International Telecommunication Union

ITU-R

Radiocommunication Sector of ITU

Recommendation ITU-R P.676-12

(08/2019)

Attenuation by atmospheric gases and related effects

P Series

Radiowave propagation

International Telecommunication Union

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SNG	Satellite news gathering
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Note: *This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R P.676-12*

Attenuation by atmospheric gases and related effects

(Question ITU-R 201/3)

(1990-1992-1995-1997-1999-2001-2005-2007-2009-2012-2013-2016-2019)

Scope

This Recommendation provides methods to estimate the attenuation of atmospheric gases on terrestrial and slant paths using:

a) a method in Annex 1 to estimate the gaseous attenuation computed by a summation of individual absorption lines that is valid for the frequency range 1-1 000 GHz;

b) two simplified approximate methods in Annex 2 to estimate the gaseous attenuation that are valid in the frequency range 1-350 GHz; and

c) other propagation effects that can be computed by a summation of functions of individual absorption lines.

Keywords

Gaseous absorption, specific attenuation, slant path attenuation, total attenuation, water vapour, oxygen, dry air, dispersion, upwelling, downwelling

The ITU Radiocommunication Assembly,

considering

the necessity of estimating the attenuation, dispersion, upwelling noise, and downwelling noise on slant paths and the attenuation on terrestrial paths due to atmospheric gases,

recommends

1 that, for general application, the procedure in Annex 1 should be used to calculate gaseous attenuation and related effects;

2 that, for approximate estimates, the computationally less intensive procedure in Annex 2 should be used to calculate gaseous attenuation.

Guide to this Recommendation

This Recommendation provides the following three methods of predicting the specific and path gaseous attenuation due to oxygen and water vapour:

1 Calculation of specific and path gaseous attenuation using the line-by-line summation in Annex 1 assuming the atmospheric pressure, temperature, and water vapour density vs. height;

2 An approximate estimate of specific and path gaseous attenuation in Annex 2 assuming the water vapour density at the surface of the Earth;

3 An approximate estimate of path attenuation in Annex 2 assuming the integrated water vapour content along the path.

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* Radiocommunication Study Group 3 made editorial amendments to this Recommendation in the years 2020 and 2021 in accordance with Resolution ITU-R 1.

These prediction methods can use local meteorological data, or, in the absence of local data, reference atmospheres or meteorological maps corresponding to a desired probability of exceedance that are provided in other ITU-R P-series Recommendations.

In addition to specific and path gaseous attenuation, this Recommendation provides methods to predict dispersion, upwelling and downwelling noise temperature, atmospheric bending, and excess atmospheric path delay using the line-by-line summation in Annex 1.

Specific attenuation

Equation (1), which is applicable to frequencies up to 1 000 GHz, may be used to predict specific attenuation. This method requires the pressure, temperature, and water vapour density at the applicable location. If local data is not available, a combination of: a) the mean annual global reference atmosphere given in Recommendation ITU-R P.835, b) the map of mean annual surface temperature in Recommendation ITU-R P.1510 and c) the maps of surface water vapour density vs. exceedance probability given in Recommendation ITU-R P.836 may be used in lieu of the standard ground-level surface water vapour density of 7.5 g/m^3.

Slant path (Earth-space) attenuation

Equation (13), or equations (40) or (41) may be used.

– Equation (13) requires knowledge of the temperature, pressure, and water vapour density profiles along the path. If local profile data is not available, the reference atmospheric profiles given in Recommendation ITU-R P.835 may be used. The surface water vapour density vs. exceedance probability given in Recommendation ITU-R P.836 may be used in lieu of the standard ground-level surface water vapour density of 7.5 g/m^3.

– Equation (40) requires knowledge of the surface pressure, surface temperature, and surface water vapour density. Equation (40) is an approximation to equation (13) applicable to frequencies up to 350 GHz assuming a mean annual global reference atmosphere and an arbitrary surface water vapour density with a negative exponential water vapour density profile vs. height. Equation (40) can be used to predict: a) the instantaneous gaseous attenuation for a specific value of surface pressure, surface temperature, and surface water vapour density or b) the gaseous attenuation corresponding to the surface water vapour density at a desired probability of exceedance. If local surface water vapour density data is not available, the surface water vapour density maps in Recommendation ITU-R P.836 may be used.

