

RECOMMENDATION ITU-R S.1559

Methodology for computing the geographical distribution of maximum downlink equivalent power flux-density levels generated by non-geostationary fixed-satellite service systems using circular orbits

(Question ITU-R 236/4)

(2002)

The ITU Radiocommunication Assembly,

considering

- a) that interference from non-geostationary (GSO) systems into GSO networks will likely have significant variations both geographically and temporally;
- b) that geographic distributions of interference levels caused by non-GSO systems vary with traffic loading, cell layouts, and constellation configurations;
- c) that interference from non-GSO systems into GSO networks can vary with beam scheduling algorithms of the non-GSO system;
- d) that non-GSO beam scheduling algorithms may likely be adjusted to meet traffic loading variations and other factors;
- e) that non-GSO interference levels are dependent on the geographic location of the GSO ground station and GSO satellite location;
- f) that information on the geographical distribution of maximum downlink equivalent power flux-density (epfd_{\downarrow}) levels generated by non-GSO systems may be useful to GSO systems designers in order to determine the expected level of non-GSO interference,

recommends

- 1 that the method described in Annex 1 could be used to construct geographic distributions of maximum epfd_{\downarrow} levels generated within a GSO satellite coverage area by a non-GSO system that uses circular orbits;
- 2 that Annex 1 could be used by administrations, including GSO/fixed-satellite service (FSS) network operators, to provide them with guidance for designing links for a specific geographical location given the following Notes:

NOTE 1 – Validation software has been specified to check compliance of a non-GSO system with epfd_{\downarrow} limits in Article 22 of the Radio Regulations (see Recommendation ITU-R S.1503). This software generates epfd_{\downarrow} levels which are meant to be an absolute envelope and are not representative of what would be generated over time by the non-GSO FSS system in operation.

NOTE 2 – If the methodology in Annex 1 is applied using the assumptions and satellite pfd mask approach as defined in the Recommendation ITU-R S.1503 validation approach, geographical distribution will be obtained of the absolute maximum epfd_{\downarrow} envelope. In the case where the method in Annex 1 is applied to simulation models more representative of the non-GSO operation, the maps generated are likely to vary during the lifetime of the non-GSO system as operating parameters will change.

ANNEX 1

Algorithm for calculating geographic distribution of maximum epfd_{\downarrow} interference levels caused by non-GSO systems**1 Introduction**

Since non-GSO interference will vary geographically it could be useful for administrations to be able to quantify this interference using software that can estimate the geographic and temporal distribution of non-GSO FSS interference.

The analysis will represent the maximum interference expected over the life of the non-GSO system. Analyses of typical scenarios provide little value since the interference levels can have extremely large variations.

The methodology presented in this Annex could be used to generate maps showing representative maximum non-GSO interference power levels that could be received on any land area. The maps will provide guidance for GSO operators for designing links. These maps would provide GSO operators with additional knowledge of maximum epfd_{\downarrow} locations.

This Annex gives a method for calculating the geographic distribution of maximum epfd levels for non-GSO systems with circular orbits that use an exclusion angle for GSO interference mitigation. For these non-GSO systems the maximum epfd_{\downarrow} interference levels (for GSO ground stations with 3 m or greater antennas) occur on or near where the non-GSO spacecraft is inline between the GSO ground station and the GSO spacecraft. For non-GSO systems where this is not the case this approach does not apply.

The maximum epfd_{\downarrow} is found from a computer simulation of the non-GSO system. However, since the value is near an inline situation, only a small number of time points need to be simulated to determine the maximum epfd_{\downarrow} . Only points that are within the 10 dB beamwidth of the GSO ground station need to be considered since in these types of non-GSO systems locations outside of the 10 dB beamwidth will almost certainly be at least 10 dB below the maximum epfd_{\downarrow} value found on the Earth's surface.

Since most non-GSO systems have unique characteristics, the proposed method does not describe how to simulate the specific non-GSO system. Instead it computes the time periods needed to be simulated.

