_{p1} \sin (\lambda_{p1} - \lambda_{s1});\ R_t \sin \psi_{p1}\right] \Big/ \left(h^2 + R_t^2 - 2h\, R_t \cos \Theta_{p1}\right)^{1/2}$$

– in satellite system $\boldsymbol{R_{s1}}$:

$$\left(-\vec{Z}_{p1}\right) = \left[X_{s1};\ Y_{s1};\ Z_{s1}\right] = \left[Z_{g1};\ Y_{g1};\ -X_{g1}\right] = {}^tT(3 \times 3)\left[X_{g1};\ Y_{g1};\ Z_{g1}\right]$$

– in antenna system $\boldsymbol{R_{a1}}(\gamma_1)$:

$$\left(-\vec{Z}_{p1}\right) = \left[X_{a1}(\gamma_1);\ Y_{a1}(\gamma_1);\ Z_{a1}(\gamma_1)\right] = {}^tM_1\,(3 \times 3) \cdot \left[X_{s1};\ Y_{s1};\ Z_{s1}\right]$$

with M_1 (3×3) the Euler transformation matrix between systems $\boldsymbol{R_{a1}}(\gamma_1)$ and $\boldsymbol{R_{s1}}$

– and in antenna system $\boldsymbol{R_{a1}}$:

$$\left(-\vec{Z}_{p1}\right) = \left[X_{a1};\ Y_{a1};\ Z_{a1}\right] = M_{za1}\,(-\gamma_1)\,(3 \times 3) \cdot \left[X_{a1}(\gamma_1);\ Y_{a1}(\gamma_1);\ Z_{a1}(\gamma_1)\right]$$

with $M_{za1}(-\gamma_1)$ (3×3) the rotation transformation matrix between systems $\boldsymbol{R_{a1}}(\gamma_1)$ and $\boldsymbol{R_{a1}}$.

2.1.1.3 Definition of linear polarization vector of a wave transmitted from the antenna A_1 of satellite S_1 to the wanted earth station P_1

Vector $[\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]$ in according with the third definition of Ludwig, (the antenna of satellite S_1 being polarized parallel to $\vec{Y}_{a1}(\gamma_1)$), is given by the following expression in system $\boldsymbol{R_{a1}}(\gamma_1)$ (see Fig. 2a):

$$[\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]_{Ra1}(\gamma_1) = \sin(\varphi_{a1} + \gamma_1) \cdot \vec{e}_{\theta a1} + \cos(\varphi_{a1} + \gamma_1) \cdot \vec{e}_{\theta a1}$$

with:

– ($\vec{e}_{\theta a1}$ and $\vec{e}_{\varphi a1}$): usual unit vectors of spherical coordinates in antenna system $\boldsymbol{R_{a1}}(\gamma_1)$:

$$\vec{e}_{\theta a1} = \left[\cos \theta_{a1}\ \cos(\varphi_{a1} + \gamma_1);\ \cos \theta_{a1}\ \sin(\varphi_{a1} + \gamma_1);\ -\sin \theta_{a1}\right]$$

$$\vec{e}_{\varphi a1} = \left[-\sin(\varphi_{a1} + \gamma_1);\ \cos(\varphi_{a1} + \gamma_1);\ 0\right]$$

– (θ_{a1} and φ_{a1}): angles for determining the direction $\overrightarrow{S_1P_1}$ or $\left(-\vec{Z}_{p1}\right)$ from S_1 to P_1 in system $\boldsymbol{R_{a1}}$:

$$\left[\theta_{a1};\ \varphi_{a1}\right] = \left[\text{arc cos}\ Z_{a1};\ \text{arc tan}\ (Y_{a1} / X_{a1})\right] \text{ in } \boldsymbol{R_{a1}}$$

X_{a1}, Y_{a1} and Z_{a1}: components of vector $\left(-\vec{Z}_{p1}\right)$ in antenna system $\boldsymbol{R_{a1}}$

– vector $[\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]$ is expressed:

  – in system $\boldsymbol{R_{s1}}$ by Euler transformation matrix M_1 (3×3) between systems $\boldsymbol{R_{a1}}(\gamma_1)$ and $\boldsymbol{R_{s1}}$:

$$[\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]_{Rs1} = M_1\,(3 \times 3) \cdot [\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]_{Ra1}(\gamma_1)$$

  – then in system $\boldsymbol{R_{g1}}$:

$$[\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]_{Rg1} = T\,(3 \times 3) \cdot [\,\vec{e}_{1\downarrow}{}^{=}(\gamma_1)]_{Rs1}$$

2.1.2 Determination of linear polarization vector of a wave transmitted off-axis from interfering satellite S_2 to wanted earth station P_1

General case: the earth station is distinct from the boresight: $P_1 \neq Pv_1$

Earth station P_1 is pointed to satellite S_1 but also receives interfering emissions of satellite S_2 (characterized by its boresight Pv_2 and the inclination γ_2 of its transmitting elliptic beam)

The off-axis transmit linear polarization vector $[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]$ of the wave transmitted from interfering satellite S_2 is calculated at first in system $\boldsymbol{R_{g2}}$ (according to the same method as from satellite S_1 in system $\boldsymbol{R_{g1}}$: see § 2.1.1).

Vector $[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]$ according the third definition of Ludwig, (the antenna of satellite S_2 being polarized parallel to $\vec{Y}_{a2}(\gamma_2)$), is given by the following expression in system $\boldsymbol{R_{a2}}(\gamma_2)$:

$$[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]_{Ra2(\gamma_2)} = \sin(\varphi_{a21} + \gamma_2) \cdot \vec{e}_{\theta a21} + \cos(\varphi_{a21} + \gamma_2) \cdot \vec{e}_{\varphi a21}$$

with:

– $(\vec{e}_{\theta a21}$ and $\vec{e}_{\varphi a21})$: usual unit vectors of spherical coordinates in antenna system $\boldsymbol{R_{a2}}(\gamma_2)$:

$$\vec{e}_{\theta a21} = [\cos\theta_{a21} \ \cos(\varphi_{a21} + \gamma_2); \ \cos\theta_{a21} \ \sin(\varphi_{a21} + \gamma_2); \ -\sin\theta_{a21}]$$

$$\vec{e}_{\varphi a21} = [-\sin(\varphi_{a21} + \gamma_2); \ \cos(\varphi_{a21} + \gamma_2); \ 0]$$

