RECOMMENDATION ITU-R P.676-6

Attenuation by atmospheric gases

(Question ITU-R 201/3)

(1990-1992-1995-1997-1999-2001-2005)

The ITU Radiocommunication Assembly,

considering

a) the necessity of estimating the attenuation by atmospheric gases on terrestrial and slant paths,

recommends

1 that, for general application, the procedures in Annex 1 be used to calculate gaseous attenuation at frequencies up to 1 000 GHz. (Software code in MATLAB is available from the Radiocommunication Bureau);

2 that, for approximate estimates of gaseous attenuation in the frequency range 1 to 350 GHz, the computationally less intensive procedure given in Annex 2 be used.

Annex 1

Line-by-line calculation of gaseous attenuation

1 Specific attenuation

The specific attenuation at frequencies up to 1 000 GHz due to dry air and water vapour, can be evaluated most accurately at any value of pressure, temperature and humidity by means of a summation of the individual resonance lines from oxygen and water vapour, together with small additional factors for the non-resonant Debye spectrum of oxygen below 10 GHz, pressure-induced nitrogen attenuation above 100 GHz and a wet continuum to account for the excess water vapour-absorption found experimentally. Figure 1 shows the specific attenuation using the model, calculated from 0 to 1 000 GHz at 1 GHz intervals, for a pressure of 1 013 hPa, temperature of 15° C for the cases of a water-vapour density of 7.5 g/m^3 (Curve A) and a dry atmosphere (Curve B).

Near 60 GHz, many oxygen absorption lines merge together, at sea-level pressures, to form a single, broad absorption band, which is shown in more detail in Fig. 2. This Figure also shows the oxygen attenuation at higher altitudes, with the individual lines becoming resolved at lower pressures.

For quick and approximate estimates of specific attenuation at frequencies up to 350 GHz, in cases where high accuracy is not required, simplified algorithms are given in Annex 2 for restricted ranges of meteorological conditions.

FIGURE 1

Specific attenuation due to atmospheric gases, calculated at 1 GHz intervals, including line centres

(Standard: 7.5 g/m^3; Dry: 0 g/m^3)

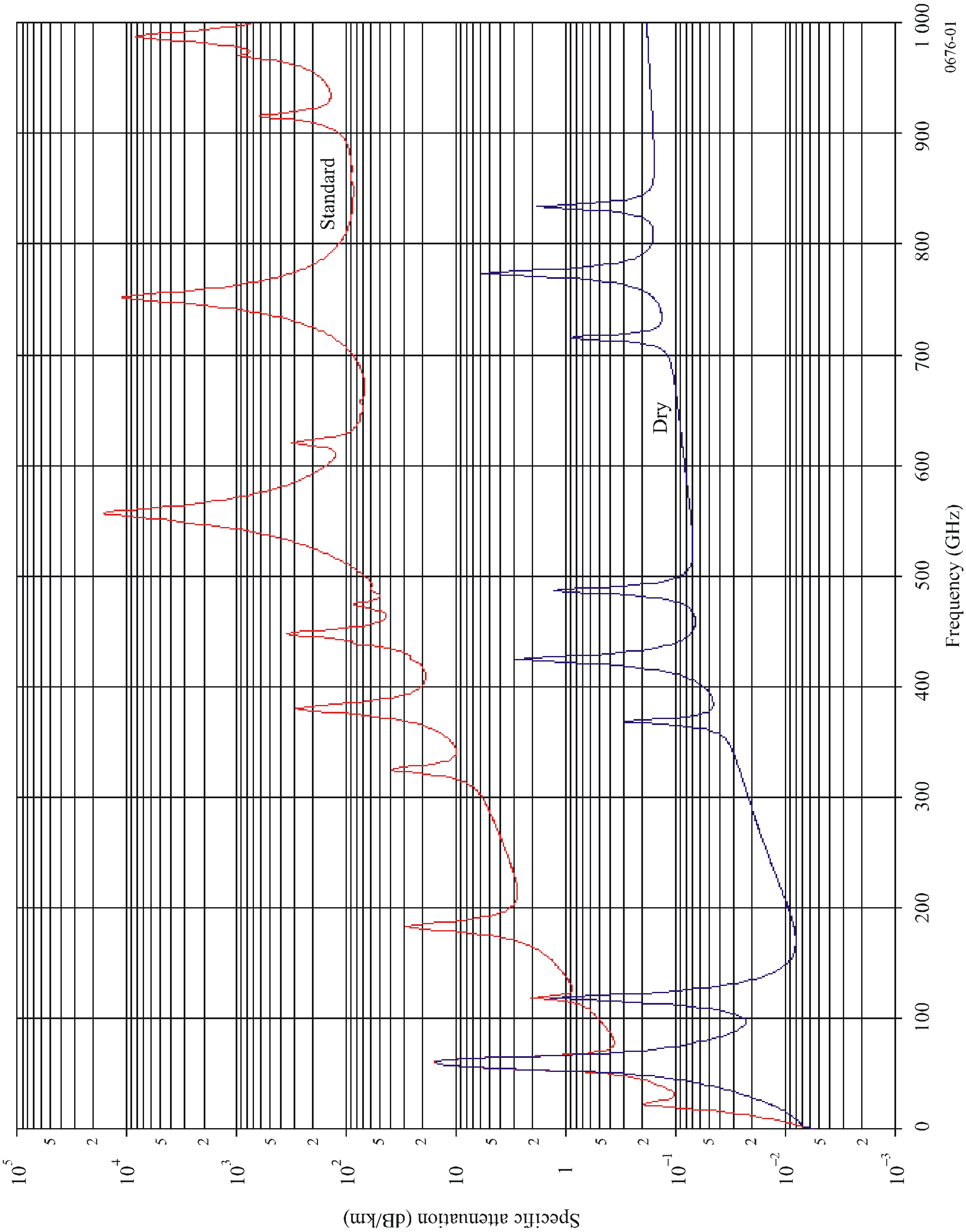
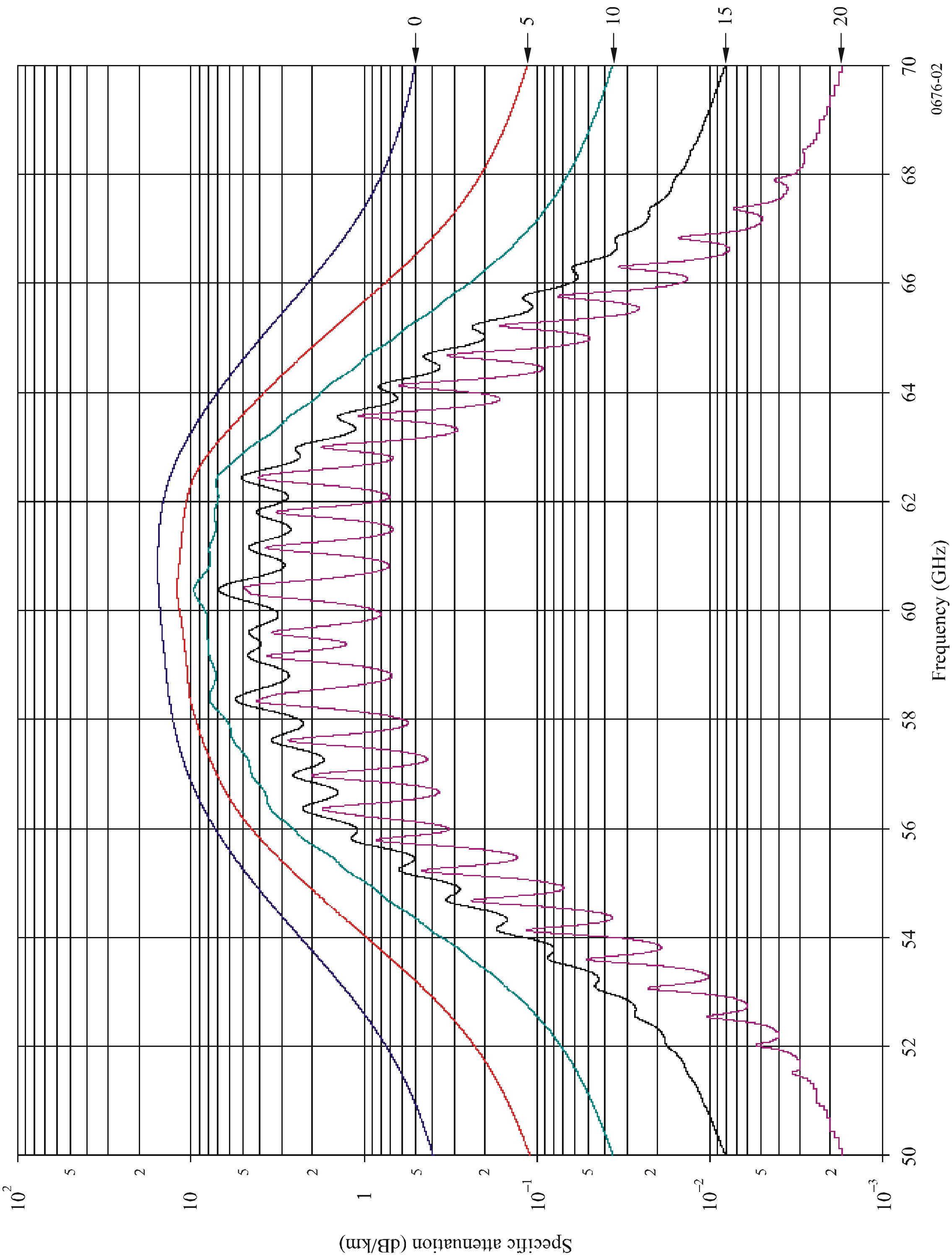