A resolution of 1° longitude by 1° latitude is sufficient resolution to show the general variations in interference levels, however finer resolution may be necessary for more detailed studies, especially in the case of larger antennas. It is noted that non-GSO systems with continuously repeating ground tracks will have maximum epfd_{\downarrow} variations over a much smaller area.

2 Outline of algorithm

The method given can be used to calculate time periods during a non-GSO simulation when a spacecraft passes through a given beamwidth of a GSO ground antenna. First a piece-wise linear approximation to the intersection of the GSO ground antenna beam with the non-GSO sphere is

computed. For each point on the intersection, exact orbital parameters of a non-GSO spacecraft are computed such that the spacecraft will intersect the point on the next orbit. By comparing a given spacecraft's orbital parameters with the orbital parameters at each point, the exact times when the satellite passes through the antenna beam can be determined.

3 Symbols used

ER:	Earth rotational coordinates. An orthogonal three-dimensional coordinate system centred on Earth's centre. ER's z-axis passes through the North Pole and the x-axis passes through the longitude 0 latitude 0 point
GT :	three dimensional location of GSO ground station in ER coordinates
G :	three dimensional location of GSO satellite in ER coordinates
G_x :	x component of vector G
G_y :	y component of vector G
G_z :	z component of vector G
\sim :	normalizes a vector (i.e. $\sim V$ is equivalent to $V / V $)
\cdot :	dot product operator
\times :	cross product operator
$\text{Min}(Q_i)$	determines the minimum of Q_i for all i
$\text{Max}(Q_i)$	Determines the maximum of Q_i for all i
A_x :	x axis of coordinate system pointing from GSO ground station to the GSO
A_y :	y axis of coordinate system pointing from GSO ground station to the GSO
A_z :	z axis of coordinate system pointing from GSO ground station to the GSO
α :	half of the -3 dB beamwidth of GSO ground station antenna
V_i :	vectors forming a cone with half angle α and centre pointing in z direction
W_i :	vectors forming a cone with half angle α and centre pointing in GSO direction
Q_i :	locations on the non-GSO sphere that intersect vectors W_i
v_0 :	mean anomaly = 0
ω_0 :	perigee argument at an initial moment
ω_r :	perigee argument precession rate
Ψ :	initial longitude
Φ_r :	ascending node longitude precession rate
Φ_e :	Earth's rotation rate

Ω_k :	k -th longitude crossing for a non-GSO spacecraft
S :	semi-major axis
T_{ia} :	number of times during an ascending orbit when the test spacecraft passes through Q_i
T_{id} :	number of times during a descending orbit when the test spacecraft passes through Q_i
Ψ_{ia} :	initial longitude of orbits that cross point Q_i while ascending in latitude
Ψ_{id} :	initial longitude of orbits that cross point Q_i while descending in latitude
t_k :	number of times when a non-GSO satellite ascends across the equatorial plane
n :	number of vectors in the cone.

4 Calculation of intersection of GSO ground terminal antenna beam with non-GSO sphere

V_i shown in Fig. 1, defines a set of vectors that sweep out a conic shape centred at the origin with half of the -3 dB beamwidth of α . The centre of the cone points toward the ER coordinate system's z axis.

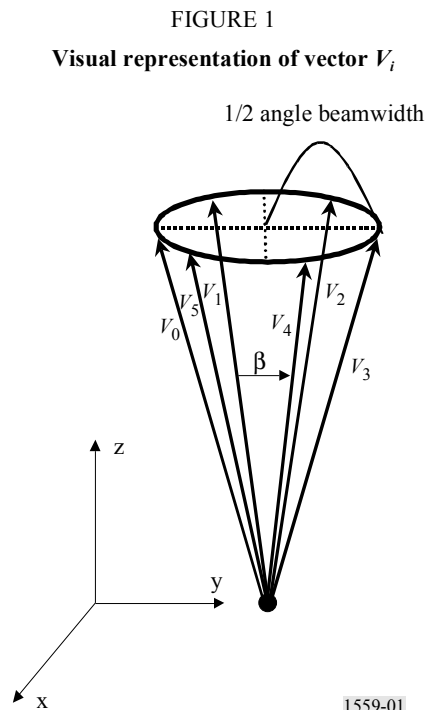
$$V_i = (\cos(\beta_i) \sin(\alpha), \sin(\beta_i) \sin(\alpha), \cos(\alpha)) \quad (1)$$

where:

$$i = 0, \dots, n$$

$$\beta_i = 2\pi i/n = \text{angle between any two adjacent vectors in Fig. 1} \quad (2)$$

α : half of the -3 dB beamwidth.



