International Telecommunication Union

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Recommendation ITU-R P.834-8
(09/2016)

Effects of tropospheric refraction on radiowave propagation

P Series
Radiowave propagation

International Telecommunication Union

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Note: *This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R P.834-8*

Effects of tropospheric refraction on radiowave propagation

(Question ITU-R 201/3)

(1992-1994-1997-1999-2003-2005-2007-2015-2016)

Scope

Recommendation ITU-R P.834 provides methods for the calculation of large-scale refractive effects in the atmosphere, including ray bending, ducting layers, the effective Earth radius, the apparent elevation and boresight angles in Earth-space paths and the effective radio path length.

Keywords

Tropospheric excess path length, Earth-space link, GNSS, numerical weather product, digital maps

The ITU Radiocommunication Assembly,

considering

a) that for the proper planning of terrestrial and Earth-space links it is necessary to have appropriate calculation procedures for assessing the refractivity effects on radio signals;

b) that procedures have been developed that allow the calculation of some refractive propagation effects on radio signals on terrestrial and Earth-space links,

recommends

1 that the information in Annex 1 should be used for the calculation of large-scale refractive effects.

Annex 1

1 Ray bending

A radio ray passing through the lower (non-ionized) layer of the atmosphere undergoes bending caused by the gradient of the refractive index. Since the refractive index varies mainly with altitude, only the vertical gradient of the refractive index is generally considered. The curvature at a point is therefore contained in the vertical plane and is expressed by:

$$\frac{1}{\rho} = -\frac{\cos\varphi}{n}\frac{\mathrm{d}n}{\mathrm{d}h} \tag{1}$$

where:

* Radiocommunication Study Group 3 made editorial amendments to this Recommendation in the year 2017 in accordance with Resolution ITU-R 1.

ρ: radius of curvature of the ray path

n: refractive index of the atmosphere

$\mathrm{d}n/\mathrm{d}h$: vertical gradient of refractive index

h: height of the point above the Earth's surface

φ: angle of the ray path with the horizontal at the point considered.

This ray curvature is defined as positive for ray bending towards the Earth's surface. This phenomenon is virtually independent of frequency, if the index gradient does not vary significantly over a distance equal to the wavelength.

2 Effective Earth radius

If the path is approximately horizontal, φ is close to zero. However, since n is very close to 1, equation (1) is simplified as follows:

$$\frac{1}{\rho} = -\frac{\mathrm{d}n}{\mathrm{d}h} \tag{2}$$

It is therefore clear that if the vertical gradient is constant, the trajectories are arcs of a circle.

If the height profile of refractivity is linear, i.e. the refractivity gradient is constant along the ray path, a transformation is possible that allows propagation to be considered as rectilinear. The transformation is to consider a hypothetical Earth of effective radius $R_e = k\,a$, with:

$$\frac{1}{ka} = \frac{1}{a} + \frac{\mathrm{d}n}{\mathrm{d}h} = \frac{1}{R_e} \tag{3}$$

where a is the actual Earth radius, and k is the effective earth radius factor (k-factor). With this geometrical transformation, ray trajectories are linear, irrespective of the elevation angle.

Strictly speaking, the refractivity gradient is only constant if the path is horizontal. In practice, for heights below 1 000 m the exponential model for the average refractive index profile (see Recommendation ITU-R P.453) can be approximated by a linear one. The corresponding k-factor is $k = 4/3$.

3 Modified refractive index

For some applications, for example for ray tracing, a modified refractive index or refractive modulus is used, defined in Recommendation ITU-R P.310. The refractive modulus M is given by:

$$M = N + \frac{h}{a} \tag{4}$$

h being the height of the point considered expressed in metres and a the Earth's radius expressed in thousands of kilometres. This transformation makes it possible to refer to propagation over a flat Earth surmounted by an atmosphere whose refractivity would be equal to the refractive modulus M.

4 Apparent boresight angle on slant paths

4.1 Introduction

In sharing studies it is necessary to estimate the apparent elevation angle of a space station taking account of atmospheric refraction. An appropriate calculation method is given below.

4.2 Visibility of space station

As described in § 1 above, a radio beam emitted from a station on the Earth's surface (h (km) altitude and θ (degrees) elevation angle) would be bent towards the Earth due to the effect of atmospheric refraction. The refraction correction, τ (degrees), can be evaluated by the following integral:

$$\tau = -\int_{h}^{\infty} \frac{n'(x)}{n(x) \cdot \tan\varphi} \, \mathrm{d}x \tag{5}$$

where φ is determined as follows on the basis of Snell's law in polar coordinates:

$$\cos\varphi = \frac{c}{(r + x) \cdot n(x)} \tag{6}$$

$$c = (r + h) \cdot n(h) \cdot \cos\theta \tag{7}$$

r: Earth's radius (6 370 km)

x: altitude (km).