– Equation (41) requires knowledge of the surface temperature, surface pressure, and integrated water vapour content along the path. Similar to equation (40), equation (41) can be used to predict: a) the instantaneous gaseous attenuation for a specific value of surface pressure, surface temperature, and integrated water vapour content, or b) the gaseous attenuation corresponding to the integrated water vapour content at a desired probability of exceedance. If local surface integrated water vapour content data is not available, the integrated water vapour maps in Recommendation ITU-R P.836 may be used.

If local surface water vapour density and integrated water vapour content data are both available, equation (41) using local integrated water vapour content is considered to be more accurate than equation (40) using local surface water vapour density data. Similarly, if local data is not available, equation (41) using the maps of integrated water vapour content in Recommendation ITU-R P.836 is considered to be more accurate than equation (40) using the maps of surface water vapour density in Recommendation ITU-R P.836.

	Equation (13)	Equation (40)	Equation (41)
Frequency range	<1 000 GHz	<350 GHz	<350 GHz
Accuracy	Best, line-by-line sum	Approximation	
Pressure vs. height	Arbitrary	Mean Annual Global Reference Atmospheric Profile	
Temperature vs. height			
Water vapour density vs. height		Surface value with negative exponential profile vs. height	Integrated water vapour content in lieu of water vapour density vs. height

Annex 1

Line-by-line calculation of gaseous attenuation

1 Specific attenuation

The specific attenuation at frequencies up to 1 000 GHz, due to dry air and water vapour, can be accurately evaluated at any value of pressure, temperature and humidity as a summation of the individual spectral lines from oxygen and water vapour, together with small additional factors for the non-resonant Debye spectrum of oxygen below 10 GHz, pressure-induced nitrogen attenuation above 100 GHz and a wet continuum to account for the excess water vapour-absorption found experimentally. Figure 1 shows the specific attenuation using the prediction method, calculated from 0 to 1 000 GHz at 1 GHz intervals, for a pressure of 1 013.25 hPa, temperature of 15°C for the cases of a water vapour density of 7.5 g/m^3 (Standard) and a dry atmosphere (Dry).

Near 60 GHz, many oxygen absorption lines merge together at sea-level pressures to form a single, broad absorption band, which is shown in more detail in Fig. 2. This Figure also shows the oxygen attenuation at higher altitudes, with the individual lines becoming resolved as the pressure decreases with increasing altitude. Some additional molecular species (e.g. oxygen isotopic species, oxygen vibrationally excited species, ozone, ozone isotopic species, and ozone vibrationally excited species, and other minor species) are not included in the line-by-line prediction method. These additional lines are insignificant for typical atmospheres but may be important for a dry atmosphere.

The specific gaseous attenuation is given by:

$$\gamma = \gamma_o + \gamma_w = 0.1820 f \left(N''_{Oxygen}(f) + N''_{Water\,Vapour}(f) \right) \qquad \text{(dB/km)} \tag{1}$$

where γ_o and γ_w are the specific attenuations (dB/km) due to dry air (oxygen, pressure-induced nitrogen and non-resonant Debye attenuation) and water vapour, respectively, f is the frequency (GHz), and $N''_{Oxygen}(f)$ and $N''_{Water\ Vapour}(f)$ are the imaginary parts of the frequency-dependent complex refractivities:

$$N''_{Oxygen}(f) = \sum_{i\,(Oxygen)} S_i F_i + N''_D(f) \tag{2a}$$

$$N''_{Water\,Vapour}(f) = \sum_{i\,(Water\,Vapour)} S_i F_i \tag{2b}$$

S_i is the strength of the i^{th} oxygen or water vapour line, F_i is the oxygen or water vapour line shape factor, and the summations extend over all the spectral lines in Tables 1 and 2;

$N''_D(f)$ is the dry continuum due to pressure-induced nitrogen absorption and the Debye spectrum as given by equation (8).

The line strength is given by:

$$S_i = a_1 \times 10^{-7} \, p \, \theta^3 \, \exp\left[a_2 (1 - \theta) \right] \quad \text{for oxygen}$$
$$= b_1 \times 10^{-1} \, e \, \theta^{3.5} \, \exp\left[b_2 (1 - \theta) \right] \quad \text{for water vapour} \tag{3}$$

where:

p: dry air pressure (hPa)

e: water vapour partial pressure (hPa) (total barometric pressure, $p_{tot} = p + e$)

θ = 300/T

T: temperature (K).

