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(01/2010)

Allowable error performance for a hypothetical reference digital path based on synchronous digital hierarchy

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Fixed-satellite service

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RECOMMENDATION ITU-R S.1521-1

Allowable error performance for a hypothetical reference digital path based on synchronous digital hierarchy

(2001-2010)

Scope

The fixed-satellite service (FSS) plays an important role in providing reliable international digital communications. Because of the integration with the terrestrial facilities, a satellite link should be designed to fulfil requirements that are compatible with terrestrial systems. ITU-T Recommendation G.828 specifies performance parameters and objectives for international synchronous digital paths which are intended to carry synchronous digital hierarchy (SDH) and asynchronous transfer mode (ATM) traffic. In response to those objectives, this Recommendation gives guidance on bit-error probability (BEP) or bit-error rate (BER) design masks which can fully comply with the requirements of ITU-T Recommendation G.828.

The ITU Radiocommunication Assembly,

considering

a) that satellites operating in the fixed-satellite service play an important role in providing reliable international digital communications;

b) that satellite link performance must be sufficient to allow compliance with overall end-to-end performance objectives and in turn end-user quality objectives;

c) that satellite link performance is generally distance independent;

d) that Recommendation ITU-R S.1062 specifies satellite link performance objectives which comply with the objectives specified in ITU-T Recommendation G.826;

e) that the error performance for international constant bit-rate synchronous digital paths which are intended to carry ATM traffic as defined in ITU-T Recommendation I.356 has been specified in ITU-T Recommendation G.828;

f) that Recommendation ITU-R S.1429 specifies the error performance allowance due to interference between different satellite systems;

g) that Recommendation ITU-R S.1323 specifies how to calculate the operating margins to allow for both fading and time-varying interference;

h) that in defining error performance criteria, it is necessary to take into account all error-inducing mechanisms, especially time-varying propagation conditions and interference;

j) that satellite systems can be designed to meet a wide range of performance requirements,

recommends

1 that satellite links within the public network and intended to carry SDH and ATM traffic should be designed to at least meet the specifications set forth in this Recommendation which is based upon ITU-T Recommendation G.828 (see Annex 1);

2 that the methodology explained in Annex 2 should be used to generate the necessary BEP design masks, see also Note 2. To fully comply with ITU-T Recommendation G.828, the BEP divided by the average number of errors per burst (BEP/α, see § 3 of Annex 2) at the output of either end of a two-way hypothetical reference digital path (HRDP) should not exceed, during the total time including the worst month, the design masks defined in Table 1 and also in the BEP masks given in Fig. 2 in Annex 2;

TABLE 1

Bit rate (kbit/s)	Percentage of total time (worst month)	BEP/α
1 664	0.2 2.0 10.0	1×10^{-9} 1×10^{-9} 1×10^{-9}
2 240	0.2 2.0 10.0	1×10^{-9} 1×10^{-9} 1×10^{-9}
6 848	0.2 2.0 10.0	1×10^{-9} 7×10^{-10} 6×10^{-10}
48 960	0.2 2.0 10.0	1×10^{-9} 2×10^{-10} 1×10^{-10}
150 336	0.2 2.0 10.0	1×10^{-9} 2×10^{-10} 9×10^{-11}
601 334	0.2 2.0 10.0	To be determined To be determined To be determined

3 that the following Notes are a part of this Recommendation:

NOTE 1 – The HRDP referred to is specified in Recommendation ITU-R S.521.

NOTE 2 – The BEP ratios given in Table 1 could be estimated by a BER measurement taken over a sufficiently long period of time. A method for measuring BERs as a function of a shorter percentage of time is given in Annex 1 of Recommendation ITU-R S.1062.

NOTE 3 – For ease of application, the values given in Table 1 are in terms of total time and represent the limits of a BEP performance model utilizing the method outlined in Annex 2. In arriving at the figures in Table 1 the errors occurring during the unavailable time have been excluded. The BEPs in Table 1 are not the only ones that meet the requirements of ITU-T Recommendation G.828. Other BEP masks may be used where appropriate to satisfy ITU-T Recommendation G.828.

NOTE 4 – This Recommendation applies to satellite systems operating below 15 GHz. The extension to systems operating at higher frequencies is the subject of further study.

NOTE 5 – A BEP value of 1×10^{-8} has been used as the unavailability threshold.

NOTE 6 – The objectives given in Table 1 are given in terms of percentage of the worst month. These monthly percentages correspond to the following yearly percentages:

- 10% of a month = 4.0% of year;
- 2% of a month = 0.6% of year;
- 0.2% of a month = 0.04% of year.

NOTE 7 – In order to comply with Table 1 at frequencies greater than 10 GHz (see also Note 4), it may be advantageous to make use of fade countermeasures including adaptive forward error correction (FEC) coding, power control or site diversity. Information on site diversity operation is given in Annex 1, Recommendation ITU-R S.1061.

NOTE 8 – The preferred method of verifying digital satellite link performance is on the basis of in-service measurements. These measurements would utilize the block error detection schemes, which are related to the inherent SDH block size and structure of the transmission system. FEC, scrambling and differential encoding have an impact on interpretation of the measurements (see Annex 2, § 3).

NOTE 9 – The error performance described in Table 1 was developed based on the use of an HRDP in the international portion of the link (e.g. switched international gateway-to-switched international gateway). Other applications of the HRDP within the connection are possible (e.g. end office-to-end office) and the error performance objectives can be adjusted accordingly.