Since the ray bending is very largely determined by the lower part of the atmosphere, for a typical atmosphere the refractive index at altitude x may be obtained from:

$$n(x) = 1 + a \cdot \exp(-bx) \tag{8}$$

where:

$a = 0.000315$

$b = 0.1361$.

This model is based on the exponential atmosphere for terrestrial propagation given in Recommendation ITU-R P.453. In addition, $n'(x)$ is the derivative of $n(x)$, i.e. $n'(x) = -ab \exp(-bx)$.

The values of $\tau(h, \theta)$ (degrees) have been evaluated under the condition of the reference atmosphere and it was found that the following numerical formula gives a good approximation:

$$\tau(h, \theta) = 1/\left[1.314 + 0.6437\,\theta + 0.02869\,\theta^2 + h\,(0.2305 + 0.09428\,\theta + 0.01096\,\theta^2) + 0.008583\,h^2\right] \tag{9}$$

The above formula has been derived as an approximation for $0 \leq h \leq 3$ km and $\theta_m \leq \theta \leq 10°$, where θ_m is the angle at which the radio beam is just intercepted by the surface of the Earth and is given by:

$$\theta_m = -\arccos\left(\frac{r}{r + h} \cdot \frac{n(0)}{n(h)}\right) \tag{10}$$

or, approximately, $\theta_m = -0.875\sqrt{h}$ (degrees).

Equation (9) also gives a reasonable approximation for $10° < \theta \leq 90°$.

Let the elevation angle of a space station be θ_0 (degrees) under free-space propagation conditions, and let the minimum elevation angle from a station on the Earth's surface for which the radio beam is not intercepted by the surface of the Earth be θ_m. The refraction correction corresponding to θ_m is τ (h, θ_m). Therefore, the space station is visible only when the following inequality holds:

$$\theta_m - \tau(h, \theta_m) \le \theta_0 \tag{11}$$

4.3 Estimation of the apparent elevation angle

When the inequality in equation (11) holds, the apparent elevation angle, θ (degrees), can be calculated, taking account of atmospheric refraction, by solving the following equation:

$$\theta - \tau(h, \theta) = \theta_0 \tag{12}$$

and the solution of equation (12) is given as follows:

$$\theta = \theta_0 + \tau_s(h, \theta_0) \tag{13}$$

where the values of τ_s (h, θ_0) are identical with those of τ (h, θ), but are expressed as a function of θ_0.

The function τ_s (h, θ_0) (degrees) can be closely approximated by the following numerical formula:

$$\tau_s(h, \theta_0) = 1/[1.728 + 0.5411\,\theta_0 + 0.03723\,\theta_0^2 + h(0.1815 + 0.06272\,\theta_0 + 0.01380\,\theta_0^2) + h^2(0.01727 + 0.008288\,\theta_0)] \tag{14}$$

The value of θ calculated by equation (13) is the apparent elevation angle.

4.4 Summary of calculations

Step 1: The elevation angle of a space station in free-space propagation conditions is designated as θ_0.

Step 2: By using equations (9) and (10), examine whether equation (11) holds or not. If the answer is no, the satellite is not visible and, therefore, no further calculations are required.

Step 3: If the answer in Step 2 is yes, calculate θ by using equations (13) and (14).

4.5 Measured results of apparent boresight angle

Table 1 presents the average angular deviation values for propagation through the total atmosphere. It summarizes experimental data obtained by radar techniques, with a radiometer and a radiotelescope. There are fluctuations about the apparent elevation angle due to local variations in the refractive index structure.

TABLE 1

Angular deviation values for propagation through the total atmosphere

Elevation angle, θ (degrees)	Average total angular deviation, Δθ (degrees)			
	Polar continental air	**Temperate continental air**	**Temperate maritime air**	**Tropical maritime air**
1	0.45	–	–	0.65
2	0.32	0.36	0.38	0.47
4	0.21	0.25	0.26	0.27
10	0.10	0.11	0.12	0.14
20		0.05	0.06	
30		0.03	0.04	
	Day-to-day variation in Δθ (for columns 1 and 4 only)			
1	0.1 r.m.s.			
10	0.007 r.m.s.			