– $(\theta_{a21}$ and $\varphi_{a21})$: angles for determining direction $\overrightarrow{S_2P_1}$ from satellite S_2 to earth station P_1 in $\boldsymbol{R_{a2}}$:

$$[\theta_{a21}; \ \varphi_{a21}] = [\text{arc cos } Z_{a21}; \ \text{arc tan}(Y_{a21} / X_{a21})] \ \text{in } \boldsymbol{R_{a2}}$$

X_{a21}, Y_{a21} and Z_{a21}: components of vector $(-\vec{Z}_{p21}) = (\overrightarrow{S_2P_1}) \,/\, \|(\overrightarrow{S_2P_1})\|$ in antenna system $\boldsymbol{R_{a2}}$

– then vector $[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]$ is expressed:

  – in system $\boldsymbol{R_{s2}}$ with the help of Euler transformation matrix M_2 (3×3) between systems $\boldsymbol{R_{s2}}$ and $\boldsymbol{R_{a2}}(\gamma_2)$:

$$[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]_{Rs2} = M_2\,(3 \times 3) \cdot [\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]_{Ra2(\gamma_2)}$$

  – in system $\boldsymbol{R_{g2}}$:

$$[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]_{Rg2} = T\,(3 \times 3) \cdot [\vec{e}_{21\downarrow}{}^{=}(\gamma_2)]_{Rs2}$$

  – then is expressed in system $\boldsymbol{R_{g1}}$:

$$\begin{array}{c}[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)] \\ \text{in } \boldsymbol{R_{g1}}\end{array} = \begin{bmatrix} \cos(\lambda_{s2} - \lambda_{s1}) & -\sin(\lambda_{s2} - \lambda_{s1}) & 0 \\ \sin(\lambda_{s2} - \lambda_{s1}) & \cos(\lambda_{s2} - \lambda_{s1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{array}{c}[\vec{e}_{21\downarrow}{}^{=}(\gamma_2)] \\ \text{in } \boldsymbol{R_{g2}}\end{array}$$

2.1.3 Calculation of relative alignment β for the linear polarization in downlink: reception by wanted earth station P_1 (see Fig. 3)

Calculation for an earth station P_1 of the alignment angle between the polarization planes of signals transmitted by a wanted satellite S_1 and an interfering satellite S_2

Polarization angles $\varepsilon_{1\downarrow}(\gamma_1)$ and $\varepsilon_{21\downarrow}(\gamma_2)$ are expressed in system $\boldsymbol{R_{p1}}$, connected with earth station P_1 and wanted satellite S_1 on which is pointed wanted earth station P_1 considered.

2.1.3.1 Above calculations can be applied for the calculation polarization angle $\varepsilon_{1\downarrow}(\gamma_1)$ of the wave transmitted with polarization vector $[\vec{e}_{1\downarrow}^{=}(\gamma_1)]$ from the antenna A_1 of wanted satellite S_1 characterized by angle γ_1:

$$\varepsilon_{1\downarrow}(\gamma_1) = \text{arc tan}\left(\left[\left(\vec{e}_{1\downarrow}^{=}(\gamma_1)\right) \cdot \vec{Y}_{p1}\right] / \left[\left(\vec{e}_{1\downarrow}^{=}(\gamma_1)\right) \cdot \vec{Y}_{p1}\right]\right) \tag{14}$$

Vectors $[\vec{e}_{1\downarrow}^{=}(\gamma_1)]$, $\vec{X}_{p1}$ and $\vec{Y}_{p1}$ are expressed in the same coordinate system $\boldsymbol{R_{g1}}$.

2.1.3.2 Similarly for calculation of polarization angle $\varepsilon_{21\downarrow}(\gamma_2)$ of the wave transmitted with polarization vector $[\vec{e}_{21\downarrow}^{=}(\gamma_2)]$ by the antenna A_1 of interfering satellite S_2 characterized by angle γ_2, hence:

$$\varepsilon_{21\downarrow}(\gamma_2) = \text{arc tan}\left(\left[\left(\vec{e}_{21\downarrow}^{=}(\gamma_2)\right) \cdot \vec{Y}_{p1}\right] / \left[\left(\vec{e}_{21\downarrow}^{=}(\gamma_2)\right) \cdot \vec{X}_{p1}\right]\right) \tag{15}$$

Vector $[\vec{e}_{21\downarrow}^{=}(\gamma_2)]$ is initially calculated in system $\boldsymbol{R_{g2}}$ then in system $\boldsymbol{R_{g1}}$.

2.1.3.3 Polarization angles $\varepsilon_{1\downarrow}(\gamma_1)$ and $\varepsilon_{21\downarrow}(\gamma_2)$ being expressed in system $\boldsymbol{R_{p1}}$, connected with earth station P_1 and wanted satellite S_1, the value of alignment angle $\beta_\downarrow$ is given in the copolar case by the following expression:

$$\beta_\downarrow = \left|\varepsilon_{1\downarrow}(\gamma_1) - \varepsilon_{2\downarrow}(\gamma_2)\right| + \delta \qquad \text{(see Note 1)} \tag{16}$$

2.2 Uplink case: reception by a wanted satellite S_1 (see Fig. 4)

Calculation for a wanted satellite S_1 of the alignment between the signals coming from a wanted earth station P_1 and an interfering earth station P_2

2.2.1 Determination of linear polarization vector and angle of a wave transmitted from earth station P_1 to satellite S_1

General case: the earth station is distinct from the boresight: $P_1 \neq Pv_1$

This earth station is pointed to satellite S_1 (characterized by its boresight Pv_1 and the inclination angle γ_1 of its receiving elliptic beam).

Hypothesis of calculation: the on-axis transmit linear polarization vector $[\vec{e}_{1\uparrow}^{=}(\gamma_1)]$ of the wave transmitted from the antenna of earth station P_1, is matched to the off-axis receive linear polarization vector of the antenna of wanted satellite S_1.

The calculation of vector $[\vec{e}_{1\uparrow}^{=}(\gamma_1)]$ from earth station P_1 to satellite S_1 is therefore expressed in system $\boldsymbol{R_{a1}}$ connected with the antenna A_1 of the wanted satellite S_1 (which is pointing towards boresight Pv_1) according to the same method as for $[\vec{e}_{1\downarrow}^{=}(\gamma_1)]$ in down-link (see § 2.1.1).

The polarization angle from earth station P_1 is given in system $\boldsymbol{R_{a1}}$ as follows:

$$\varepsilon_{1\downarrow}(\gamma_1) = \text{arc tan}\left(\left[\left(\vec{e}_{1\downarrow}^{=}(\gamma_1)\right) \cdot \vec{X}_{a1}\right] / \left[\left(\vec{e}_{1\downarrow}^{=}(\gamma_1)\right) \cdot \vec{Y}_{a1}\right]\right)$$

Vectors $[\vec{e}_{1\uparrow}^{=}(\gamma_1)]$, $\vec{X}_{a1}$ and $\vec{Y}_{a1}$ are expressed in the same system ($\boldsymbol{R_{a1}}$ for instance).