FIGURE 2

Specific attenuation in the range 50-70 GHz at the altitudes indicated

(0 km, 5 km, 10 km, 15 km and 20 km)



The specific gaseous attenuation is given by:

$$\gamma = \gamma_o + \gamma_w = 0.1820 f \; N''(f) \qquad \text{dB/km} \tag{1}$$

where γ_o and γ_w are the specific attenuations (dB/km) due to dry air (oxygen, pressure-induced nitrogen and non-resonant Debye attenuation) and water vapour, respectively, and where f is the frequency (GHz) and $N''(f)$ is the imaginary part of the frequency-dependent complex refractivity:

$$N''(f) = \sum_i S_i \, F_i + N''_D(f) \tag{2}$$

S_i is the strength of the i-th line, F_i is the line shape factor and the sum extends over all the lines; $N''_D(f)$ is the dry continuum due to pressure-induced nitrogen absorption and the Debye spectrum.

The line strength is given by:

$$\begin{aligned} S_i &= a_1 \times 10^{-7} \; p \; \theta^3 \; \exp\left[a_2 \, (1 - \theta) \right] && \text{for oxygen} \\ &= b_1 \times 10^{-1} \; e \; \theta^{3.5} \; \exp\left[b_2 \, (1 - \theta) \right] && \text{for water vapour} \end{aligned} \tag{3}$$

where:

p: dry air pressure (hPa)

e: water vapour partial pressure in hPa (total barometric pressure $P = p + e$)

θ = $300/T$

T: temperature (K).

Local values of p, e and T measured profiles (e.g. using radiosondes) should be used; however, in the absence of local information, the reference standard atmospheres described in Recommendation ITU-R P.835 should be used. (Note that where total atmospheric attenuation is being calculated, the same-water vapour partial pressure is used for both dry-air and water-vapour attenuations.)

The water-vapour partial pressure, e, may be obtained from the water-vapour density ρ using the expression:

$$e = \frac{\rho T}{216.7} \tag{4}$$

The coefficients a_1, a_2 are given in Table 1 for oxygen, those for water vapour, b_1 and b_2, are given in Table 2.

The line-shape factor is given by:

$$F_i = \frac{f}{f_i} \left[\frac{\Delta f - \delta \left(f_i - f \right)}{\left(f_i - f \right)^2 + \Delta f^2} + \frac{\Delta f - \delta \left(f_i + f \right)}{\left(f_i + f \right)^2 + \Delta f^2} \right] \tag{5}$$

where f_i is the line frequency and Δf is the width of the line:

$$\begin{aligned} \Delta f &= a_3 \times 10^{-4} \left(p \; \theta^{(0.8 - a_4)} + 1.1 \, e \, \theta \right) && \text{for oxygen} \\ &= b_3 \times 10^{-4} \left(p \; \theta^{b_4} + b_5 \, e \, \theta^{b_6} \right) && \text{for water vapour} \end{aligned} \tag{6a}$$

The line width Δf is modified to account for Doppler broadening:

$$\Delta f = \sqrt{\Delta f^2 + 2.25 \times 10^{-6}} \qquad \text{for oxygen}$$
$$= 0.535 \Delta f + \sqrt{0.217 \Delta f^2 + \frac{2.1316 \times 10^{-12} f_i^2}{\theta}} \qquad \text{for water vapour} \qquad (6b)$$

δ is a correction factor which arises due to interference effects in oxygen lines:

$$\delta = (a_5 + a_6 \theta) \times 10^{-4} (p + e) \theta^{0.8} \qquad \text{for oxygen}$$
$$= 0 \qquad \text{for water vapour} \qquad (7)$$

The spectroscopic coefficients are given in Tables 1 and 2.

TABLE 1

Spectroscopic data for oxygen attenuation

f_0	a_1	a_2	a_3	a_4	a_5	a_6
50.474238	.94	9.694	8.90	.0	2.400	7.900
50.987749	2.46	8.694	9.10	.0	2.200	7.800
51.503350	6.08	7.744	9.40	.0	1.970	7.740
52.021410	14.14	6.844	9.70	.0	1.660	7.640
52.542394	31.02	6.004	9.90	.0	1.360	7.510
53.066907	64.10	5.224	10.20	.0	1.310	7.140
53.595749	124.70	4.484	10.50	.0	2.300	5.840
54.130000	228.00	3.814	10.70	.0	3.350	4.310
54.671159	391.80	3.194	11.00	.0	3.740	3.050
55.221367	631.60	2.624	11.30	.0	2.580	3.390
55.783802	953.50	2.119	11.70	.0	–1.660	7.050
56.264775	548.90	.015	17.30	.0	3.900	–1.130
56.363389	1 344.00	1.660	12.00	.0	–2.970	7.530
56.968206	1 763.00	1.260	12.40	.0	–4.160	7.420
57.612484	2 141.00	.915	12.80	.0	–6.130	6.970
58.323877	2 386.00	.626	13.30	.0	–2.050	.510
58.446590	1 457.00	.084	15.20	.0	7.480	–1.460
59.164207	2 404.00	.391	13.90	.0	–7.220	2.660