5 Focusing and defocusing of a wave for propagation through the atmosphere

Changes in signal level may also result from spreading or narrowing of the antenna beam caused by the variation of atmospheric refraction with the elevation angle. This effect should be negligible for elevation angles above 3°.

The equation below can be used to calculate the signal loss or gain due to refraction effects for a wave passing through the total atmosphere

$$b = \pm 10 \log(B)$$

where:

$$B = 1 - \frac{0.5411 + 0.07446\theta_0 + h(0.06272 + 0.0276\theta_0) + h^2 0.008288}{\left[1.728 + 0.5411\theta_0 + 0.03723\theta_0^2 + h(0.1815 + 0.06272\theta_0 + 0.0138\theta_0^2) + h^2(0.01727 + 0.008288\theta_0)\right]^2}$$

- θ_0: elevation angle of the line connecting the transmitting and receiving points, (degrees) ($\theta_0 < 10°$)
- h: altitude of the lower point above sea level, (km) ($h < 3$ km)
- b: change in signal level for the wave passing through the atmosphere, compared to free-space conditions, (dB)

  the sign in the equation for b will be negative "–" for a transmitting source located near the Earth's surface and positive "+" for a source located outside the atmosphere.

6 Excess radio path length and its variations

Since the tropospheric refractive index is higher than unity and varies as a function of altitude, a wave propagating between the ground and a satellite has a radio path length exceeding the geometrical path length. The difference in length can be obtained by the following integral:

$$\Delta L = \int_A^B (n - 1)\, \mathrm{d}s \qquad (15)$$

where:

- s: length along the path
- n: refractive index
- A and B: path ends.

Equation (15) can be used only if the variation of the refractive index n along the path is known.

When the temperature T, the atmospheric pressure P and the relative humidity H are known at the ground level, the excess path length ΔL can be computed using the semi-empirical method explained below, which has been derived using the atmospheric radio-sounding profiles provided by a one-year campaign at 500 meteorological stations in 1979. In this method, the general expression of the excess path length ΔL is:

$$\Delta L = \frac{\Delta L_V}{\sin \varphi_0 (1 + k \cot^2 \varphi_0)^{1/2}} + \delta(\varphi_0, \Delta L_V) \tag{16}$$

where:

- φ_0: elevation angle at the observation point
- ΔL_V: vertical excess path length
- k and $\delta(\varphi_0, \Delta L_V)$: corrective terms, in the calculation of which the exponential atmosphere model is used.

The k factor takes into account the variation of the elevation angle along the path. The $\delta(\varphi_0, \Delta L_V)$ term expresses the effects of refraction (the path is not a straight line). This term is always very small, except at very low elevation angle and is neglected in the computation; it involves an error of only 3.5 cm for a φ_0 angle of 10° and of 0.1 mm for a φ_0 angle of 45°. It can be noted, moreover, that at very low elevation for which the δ term would not be negligible, the assumption of a plane stratified atmosphere, which is the basis of all methods of computation of the excess path length, is no longer valid.

The vertical excess path length (m) is given by:

$$\Delta L_V = 0.00227\, P + f(T)\, H \tag{17}$$

In the first term of the right-hand side of equation (17), P is the atmospheric pressure (hPa) at the observation point.

In the empirical second term, H is the relative humidity (%); the function of temperature $f(T)$ depends on the geographical location and is given by:

$$f(T) = a\, 10^{bT} \tag{18}$$

where:

- T is in °C
- a is in m/% of relative humidity
- b is in °C^{-1}.

Parameters a and b are given in Table 2 according to the geographical location.

TABLE 2

Location	***a*** **(m/%)**	***b*** **(°C^{-1})**
Coastal areas (islands, or locations less than 10 km away from sea shore)	5.5×10^{-4}	2.91×10^{-2}
Non-coastal equatorial areas	6.5×10^{-4}	2.73×10^{-2}
All other areas	7.3×10^{-4}	2.35×10^{-2}

To compute the corrective factor k of equation (16), an exponential variation with height h of the atmospheric refractivity N is assumed:

$$N(h) = N_s \exp(-h / h_0) \tag{19}$$

where N_s is the average value of refractivity at the Earth surface (see Recommendation ITU-R P.453) and h_0 is given by:

$$h_0 = 10^6 \frac{\Delta L_V}{N_s} \tag{20}$$

k is then computed from the following expression:

$$k = 1 - \left[\frac{n_s \, r_s}{n(h_0) \, r(h_o)} \right]^2 \tag{21}$$

where n_s and $n(h_0)$ are the values of the refractive index at the Earth surface and at height h_0 (given by equation (20)) respectively, and r_s and $r(h_0)$ are the corresponding distances to the centre of the Earth.