NOTE 10 – The methods described in this Recommendation may be applied to the design of satellite links in private networks.

NOTE 11 – The performance objectives shall be met for the required transmission rate not for any higher rate created to support multiplexing or error correction. For instance, if the transmission rate over a satellite link is 6 Mbit/s and the required transmission rate between the end points is 2 Mbit/s, then the performance objectives for 2 Mbit/s transmission apply.

Annex 1

1 General

ITU-T Recommendation G.828 defines error performance parameters and objectives for international synchronous digital paths which are intended to carry SDH and ATM traffic. This Recommendation adopts all the definitions parameters and objectives defined therein. The objectives given in ITU-T Recommendation G.828 are defined as being independent of the physical network supporting the path.

In-service measurement of error rates at the SDH layer is supported by bit interleaved parity (BIP) codes carried in the SDH container headers.

1.1 Definitions

For convenience, the definitions in ITU-T Recommendation G.828 are repeated here.

Error performance measurements are based upon blocks whose size is consistent with the SDH frame structure and varies according to the bit rate, see Table 2.

1.1.1 A block

A block is a set of consecutive bits associated with the path; each bit belongs to one and only one block. Consecutive bits may not be contiguous in time.

1.1.2 Error events

- *Errored block (EB)*: A block in which one or more bits are in error.
- *Errored second (ES)*: A 1 s period with one or more EBs or at least one defect. Defects and related performance criteria are listed in Annex B of ITU-T Recommendation G.828.
- *Severely errored second (SES)*: A 1 s period which contains ≥30% EBs or at least one defect. SES is a subset of ES. To simplify measurements, the defect is used in the definition of SES instead of defining SES directly in terms of severe bit errors. However, it should be appreciated that there may exist error patterns that would not trigger a defect. Field experience will establish if this is a major problem in measuring errors.
- *Background block error (BBE)*: An EB not occurring as part of an SES.
- *Severely errored period (SEP)*: A sequence of between 3 and 9 consecutive SESs. The period is terminated by a second that is not a SES. Thus the SEP event is the same as the consecutive SES (CSES) event, as defined in ITU-T Recommendation G.784, with the lower threshold set to 3 s.

Table 2 gives the relationships between block sizes, bit rates, error detection code (EDC) and path types.

TABLE 2

Block sizes versus bit rates

Bit rate (kbit/s)	Path type	SDH block size used in ITU-T Recommendation G.828 (bits)	EDC
1 664	VC-11, TC-11	832	BIP-2
2 240	VC-12, TC-12	1 120	BIP-2
6 848	VC-2, TC-2	3 424	BIP-2
48 960	VC-3, TC-3	6 120	BIP-8
150 336	VC-4, TC-4	18 792	BIP-8
601 344	VC-4-4c, TC-4-4c	75 168	BIP-8
2 405 376	VC-4-16c, TC-4-16c	300 672	BIP-8
9 621 504	VC-4-64c, TC-4-64c	1 202 688	BIP-8

1.1.3 Error performance parameters

Error performance should only be evaluated whilst the path is in the available state. For a definition of the entry/exit criteria for the unavailable state, see ITU-T Recommendation G.828, Annex A.

- *Errored second ratio (ESR)*: The ratio of ES to total seconds in available time during a fixed measurement interval.
- *Severely errored second ratio (SESR)*: The ratio of SES to total seconds in available time during a fixed measurement interval.

- *Background block error ratio (BBER)*: The ratio of BBE to total blocks in available time during a fixed measurement interval. The count of total blocks excludes all blocks during SESs.
- *Severely errored period intensity (SEPI)*: The number of SEP events in available time, divided by the total available time in seconds. Thus the SEPI parameter has a unit of 1/s.

1.1.4 Block based measurements

Each block is monitored by means of a BIP EDC carried in the SDH header. When an error state is detected, it is not possible to determine whether a block or its controlling EDC bits are in error. Therefore if there is a discrepancy between the EDC and its controlled block, it shall always be assumed that the block is in error.

2 Error performance objectives

2.1 End-to-end objectives

Table 3 specifies the end-to-end objectives for a 27 500 km hypothetical reference path (HRP). The objectives applicable to a real path are derived from Table 3 using the allocation principles detailed in § 6.2 of ITU-T Recommendation G.828. Each direction of the path shall independently satisfy the allocated objectives for all parameters. The objectives are long-term objectives to be met over an evaluation period of typically 30 consecutive days.

TABLE 3

End-to-end error performance objectives for a 27 500 km international synchronous digital HRP as defined in ITU-T Recommendation G.828

Bit rate (kbit/s)	Path type	Blocks/s	ESR	SESR	BBER(1)	SEPI(2)
1 664	VC-11, TC-11	2 000	0.01	0.002	5×10^{-5}	0.0002/s
2 240	VC-12, TC-12	2 000	0.01	0.002	5×10^{-5}	0.0002/s
6 848	VC-2, TC-2	2 000	0.01	0.002	5×10^{-5}	0.0002/s
48 960	VC-3, TC-3	8 000	0.02	0.002	5×10^{-5}	0.0002/s
150 336	VC-4, TC-4	8 000	0.04	0.002	1×10^{-4}	0.0002/s
601 344	VC-4-4c, TC-4-4c	8 000	(3)	0.002	1×10^{-4}	0.0002/s

(1) This BBER objective corresponds to an equivalent BER of 8.3×10^{-10}, an improvement over the BER of 5.3×10^{-9} for the VC-4 rate. Equivalent BER is valuable as a rate-independent indication of error performance, as BBER objectives cannot remain constant as block sizes increase.