Next an orthogonal coordinate system A_x, A_y, A_z is computed such that the A_z axis points from the GSO ground terminal toward the GSO.

$$A_z = \sim(G - GT) \tag{3}$$

$$T = (0, 0, 1) \tag{4}$$

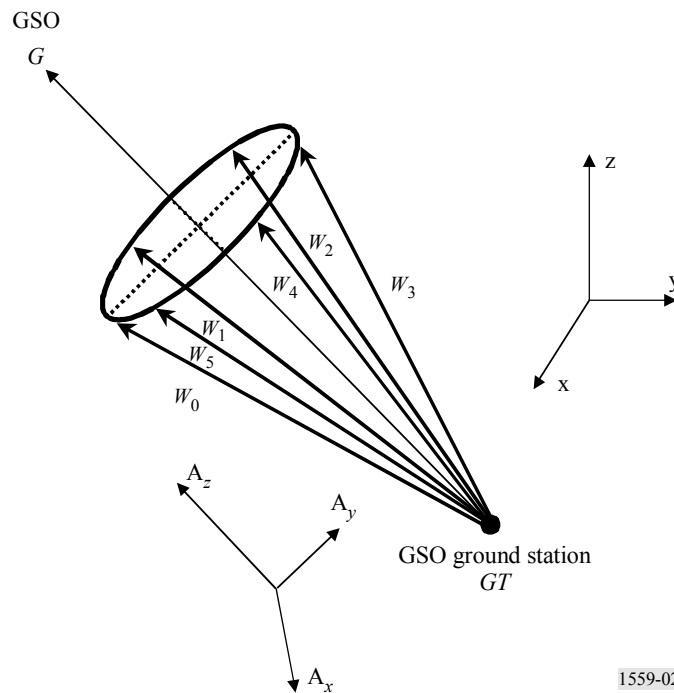
$$A_x = \sim(T \times A_z) \tag{5}$$

$$A_y = A_z \times A_x \tag{6}$$

The cone is pointed toward the GSO yielding the vectors W_i shown in Fig. 2. This is done using a simple coordinate transform.

$$W_i = A_x (V_{ix}) + A_y (V_{iy}) + A_z (V_{iz}) \tag{7}$$

FIGURE 2
Visual representation of vector W_i



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The vectors W_i are extended from GT toward G , using the methods defined in equations (8) to (14), until they intersect the non-GSO sphere at points Q_i as shown in Fig. 3.

L_i in equation (8), represents a line beginning at GT and extending in the direction of W_i . L_i will intersect the non-GSO sphere when $|L_i|$ is equal to the non-GSO semi-major axis. Equations (10) to (13) solve the quadratic equation shown in equation (9) to find the intersection point Q_i .