2.2.2 Determination of linear polarization vector and angle of a wave transmitted from earth station P_2 to satellite S_2

General case: the earth station is distinct from the boresight: $P_2 \neq Pv_2$

This earth station is pointed to satellite S_2 (characterized by its boresight Pv_2 and the inclination angle γ_2 of its receiving elliptic beam).

Hypothesis of calculation: the on-axis transmit linear polarization vector $[\vec{e}_{2\uparrow}^{=}(\gamma_2)]$ of the wave transmitted from the antenna of earth station P_2, is matched to the off-axis receive linear polarization vector of the antenna of satellite S_2.

The calculation of vector $[\vec{e}_{2\uparrow}^{=}(\gamma_2)]$ from earth station P_2 to satellite S_2 is therefore expressed in system $\boldsymbol{R_{p2}}$ connected with earth station P_2 pointed to satellite S_2 (which is pointing towards boresight Pv_2) according to the same method as for $[\vec{e}_{2\downarrow}^{=}(\gamma_2)]$ in down-link (see § 2.1.1).

The polarization angle from earth station P_2 is given in system $\boldsymbol{R_{p2}}$ as follows:

$$\varepsilon_{2\uparrow}(\gamma_2) = \arc\tan\left([(\vec{e}_{2\uparrow}^{=}(\gamma_2)) \cdot \vec{Y}_{p2}] \; / \; [(\vec{e}_{2\uparrow}^{=}(\gamma_2)) \cdot \vec{X}_{p2}]\right)$$

Vectors $[\vec{e}_{2\uparrow}^{=}(\gamma_2)]$, $\vec{X}_{p2}$ and $\vec{Y}_{p2}$ are expressed in the same system ($\boldsymbol{R_{p2}}$ for instance).

2.2.3 Determination of off-axis linear polarization vector and angle of a wave transmitted from earth station P_2 to wanted satellite S_1

This earth station P_2 which is pointed to satellite S_2 (characterized by its boresight Pv_2 and the inclination angle γ_2 of its receiving elliptic beam), transmits also interfering emissions to wanted satellite S_1 (characterized by its boresight Pv_1 and the inclination angle γ_1 of its receiving elliptic beam) (see Fig. 2b).

The off-axis transmit linear polarization vector $[\vec{e}_{21\uparrow}^{=}(\varepsilon_{2\uparrow}\ (\gamma_2)]$ of the wave transmitted from the antenna of earth station P_2 is given by the following expression in system $\boldsymbol{R_{p2}}$ (in this case of a linearly polarized field parallel to $\vec{X}'_{p2}$ (with $\vec{X}'_{p2} = Rot_{zp2}(\varepsilon_2)\ (\vec{X}_{p2})$)):

$$[\vec{e}_{21\uparrow}^{=}(\varepsilon_{2\uparrow}(\gamma_2))]_{Rp2} = [\vec{e}_{21\uparrow}^{=}(\varepsilon_2)]_{Rp2} = \cos(\varphi_{p21} - \varepsilon_2) \cdot \vec{e}_{\theta p21} - \sin(\varphi_{p21} - \varepsilon_2) \cdot \vec{e}_{\psi p21}$$

with: $\varepsilon_{2\uparrow}(\gamma_2) = \varepsilon_2$

with:

– ($\vec{e}_{\theta p2,1}$: $\vec{e}_{\varphi p2,1}$): usual unit vectors of spherical coordinates in antenna coordinate system $\boldsymbol{R_{p2}}$:

$$\vec{e}_{\theta p21} = [\cos\theta_{p21}\ \cos\varphi_{p21};\ \cos\theta_{p21} \cdot \sin\varphi_{p21};\ -\sin\theta_{p21}]$$

$$\vec{e}_{\varphi p21} = [-\sin\varphi_{a21};\ \cos\varphi_{a21};\ 0]$$

– (θ_{p21} and φ_{p21}): angles for determining direction $\overrightarrow{P_2S_1}$ from earth station P_2 to satellite S1 in $\boldsymbol{R_{p2}}$:

$$\cos\theta_{p21} = \left((\overrightarrow{P_2S_2}) \cdot (\overrightarrow{P_2S_1})\right) / \left(\|\overrightarrow{P_2S_2}\| \cdot \|\overrightarrow{P_2S_1}\|\right) \text{ in } \boldsymbol{R_{p2}}$$

and

$$\tan\varphi_{p21} = \left((\overrightarrow{P_2S_1}) \cdot \vec{Y}_{p2}\right) / \left((\overrightarrow{P_2S_1}) \cdot \vec{X}_{p2}\right)$$

– Direction earth station-satellite $\overrightarrow{P_2S_2}$ is represented by vector $\vec{Z}_{p2} = (\overrightarrow{P_2S_2}) / \|\overrightarrow{P_2S_2}\|$.

Vectors $\vec{X}_{p2}$, $\vec{Y}_{p2}$, $\vec{Z}_{p2}$ are expressed in system $\boldsymbol{R_{g2}}$ then system $\boldsymbol{R_{g1}}$ and vector $(\overrightarrow{P_2S_1})$ is expressed in system $\boldsymbol{R_{g1}}$.

2.2.3.1 Calculation of $\vec{X}_{p2}$, $\vec{Y}_{p2}$, $\vec{Z}_{p2}$ in system $\boldsymbol{R_{g2}}$:

Unit vectors of coordinate system $\boldsymbol{R_{p2}}$ of earth station P_2 (which is pointed to satellite S_2) are defined in system $\boldsymbol{R_{g2}}$ by the following expressions:

$$\vec{Z}_{p2} = \left[h - R_t\ \cos\psi_{p2}\ \cos(\lambda_{p2} - \lambda_{s2});\ -R_t\ \cos\psi_{p2}\ \sin(\lambda_{p2} - \lambda_{s2});\ -R_t\ \sin\psi_{p2}\right] / \|norm\,\vec{Z}_{p2}\|$$

with:

$$\|norm\,\vec{Z}_{p2}\| = (h^2 + R_t^2 - 2h\ R_t\ \cos\Theta_{p2})^{1/2}$$

$$\vec{Z}_{p2} = (\overrightarrow{P_2S_2}) / \|\overrightarrow{P_2S_2}\|$$

$$\vec{X}_{p2} = \left[0;\ R_t\ \sin\psi_{p2};\ -R_t\ \cos\psi_{p2}\ \sin(\lambda_{p2} - \lambda_{s2})\right] / \|norm\,\vec{X}_{p2}\|$$