TABLE 1 (*end*)

f_0	a_1	a_2	a_3	a_4	a_5	a_6
59.590983	2 112.00	.212	14.30	.0	7.650	–.900
60.306061	2 124.00	.212	14.50	.0	–7.050	.810
60.434776	2 461.00	.391	13.60	.0	6.970	–3.240
61.150560	2 504.00	.626	13.10	.0	1.040	–.670
61.800154	2 298.00	.915	12.70	.0	5.700	–7.610
62.411215	1 933.00	1.260	12.30	.0	3.600	–7.770
62.486260	1 517.00	.083	15.40	.0	–4.980	.970
62.997977	1 503.00	1.665	12.00	.0	2.390	–7.680
63.568518	1 087.00	2.115	11.70	.0	1.080	–7.060
64.127767	733.50	2.620	11.30	.0	–3.110	–3.320
64.678903	463.50	3.195	11.00	.0	–4.210	–2.980
65.224071	274.80	3.815	10.70	.0	–3.750	–4.230
65.764772	153.00	4.485	10.50	.0	–2.670	–5.750
66.302091	80.09	5.225	10.20	.0	–1.680	–7.000
66.836830	39.46	6.005	9.90	.0	–1.690	–7.350
67.369598	18.32	6.845	9.70	.0	–2.000	–7.440
67.900867	8.01	7.745	9.40	.0	–2.280	–7.530
68.431005	3.30	8.695	9.20	.0	–2.400	–7.600
68.960311	1.28	9.695	9.00	.0	–2.500	–7.650
118.750343	945.00	.009	16.30	.0	–.360	.090
368.498350	67.90	.049	19.20	.6	.000	.000
424.763124	638.00	.044	19.30	.6	.000	.000
487.249370	235.00	.049	19.20	.6	.000	.000
715.393150	99.60	.145	18.10	.6	.000	.000
773.839675	671.00	.130	18.20	.6	.000	.000
834.145330	180.00	.147	18.10	.6	.000	.000

TABLE 2

Spectroscopic data for water-vapour attenuation

f_0	b_1	b_2	b_3	b_4	b_5	b_6
22.235080	0.1130	2.143	28.11	.69	4.800	1.00
67.803960	0.0012	8.735	28.58	.69	4.930	.82
119.995940	0.0008	8.356	29.48	.70	4.780	.79
183.310091	2.4200	.668	30.50	.64	5.300	.85
321.225644	0.0483	6.181	23.03	.67	4.690	.54
325.152919	1.4990	1.540	27.83	.68	4.850	.74
336.222601	0.0011	9.829	26.93	.69	4.740	.61
380.197372	11.5200	1.048	28.73	.54	5.380	.89
390.134508	0.0046	7.350	21.52	.63	4.810	.55
437.346667	0.0650	5.050	18.45	.60	4.230	.48
439.150812	0.9218	3.596	21.00	.63	4.290	.52
443.018295	0.1976	5.050	18.60	.60	4.230	.50
448.001075	10.3200	1.405	26.32	.66	4.840	.67
470.888947	0.3297	3.599	21.52	.66	4.570	.65
474.689127	1.2620	2.381	23.55	.65	4.650	.64
488.491133	0.2520	2.853	26.02	.69	5.040	.72
503.568532	0.0390	6.733	16.12	.61	3.980	.43
504.482692	0.0130	6.733	16.12	.61	4.010	.45
547.676440	9.7010	.114	26.00	.70	4.500	1.00
552.020960	14.7700	.114	26.00	.70	4.500	1.00
556.936002	487.4000	.159	32.10	.69	4.110	1.00
620.700807	5.0120	2.200	24.38	.71	4.680	.68
645.866155	0.0713	8.580	18.00	.60	4.000	.50
658.005280	0.3022	7.820	32.10	.69	4.140	1.00
752.033227	239.6000	.396	30.60	.68	4.090	.84
841.053937	0.0140	8.180	15.90	.33	5.760	.45
859.962313	0.1472	7.989	30.60	.68	4.090	.84
899.306675	0.0605	7.917	29.85	.68	4.530	.90
902.616173	0.0426	8.432	28.65	.70	5.100	.95
906.207325	0.1876	5.111	24.08	.70	4.700	.53
916.171582	8.3400	1.442	26.70	.70	4.780	.78
923.118427	0.0869	10.220	29.00	.70	5.000	.80
970.315022	8.9720	1.920	25.50	.64	4.940	.67
987.926764	132.1000	.258	29.85	.68	4.550	.90
1 780.000000	22 300.0000	.952	176.20	.50	30.500	5.00

The dry air continuum arises from the non-resonant Debye spectrum of oxygen below 10 GHz and a pressure-induced nitrogen attenuation above 100 GHz.

$$N''_D(f) = f\,p\,\theta^2 \left[\frac{6.14 \times 10^{-5}}{d\left[1 + \left(\frac{f}{d}\right)^2\right]} + \frac{1.4 \times 10^{-12}\,p\,\theta^{1.5}}{1 + 1.9 \times 10^{-5}\,f^{1.5}} \right] \tag{8}$$

where d is the width parameter for the Debye spectrum:

$$d = 5.6 \times 10^{-4}\,p\theta^{0.8} \tag{9}$$

2 Path attenuation

2.1 Terrestrial paths

For a terrestrial path, or for slightly inclined paths close to the ground, the path attenuation, A, may be written as:

$$A = \gamma r_0 = (\gamma_o + \gamma_w) r_0 \qquad \text{dB} \tag{10}$$

where r_0 is path length (km).

2.2 Slant paths

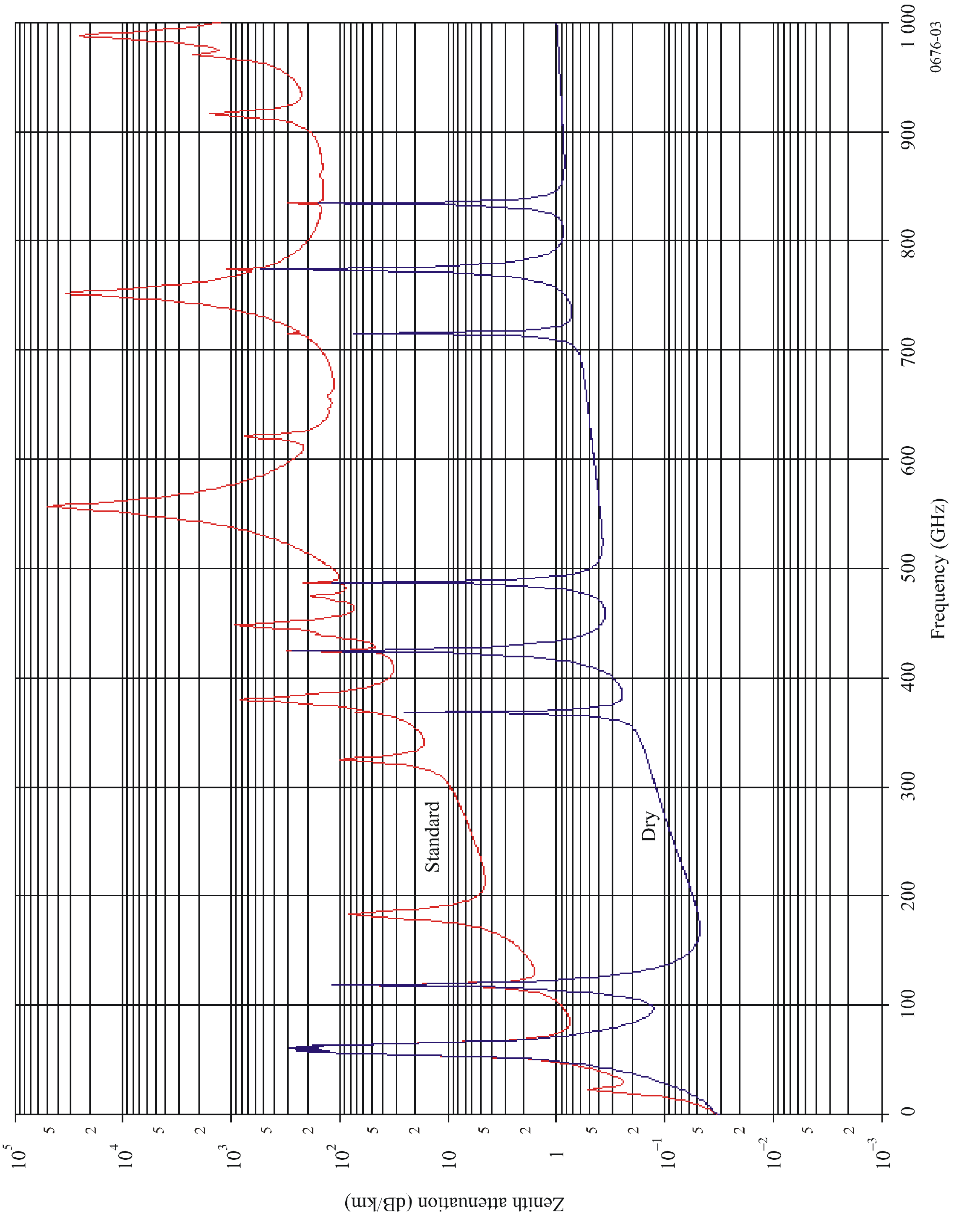
This section gives a method to integrate the specific attenuation calculated using the line-by-line model given above, at different pressures, temperatures and humidities through the atmosphere. By this means, the path attenuation for communications systems with any geometrical configuration within and external to the Earth's atmosphere may be accurately determined simply by dividing the atmosphere into horizontal layers, specifying the profile of the meteorological parameters pressure, temperature and humidity along the path. In the absence of local profiles, from radiosonde data, for example, the reference standard atmospheres in Recommendation ITU-R P.835 may be used, either for global application or for low (annual), mid (summer and winter) and high latitude (summer and winter) sites.