$$L_i = \delta W_i + GT \quad (8)$$

$$L_i \cdot L_i = S^2 \quad (9)$$

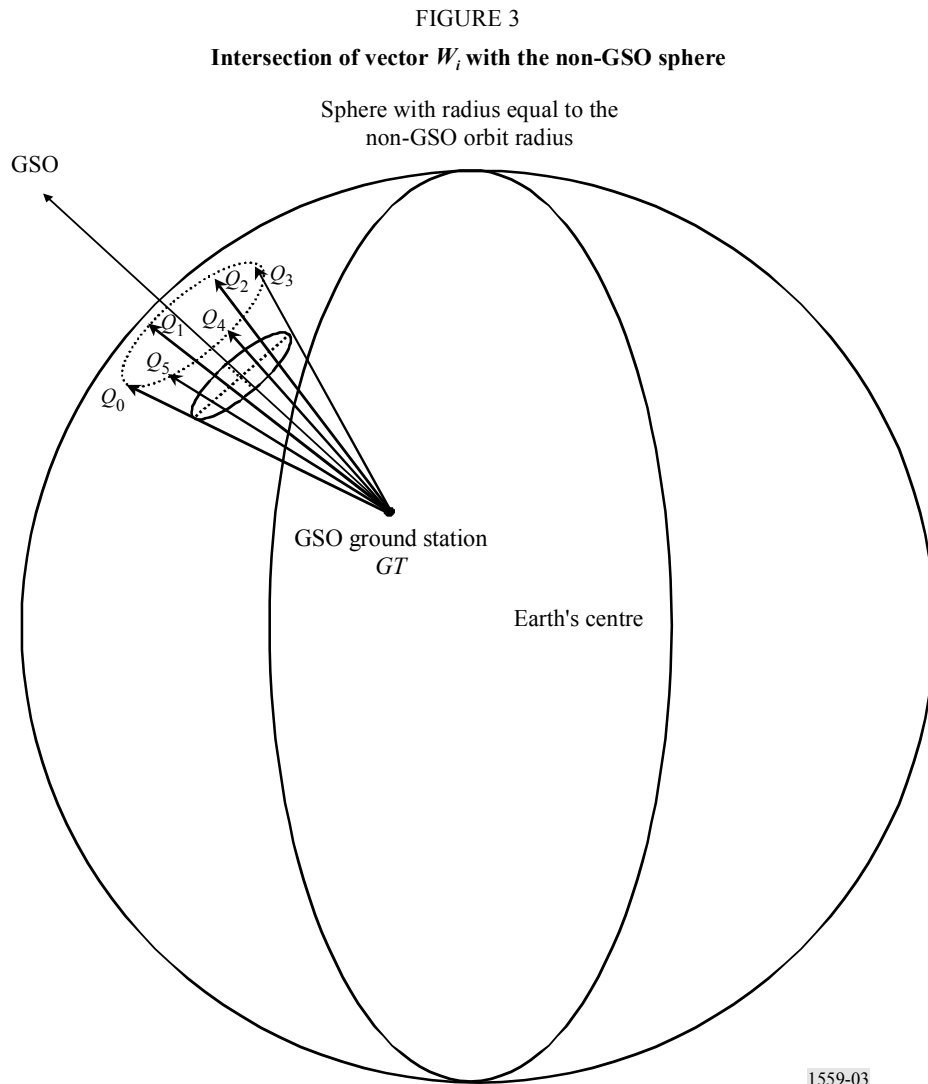
$$a_i = W_i \cdot W_i \quad (10)$$

$$b_i = 2 W_i \cdot GT \quad (11)$$

$$c_i = GT \cdot GT - S^2 \quad (12)$$

$$\delta_i = (-b_i + (b_i^2 - 2 a_i c_i)^{0.5}) / (2 a_i) \quad (13)$$

$$Q_i = \delta_i W_i + G \quad (14)$$



5 Calculation of non-GSO orbital parameters and times when satellite and beam intersection points (Q_i) are co-located

A test spacecraft with the orbital parameters of a non-GSO satellite and a mean anomaly of zero is chosen such that it will intersect Q_i on the next orbit. Equation (15) gives an expression for the orbit location of the test spacecraft in ER coordinates.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} S(\cos(\theta) \cos(\Omega) - \sin(\theta) \sin(\Omega) \cos(i)) \\ S(\cos(\theta) \cos(\Omega) - \sin(\theta) \cos(\Omega) \cos(i)) \\ S \sin(\theta) \sin(i) \end{bmatrix} \quad (15)$$

where:

$$\theta = v_0 + \omega_0 + (\omega_r + 2\pi/T) t$$

$$\Omega = \Psi + (\Phi_r - \Phi_e) t$$

$$v_0: \text{ mean anomaly} = 0$$

$$\omega_0: \text{ perigee argument at an initial moment}$$

$$\omega_r: \text{ perigee argument precession rate}$$

$$\Psi: \text{ initial longitude}$$

$$\Phi_r: \text{ ascending node longitude precession rate}$$

$$\Phi_e: \text{ Earth's rotation rate}$$

$$S: \text{ semi-major axis}$$

$$i: \text{ inclination angle.}$$

For a Q_i point that is lower in latitude than the inclination angle, there will be two unique orbits of the test spacecraft that intersect Q_i . One occurs when the spacecraft is ascending in latitude and the other when the spacecraft is descending. For each of these orbits, both the initial longitude (time = 0) and intersection times with Q_i are computed.