(2) Provisional value requiring further study.

(3) ESR objectives tend to lose significance for high bit rates and are therefore not specified for paths operating above 160 Mbit/s. However, a significant increase in ESR indicates a degrading transmission system. Therefore, for maintenance purposes ES monitoring should be implemented.

Synchronous digital paths operating at bit rates covered by this Recommendation may be carried by digital sections operating at higher bit rates. Such systems must meet their end-to-end objectives. For example, in SDH, an STM-1 section may carry a VC-4 path and therefore the STM-1 section should be designed to ensure compliance with the objectives of the VC-4 path.

Objectives are allocated to the national and international portions of a path. In the above example, if the STM-1 section does not form a complete national or international portion, the corresponding national/international allocation must be subdivided to determine the appropriate allocation for the digital section. This is outside the scope of this Recommendation.

2.2 Apportionment of end-to-end objectives

The levels of performance expected are apportioned between international and national portions of an HRP.

Further subdivision of these objectives is beyond the scope of this Recommendation.

2.2.1 Allocation to the national portion

Each national portion is allocated a fixed allowance of 17.5% of the end-to-end objective plus a distance-based allocation.

When a national portion includes a satellite hop, a total allowance of 42% of the end-to-end objectives in Table 3 is allocated to this national portion. The 42% allowance completely replaces both the distance-based allowance and the 17.5% block allowance.

2.2.2 Allocation to the international portion

Independent of the distance spanned, any satellite hop in the international portion receives a 35% allocation of the objectives in Table 3. The 35% allowance completely replaces all distance-based and block allowances given to parts of the international portion spanned by the satellite hop.

3 Satellite HRDP performance objectives

TABLE 4

Satellite HRDP performance objectives for an international SDH link

Rate (kbit/s)	**1 664 (VC-11)**	**2 240 (VC-12)**	**6 848 (VC-2)**	**48 960 (VC-3)**	**150 336 (VC-4)**	**601 334 (VC-4-4c)**
ESR	0.0035	0.0035	0.0035	0.007	0.014	(1)
SESR	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
BBER	1.75×10^{-5}	1.75×10^{-5}	1.75×10^{-5}	1.75×10^{-5}	0.35×10^{-4}	0.35×10^{-4}

(1) Due to the lack of information on the performance of paths operating above 160 Mbit/s, no ESR objectives are recommended at this time. Nevertheless, ESR processing should be implemented within any system operating at these rates for maintenance or monitoring purposes.

Annex 2

BEP mask derivation

1 Introduction

The parameters and objectives defined in ITU-T Recommendation G.828 are not directly appropriate for satellite transmission system design. They must be transformed into a BEP versus percentage-of-time distribution, also called a BEP mask, in such a way that any digital transmission system designed to meet the mask would also meet the objectives of the Recommendation. The transform explained in this Annex, however, does not result in one unique mask.

This annex explains the methodology for creating a BEP mask.

2 Probability of the basic events

It is well known that transmission errors over satellite links occur in bursts where the average number of errors per burst is, among other factors, a function of the scrambler and the FEC code. Consequently, a successful model of the digital performance over satellite links has to take into account this bursty nature.

One statistical model that can adequately represent the random occurrence of bursts is the Neyman-A contiguous distribution, where the probability of *k* errors occurring in *N* bits, $P(k)$, is:

$$P(k) = \frac{\alpha^k}{k!}\, e^{-\frac{BEP \cdot N}{\alpha}} \sum_{j=0}^{\infty} \frac{j^k}{j!} \left(\frac{BEP \cdot N}{\alpha} \right)^j e^{-j\alpha} \tag{1}$$

where:

α: average number of errored bits in a burst of errors

BEP: bit-error probability.

If $N = N_B$ is taken as the number of bits in a block of data, then the probability of zero errors in a block is:

$$P(0) = e^{-\frac{BEP \cdot N_B}{\alpha}} \sum_{j=0}^{\infty} \left[\left(\frac{BEP \cdot N_B}{\alpha} \right)^j \Big/ j! \right] e^{-j\alpha} \cong e^{-\frac{BEP \cdot N_B}{\alpha}} \tag{2}$$

for all practical values of α.

The probability of an errored block, P_{EB}, is then given by:

$$P_{EB} = 1 - P(0) = 1 - e^{-\frac{BEP \cdot N_B}{\alpha}} = 1 - e^{-N_B \cdot BEP_{CRC}(t)} \tag{3}$$

where $BEP_{CRC}(t) = BEP/\alpha$, and the BEP_{CRC} is explicitly shown as a function of time. The probability of an ES, $P_{ES}(t)$, can then be expressed as:

$$P_{ES}(t) = 1 - e^{\,n \cdot P_{EB}(t)} \tag{4}$$

where *n* is the number of blocks per second.