with:

$$\|norm\,\vec{X}_{p2}\| = |R_t \sin \Theta_{p2}|$$

$$\vec{Y}_{p2} = \left[R_t^2;\ R_t\ (k - \cos \Theta_{p2})\ \cos \psi_{p2}\ \sin (\lambda_{p2} - \lambda_{s2});\ R_t^2\ (k - \cos \psi_{p2})\ \sin \psi_{p2}\right] / \|norm\,\vec{Y}_{p2}\|$$

with:

$$\|norm\,\vec{Y}_{p2}\| = R_t^2\ |\sin \Theta_{p2}|\ \left(\sin^2 \Theta_{p2} + (k - \cos \Theta_{p2})^2\right)^{1/2}$$

moreover $\cos \Theta_{p2} = \cos \psi_{p2}\ \cos (\lambda_{p2} - \lambda_{s2})$.

2.2.3.2 Calculation of vector $(\overrightarrow{P_2S_1})$ in system $\boldsymbol{R_{g1}}$:

$$(\overrightarrow{P_2S_1}) = \left[h - R_t\ \cos \psi_{p2}\ \cos (\lambda_{p2} - \lambda_{s1});\ -R_t\ \cos \psi_{p2}\ \sin (\lambda_{p2} - \lambda_{s1});\ -R_t\ \sin \psi_{p2}\right]$$

with:

$$\|(\overrightarrow{P_2S_1})\| = (h^2 + R_t^2 - 2h\ R_t\ \cos \Theta'_{p2})^{1/2}$$

and $\cos \Theta'_{p2} = \cos \psi_{p2}\ \cos (\lambda_{p2} - \lambda_{s1})$.

2.2.3.3 Calculation of off-axis transmit linear polarization vector $[\vec{e}_{21}{\uparrow}^{=}(\varepsilon_2)]$ in systems $\boldsymbol{R_{p2}}$ and $\boldsymbol{R_{g1}}$:

Vector $[\vec{e}_{21}{\uparrow}^{=}(\varepsilon_2{\uparrow}(\gamma_2))]$ is given by the following expression:

– in system $\boldsymbol{R_{g2}}$:

$$\underset{\text{in } \boldsymbol{R_{p2}}}{[\vec{e}{\uparrow}_{21}{}^{=}(\varepsilon_2)]} = \begin{bmatrix} \cos \theta_{p21}\ \cos \varphi_{p21}\ \cos (\varphi_{p21} - \varepsilon_2) + \sin \varphi_{p21}\ \sin (\varphi_{p21} - \varepsilon_2) \\ \cos \theta_{p21}\ \sin \varphi_{p21}\ \cos (\varphi_{p21} - \varepsilon_2) - \cos \varphi_{p21}\ \sin (\varphi_{p21} - \varepsilon_2) \\ -\sin \theta_{p21}\ \cos (\varphi_{p21} - \varepsilon_2) \end{bmatrix} = \begin{bmatrix} e_{21}(1)_{Rp2} \\ e_{21}(2)_{Rp2} \\ e_{21}(3)_{Rp2} \end{bmatrix}$$

– then in systems $\boldsymbol{R_{g2}}$ and $\boldsymbol{R_{g1}}$:

– in system $\boldsymbol{R_{g2}}$ through the above expressions of unit vectors $\vec{X}_{p2}$, $\vec{Y}_{p2}$, $\vec{Z}_{p2}$ of system $\boldsymbol{R_{p2}}$ which are a function of unit vectors $\vec{X}_{g2}$, $\vec{Y}_{g2}$, $\vec{Z}_{g2}$ of system $\boldsymbol{R_{g2}}$

$$[\vec{e}_{21}{\uparrow}^{=}(\varepsilon_2)]_{Rg2} = [e_{21}(1)_{Rg2};\ e_{21}(2)_{Rg2};\ e_{21}(3)_{Rg2}]$$

– then in system $\boldsymbol{R_{g1}}$ through the following transformation matrix to go from coordinate system $\boldsymbol{R_{g2}}$ to coordinate system $\boldsymbol{R_{g1}}$:

$$\underset{\text{in } \boldsymbol{R_{g1}}}{[\vec{e}{\uparrow}_{21}{}^{=}(\varepsilon_2)]} = \begin{bmatrix} \cos (\lambda_{s2} - \lambda_{s1}) & -\sin (\lambda_{s2} - \lambda_{s1}) & 0 \\ \sin (\lambda_{s2} - \lambda_{s1}) & \cos (\lambda_{s2} - \lambda_{s1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \underset{\text{in } \boldsymbol{R_{g2}}}{[\vec{e}{\uparrow}_{21}{}^{=}(\varepsilon_2)]} = \begin{bmatrix} e_{21}(1)_{Rg1} \\ e_{21}(2)_{Rg1} \\ e_{21}(3)_{Rg1} \end{bmatrix}$$

– Then in systems $\boldsymbol{R_{s1}}$ and $\boldsymbol{R_{a1}}$:

– in satellite system $\boldsymbol{R_{s1}}$:

$$[\vec{e}_{21}{\uparrow}^{=}(\varepsilon_2)]_{Rs1} = [e_{21}(1)_{Rs1};\ e_{21}(2)_{Rs1};\ e_{21}(3)_{Rs1}] = [e_{21}(3)_{Rg1};\ e_{21}(2)_{Rg1};\ -e_{21}(1)_{Rg1}]$$

– in antenna system $\boldsymbol{R_{a1}}$:

$$\begin{aligned} [\vec{e}_{21}{\uparrow}^{=}(\varepsilon_2)]_{Ra1} &= [e_{21}(1)_{Ra1};\ e_{21}(2)_{Ra1};\ e_{21}(3)_{Ra1}] \\ &= {}^tM_1\ (\gamma_1 = 0)\ (3 \times 3) \cdot [e_{21}(1)_{Rs1};\ e_{21}(2)_{Rs1};\ e_{21}(3)_{Rs1}] \end{aligned}$$

with M_1 ($\gamma_1 = 0$) (3×3) the Euler transformation matrix (with $\gamma_1 = 0$) between systems $\boldsymbol{R_{a1}}$ and $\boldsymbol{R_{s1}}$.