Since the z coordinate of the test spacecraft is $S \sin(\theta) \sin(i)$, θ can have two possible solutions. One when the spacecraft is ascending in latitude and the other when the spacecraft is descending in latitude.

$$\theta_{i1} = \sin^{-1}(\theta_{iz}/(S \sin(i))) \quad (16)$$

$$\theta_{i2} = \pi - \theta_{i1}$$

T_{ij} , the number of times during the orbit when the test spacecraft passes through Q_i is given by:

$$T_{ij} = (\theta_{ij} - \omega_0) / (\omega_r + 2\pi/P) \quad \text{for } j = 1, 2 \quad (17)$$

where P is orbit period.

From T_{ij} the initial longitude Ψ is computed by solving the simultaneous equations for x and y in equation (15).

$$\Psi_{ij} = \cos^{-1}[(Q_{ix} + Q_{iy}) / (R(\cos^2(\theta_{ij}) + \sin^2(\theta_{ij})))] \pm \Phi_r T_i - \Phi_e T_i \quad (18)$$

where R is orbit radius.

6 Calculating longitude crossings for a given simulated satellite

The times at which a satellite crosses the equatorial plane is calculated as follows:

$$t_k = (2k\pi - v_0 - \omega_0) / (\omega_r + 2\pi/P) \quad (19)$$

where:

$$k = 1, \dots, M$$

M : number of orbits before satellite repeats

v_0 : mean anomaly = 0

ω_0 : perigee argument at an initial moment

ω_r : perigee argument precession rate

P : orbit period.

The corresponding longitude ascending crossings are given by:

$$\Omega_k = \Psi + (\Phi_r - \Phi_e)t_k \quad (20)$$

7 Determining if an orbit crosses through the beam of the GSO ground station antenna

If a satellite's ascending longitude crossing Ω is in-between the $\min(\Psi_{ij})$ and $\max(\Psi_{ij})$ for any j , then the satellite will cross the beam of the GSO antenna at some point during the next orbit. It is important to note that the modulo nature of longitude is taken into account when making this calculation.

8 Determining the simulation time of a spacecraft crossing through the beam of the GSO ground station antenna

The simulated time of an orbit crossing an antenna beam is calculated by intersecting the orbit with the piecewise line segments of Q_i as shown in Fig. 4. If an ascending longitude crossing Ω is in-between the $\min(\Psi_{ij})$ and $\max(\Psi_{ij})$ then the orbit will cross the beam of the GSO antenna. The orbit will intersect the Q_i at two locations. By interpolating the corresponding T_{ij} the intersection times are approximated.

Let:

$$N_i = (i + 1) \bmod n \quad (21)$$

For any non-zero duration crossing there will be one value of $i = i1$ such that $\Psi_{ij} \geq \Omega_k \geq \Psi_{Nij}$. Also there will be one value of $i = i2$ such that $\Psi_{ij} \leq \Omega_k \leq \Psi_{Nij}$.

NOTE 1 – Points near the equator might have only one intersection since there may be a crossing that begins on one orbit and ends on the next.

The corresponding intersection time after the equatorial crossing can be approximated from a linear interpolation of the known intersection times T_i .