For Earth-space paths with elevation angles, θ the tropospheric excess path length, $\Delta L(\theta)$, (m) can be expressed as the sum of hydrostatic and wet components, $\Delta L_H(\theta)$ and $\Delta L_W(\theta)$.

The excess path length along a vertical path, ΔL_{Hv} and ΔL_{Wv} can be projected to the elevation angle, θ, greater than 3°, using two separate mapping function for the hydrostatic and wet components, $m_H(\theta)$ and $m_W(\theta)$:

$$\Delta L(\theta) = \Delta L_H(\theta) + \Delta L_W(\theta) = \Delta L_{Hv} \cdot m_H(\theta) + \Delta L_{Wv} \cdot m_W(\theta) \qquad \text{m} \tag{22}$$

The hydrostatic vertical component at the Earth surface, ΔL_{Hvs}, can be derived using:

$$\Delta L_{Hvs} = 10^{-6} \frac{R_d}{g_{ms}} k_1 \cdot p_s \qquad \text{m} \tag{22a}$$

The wet vertical component at the Earth surface, ΔL_{Wvs}, can be derived using:

$$\Delta L_{Wvs} = 10^{-6} \frac{R_d}{g_{ms}} \frac{k_2}{(\lambda + 1)} \cdot \frac{e_s}{T_{ms}} \qquad \text{m} \tag{22b}$$

where:

p_s, e_s: air total pressure and water vapour partial pressure at the Earth surface (hPa)

T_{ms}: mean temperature of the water vapour column above the surface (K)

λ: vapour pressure decrease factor

R_d : R/M_d = 287.0 (J/kg K)

R: molar gas constant = 8.314 (J/mol K)

M_d: dry air molar mass = 28.9644 (g/mol)

k_1 = 77.604 (K/hPa)

k_2 = 373 900 (K^2/hPa)

g_{ms}= $g_m(h_s)$

$g_m(h)$ = $9.784 \cdot (1 - 0.00266 \cdot \cos(2 \cdot \text{lat}) - 0.00028 \cdot h)$

= gravity acceleration at the mass centre of air from height h (m/s^2)

lat: Latitude of the location (radians)

h_s: height of the Earth surface above mean sea level (a.m.s.l.) (km)

h: height of the receiver above mean sea level (a.m.s.l.) (km).

For receivers located at a height, h different than the surface height, h_s, the hydrostatic and wet vertical component, $\Delta L_{Hv}(h)$ and $\Delta L_{Wv}(h)$, are given by:

$$\Delta L_{Hv}(h) = 10^{-6} \frac{R_d}{g_m(h)} k_1 \cdot p(h) \qquad \text{m} \qquad (23a)$$

$$\Delta L_{Wv}(h) = 10^{-6} \frac{R_d}{g_m(h)} \frac{k_2}{(\lambda+1)} \cdot \frac{e(h)}{T_m(h)} \qquad \text{m} \qquad (23b)$$

where:

The values of the input meteorological parameters at height h, $T_m(h)$, $e(h)$ and $p(h)$, can be derived from values at the Earth surface, T_{ms}, e_s and p_s, using the following equations:

$$T_m(h) = T_{ms} - \alpha_m \cdot (h - h_s) \qquad \text{K} \qquad (24a)$$

$$p(h) = p_s \left[1 - \frac{\alpha \cdot (h - h_s)}{T_s} \right]^{\frac{g}{R'_d \alpha}} \qquad \text{hPa} \qquad (24b)$$

$$e(h) = e_s \cdot \left[\frac{p(h)}{p_s} \right]^{\lambda+1} \qquad \text{hPa} \qquad (24c)$$

where:

α_m: lapse rate of the mean temperature of water vapour from the Earth surface (K/km);

T_s = air temperature at the Earth surface (K) = $T_{ms} \Big/ \left[1 - \frac{\alpha R'_d}{(\lambda+1) g} \right]$ K (24d)

α = lapse rate of air temperature from the Earth surface

$$\alpha = 0.5 \cdot \left[\frac{(\lambda+1)\cdot g}{R'_d} - \sqrt{\frac{(\lambda+1)\cdot g}{R'_d}\left[\frac{(\lambda+1)\cdot g}{R'_d} - 4\alpha_m\right]} \right] \quad \text{K/km} \tag{24e}$$

R'_d = Rd /1000 = 0.287 J/(g K) (24f)