Figure 3 shows the zenith attenuation calculated at 1 GHz intervals with this model for the global reference standard atmosphere in Recommendation ITU-R P.835, with horizontal layers 1 km thick and summing the attenuations for each layer, for the cases of a moist atmosphere (Curve A) and a dry atmosphere (Curve B).

FIGURE 3

Zenith attenuation due to atmospheric gases, calculated at 1 GHz intervals, including line centres

(Standard: 7.5 g/m^3 at sea level; Dry: 0 g/m^3)



The total slant path attenuation, $A(h, \varphi)$, from a station with altitude, h, and elevation angle, φ, can be calculated as follows when $\varphi \geq 0$:

$$A(h, \varphi) = \int_h^\infty \frac{\gamma(H)}{\sin \Phi} \mathrm{d}H \tag{11}$$

where the value of Φ can be determined as follows based on Snell's law in polar coordinates:

$$\Phi = \arccos\left(\frac{c}{(r+H) \times n(H)}\right) \tag{12}$$

where:

$$c = (r+h) \times n(h) \times \cos \varphi \tag{13}$$

where $n(h)$ is the atmospheric radio refractive index, calculated from pressure, temperature and water-vapour pressure along the path (see Recommendation ITU-R P.835) using Recommendation ITU-R P.453.

On the other hand, when $\varphi < 0$, there is a minimum height, h_{min}, at which the radio beam becomes parallel with the Earth's surface. The value of h_{min} can be determined by solving the following transcendental equation:

$$(r + h_{min}) \times n(h_{min}) = c \tag{14}$$

This can be easily solved by repeating the following calculation, using $h_{min} = h$ as an initial value:

$$h'_{min} = \frac{c}{n(h_{min})} - r \tag{15}$$

Therefore, $A(h, \varphi)$ can be calculated as follows:

$$A(h, \varphi) = \int_{h_{min}}^\infty \frac{\gamma(H)}{\sin \Phi} \mathrm{d}H + \int_{h_{min}}^h \frac{\gamma(H)}{\sin \Phi} \mathrm{d}H \tag{16}$$

In carrying out the integration of equations (11) and (16), care should be exercised in that the integrand becomes infinite at $\Phi = 0$. However, this singularity can be eliminated by an appropriate variable conversion, for example, by using $u^4 = H - h$ in equation (11) and $u^4 = H - h_{min}$ in equation (16).

A numerical solution for the attenuation due to atmospheric gases can be implemented with the following algorithm.

To calculate the total attenuation for a satellite link, it is necessary to know not only the specific attenuation at each point of the link but also the length of path that has that specific attenuation. To derive the path length it is also necessary to consider the ray bending that occurs in a spherical Earth.

Using Fig. 4 as a reference, a_n is the path length through layer n with thickness δ_n that has refractive index n_n. α_n and β_n are the entry and exiting incidence angles. r_n are the radii from the centre of the Earth to the beginning of layer n. a_n can then be expressed as:

$$a_n = -r_n \cos \beta_n + \frac{1}{2}\sqrt{4 r_n^2 \cos^2 \beta_n + 8 r_n \delta_n + 4 \delta_n^2} \tag{17}$$

The angle α_n can be calculated from:

$$\alpha_n = \pi - \arccos\left(\frac{-a_n^2 - 2 r_n \delta_n - \delta_n^2}{2 a_n r_n + 2 a_n \delta_n}\right) \tag{18}$$

β_1 is the incidence angle at the ground station (the complement of the elevation angle θ). β_{n+1} can be calculated from α_n using Snell's law that in this case becomes:

$$\beta_{n+1} = \arcsin\left(\frac{n_n}{n_{n+1}} \sin \alpha_n\right) \tag{19}$$

where n_n and n_{n+1} are the refractive indexes of layers n and $n + 1$.

The remaining frequency dependent (dispersive) term has a marginal influence on the result (around 1%) but can be calculated from the method shown in the ITU-R Handbook on Radiometeorology.

The total attenuation can be derived using:

$$A_{gas} = \sum_{n=1}^{k} a_n \gamma_n \qquad \text{dB} \tag{20}$$

where γ_n is the specific attenuation derived from equation (1).

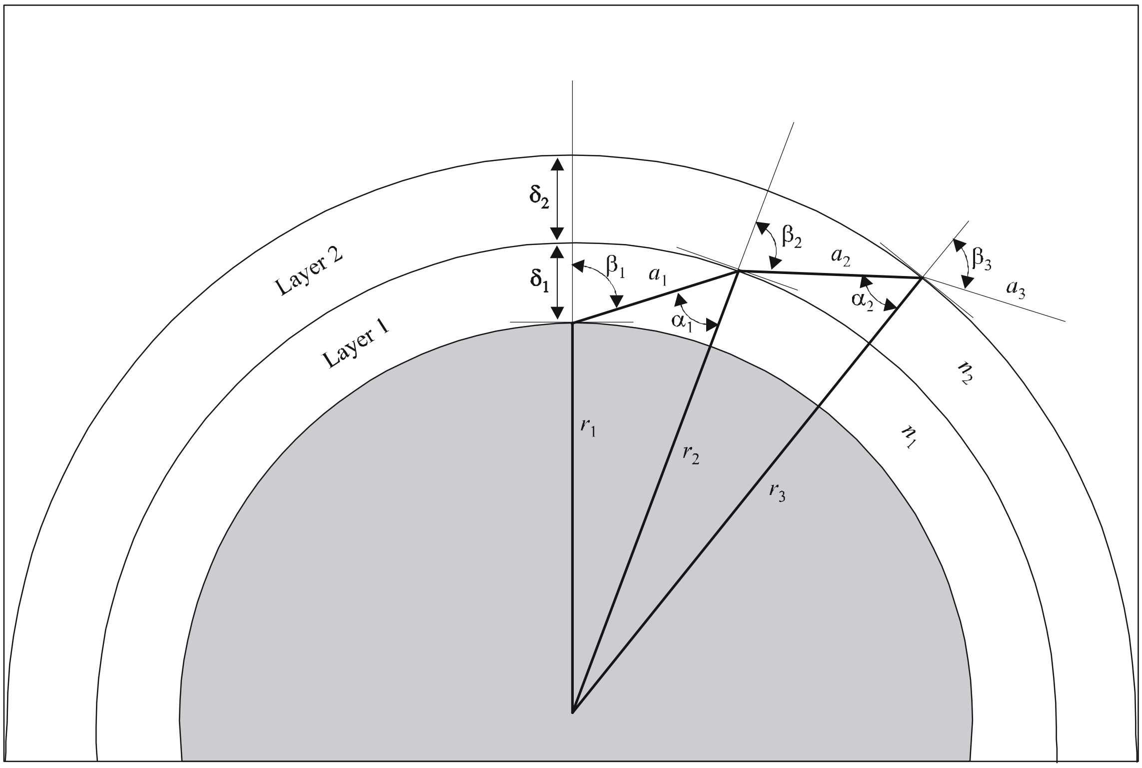
To ensure an accurate estimate of the path attenuation, the thickness of the layers should increase exponentially, from 10 cm at the lowest layer (ground level) to 1 km at an altitude of 100 km, according to the following equation:

$$\delta_i = 0.0001 \exp\left\{\frac{i - 1}{100}\right\} \qquad \text{km} \tag{21}$$

from $i = 1$ to 922, noting that $\delta_{922} \cong 1.0$ km and $\sum_{i=1}^{922} \delta_i \cong 100$ km.