Let:

$$\chi_1 = (\Omega_k - \Psi_{i1j}) / (\Psi_{Ni1j} - \Psi_{i1j}) \quad (22)$$

$$\text{int_}t_1 = \chi_1 T_{Ni1j} + (1 - \chi_1) T_{i1j} \quad (23)$$

$$\chi_2 = (\Omega_k - \Psi_{i2j}) / (\Psi_{Ni2j} - \Psi_{i2j}) \quad (24)$$

$$\text{int_}t_2 = \chi_2 T_{Ni2j} + (1 - \chi_2) T_{i2j} \quad (25)$$

Since $\text{int_}t_1$ and $\text{int_}t_2$ represent the times since a longitude crossing the time in the simulation is computed as follows:

$$\text{Start_time}_k = \min(\text{int_}t_1, \text{int_}t_2) + t_k \quad (26)$$

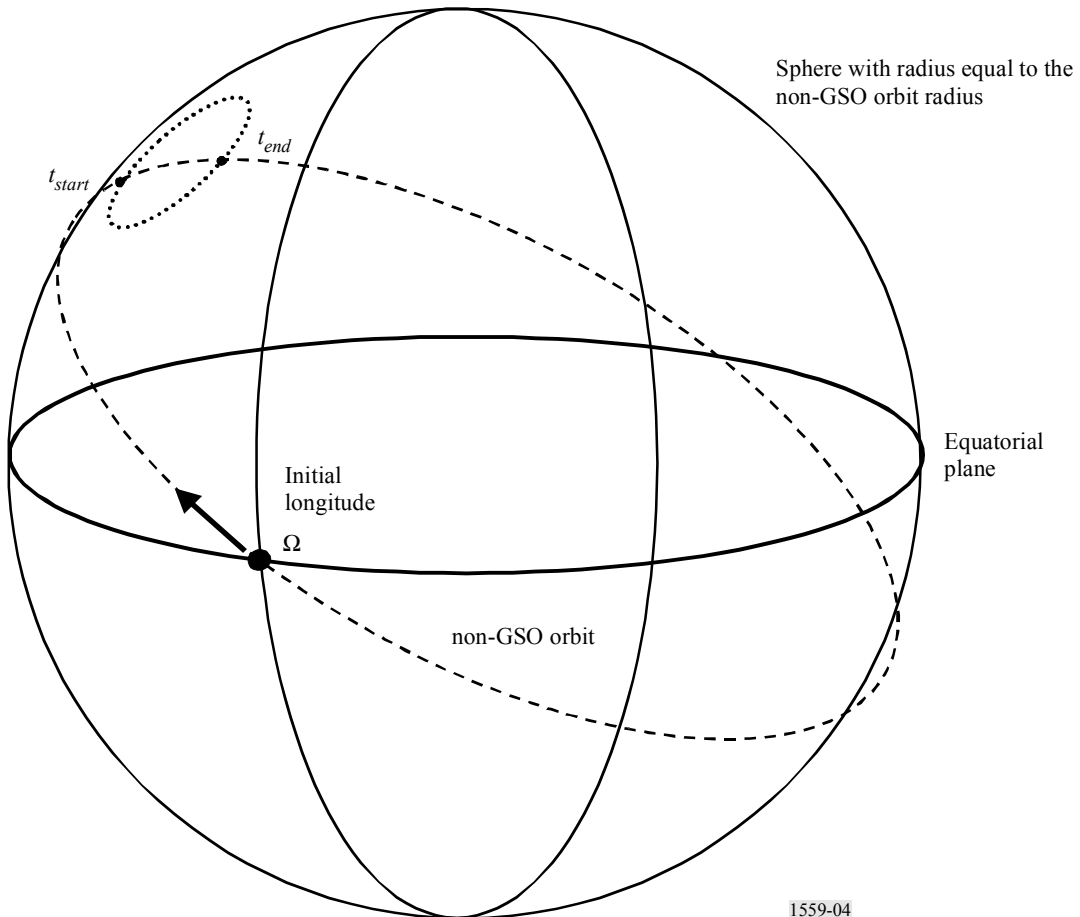
$$\text{End_time}_k = \max(\text{int_}t_1, \text{int_}t_2) + t_k \quad (27)$$

for all k such that

$$\min(\Psi_{ij}) \leq \Omega_k \leq \max(\Psi_{ij}) \quad (28)$$

These Start and End times are then computed for all of the non-GSO spacecraft.