$g = 9.806 \cdot (1 - 0.002637 \cdot \cos(2 \cdot lat) - 0.00031 \cdot h_s)$ = gravity acceleration at Earth surface m/s^2

All the input parameters of the model, p_s, e_s, T_{ms}, λ, and α_m, can be derived by assuming the meteorological parameters are characterized by the seasonal fluctuation:

$$X_i(D_y) = a1_i - a2_i \cos\left[2\pi \frac{(D_y - a3_i)}{365.25}\right] \tag{25a}$$

where:

X_i: p_s, e_s, T_{ms}, λ or α_m. Index i, 1 designates p_s, 2 designates e_s, 3 designates T_{ms}, 4 designates λ, 5 designates α_m

$a1_i$: average value of the parameter

$a2_i$: seasonal fluctuation of the parameter

$a3_i$: day of the minimum value of the parameter

D_y: day of the year (1, 2, ... , 365.25), 1 = 1 January, 32 = 1 February, 60.25 = 1 March.

The coefficients $a1$, $a2$ and $a3$ of the parameters p_s, e_s, T_{ms}, λ, and α_m, and the height of the reference level, h_{ref}, at which these coefficients have been calculated are an integral part of this Recommendation and are available in the form of digital maps provided in the file R-REC-P.834-8-201609-I!!ZIP-E.

The data is from 0° to 360° in longitude and from +90° to −90° in latitude, with a resolution of 1.5° in both latitude and longitude. The excess path length at any desired location and at any height above the surface of the Earth, h, can be derived by the following method:

a) For each of the five parameters, p_s, e_s, T_{ms}, λ, α_m, and the reference height, h_{ref}, determine the coefficients $a1_i$, $a2_i$ and $a3_i$ from the digital maps at the four grid points closest to the desired location.

b) Calculate the values of the five parameters, p_s, e_s, T_{ms}, λ or α_m, at the reference height, h_{ref}, for the day of the year D_y, X_{i1}, X_{i2}, X_{i3} and X_{i4} at the four closest grid points, using equation (25) with the coefficients $a1_i$, $a2_i$ and $a3_i$ of each grid point; i.e. $p^i(h^i_{ref})$, $e^i(h^i_{ref})$, $T^i_m(h^i_{ref})$, λ^i, α^i_m, and h^i_{ref}, where i = {1, 2, 3, and 4}. Note that the superscript i represents the number of the gridpoint rather than a power.

c) Calculate the value of the three parameters, $p^i(h)$, $e^i(h)$, $T^i_m(h)$ at the four grid points and height h as follows:

$$\mathrm{T}^i_m(h) = T^i_m(h^i_{ref}) - \alpha^i_m \cdot (h - h^i_{ref}) \quad \text{K} \tag{25b}$$

$$p^i(h) = p^i(h^i_{ref})\left[1 - \frac{\alpha^i \cdot (h - h^i_{ref})}{T^i(h)}\right]^{\frac{g^i}{R'_d \alpha^i}} \qquad \text{hPa} \qquad (25c)$$

$$e^i(h) = e^i\left(h^i_{ref}\right) \cdot \left[\frac{p^i(h)}{p^i\left(h^i_{ref}\right)}\right]^{\lambda^i + 1} \qquad \text{hPa} \qquad (25d)$$

$$T^i(h) = \frac{T^i_m(h)}{\left[1 - \frac{\alpha^i R'_d}{(\lambda^i + 1) g^i}\right]} \qquad \text{K} \qquad (25e)$$

$$\alpha^i = 0.5 \cdot \left[\frac{\left(\lambda^i + 1\right) \cdot g^i}{R'_d} - \sqrt{\frac{\left(\lambda^i + 1\right) \cdot g^i}{R'_d}\left[\frac{\left(\lambda^i + 1\right) \cdot g^i}{R'_d} - 4\alpha^i_m\right]}\right] \qquad \text{K/km} \qquad (25f)$$

$$g^i = 9.806 \cdot \left(1 - 0.002637 \cdot \cos\left(2 \cdot lat^i\right) - 0.00031 \cdot h^i_{ref}\right) \qquad \text{m/s}^2 \qquad (25g)$$

d) Calculate the values of $\Delta L^i_{Hv}(h)$ and $\Delta L^i_{Wv}(h)$, at the four grid points at height h using equations (23a) and (23b) with the values of $p^i(h)$, $e^i(h)$ and $T^i_m(h)$ of each grid point.