For Earth-to-space applications, the integration should be performed at least up to 30 km, and up to 100 km at the oxygen line-centre frequencies.

FIGURE 4



0676-04

3 Dispersive effects

The effects of dispersion are discussed in the ITU-R Handbook on Radiometeorology, which contains a model for calculating dispersion based on the line-by-line calculation. For practical purposes, dispersive effects should not impose serious limitations on millimetric terrestrial communication systems operating with bandwidths of up to a few hundred MHz over short ranges (for example, less than about 20 km), especially in the window regions of the spectrum, at frequencies removed from the centres of major absorption lines. For satellite communication systems, the longer path lengths through the atmosphere will constrain operating frequencies further to the window regions, where both atmospheric attenuation and the corresponding dispersion are low.

Annex 2

Approximate estimation of gaseous attenuation in the frequency range 1-350 GHz

This Annex contains simplified algorithms for quick, approximate estimation of gaseous attenuation for a limited range of meteorological conditions and a limited variety of geometrical configurations.

1 Specific attenuation

The specific attenuation due to dry air and water vapour, from sea level to an altitude of 10 km, can be estimated using the following simplified algorithms, which are based on curve-fitting to the line-by-line calculation, and agree with the more accurate calculations to within an average of about ±10% at frequencies removed from the centres of major absorption lines. The absolute difference between the results from these algorithms and the line-by-line calculation is generally less than 0.1 dB/km and reaches a maximum of 0.7 dB/km near 60 GHz. For altitudes higher than 10 km, and in cases where higher accuracy is required, the line-by-line calculation should be used.

For dry air, the attenuation γ_o (dB/km) is given by the following equations:

For $f \leq 54$ GHz:

$$\gamma_o = \left[\frac{7.2\ r_t^{2.8}}{f^2 + 0.34\ r_p^2\ r_t^{1.6}} + \frac{0.62\,\xi_3}{(54 - f)^{1.16\xi_1} + 0.83\ \xi_2} \right] f^2 r_p^2 \times 10^{-3} \tag{22a}$$

For 54 GHz $< f \leq 60$ GHz:

$$\gamma_o = \exp\left[\frac{\ln \gamma_{54}}{24}(f - 58)(f - 60) - \frac{\ln \gamma_{58}}{8}(f - 54)(f - 60) + \frac{\ln \gamma_{60}}{12}(f - 54)(f - 58) \right] \tag{22b}$$

For 60 GHz $< f \leq 62$ GHz:

$$\gamma_o = \gamma_{60} + (\gamma_{62} - \gamma_{60}) \frac{f - 60}{2} \tag{22c}$$

For 62 GHz $< f \leq 66$ GHz:

$$\gamma_o = \exp\left[\frac{\ln \gamma_{62}}{8}(f - 64)(f - 66) - \frac{\ln \gamma_{64}}{4}(f - 62)(f - 66) + \frac{\ln \gamma_{66}}{8}(f - 62)(f - 64) \right] \tag{22d}$$

For 66 GHz $< f \leq 120$ GHz:

$$\gamma_o = \left\{ 3.02 \times 10^{-4} r_t^{3.5} + \frac{0.283\, r_t^{3.8}}{(f - 118.75)^2 + 2.91 r_p^2 r_t^{1.6}} + \frac{0.502\xi_6 [1 - 0.0163\xi_7 (f - 66)]}{(f - 66)^{1.4346\xi_4} + 1.15\xi_5} \right\} f^2 r_p^2 \times 10^{-3} \tag{22e}$$

For 120 GHz $< f \leq 350$ GHz:

$$\gamma_o = \left[\frac{3.02 \times 10^{-4}}{1 + 1.9 \times 10^{-5} f^{1.5}} + \frac{0.283 r_t^{0.3}}{(f - 118.75)^2 + 2.91 r_p^2 r_t^{1.6}} \right] f^2 r_p^2 r_t^{3.5} \times 10^{-3} + \delta \tag{22f}$$

with:

$$\xi_1 = \varphi(r_p, r_t, 0.0717, -1.8132, 0.0156, -1.6515) \tag{22g}$$

$$\xi_2 = \varphi(r_p, r_t, 0.5146, -4.6368, -0.1921, -5.7416) \tag{22h}$$

$$\xi_3 = \varphi(r_p, r_t, 0.3414, -6.5851, 0.2130, -8.5854) \tag{22i}$$

$$\xi_4 = \varphi(r_p, r_t, -0.0112, 0.0092, -0.1033, -0.0009) \tag{22j}$$

$$\xi_5 = \varphi(r_p, r_t, 0.2705, -2.7192, -0.3016, -4.1033) \tag{22k}$$

$$\xi_6 = \varphi(r_p, r_t, 0.2445, -5.9191, 0.0422, -8.0719) \tag{22l}$$

$$\xi_7 = \varphi(r_p, r_t, -0.1833, 6.5589, -0.2402, 6.131) \tag{22m}$$

$$\gamma_{54} = 2.192\varphi(r_p, r_t, 1.8286, -1.9487, 0.4051, -2.8509) \tag{22n}$$

$$\gamma_{58} = 12.59\varphi(r_p, r_t, 1.0045, 3.5610, 0.1588, 1.2834) \tag{22o}$$

$$\gamma_{60} = 15.0\varphi(r_p, r_t, 0.9003, 4.1335, 0.0427, 1.6088) \tag{22p}$$

$$\gamma_{62} = 14.28\varphi(r_p, r_t, 0.9886, 3.4176, 0.1827, 1.3429) \tag{22q}$$

$$\gamma_{64} = 6.819\varphi(r_p, r_t, 1.4320, 0.6258, 0.3177, -0.5914) \tag{22r}$$

$$\gamma_{66} = 1.908\varphi(r_p, r_t, 2.0717, -4.1404, 0.4910, -4.8718) \tag{22s}$$

$$\delta = -0.00306\varphi(r_p, r_t, 3.211, -14.94, 1.583, -16.37) \tag{22t}$$

$$\varphi(r_p, r_t, a, b, c, d) = r_p^a r_t^b \exp[c(1 - r_p) + d(1 - r_t)] \tag{22u}$$

where:

f: frequency (GHz)

$r_p =$ $p/1013$

$r_t =$ $288/(273 + t)$

p: pressure (hPa)

t: temperature (°C), where mean temperature values can be obtained from maps given in Recommendation ITU-R P.1510, when no adequate temperature data are available.