FIGURE 4
Intersection of non-GSO orbit with antenna beam



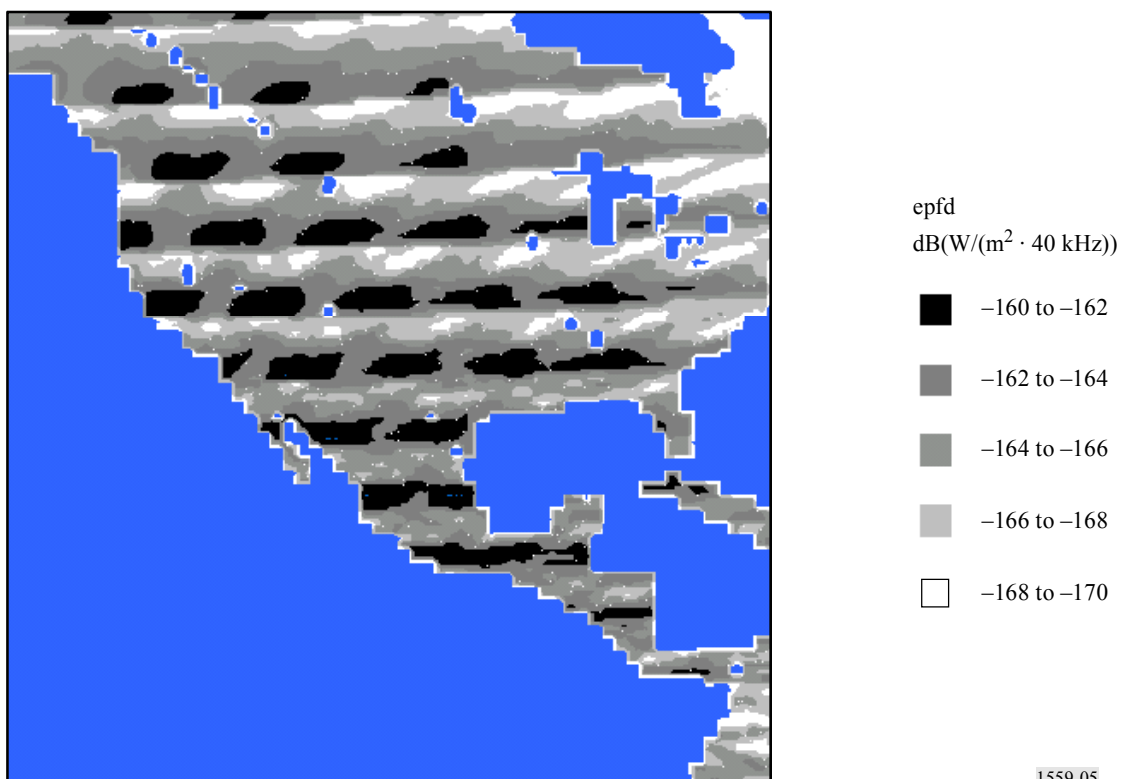
9 Example computation

Non-GSO systems will naturally have a variation in maximum epfd_{\downarrow} levels based on latitude, distance from the nearest non-GSO gateway and elevation angle from the GSO ground station to the supporting GSO spacecraft.

An example of such a map is shown in Fig. 5. This map assumes the use of hypothetical pfd masks according to Recommendation ITU-R S.1503 (a fully loaded system, an envelope of all possible scheduling algorithms), and non-repeating ground tracks. It should be noted that the methodology is also valid for repeating ground-tracks and other operating assumptions. These maps may be used for sensitive GSO link planning purposes, as they represent an upper bound on the interference distribution, even though the maps generated by the system in operation may well show generally lower levels of maximum epfd_{\downarrow} for any geographical location.

FIGURE 5

Example of geographic distribution of maximum epfd levels



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10 Future work

This Recommendation only treats the case of non-GSO systems that use circular orbits. It would be useful to further develop the methodology so that it could be used for the case of non-GSO systems that use elliptical orbits.