The polarization angle from earth station P_2 is given in system $\boldsymbol{R_{a1}}$ as follows:

$$\varepsilon_{21}\uparrow(\varepsilon_2) = \text{arc tan}\left(\left[\left(\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right) \cdot \vec{X}_{a1}\right] \,/\, \left[\left(\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right) \cdot \vec{Y}_{a1}\right]\right)$$

Vectors $\left[\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right]$, $\vec{X}_{a1}$ and $\vec{Y}_{a1}$ are expressed in the same system ($\boldsymbol{R_{a1}}$ for instance).

2.2.4 Calculation of the alignment angle in up-link: reception by wanted satellite S_1 (see Fig. 4)

Calculation for a wanted satellite S_1 of the alignment angle between the polarization planes of signals transmitted by a wanted earth station P_1 and an interfering earth station P_2

Polarization angles $\varepsilon_1\uparrow(\gamma_1)$ and $\varepsilon_{21}\uparrow(\varepsilon_2)$ should be, in this case, expressed in satellite antenna system $\boldsymbol{R_{a1}}$ connected with the antenna A_1 of wanted satellite S_1 which is pointed towards boresight Pv_1.

2.2.4.1 Above calculations are applied for the calculation of the polarization angle $\varepsilon_1\uparrow(\gamma_1)$ of the wave transmitted with polarization vector $\left[\vec{e}_1\uparrow^{=}(\gamma_1)\right]$ from wanted earth station P_1 towards the receiving antenna A_1 of wanted satellite S_1 characterized by angle γ_1:

$$\varepsilon_1\uparrow(\gamma_1) = \text{arc tan}\left(\left[\left(\vec{e}_1\uparrow^{=}(\gamma_1)\right) \cdot \vec{X}_{a1}\right] \,/\, \left[\left(\vec{e}_1\uparrow^{=}(\gamma_1)\right) \cdot \vec{Y}_{a1}\right]\right) \tag{17}$$

Vectors $\left[\vec{e}_1\uparrow^{=}(\gamma_1)\right]$, $\vec{X}_{a1}$ and $\vec{Y}_{a1}$ are expressed in the same system $\boldsymbol{R_{a1}}$.

2.2.4.2 Similarly for the calculation of the polarization angle $\varepsilon_{21}\uparrow(\varepsilon_2)$ of the wave transmitted with the polarization vector $\left[\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right]$ from interfering earth station P_2 towards the receiving antenna A_1 of wanted satellite S_1, (earth station P_2 is pointed to the receiving antenna A_2 of interfering satellite S_2 characterized by angle γ_2), hence:

$$\varepsilon_{21}\uparrow(\varepsilon_2) = \text{arc tan}\left(\left[\left(\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right) \cdot \vec{X}_{a1}\right] \,/\, \left[\left(\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right) \cdot \vec{Y}_{a1}\right]\right) \tag{18}$$

Vectors $\left[\vec{e}_{21}\uparrow^{=}(\varepsilon_2)\right]$, $\vec{X}_{a1}$ and $\vec{Y}_{a1}$ are also expressed in the same system $\boldsymbol{R_{a1}}$.

2.2.4.3 Polarization angles $\varepsilon_1\uparrow(\gamma_1)$ and $\varepsilon_{21}\uparrow(\varepsilon_2)$ being expressed in antenna system $\boldsymbol{R_{a1}}$ connected with the wanted satellite S_1, the value of alignment angle $\beta\uparrow$ is given in the copolar case by the expression:

$$\beta\uparrow = \left|\varepsilon_1\uparrow(\gamma_1) - \varepsilon_{21}\uparrow(\varepsilon_2)\right| + \delta \qquad \text{(see Note 1)} \tag{19}$$

NOTE 1 – Validity area of calculations:

Two following simplifying assumptions, suitable for use of the calculations of this Recommendation, have been devised:

– the linear polarization angle of each of the two wanted and interfering signals received by the antenna of the wanted satellite or wanted earth station is calculated in the plane $(\vec{X}, \vec{Y})_{rec}$ perpendicular to the main direction of the receiving antenna instead of being calculated in a plane $(\vec{e}_\theta, \vec{e}_\varphi)_{rec}$ perpendicular to the respective propagation direction of each of the two signals.

  The off-axis angles of antennas must be below about 40° in order that the above approximation is valid;

– the co- and cross-polarized radiation patterns are with revolution symmetry around the main axis of the antenna (in the case of reflector antenna or elliptical beam antenna).

FIGURE 2

Earth station Pv-satellite S link

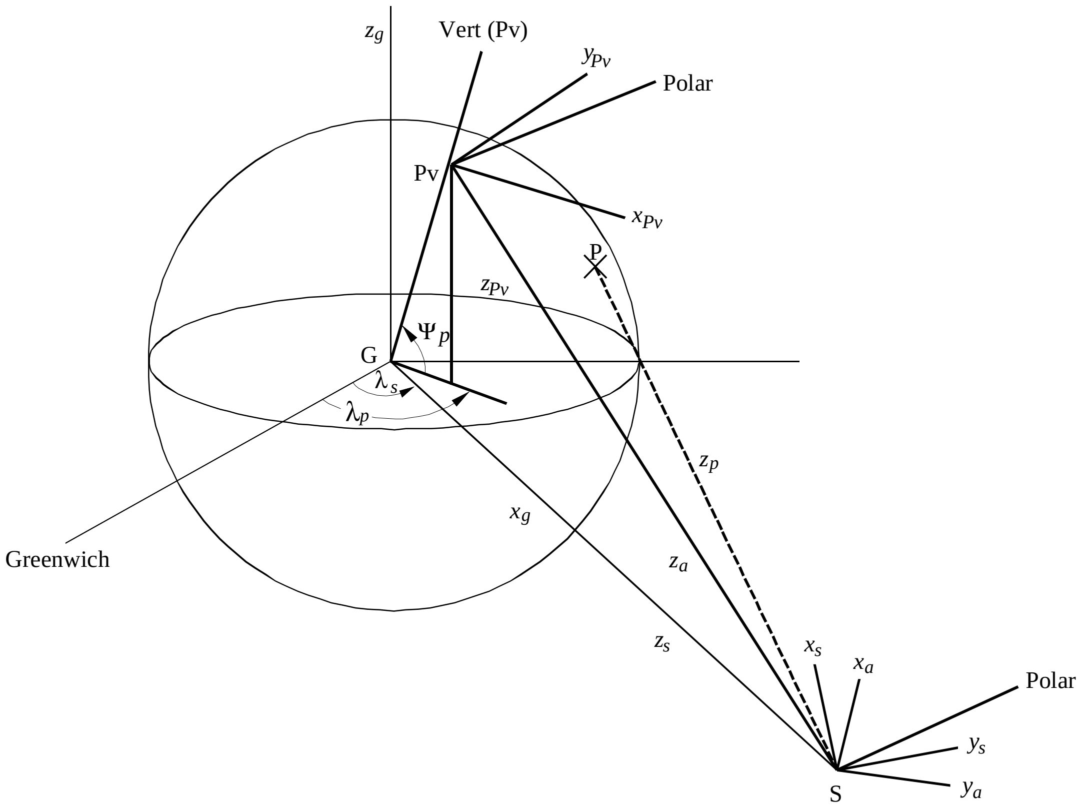


FIGURE 2a

Linear main polarization parallel to $y_a(\gamma)$ from satellite S (on-axis $\overrightarrow{SPv}$ and off-axis $\overrightarrow{SP}$)