$$\Delta L^i_{Hv}\left(h\right) = 10^{-6} \frac{R_d}{g^i_m(h)} k_1 \times p^i\left(h\right) \qquad \text{m} \qquad (25h)$$

$$\Delta L^i_{Wv}\left(h\right) = 10^{-6} \frac{R_d}{g^i_m(h)} \frac{k_2}{(\lambda^i + 1)} \cdot \frac{e^i\left(h\right)}{T^i_m\left(h\right)} \qquad \text{m} \qquad (25i)$$

where:

$$g^i_m(h) = 9.784 \cdot (1 - 0.00266 \cdot \cos(2 \cdot lat^i) - 0.00028 \cdot h) \qquad \text{m/s}^2 \qquad (25j)$$

e) Calculate the values of $\Delta L_{Hv}(h)$ and $\Delta L_{Wv}(h)$ at height h at the desired location by performing a bi-linear interpolation of the four values of $\Delta L^i_{Hv}(h)$ and $\Delta L^i_{Wv}(h)$ at the four grid points as described in Recommendation ITU-R P.1144.

f) Calculate the value of tropospheric excess path length at elevation θ at the height h at the desired location, $\Delta L(h,\theta)$, using equation (22).

The accuracy of the proposed model has been tested using radiosonde, GNSS and radiometric measurements to determine ΔL_{vs} and the worldwide uncertainty is between 2 and 6 cm. Where a higher accuracy is needed, concurrent local measurements of air total pressure and water vapour pressure can be used as inputs to the model.

The mapping function of the hydrostatic and wet components, $m_h(\theta)$ and $m_w(\theta)$ are given by:

$$m_h(\theta) = m(\theta, a_h, b_h, c_h) \qquad (26a)$$

$$m_w(\theta) = m(\theta, a_w, b_w, c_w) \qquad (26b)$$

where:

$$m(\theta)=\frac{1+\left[\dfrac{a}{1+\left(\dfrac{b}{1+c}\right)}\right]}{\sin\theta+\left[\dfrac{a}{\sin\theta+\left(\dfrac{b}{\sin\theta+c}\right)}\right]}$$

$$b_h = 0.0029$$

$$b_w = 0.00146$$

$$c_w = 0.04391$$

$$c_h = c_1 + \left[\left(\cos\left(\frac{D_y - 28}{365.25}\cdot 2\pi + \psi\right) + 1\right)\cdot c_{11} + c_{10}\right]\cdot\left[1-\cos(lat)\right] \tag{26c}$$

Hemisphere	c_1	c_{10}	c_{11}	ψ
Northern	0.062	0.001	0.005	0
Southern	0.062	0.002	0.007	π

$$a_h = A_{0h} + A_{1h}\cdot\cos\left(2\pi\cdot\frac{D_y}{365.25}\right) + B_{1h}\cdot\sin\left(2\pi\cdot\frac{D_y}{365.25}\right) + A_{2h}\cdot\cos\left(4\pi\cdot\frac{D_y}{365.25}\right) + B_{2h}\cdot\sin\left(4\pi\cdot\frac{D_y}{365.25}\right) \tag{26d}$$

$$a_w = A_{0w} + A_{1w}\cdot\cos\left(2\pi\cdot\frac{D_y}{365.25}\right) + B_{1w}\cdot\sin\left(2\pi\cdot\frac{D_y}{365.25}\right) + A_{2w}\cdot\cos\left(4\pi\cdot\frac{D_y}{365.25}\right) + B_{2w}\cdot\sin\left(4\pi\cdot\frac{D_y}{365.25}\right) \tag{26e}$$

The coefficients A_{0h}, A_{1h}, A_{2h}, B_{1h}, B_{2h}, A_{0w}, A_{1w}, A_{2w}, B_{1w} and B_{2w} are an integral part of this Recommendation and are available in the form of digital maps in the file R-REC-P.834-8-201609-I!!ZIP-E. Calculate the values of the parameters a_h and a_w at the desired location by performing a bi-linear interpolation of the four values of these coefficients at the four grid points as described in Recommendation ITU-R P.1144.