For water vapour, the attenuation γ_w (dB/km) is given by:

$$\gamma_w = \left\{ \frac{3.98\eta_1 \exp[2.23(1-r_t)]}{(f-22.235)^2 + 9.42\eta_1^2} g(f,22) + \frac{11.96\eta_1 \exp[0.7(1-r_t)]}{(f-183.31)^2 + 11.14\eta_1^2} \right.$$
$$+ \frac{0.081\eta_1 \exp[6.44(1-r_t)]}{(f-321.226)^2 + 6.29\eta_1^2} + \frac{3.66\eta_1 \exp[1.6(1-r_t)]}{(f-325.153)^2 + 9.22\eta_1^2}$$
$$+ \frac{25.37\eta_1 \exp[1.09(1-r_t)]}{(f-380)^2} + \frac{17.4\eta_1 \exp[1.46(1-r_t)]}{(f-448)^2} \tag{23a}$$
$$+ \frac{844.6\eta_1 \exp[0.17(1-r_t)]}{(f-557)^2} g(f,557) + \frac{290\eta_1 \exp[0.41(1-r_t)]}{(f-752)^2} g(f,752)$$
$$\left. + \frac{8.3328\times10^4 \eta_2 \exp[0.99(1-r_t)]}{(f-1\,780)^2} g(f,1\,780) \right\} f^2 r_t^{2.5} \rho \times 10^{-4}$$

with:

$$\eta_1 = 0.955 r_p r_t^{0.68} + 0.006\rho \tag{23b}$$

$$\eta_2 = 0.735\ r_p r_t^{0.5} + 0.0353\ r_t^4 \rho \tag{23c}$$

$$g(f, f_i) = 1 + \left(\frac{f - f_i}{f + f_i} \right)^2 \tag{23d}$$

where ρ is the water-vapour density (g/m^3).

Figure 5 shows the specific attenuation from 1 to 350 GHz at sea-level for dry air and water vapour with a density of 7.5 g/m^3.

2 Path attenuation

2.1 Terrestrial paths

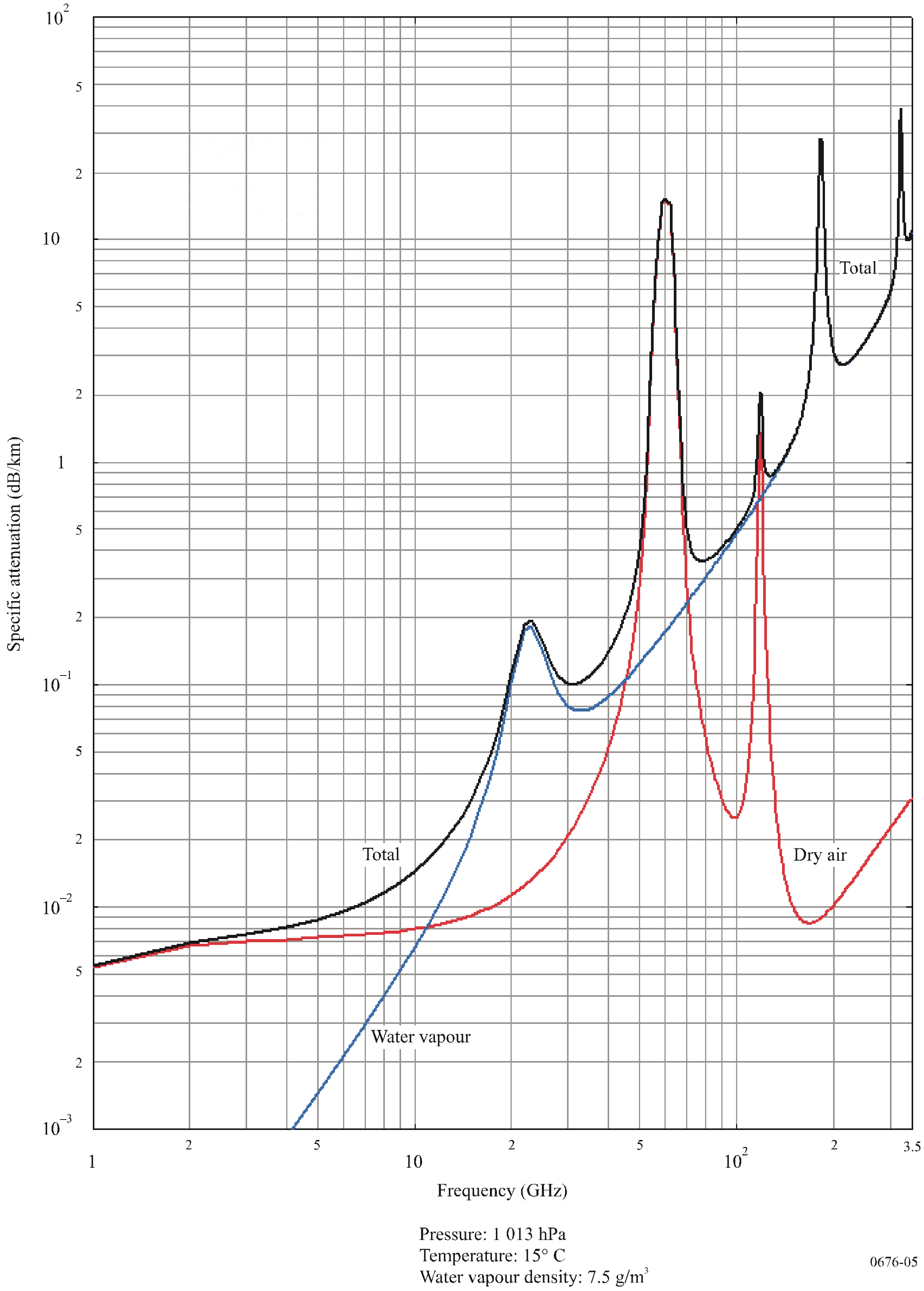
For a horizontal path, or for slightly inclined paths close to the ground, the path attenuation, A, may be written as:

$$A = \gamma r_0 = (\gamma_o + \gamma_w) r_0 \quad \text{dB} \tag{24}$$

where r_0 is the path length (km).

FIGURE 5

Specific attenuation due to atmospheric gases



2.2 Slant paths

This section contains simple algorithms for estimating the gaseous attenuation along slant paths through the Earth's atmosphere, by defining an equivalent height by which the specific attenuation calculated in § 1 may be multiplied to obtain the zenith attenuation. The equivalent heights are dependent on pressure, and can hence be employed for determining the zenith attenuation from sea level up to an altitude of about 10 km. The resulting zenith attenuations are accurate to within ±10% for dry air and ±5% for water vapour from sea level up to altitudes of about 10 km, using the pressure, temperature and water-vapour density appropriate to the altitude of interest. For altitudes higher than 10 km, and particularly for frequencies within 0.5 GHz of the centres of resonance lines at any altitude, the procedure in Annex 1 should be used. Note that the Gaussian function in equation (25b) describing the oxygen equivalent height in the 60 GHz band can yield errors higher than 10% at certain frequencies, since this procedure cannot reproduce the structure shown in Fig. 7. The expressions below were derived from zenith attenuations calculated with the procedure in Annex 1, integrating the attenuations numerically over a bandwidth of 500 MHz; the resultant attenuations hence effectively represent approximate minimum values in the 50-70 GHz band. The path attenuation at elevation angles other than the zenith may then be determined using the procedures described later in this section.