Since the probability of k errored blocks in a total of n blocks, $P_{n,k}(t)$, is given by:

$$P_{n,k}(t) = \frac{n!}{(n-k)!k!}(1-P_{EB}(t))^{n-k}\ P_{EB}^{k}(t) \tag{5}$$

then, the probability of an SES, $P_{SES}(t)$, is:

$$P_{SES}(t) = \sum_{k=0.3n}^{n} P_{n,k}(t) = 1 - \sum_{k=0}^{0.3n-1} P_{n,k}(t) = 1 - \sum_{k=0}^{0.3n-1} \frac{n!}{(n-k)!k!}(1-P_{EB}(t))^{n-k}\ P_{EB}^{k}(t) \tag{6}$$

2.1 Generation of masks

Assuming a general form of the mask, as in Fig. 1, and using the probability formula the ESR (defined as the total ES, i.e. seconds with one or more errored blocks) divided by the total available seconds, T_a, is given by:

$$ESR = \frac{\int_{T_a} P_{ES}(t)}{T_a} \tag{7}$$

Similarly, the SESR is given by:

$$SESR = \frac{\int_{T_a} P_{SES}(t)}{T_a} \tag{8}$$

If $P_{ES}(t)$ and $P_{SES}(t)$ are assumed to be piece-wise constant in time, then ESR and SESR can be expressed as:

$$ESR = \sum_{i=1}^{M} P_{ES_i} \cdot \Delta t_i \tag{9}$$

and

$$SESR = \sum_{i=1}^{M} P_{ES_i} \cdot \Delta t_i \tag{10}$$

where M is the total number of time intervals, $P_{ES_i}(t)$ and $P_{SES_i}(t)$ are the probability of an ES and SES respectively in the i-th time interval divided by T_a.

BBER is defined as the ratio between EBs to the total blocks during available seconds, excluding all blocks during SES. Thus:

$$BBER = \frac{\int_{T_a} \left(\sum_{k=1}^{0.3n} P_{n,k}(t) \cdot k \right) \mathrm{d}t}{n \cdot \left(T_a - \int_{T_a} P_{SES}(t) \cdot \mathrm{d}t \right)} = \frac{\sum_{k=1}^{0.3n} \left(\frac{1}{T_a} \int_{T_a} P_{n,k}(t) \cdot \mathrm{d}t \right) \cdot k}{n \cdot (1 - SESR)} \tag{11}$$

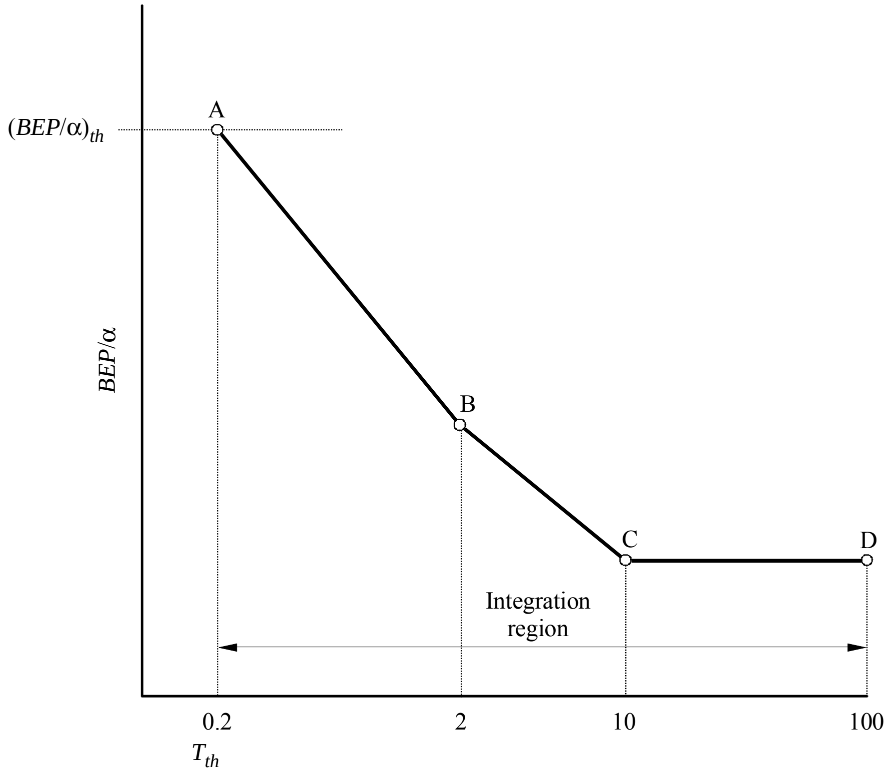
If $\overline{P_{n,k}} = \dfrac{\int_{T_a} P_{n,k} \cdot \mathrm{d}t}{T_a}$ is set, then BBER can be expressed as:

$$BBER = \frac{\sum_{k=1}^{0.3n} \overline{P_{n,k}} \cdot k}{n \cdot (1 - SESR)} \tag{12}$$

In selecting the value of BEP_{th}/α for the generation of the masks, however, the occurrence of incorrect pointer interpretations (IPI), which is crucial to the proper operation of SDH links, must be considered. Measurements have shown that IPI increases significantly at or beyond a BEP of approximately 1×10^{-7}, which is significantly lower than that at which satellite modems experience loss of synchronization. In view of this, further study will be required to define a BEP threshold for IPI, denoted here by BEP_{IPI}, where the SDH satellite link becomes unavailable since this will be the limiting factor. For the purposes of this Recommendation, a BEP_{th}/α (equal to BEP_{IPI}/α) value of 1×10^{-9} has been used.