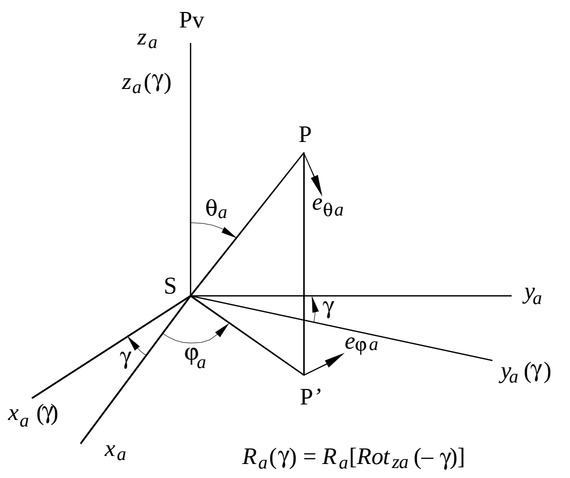


FIGURE 2b

Linear main polarization parallel to $x_{p2'}$ from earth station P_2 (on-axis $\overrightarrow{P_2S_2}$ and off-axis $\overrightarrow{P_2S_1}$)

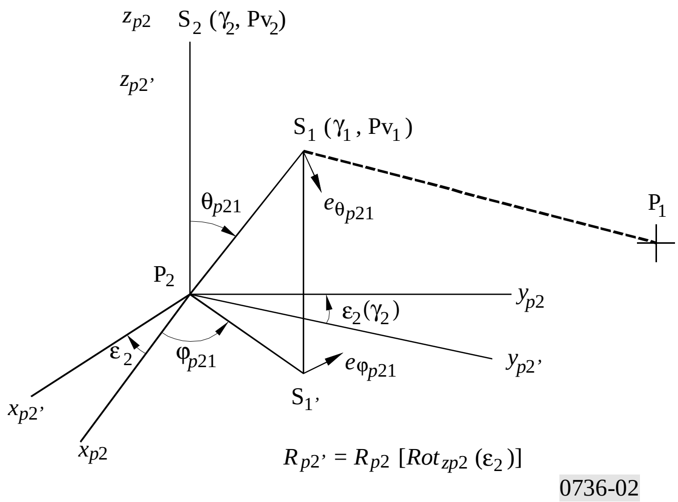


FIGURE 3

Down-link: wanted receiving earth station P_1

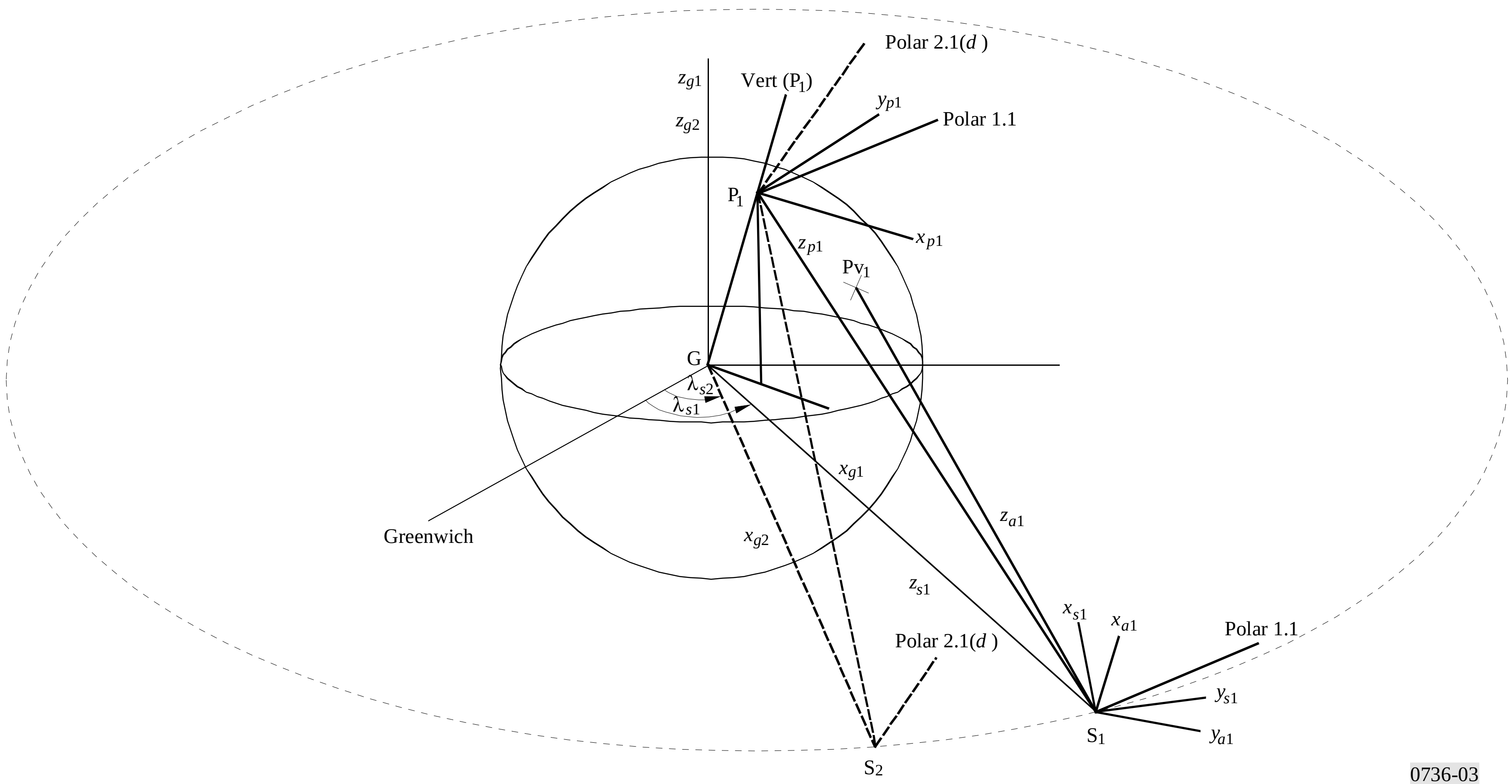
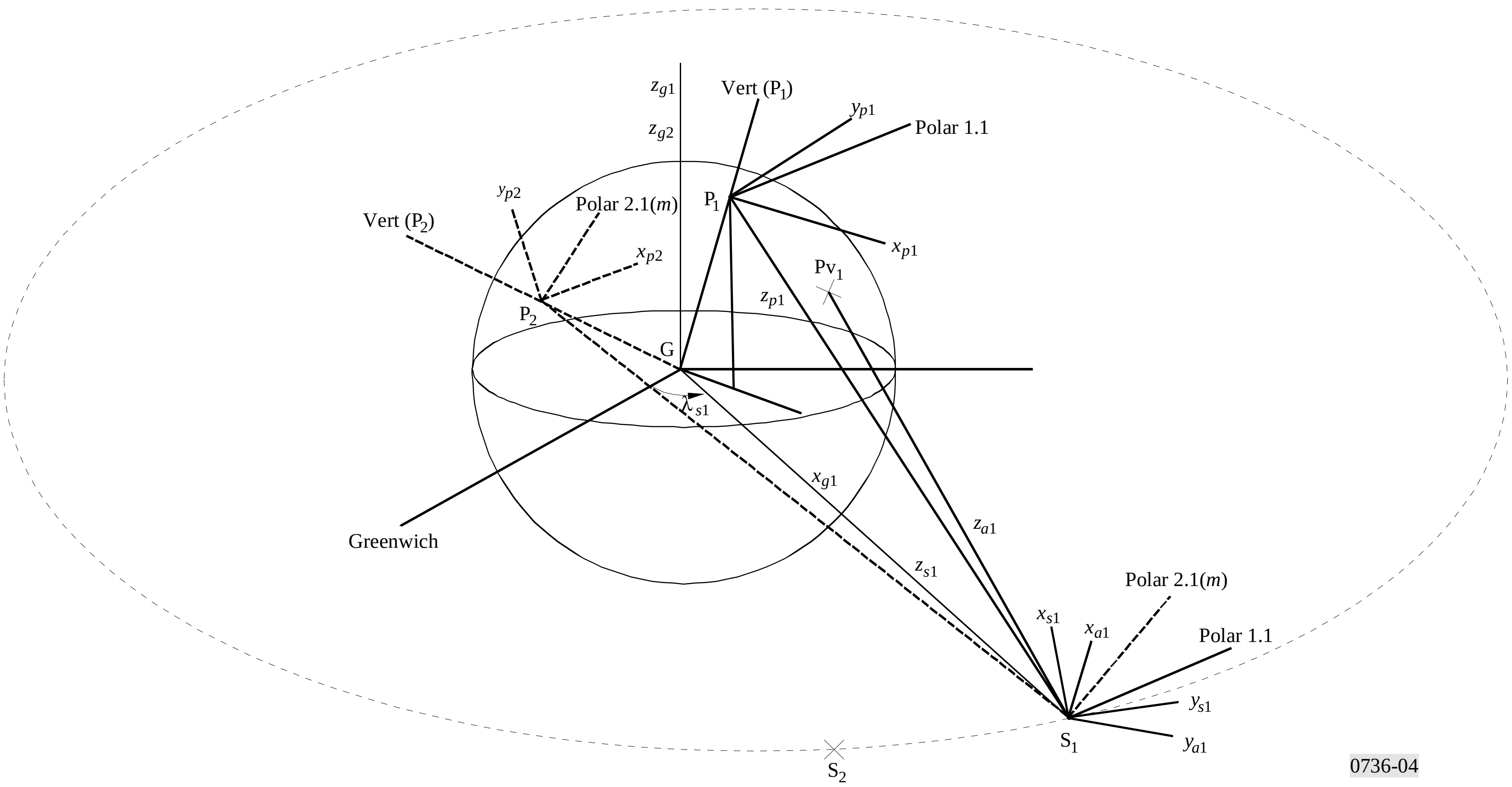


FIGURE 4

Up-link: wanted receiving satellite S_1



APPENDIX 3

TO ANNEX 1

Depolarization due to rain: Cross-polarization of waves in the troposphere for Earth-space telecommunications systems

1 Definition of cross-polarization discrimination due to rain

The cross-polarization discrimination due to rain, Dx_{rain}, is the ratio of the received power on the transmitted polarization over the received power on the orthogonal polarization.

$$Dx_{rain} = U - V(f) \log A_p \qquad \text{dB}$$

where:

A_p: rain attenuation (dB) exceeded for the required percentage of time p, for the path in question, commonly called co-polar attenuation (i.e. on the transmitted polarization transmitted)

$V(f)$: value near 20 between 8 and 15 GHz

$U =$ $U(f, \varepsilon_s, \tau, \sigma)$

where:

f: frequency (GHz)

ε_s: path elevation angle (degrees)

τ: tilt angle (degrees) of the polarization of the linearly polarized vector with respect to the horizontal local plan, (for circular polarization, use $\tau = 45°$)