For dry air, the equivalent height is given by:

$$h_o = \frac{6.1}{1 + 0.17\ r_p^{-1.1}} (1 + t_1 + t_2 + t_3) \tag{25a}$$

where:

$$t_1 = \frac{4.64}{1 + 0.066\ r_p^{-2.3}} \ \exp\left[-\left(\frac{f - 59.7}{2.87 + 12.4 \exp\ (-7.9\ r_p)}\right)^2\right] \tag{25b}$$

$$t_2 = \frac{0.14 \exp\ (2.12\ r_p)}{(f - 118.75)^2 + 0.031\ \exp(2.2\ r_p)} \tag{25c}$$

$$t_3 = \frac{0.0114}{1 + 0.14\ r_p^{-2.6}}\ f\ \frac{-0.0247 + 0.0001 f + 1.61 \times 10^{-6} f^2}{1 - 0.0169 f + 4.1 \times 10^{-5} f^2 + 3.2 \times 10^{-7} f^3} \tag{25d}$$

with the constraint that:

$$h_o \le 10.7\ r_p^{0.3} \quad \text{when} \quad f < 70\ \text{GHz} \tag{25e}$$

and for water vapour, the equivalent height is:

$$h_w = 1.66 \left(1 + \frac{1.39\sigma_w}{(f - 22.235)^2 + 2.56\sigma_w} + \frac{3.37\sigma_w}{(f - 183.31)^2 + 4.69\sigma_w} + \frac{1.58\sigma_w}{(f - 325.1)^2 + 2.89\sigma_w}\right) \tag{26a}$$

$$\text{for } f \le 350 \text{ GHz}$$

$$\sigma_w = \frac{1.013}{1 + \exp[-8.6\ (r_p - 0.57)]} \tag{26b}$$

The zenith attenuation between 50 to 70 GHz is a complicated function of frequency, as shown in Fig. 7, and the above algorithms for equivalent height can provide only an approximate estimate, in general, of the minimum levels of attenuation likely to be encountered in this frequency range. For greater accuracy, the procedure in Annex 1 should be used.

The concept of equivalent height is based on the assumption of an exponential atmosphere specified by a scale height to describe the decay in density with altitude. Note that scale heights for both dry air and water vapour may vary with latitude, season and/or climate, and that water vapour distributions in the real atmosphere may deviate considerably from the exponential, with corresponding changes in equivalent heights. The values given above are applicable up to altitudes of about 10 km.

The total zenith attenuation is then:

$$A = \gamma_o \, h_o + \gamma_w \, h_w \quad \text{dB} \tag{27}$$

Figure 6 shows the total zenith attenuation at sea level, as well as the attenuation due to dry air and water vapour, using the mean annual global reference atmosphere given in Recommendation ITU-R P.835. Between 50 and 70 GHz greater accuracy can be obtained from the 0 km curve in Fig. 7 which was derived using the line-by-line calculation as described in Annex 1.

2.2.1 Elevation angles between 5° and 90°

2.2.1.1 Earth-space paths

For an elevation angle, φ, between 5° and 90°, the path attenuation is obtained using the cosecant law, as follows:

For path attenuation based on surface meteorological data:

$$A = \frac{A_o + A_w}{\sin \varphi} \quad \text{dB} \tag{28}$$

where $A_o = h_o \, \gamma_o$ and $A_w = h_w \, \gamma_w$

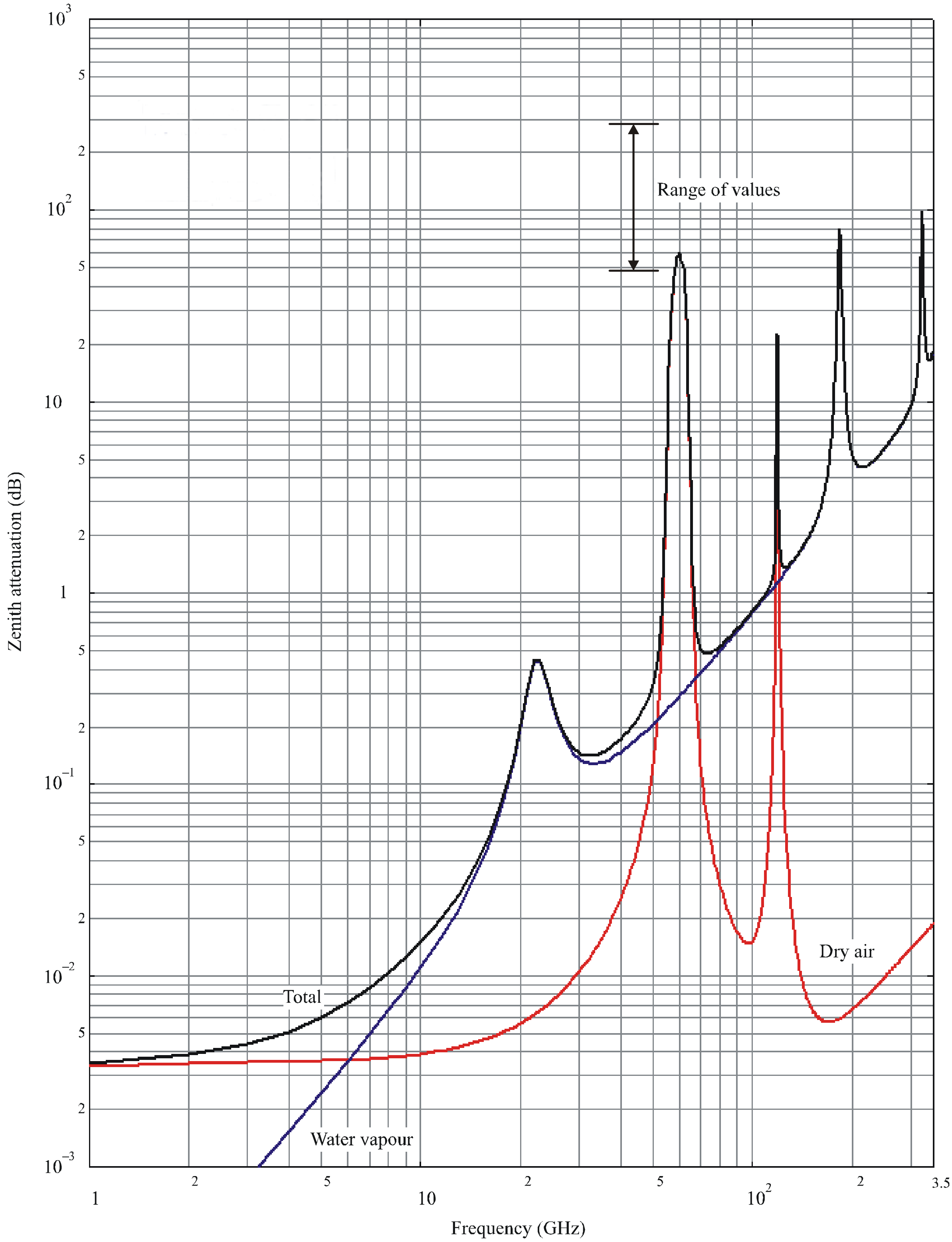
and for path attenuation based on integrated water vapour content:

$$A(p) = \frac{A_o + A_w(p)}{\sin \varphi} \quad \text{dB} \tag{29}$$

where $A_w(p)$ is given in § 2.3.