FIGURE 1

Specific attenuation due to atmospheric gases, calculated at 1 GHz intervals, including line centres

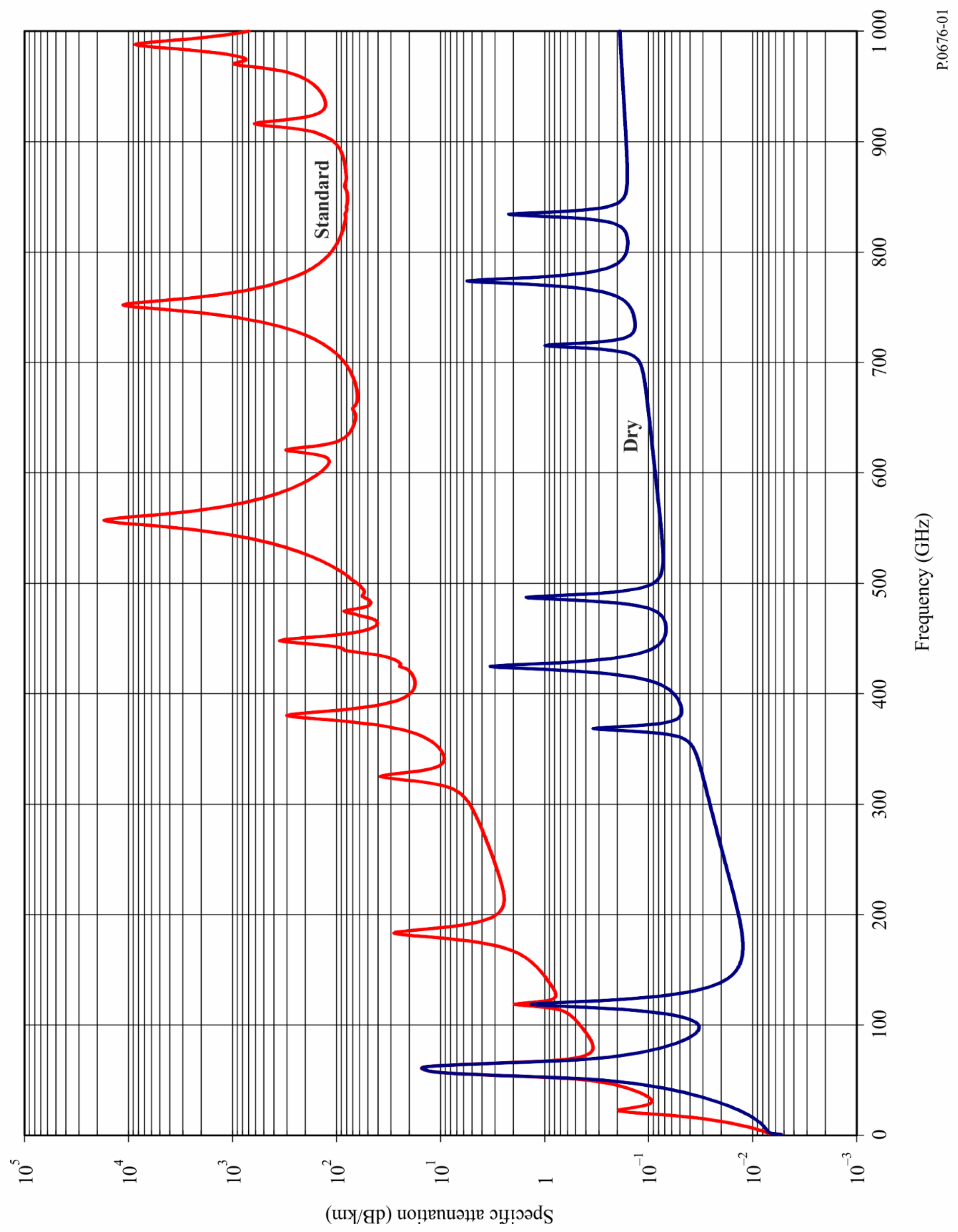
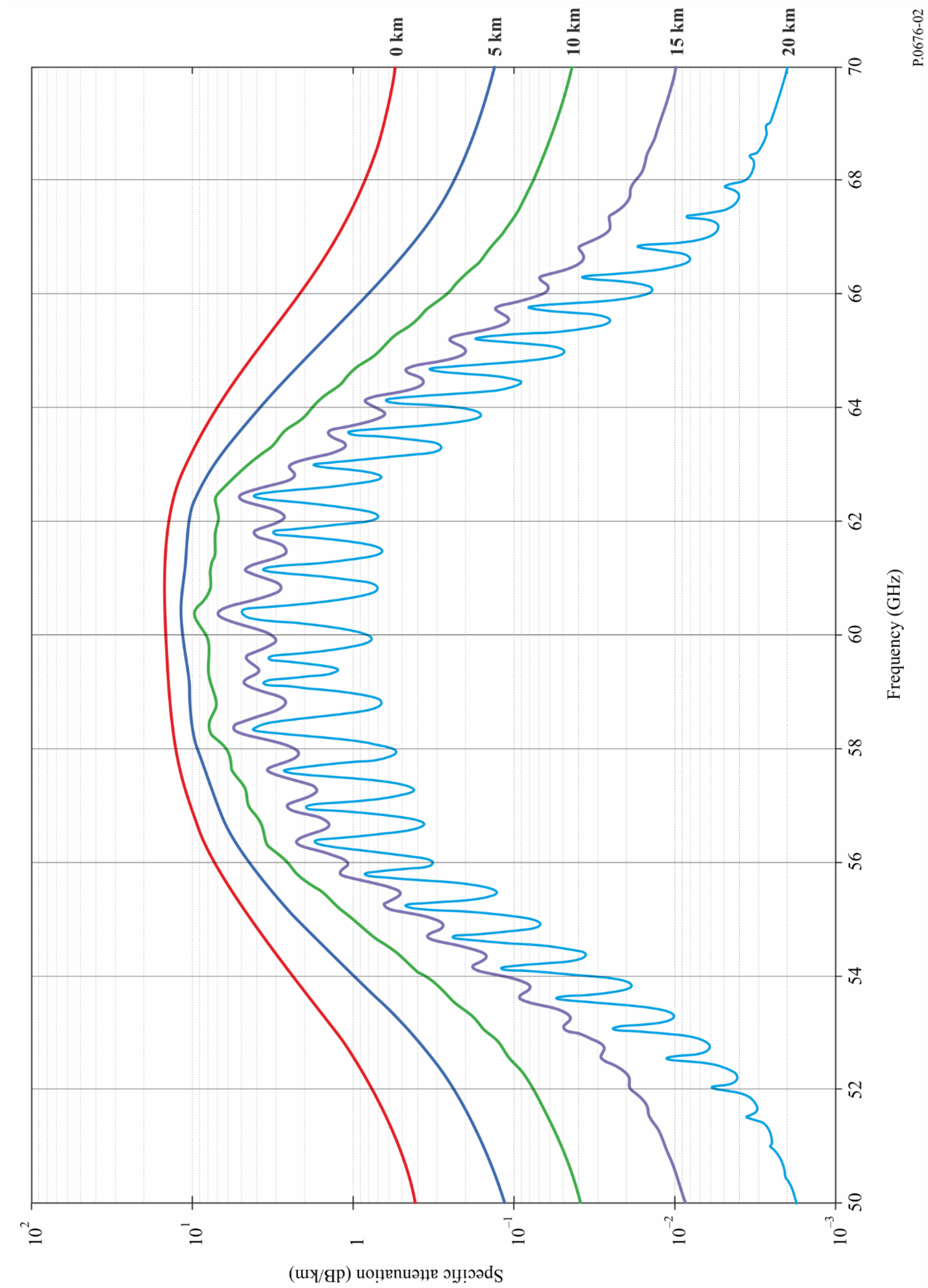


FIGURE 2

Specific attenuation in the range 50-70 GHz at the altitudes indicated, calculated at intervals of 10 MHz, including line centres (0 km, 5 km, 10 km, 15 km and 20 km)



If available, local altitude profiles of p, e and T (e.g. using radiosondes) should be used. In the absence of local data, an appropriate reference standard atmosphere given in Recommendation ITU-R P.835 should be used. (Note that where total atmospheric attenuation is calculated, the same-water vapour partial pressure is used for the attenuation attributable to oxygen and the attenuation attributable to water vapour.)