FIGURE 1

General form of mask



Total time, T (% of the worst month)

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This method will result in an infinite number of masks meeting the ITU-T Recommendation G.828 performance objectives. Therefore, the following process is used to define a mask and to determine points B, C and D of the mask (see Fig. 2):

Step 1: Set the value $BEP_{th}/\alpha = 1 \times 10^{-9}$.

Step 2: Set the unavailability threshold time value, T_{th}, (T_{th} = 0.2%) such that point A corresponds to the value BEP_{th} / α.

Step 3: Set the mask values at 2%, 10% and 100% of the time (points B, C and D).

Step 4: Calculate ESR, SESR and BBER by integrating over the region between T_{th} (0.2%) and 100%. In deriving these masks, it is assumed that the satellite link is unavailable for BEP values above BEP_{th} / α

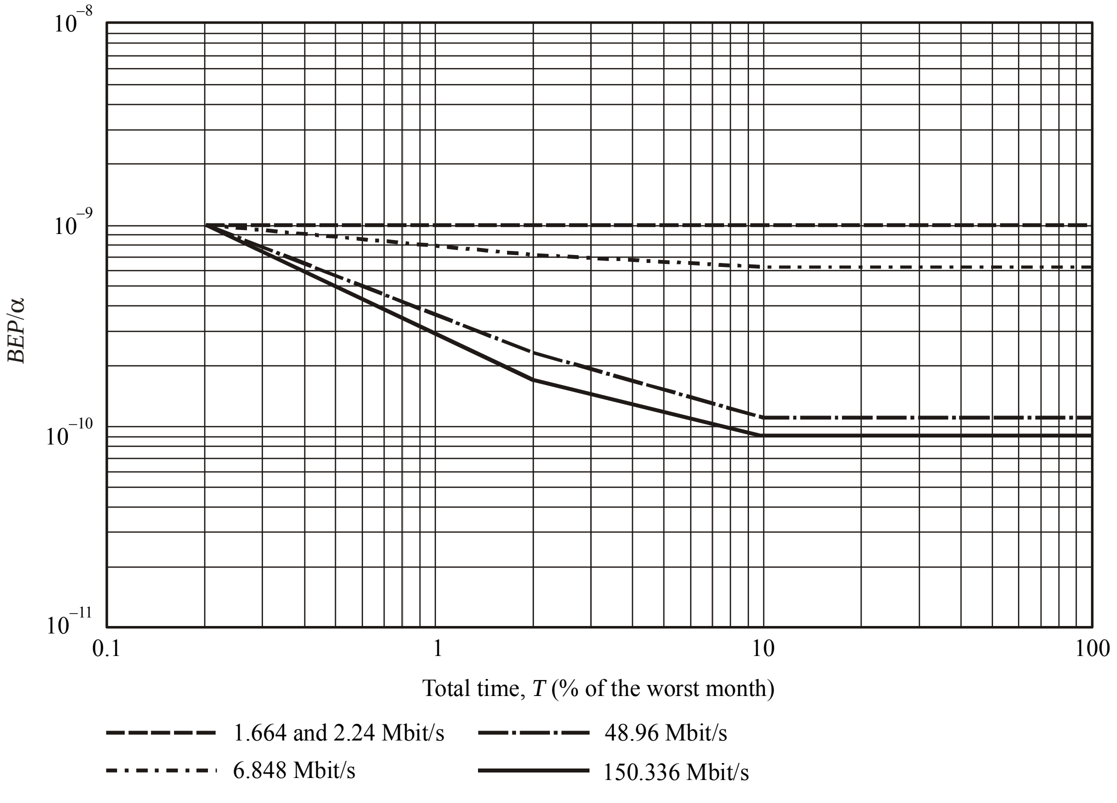
Step 5: Repeat Step 3 and 4 until all of the parameters (ESR, SESR and BBER) meet the objectives in Table 3.

The above process ensures a link unavailability of 0.2% of the time.

Using the above process with the additional assumptions that BEP/α corresponding to points C and D are the same, an example set of masks for various transmission rates was generated and is shown in Fig. 2.

FIGURE 2

Generated masks for satellite hops



$\alpha = 10$
BEP threshold time is 1×10^{-8}
Available time from 0.002 to 1
All models outperform BBER criteria
Model for 1.664 and 2.24 Mbit/s also outperform ESR criteria

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3 Relationship between BER and error-event ratio

It is well known that errors on satellite links employing FEC and scrambler schemes tend to occur in clusters. The appearance of the clusters, which can also be called error events, is random following a Poisson distribution. The resulting block error rate is the same as if it were caused by randomly (Poisson distributed) occurring bit errors with a BER, BER/α, where α (used in § 2.1 to account for the burstiness of errors) is the average number of errored bits within a cluster, α also represents the ratio between the BER and error-event ratio. For example, in a random binary error channel without FEC and scrambler, α is considered to be one. With higher order modulation schemes, however, α may be larger than one.

In a given FEC scheme, theoretical values of α can be estimated using the weight distribution of the FEC scheme. The background of the theoretical value derivation is given in § 3.1. Statistical properties of the clusters of errors are dependent on the FEC/scrambler scheme used. Computer simulations and measurements of various FEC schemes (without scrambler or differential encoding) were used to determine the factor α. An additive white Gaussian channel is assumed in the simulation. These results are given in § 3.2 to § 3.6.