σ: deviation of the raindrop canting angle distribution.

From the cross-polarization discrimination, it is possible to calculate cross-polarization angle Ψx_{rain} (rotation angle of the polarization vector) for calculating the level of depolarization.

$$\text{tg}^2(\Psi x_{rain}) = 10^{-(Dx_{rain} / 10)}$$

2 Executive summary of a method for calculating long-term statistics of hydrometeor-induced cross-polarization (see Recommendation ITU-R P.618)

Different depolarization mechanisms, particularly hydrometeor effects, are important in the troposphere. Cross-polarization effects are given in Recommendation ITU-R P.618, as also are the calculation of long-term statistics of hydrometeor induced cross-polarization. The method described allows the calculation of cross-polarization statistics from rain attenuation statistics for a same path for $8 \le f \le 35$ GHz and $\varepsilon_s \le 60°$, the method allowing scaling of results at similar frequencies is also presented.

$$Dx_{rain} = U - V(f) \log A_p \qquad \text{dB}$$

with:

$$U = C_f + C_\tau + C_{\varepsilon_s} + C_\sigma \quad \text{and} \quad C_A = V(f) \log A_p$$

$$Dx_{rain} = C_f + C_\tau + C_{\varepsilon_s} + C_\sigma - C_A \qquad \text{dB}$$

with the following parameters:

C_f: frequency-dependent term

$$C_f = 30 \log f \qquad \text{for } 8 \leq f \leq 35 \text{ GHz}$$

C_τ: polarization improvement factor dependent on tilt angle τ:

$$C_\tau = -10 \log \left[1 - 0.484\,(1 + \cos 4\tau)\right]$$

if $\tau = 45°$, $C_\tau = 0$ and if $\tau = 0°$ or $90°$, C_τ is maximum and reaches the value of 15 dB (in the case of circular polarization, $\tau = 45°$);

C_{ε_s}: elevation angle dependent term

$$C_{\varepsilon_s} = -40 \log (\cos \varepsilon_s) \qquad \text{for } \varepsilon_s \leq 60°$$

C_σ: rain drop canting angle dependent term:

$$C_\sigma = 0.0052\,\sigma^2$$

with σ (degrees) the effective standard deviation of the distribution of the raindrop canting angle. The values of σ are 0°, 5°, 10° and 15° for respectively 1%, 0.1%, 0.01% and 0.001% of the time

C_A: rain attenuation dependant term:

$$C_A = V(f) \log A_p$$

where:

$$V(f) = 12.8\,f^{0.19} \qquad \text{for } 8 \leq f \leq 20 \text{ GHz}$$

$$V(f) = 22.6 \qquad \text{for } 20 < f \leq 35 \text{ GHz}$$

3 Executive summary of a method for calculating long-term rain attenuation statistics from point rainfall rate (see Recommendation ITU-R P.618)

The attenuation due to the precipitation is also given in Recommendation ITU-R P.618, the general method described allows prediction of the attenuation from precipitation and clouds along a slant propagation path. The parameter $R_{0.01}$ (mm/h) is the point rainfall rate for the location and for 0.01% of an average year.

The attenuation A_p exceeded for a percentage p% of an average year, in the range 0.001% to 1% is determined from the attenuation $A_{0.01}$ exceeded for a percentage 0.01% of an average year by using the following formulae:

$$\frac{A_p}{A_{0.01}} = 0.12\,p^{-(0.546 + 0.043 \log p)} \qquad \text{for } p\text{\% of the time of an average year}$$

and

$$A_{0.01} = \gamma_R\,L_s\,r_{0.01} \qquad \text{for 0.01\% of the time of an average year}$$

The parameters are the following:

γ_R: specific attenuation due to precipitation:

$$\gamma_R = k(f)\,(R_{0.01})^{\alpha(f)} \text{ (dB/km) (see Recommendation ITU-R P.838)}$$

$k(f)$ and $\alpha(f)$ are coefficients which depend on frequency among others factors

$R_{0.01}$: rainfall rate (mm/h) exceeded for 0.01% of an average year (integration time of 1 mn)

$r_{0.01}$: reduction factor of the length of the precipitation path:

$$r_{0.01} = \frac{1}{1 + L_G / L_0}$$

where $L_0 = 35 \exp(-0.015\ R_{0.01})$

L_s : slant path length above the rain height:

$$L_s = \frac{(h_R - h_s)}{\sin \varepsilon_s}$$

where the horizontal projection $L_G = L_s \cos \varepsilon_s$ (km)

h_s : height above mean sea level of the earth station (km)

h_R : effective rain height for the latitude of the earth station (km)

$$h_R = \begin{cases} 3 + 0.028\ \psi_p & \text{for } 0 \leq \psi_p < 36° \\ 4 - 0.075\ (\psi_p - 36) & \text{for } \psi_p \geq 36° \end{cases}$$

where ψ_p is the absolute value of the earth station latitude.

4 Degradations of polarization

Rain and snow may deteriorate the polarization vector direction. The drops of rain have a shape, not spherical but generally rather ellipsoidal.

When a linearly or circularly polarized wave goes through such drops of rain, the components of polar vector have various attenuations and phase-shift according to the axis of the ellipsoid of drops. Consequently, the wave is linearly polarized, and has therefore a component in the orthogonal direction at that of the transmit wave. It is the phenomenon of cross-polarization or depolarization. The orthogonality of two waves polarized perpendicularly is kept with the differential phase-shift effect; on the other hand, it is not kept with the differential attenuation effect.

The differential phase-shift effect is preponderant over the differential attenuation effect. The differential attenuation effect is low particularly at 6/4 GHz, but its effect is not negligible for the higher frequencies.

The levels of depolarization are functions of the precipitation rate. In the not very rainy regions, the cross-polarization effects are relatively light and decrease only lightly the cross-polarization discrimination. On the other hand, in the very rainy regions, cross-polarization effects are strong and decrease in an unacceptable way the cross-polarization discrimination.

In the case of a wave with a circular polarization or in a linear polarization with a tilt angle $\tau = 45°$, the degradation of the cross-polarization discrimination is maximum. On the other hand, the degradation is reduced if the polarization is near a horizontal or vertical polarization due to the symmetry of the drops of rain.

5 Effects of rain on the attenuation of waves

5.1 Estimations of attenuations

The structure of precipitations is not well known for the rain rate and for its horizontal and vertical extent. The effect of rain is relatively low below 10 GHz, on the other hand, its effect is significant above 10 GHz. The calculation of the attenuation due to rain is based essentially on the knowledge of rainfall rates.

The strong attenuations during a very small percentage of the time correspond to rare events with a time span of a few min. A percentage of 0.01% of an average year corresponds to a time span of 50 min. These relatively short time-scale events have a periodicity of several years, it is therefore necessary to take measures during several years to obtain statistically significant data.

However, the effect of rain is still a new and incomplete area of study, and the methods of calculation are not fully defined.

The main factors are the following:

a) *Elevation angle*

Strong rains have a structure which is more vertical than horizontal. For angles greater than 15°, strong attenuations depend very little on the elevation angle (compact regions with intense rainfall). For very small intense rains, the attenuation is light and with a cosecant law.

b) *Frequency*

The frequency effect on the determination is complicated, but there are empirical laws independent of the rain intensity. There are also semi-empirical laws connecting the attenuations A_{p1} and A_{p2} at frequencies f_1 and f_2 for frequencies lower than 50 GHz (similarity in frequency: Recommendation ITU-R P.618).

c) *Climate*

Attenuations depend mainly on the statistic distribution of the rain intensity in a point. From this distribution, the calculation of attenuations may begin (see Recommendations ITU-R P.837, ITU-R P.838 and ITU-R P.839).