The water vapour partial pressure, e, at any altitude may be obtained from the water vapour density, ρ, and the temperature, T, at that altitude using the expression:

$$e = \frac{\rho T}{216.7} \tag{4}$$

Spectroscopic data for oxygen is given in Table 1, and spectroscopic data for water vapour is given in Table 2. The last entry in Table 2 is a pseudo-line centred at 1 780 GHz whose lower wing represents the combined contribution below 1 000 GHz of water vapour resonances not included in the line-by-line prediction method (i.e. the wet continuum). The pseudo-line's parameters are adjusted to account for the difference between the measured absorption in the atmospheric windows and the calculated local-line absorption.

The line-shape factor is given by:

$$F_i = \frac{f}{f_i}\left[\frac{\Delta f - \delta\left(f_i - f\right)}{\left(f_i - f\right)^2 + \Delta f^2} + \frac{\Delta f - \delta\left(f_i + f\right)}{\left(f_i + f\right)^2 + \Delta f^2}\right] \tag{5}$$

where f_i is the oxygen or water vapour line frequency and Δf is the width of the line:

$$\begin{aligned} \Delta f &= a_3 \times 10^{-4} \, (p \;\, \theta^{(0.8 \; - \; a_4)} \; + \; 1.1\, e\, \theta) && \text{for oxygen} \\ &= b_3 \times 10^{-4} \, (p \;\, \theta^{b_4} \; + \; b_5 \, e\, \theta^{b_6}) && \text{for water vapour} \end{aligned} \tag{6a}$$

The line width Δf is modified to account for Zeeman splitting of oxygen lines and Doppler broadening of water vapour lines:

$$\begin{aligned} \Delta f &= \sqrt{\Delta f^2 + 2.25 \times 10^{-6}} && \text{for oxygen} \\ &= 0.535 \Delta f + \sqrt{0.217 \Delta f^2 + \frac{2.1316 \times 10^{-12} f_i^2}{\theta}} && \text{for water vapour} \end{aligned} \tag{6b}$$

δ is a correction factor that arises due to interference effects in oxygen lines:

$$\begin{aligned} \delta &= (a_5 + a_6 \theta) \times 10^{-4} (p + e)\, \theta^{0.8} && \text{for oxygen} \\ &= 0 && \text{for water vapour} \end{aligned} \tag{7}$$

TABLE 1

Spectroscopic data for oxygen attenuation

f_0	a_1	a_2	a_3	a_4	a_5	a_6
50.474214	0.975	9.651	6.690	0.0	2.566	6.850
50.987745	2.529	8.653	7.170	0.0	2.246	6.800
51.503360	6.193	7.709	7.640	0.0	1.947	6.729
52.021429	14.320	6.819	8.110	0.0	1.667	6.640
52.542418	31.240	5.983	8.580	0.0	1.388	6.526
53.066934	64.290	5.201	9.060	0.0	1.349	6.206
53.595775	124.600	4.474	9.550	0.0	2.227	5.085
54.130025	227.300	3.800	9.960	0.0	3.170	3.750
54.671180	389.700	3.182	10.370	0.0	3.558	2.654
55.221384	627.100	2.618	10.890	0.0	2.560	2.952
55.783815	945.300	2.109	11.340	0.0	–1.172	6.135
56.264774	543.400	0.014	17.030	0.0	3.525	–0.978
56.363399	1331.800	1.654	11.890	0.0	–2.378	6.547
56.968211	1746.600	1.255	12.230	0.0	–3.545	6.451
57.612486	2120.100	0.910	12.620	0.0	–5.416	6.056
58.323877	2363.700	0.621	12.950	0.0	–1.932	0.436
58.446588	1442.100	0.083	14.910	0.0	6.768	–1.273
59.164204	2379.900	0.387	13.530	0.0	–6.561	2.309
59.590983	2090.700	0.207	14.080	0.0	6.957	–0.776
60.306056	2103.400	0.207	14.150	0.0	–6.395	0.699
60.434778	2438.000	0.386	13.390	0.0	6.342	–2.825
61.150562	2479.500	0.621	12.920	0.0	1.014	–0.584
61.800158	2275.900	0.910	12.630	0.0	5.014	–6.619
62.411220	1915.400	1.255	12.170	0.0	3.029	–6.759
62.486253	1503.000	0.083	15.130	0.0	–4.499	0.844
62.997984	1490.200	1.654	11.740	0.0	1.856	–6.675
63.568526	1078.000	2.108	11.340	0.0	0.658	–6.139
64.127775	728.700	2.617	10.880	0.0	–3.036	–2.895
64.678910	461.300	3.181	10.380	0.0	–3.968	–2.590
65.224078	274.000	3.800	9.960	0.0	–3.528	–3.680
65.764779	153.000	4.473	9.550	0.0	–2.548	–5.002
66.302096	80.400	5.200	9.060	0.0	–1.660	–6.091
66.836834	39.800	5.982	8.580	0.0	–1.680	–6.393
67.369601	18.560	6.818	8.110	0.0	–1.956	–6.475
67.900868	8.172	7.708	7.640	0.0	–2.216	–6.545
68.431006	3.397	8.652	7.170	0.0	–2.492	–6.600
68.960312	1.334	9.650	6.690	0.0	–2.773	–6.650
118.750334	940.300	0.010	16.640	0.0	–0.439	0.079