3.1 Derivation of the average number of errored bits in a cluster

Given an (n,k) systematic block code C, its well-known weight enumerating function (WEF) is:

$$B^C(H) \triangleq \sum_{i=0}^{n} B_i H^i \tag{13}$$

where:

B_i: (integer) number of code words with Hamming weight (number of ones) i

H: dummy variable.

The WEF of a code can be used to compute the exact expression of the probability of undetected errors and an upper bound to the word error probability.

The input-redundancy weight enumerating function (IRWEF) of the code can be defined as:

$$A^C(W,Z) \triangleq \sum_{w,j} A_{w,j} W^w Z^j \tag{14}$$

where $A_{w,j}$ denotes the (integer) number of code words generated by an input information word of Hamming weight w whose parity check bits have Hamming weight j, so that the overall Hamming weight is $w + j$. The IRWEF shows the separate contributions of the information and of the parity check bits to the total Hamming weight of the code words, and thus provides additional information on the (Hamming) weight profile of the code.

By using the above expression, the BEP, P_b, can be upper-bounded by:

$$P_b \le \sum_{m=d_{min}}^{\infty} D_m P(R''_m | C_0) \tag{15}$$

where d_{min} is the minimum distance of the code, $P(R''_m | C_0)$ is the probability of the decoder selecting the code word of weight m provided the transmitted code word is all-zero code word, and:

$$D_m = \sum_{j+w=m} \frac{w}{k} A_{w,j} \tag{16}$$

Therefore, the average number of bits in a cluster α will be the mean value of w, and leading to:

$$\overline{w} = \sum_{m=d_{min}}^{\infty} \sum_{m=w+j} w A_{w,j} P_m \tag{17}$$

where P_m is the probability of error events with m errors in all error events. Because P_m decreases rapidly with m, especially in low BEP values, $\overline{w}$ can be approximated by:

$$\overline{w} \approx \sum_{d_{min}=w+j} w\ A_{w,j} P_{d_{min}} \tag{18}$$

3.2 Factors for binary BCH codes

Using equation (19), α values for systematic BCH codes can be estimated. Table 5 shows weight distribution of the (7,4) BCH code, and the minimum distance of the (7,4) code is 3. Therefore, α for the code can be estimated as follows:

$$\overline{w}_{(7,4)} = \alpha_{(7,4)} \approx 1 \times \frac{3}{7} + 2 \times \frac{3}{7} + 3 \times \frac{1}{7} \cong 1.7 \tag{19}$$

TABLE 5

Weight distribution of the (7,4) BCH code

w	***j***	$A_{w,j}$
0	0	1
1	2	3
1	3	1
2	1	3
2	2	3
3	0	1
3	1	3
4	3	1

Table 6 shows the estimated α for various systematic BCH codes, and Table 7 compares the simulation results for the (15,11) BCH code to the estimated results. As BER gets lower, the estimated value approximates to the simulation value.

For non-systematic codes, when decoding fails, approximately half of the information word will be in error. In this case, α can be approximated to *k*/2.

TABLE 6

Theoretical α values estimated for various BCH codes

(*n*,*k*) BCH code	α	**(*n*,*k*) extended code**	α	**(*n*,*k*) expurgated code**	α
(15,11)	2.20	(16,11)	2.75	(15,10)	2.67
(31,26)	2.52	(32,26)	3.25	(31,25)	3.23
(31,21)	3.73	(32,21)	4.56	(31,20)	4.53
(63,57)	2.06	(64,57)	2.96	(63,56)	2.96
(63,51)	4.07	(64,51)	4.50		

TABLE 7

Comparison of theoretical and simulated α values for the (15,11) BCH code

BER	Simulated α	Theoretical α
2.88×10^{-2}	2.60	2.2
4.69×10^{-3}	2.37	
5.57×10^{-4}	2.36	
2.36×10^{-5}	2.33	

3.3 Factors for convolutional codes

A similar approach can be applied to convolutional codes. For known convolutional codes, various studies identified their weight distributions in terms of a_d, the number of code words of distance d, and c_d, the sum of bit errors (the information error weight) for code words of distance d. With the same approximation to the binary BCH codes, $\overline{w}\,(= \alpha)$ for the convolutional codes can be approximated to $(c_{d_f})/(a_{d_f})$, where d_f is the free distance of the code.

Table 8 shows weight distributions of popular convolutional codes, and Table 9 compares the theoretically estimated α values and simulated values. As was confirmed in the binary BCH codes, the estimated α values are nearly equal to the simulated values in the low BER ranges.

TABLE 8

Weight distribution of convolutional codes

Code rate R	Constraint length K	Generator (octal)	d_f	$(a_d, d = d_f, d = d_f + 1, d = d_f + 2, \ldots)$ $(c_d, d = d_f, d = d_f + 1, d = d_f + 2, \ldots)$
1/2	7	133, 171	10	(11, 0, 38, 0, 193, 0, 1 331, 0, 7 275, ...) (36, 0, 211, 0, 1 404, 0, 11 633, ...)
	9	561, 753	12	(11, 0, 50, 0, 286, 0, 1 630, 0, 9 639, ...) (33, 0, 281, 0, 2 179, 0, 15 035, ...)
2/3(1)	7	133, 171	6	(1, 16, 48, 158, 642, 2 435, 9 174 ...) (3, 70, 285, 1 276, 6 160, 27 128, ...)
7/8(1)	7	133, 171	3	(2, 42, 468, 4 939, 52 821 ...) (14, 389, 6 792, 97 243, 1 317 944 ...)