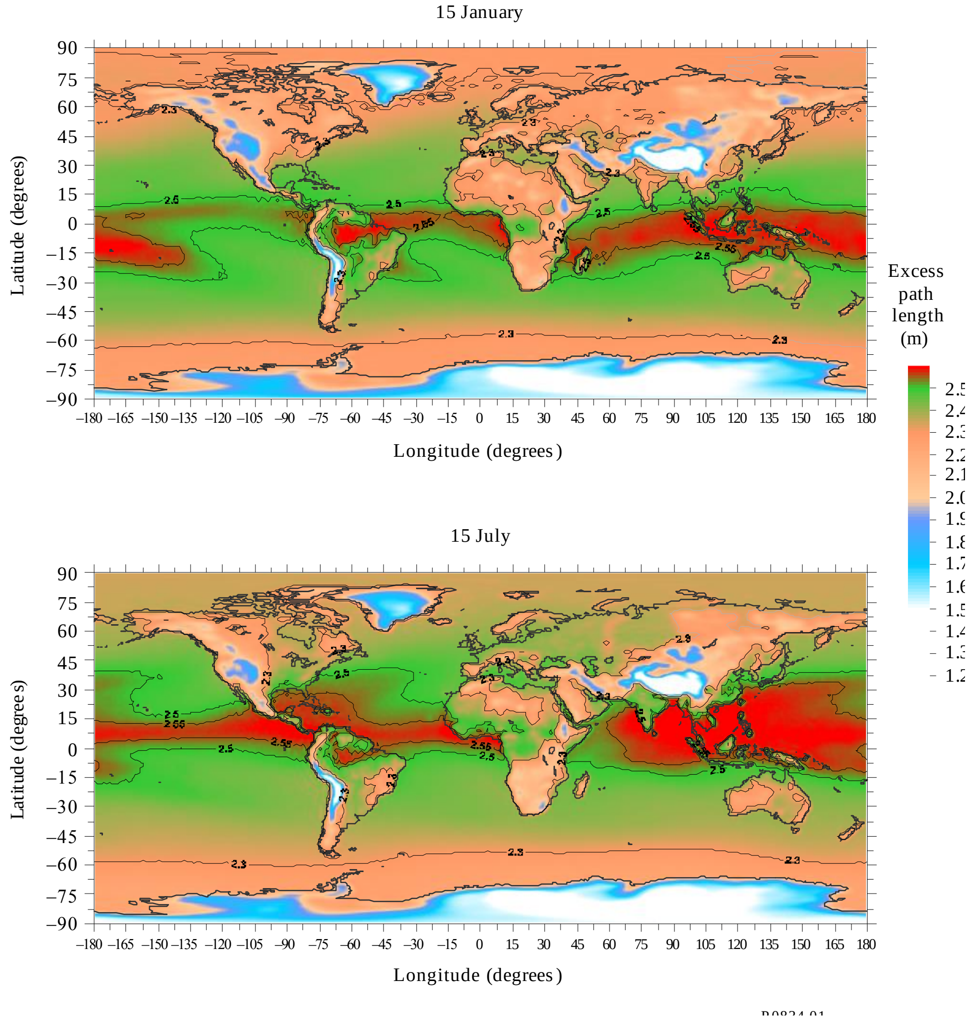
For the case of an Earth-space link with elevation angles, θ, greater than 20°, the mapping functions given by equations (26a) and (26b) can be approximated by:

$$m_h(\theta) = m_w(\theta) = \frac{1}{\sin(\theta)} \tag{26f}$$

In the application of this model it is recommended to use either equations (26a) and (26b) or equation (26f) consistently along all the elevation angles.

FIGURE 1

Maps of the average excess path delay at reference level in January and July



7 Propagation in ducting layers

Ducts exist whenever the vertical refractivity gradient at a given height and location is less than −157 N/km.

The existence of ducts is important because they can give rise to anomalous radiowave propagation, particularly on terrestrial or very low angle Earth-space links. Ducts provide a mechanism for radiowave signals of sufficiently high frequencies to propagate far beyond their normal line-of-sight range, giving rise to potential interference with other services (see Recommendation ITU-R P.452). They also play an important role in the occurrence of multipath interference (see Recommendation ITU-R P.530) although they are neither necessary nor sufficient for multipath propagation to occur on any particular link.

7.1 Influence of elevation angle

When a transmitting antenna is situated within a horizontally stratified radio duct, rays that are launched at very shallow elevation angles can become "trapped" within the boundaries of the duct. For the simplified case of a "normal" refractivity profile above a surface duct having a fixed refractivity gradient, the critical elevation angle α (rad) for rays to be trapped is given by the expression:

$$\alpha = \sqrt{2 \times 10^{-6} \left| \frac{\mathrm{d}M}{\mathrm{d}h} \right| \Delta h} \tag{27}$$

where dM/dh is the vertical gradient of modified refractivity $\left(\frac{\mathrm{d}M}{\mathrm{d}h} < 0 \right)$ and Δh is the thickness of the duct which is the height of duct top above transmitter antenna.