FIGURE 6

Total, dry air and water-vapour zenith attenuation from sea level

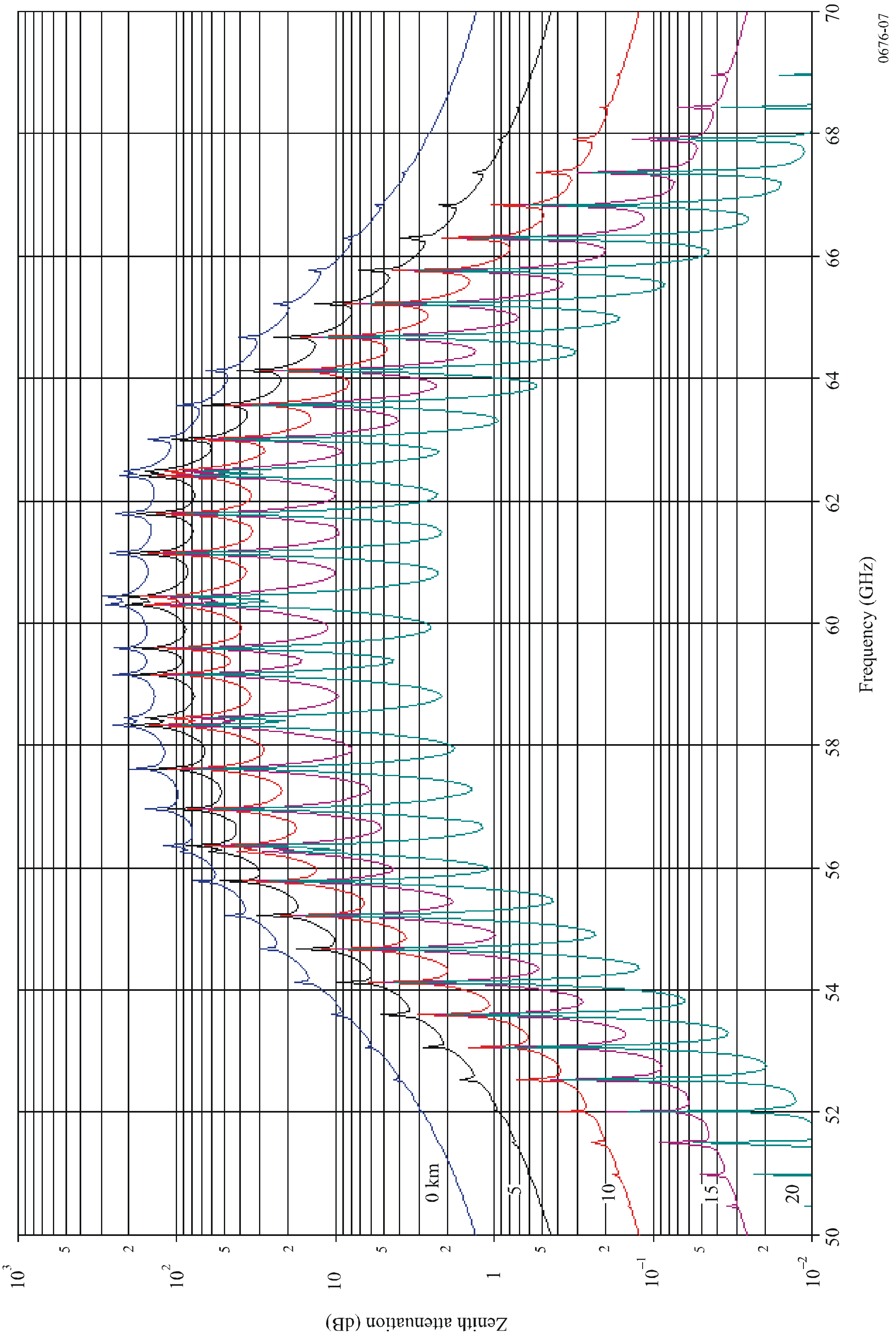


Surface pressure: 1 013 hPa
Surface temperature: 15° C
Surface water-vapour density: 7.5 g/m^3

0676-06

FIGURE 7

Zenith oxygen attenuation from the altitudes indicated, calculated at intervals of 50 MHz, including line centres (0 km, 5 km, 10 km, 15 km and 20 km)



2.2.1.2 Inclined paths

To determine the attenuation values on an inclined path between a station situated at altitude h_1 and another at a higher altitude h_2, where both altitudes are less than 10 km above mean sea level, the values h_o and h_w in equation (28) must be replaced by the following h'_o and h'_w values:

$$h'_o = h_o \left[\mathrm{e}^{-h_1/h_o} - \mathrm{e}^{-h_2/h_o} \right] \qquad \text{km} \qquad (30)$$

$$h'_w = h_w \left[\mathrm{e}^{-h_1/h_w} - \mathrm{e}^{-h_2/h_w} \right] \qquad \text{km} \qquad (31)$$

it being understood that the value ρ of the water-vapour density used in equation (23) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \times \exp(h_1 / 2) \qquad (32)$$

where ρ_1 is the value corresponding to altitude h_1 of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).

Equations (30), (31) and (32) use different normalizations for the dry air and water-vapour equivalent heights. While the mean air pressure referred to sea level can be considered constant around the world (equal to 1 013 hPa), the water-vapour density not only has a wide range of climatic variability but is measured at the surface (i.e. at the height of the ground station). For values of surface water-vapour density, see Recommendation ITU-R P.836.

2.2.2 Elevation angles between 0° and 5°

2.2.2.1 Earth-space paths

In this case, Annex 1 of this Recommendation should be used. The same Annex should also be used for elevations less than zero.

2.2.2.2 Inclined paths

The attenuation on an inclined path between a station situated at altitude h_1 and a higher altitude h_2 (where both altitudes are less than 10 km above mean sea level), can be determined from the following:

$$A = \gamma_o \sqrt{h_o} \left[\frac{\sqrt{R_e + h_1} \cdot \mathrm{F}(x_1)\, \mathrm{e}^{-h_1/h_o}}{\cos \varphi_1} - \frac{\sqrt{R_e + h_2} \cdot \mathrm{F}(x_2)\, \mathrm{e}^{-h_2/h_o}}{\cos \varphi_2} \right]$$

$$+ \gamma_w \sqrt{h_w} \left[\frac{\sqrt{R_e + h_1} \cdot \mathrm{F}(x'_1)\, \mathrm{e}^{-h_1/h_w}}{\cos \varphi_1} - \frac{\sqrt{R_e + h_2} \cdot \mathrm{F}(x'_2)\, \mathrm{e}^{-h_2/h_w}}{\cos \varphi_2} \right] \qquad \text{dB} \qquad (33)$$

where:

R_e: effective Earth radius including refraction, given in Recommendation ITU-R P.834, expressed in km (a value of 8 500 km is generally acceptable for the immediate vicinity of the Earth's surface)

φ_1: elevation angle at altitude h_1

F : function defined by:

$$F(x) = \frac{1}{0.661\,x + 0.339\sqrt{x^2 + 5.51}} \tag{34}$$

$$\varphi_2 = \arccos\left(\frac{R_e + h_1}{R_e + h_2}\cos\varphi_1\right) \tag{35a}$$

$$x_i = \tan\varphi_i\sqrt{\frac{R_e + h_i}{h_o}} \qquad \text{for } i = 1, 2 \tag{35b}$$

$$x_i' = \tan\varphi_i\sqrt{\frac{R_e + h_i}{h_w}} \qquad \text{for } i = 1, 2 \tag{35c}$$

it being understood that the value ρ of the water vapour density used in equation (23) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \cdot \exp\left(h_1 / 2\right) \tag{36}$$

where ρ_1 is the value corresponding to altitude h_1 of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).

Values for ρ_1 at the surface can be found in Recommendation ITU-R P.836. The different formulation for dry air and water vapour is explained at the end of § 2.2.

2.3 Slant path water-vapour attenuation

The above method for calculating slant path attenuation by water vapour relies on the knowledge of the profile of water-vapour pressure (or density) along the path. In cases where the integrated water vapour content along the path, V_t, is known, an alternative method may be used. The total water-vapour attenuation can be estimated as:

$$A_w(f, \theta, P) = \frac{0.0173\, V_t(P)}{\sin\theta}\,\frac{\gamma_W(f, p_{ref}, \rho_{v,ref}, t_{ref})}{\gamma_W(f_{ref}, p_{ref}, \rho_{v,ref}, t_{ref})} \qquad \text{dB} \tag{37}$$

where:

f : frequency (GHz)

θ: elevation angle ($\geq 5°$)

f_{ref} : 20.6 (GHz)

p_{ref} = 780 (hPa)

$\rho_{v,ref}$ = $\frac{V(P)}{4}$ (g/m^3)

$$t_{ref} = 14 \ln \left(\frac{0.22\, V_t(P)}{4} \right) + 3 \ (°C)$$

$V_t(P)$: integrated water vapour content at the required percentage of time (kg/m^2 or mm), which can be obtained either from radiosonde profiles, radiometric measurements, or Recommendation ITU-R P.836 (kg/m^2 or mm)

$\gamma_W(f, p, \rho, t)$: specific attenuation as a function of frequency, pressure, water-vapour density, and temperature calculated from equation (23a) (dB/km).