(1) Punctured codes from R 1/2 code with $K = 7$.

3.4 Factors for concatenated codes

For a concatenated code with a Reed-Solomon (RS) outer code and a convolutional inner code, the α value is directly related to the weight distribution of the RS code because the RS code is outer code. The α value for the RS codes can be found using the same rule as used in the binary BCH code, if maximum likelihood decoding is used. In this case, the binary weight distribution of the RS codes should be found.

TABLE 9

Comparison of theoretical and simulated α values for convolutional codes

Code rate R	**Constraint length K**	**Generator (octal)**	d_f	**α (estimated)**	**BER**	**α (simulated)**
1/2	7	133, 171	10	3.27	1.74×10^{-2}	7.21
					1.91×10^{-3}	5.68
					1.05×10^{-4}	3.74
					5.05×10^{-6}	3.48
					1.07×10^{-7}	3.00
	9	561, 753	12	3.00	1.22×10^{-2}	13.00
					1.77×10^{-3}	11.56
					2.10×10^{-5}	4.38
					4.20×10^{-7}	3.96
2/3	7	133, 171	6	3.00	3.61×10^{-2}	8.00
					7.86×10^{-4}	7.14
					2.96×10^{-6}	5.32
					2.14×10^{-7}	5.67
7/8	7	133, 171	3	7.00	6.24×10^{-2}	9.08
					2.68×10^{-2}	8.85
					9.82×10^{-3}	7.77
					1.77×10^{-5}	7.57
					1.49×10^{-6}	7.29

Table 10 shows simulated α values for the RS codes in the concatenated coding scheme specified in Recommendations ITU-R BO.1724 and ITU-R S.1709. The RS (204,188) code shortened from the original RS (255,239) code is used. The RS (71,55) shortened code is also used for a different packet size.

TABLE 10

Simulated α values for RS codes in the concatenated coding scheme

(n,k) RS code	**BER**	**α**	**(n,k) RS code**	**BER**	**α**
(204,188)	7.74×10^{-3}	12.80	(71,55)	6.17×10^{-3}	8.47
	5.19×10^{-4}	9.14		2.03×10^{-4}	7.74
	1.02×10^{-6}	8.58		2.02×10^{-7}	7.32

3.5 Factors for turbo codes

For turbo codes, a similar approach to convolutional codes can be used because they are based on convolutional codes. Table 11 shows weight distributions of turbo codes specified in Recommendations ITU-R BO.1724 and ITU-R S.1709, and Table 12 shows corresponding estimated α values. Table 13 shows the simulated α values for the packet size of 53 bytes. Because the turbo codes use an iterative decoding algorithm, α values and BER depend on the decoding algorithm and the number of iterations. In the simulation, a max-log MAP decoding algorithm was used and α values were estimated at iterations of 6 and 15. Because the theoretical values estimated in Table 14 can be considered as a lower bound, they are smaller than the simulated values in Table 15.

TABLE 11

Weight distribution of turbo codes ($d_f/a_d/c_d$)

Packet size (bytes)	R = 1/3	R = 1/2	R = 2/3	R = 3/4	R = 6/7
53	31/106/954	18/159/954	11/159/901	7/10/50	4/9/27
	32/265/1643	19/159/1431	12/265/1325	8/85/375	5/194/719
	33/106/901	20/530/3551	13/1802/11342	9/486/2335	6/1228/5371
188	33/3476/3384	19/376/3384	12/188/1316	9/27/171	6/199/826
	35/376/3760	20/376/3008	14/752/5264	10/148/1025	7/1578/7269
	36/752/6392	22/752/6768	15/1504/12220	11/1462/9674	8/9144/49558

TABLE 12

Theoretically approximated α values for turbo codes

Packet size (bytes)	R = 1/3	R = 1/2	R = 2/3	R = 3/4	R = 6/7
53	9.00	6.00	5.67	5.00	3.00
	6.20	9.00	5.00	4.41	3.70
	8.50	6.70	6.29	4.80	4.37
752	9.00	9.00	7.00	6.33	4.15
	10.00	8.00	7.00	6.93	4.60
	8.50	9.00	8.13	6.62	5.42