Figure 2 gives the maximum angle of elevation for rays to be trapped within the duct. The maximum trapping angle increases rapidly for decreasing refractivity gradients below −157 N/km (i.e. increasing lapse rates) and for increasing duct thickness.

7.2 Minimum trapping frequency

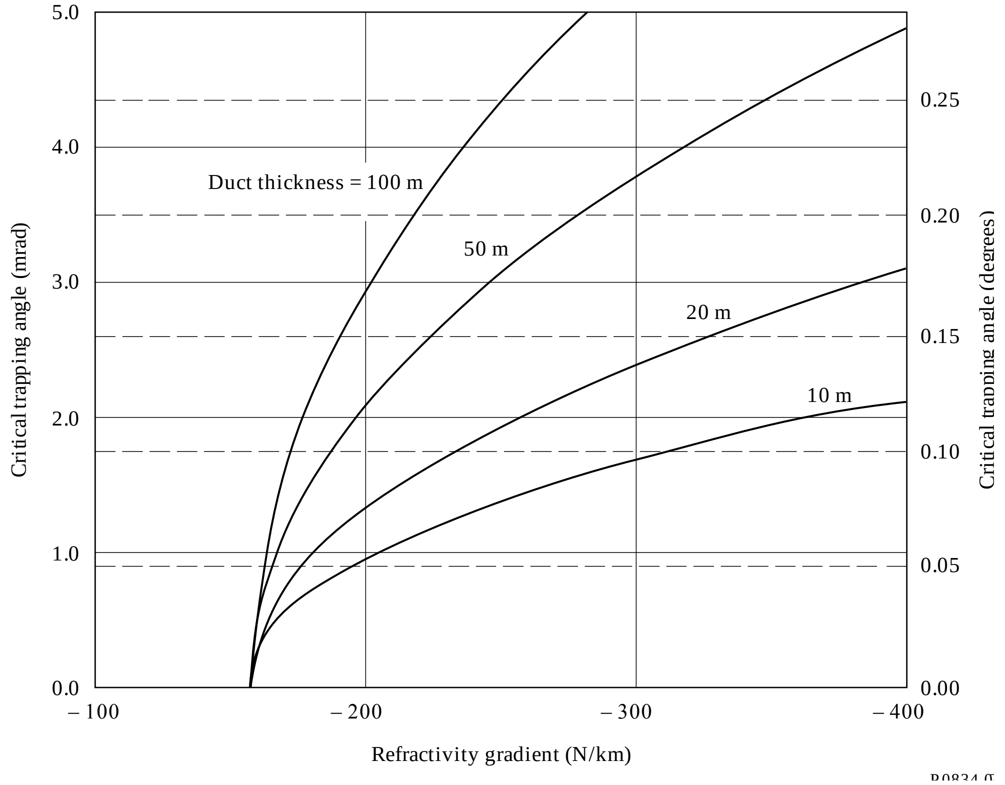
The existence of a duct, even if suitably situated, does not necessarily imply that energy will be efficiently coupled into the duct in such a way that long-range propagation will occur. In addition to satisfying the maximum elevation angle condition above, the frequency of the wave must be above a critical value determined by the physical depth of the duct and by the refractivity profile. Below this minimum trapping frequency, ever-increasing amounts of energy will leak through the duct boundaries.

The minimum frequency for a wave to be trapped within a tropospheric duct can be estimated using a phase integral approach. Figure 3 shows the minimum trapping frequency for surface ducts (solid curves) where a constant (negative) refractivity gradient is assumed to extend from the surface to a given height, with a standard profile above this height. For the frequencies used in terrestrial systems (typically 8-16 GHz), a ducting layer of about 5 to 15 m minimum thickness is required and in these instances the minimum trapping frequency, f_{min}, is a strong function of both the duct thickness and the refractive index gradient.

In the case of elevated ducts an additional parameter is involved even for the simple case of a linear refractivity profile. That parameter relates to the shape of the refractive index profile lying below the ducting gradient. The dashed curves in Fig. 3 show the minimum trapping frequency for a constant gradient ducting layer lying above a surface layer having a standard refractivity gradient of −40 N/km.

FIGURE 2

Maximum trapping angle for a surface duct of constant refractivity gradient over a spherical Earth



For layers having lapse rates that are only slightly greater than the minimum required for ducting to occur, the minimum trapping frequency is actually increased over the equivalent surface-duct case. For all ducting gradients, trapping by an elevated duct requires a much thinner layer than a surface duct of equal gradient for any given frequency.

FIGURE 3

Minimum frequency for trapping in atmospheric radio ducts of constant refractivity gradients