TABLE 13

Simulated α values for turbo codes

Iteration number	R = 1/3 BER/α	R = 2/5 BER/α	R = 1/2 BER/α	R = 3/4 BER/α	R = 6/7 BER/α
6	$5.58 \times 10^{-5}/16.8$	$3.79 \times 10^{-5}/16.6$	$1.39 \times 10^{-4}/21.5$	$9.53 \times 10^{-4}/15.9$	$3.44 \times 10^{-5}/6.8$
	$9.28 \times 10^{-6}/14.0$	$5.56 \times 10^{-6}/12.8$	$2.24 \times 10^{-5}/17.1$	$3.47 \times 10^{-5}/11.3$	$2.34 \times 10^{-6}/5.2$
	$1.42 \times 10^{-6}/10.6$	$9.68 \times 10^{-7}/10.6$	$5.69 \times 10^{-7}/9.0$	$9.89 \times 10^{-7}/7.8$	$2.53 \times 10^{-7}/4.1$
15	$2.25 \times 10^{-5}/23.7$	$1.57 \times 10^{-5}/20.8$	$6.36 \times 10^{-5}/26.6$	$6.46 \times 10^{-4}/18.3$	$2.67 \times 10^{-5}/7.0$
	$3.28 \times 10^{-6}/16.5$	$2.41 \times 10^{-6}/14.5$	$9.30 \times 10^{-6}/18.9$	$1.89 \times 10^{-5}/12.2$	$1.74 \times 10^{-6}/4.8$
	$5.62 \times 10^{-7}/11.6$	$4.25 \times 10^{-7}/10.8$	$3.02 \times 10^{-7}/8.9$	$6.02 \times 10^{-7}/7.9$	$1.78 \times 10^{-7}/4.3$

3.6 Factors for block turbo codes

Block turbo codes (BTCs) are product codes that are decoded iteratively. The minimum distance of a product code is the product of the minimum distances of its constituent codes. For example, the minimum distance of m-dimensional product code with the same constituent code with minimum distance of d_{min} will be $(d_{min})^m$. Using the same principle, the α value for a BTC α_{BTC} can be represented as follows:

$$\alpha_{BTC} = \alpha_{c_1} \cdot \alpha_{c_2} \cdots \alpha_{c_m} \tag{20}$$

where α_{c_i} is the α value for the i-th constituent code. The binary systematic codes demonstrated in § 3.2 are usually used as constituent codes.

Table 14 shows theoretically estimated α_{BTC} using equation (20), where the same constituent codes previously used are assumed in the BTC. Therefore, the α_c in Table 14 is the same as the values in Table 6. Tables 15 and 16 compare the theoretically estimated values and simulated values for two-dimensional BTCs. As confirmed in § 3.2 and 3.3, the estimated values are nearly equal to the simulated values in low BER ranges.

TABLE 14

Theoretically approximated values for block turbo codes

(n,k) extended code	d_{min}	α_c	**2-dimensional** α_{BTC}	**3-dimensional** α_{BTC}
(16,11)	4	2.75	7.56	20.80
(32,26)	4	3.25	10.56	34.33
(32,21)	6	4.56	20.79	94.82
(64,57)	4	2.96	8.76	25.93
(64,51)	6	4.50	20.25	91.13

TABLE 15

Comparison of theoretical and simulated α values for the (16,11) × (16,11) BTC

E_b/N_0 **(dB)**	**BER**	α_{BTC}	**Constituent code**	
			BER	α_c
1.0	4.41×10^{-2}	14.50	1.25×10^{-1}	2.82
2.0	3.43×10^{-3}	10.35	7.82×10^{-2}	2.88
2.5	4.24×10^{-4}	7.46	5.97×10^{-2}	2.52
3.0	8.30×10^{-5}	7.25	4.31×10^{-2}	2.82
3.5	8.51×10^{-6}	7.31	2.97×10^{-2}	2.99

TABLE 16

Comparison of theoretical and simulated α values for the (32,26) × (32,26) BTC

E_b/N_0 (dB)	BER	α_{BTC}	Constituent code	
			BER	α_c
2.0	4.19×10^{-3}	31.57	5.96×10^{-2}	3.88
3.0	7.80×10^{-6}	11.21	3.10×10^{-2}	3.33
3.3	2.10×10^{-6}	9.76	2.35×10^{-2}	3.15

3.7 Other measurement results and summary

Laboratory measurements of the INTELSAT IDR type digital transmissions (FEC $R = 3/4$ plus scrambler) led to an $\alpha = 10$ over the range of BER 1×10^{-4} to 1×10^{-11}. An $\alpha = 5$ was determined in the same measurements for the INTELSAT IBS-type digital transmissions (FEC $R = 1/2$ plus scrambler).

From the results investigated, it is shown that α is the function of the weight distribution of the FEC scheme and the BER. The impact of parameter α on the performance model could be assessed as follows.

The masks in Fig. 2 were generated using $\alpha = 10$. If, for example, no FEC/scrambler ($\alpha = 1$) were used, the models would be shifted by one decade and the BER requirements would be more stringent by one decade.

4 Conclusions

Studies have shown that the masks required to meet the requirements of ITU-T Recommendation G.828 are transmission rate dependent.

The design masks are also dependent on the error distribution, which in turn are influenced by the FEC/scrambler scheme employed.

Service requirements also need to be taken into account in deriving the design masks.

5 List of acronyms/abbreviations

ATM	Asynchronous transfer mode
BBE	Background block error
BBER	Background block error ratio
BEP	Bit error probability
BER	Bit error ratio
BIP	Bit interleaved parity
BTC	Block turbo code
CSES	Consecutive severely errored second
EB	Errored block
EDC	Error detection code
ES	Errored second

ESR	Errored second ratio
FEC	Forward error-correction
FSS	Fixed-satellite service
HRDP	Hypothetical reference digital path
HRP	Hypothetical reference path
IPI	Incorrect pointer interpretations
SDH	Synchronous digital hierarchy
SEP	Severely errored period
SEPI	Severely errored period intensity
SES	Severely errored second
SESR	Severely errored seconds ratio
STM	Synchronous transfer module
TC	Tandem connection
VC	Virtual container