TABLE 1 (*end*)

f_0	a_1	a_2	a_3	a_4	a_5	a_6
368.498246	67.400	0.048	16.400	0.0	0.000	0.000
424.763020	637.700	0.044	16.400	0.0	0.000	0.000
487.249273	237.400	0.049	16.000	0.0	0.000	0.000
715.392902	98.100	0.145	16.000	0.0	0.000	0.000
773.839490	572.300	0.141	16.200	0.0	0.000	0.000
834.145546	183.100	0.145	14.700	0.0	0.000	0.000

TABLE 2

Spectroscopic data for water vapour attenuation

f_0	b_1	b_2	b_3	b_4	b_5	b_6
22.235080	.1079	2.144	26.38	.76	5.087	1.00
67.803960	.0011	8.732	28.58	.69	4.930	.82
119.995940	.0007	8.353	29.48	.70	4.780	.79
183.310087	2.273	.668	29.06	.77	5.022	.85
321.225630	.0470	6.179	24.04	.67	4.398	.54
325.152888	1.514	1.541	28.23	.64	4.893	.74
336.227764	.0010	9.825	26.93	.69	4.740	.61
380.197353	11.67	1.048	28.11	.54	5.063	.89
390.134508	.0045	7.347	21.52	.63	4.810	.55
437.346667	.0632	5.048	18.45	.60	4.230	.48
439.150807	.9098	3.595	20.07	.63	4.483	.52
443.018343	.1920	5.048	15.55	.60	5.083	.50
448.001085	10.41	1.405	25.64	.66	5.028	.67
470.888999	.3254	3.597	21.34	.66	4.506	.65
474.689092	1.260	2.379	23.20	.65	4.804	.64
488.490108	.2529	2.852	25.86	.69	5.201	.72
503.568532	.0372	6.731	16.12	.61	3.980	.43
504.482692	.0124	6.731	16.12	.61	4.010	.45
547.676440	.9785	.158	26.00	.70	4.500	1.00
552.020960	.1840	.158	26.00	.70	4.500	1.00
556.935985	497.0	.159	30.86	.69	4.552	1.00
620.700807	5.015	2.391	24.38	.71	4.856	.68
645.766085	.0067	8.633	18.00	.60	4.000	.50
658.005280	.2732	7.816	32.10	.69	4.140	1.00
752.033113	243.4	.396	30.86	.68	4.352	.84
841.051732	.0134	8.177	15.90	.33	5.760	.45
859.965698	.1325	8.055	30.60	.68	4.090	.84
899.303175	.0547	7.914	29.85	.68	4.530	.90
902.611085	.0386	8.429	28.65	.70	5.100	.95
906.205957	.1836	5.110	24.08	.70	4.700	.53

TABLE 2 (*end*)

f_0	b_1	b_2	b_3	b_4	b_5	b_6
916.171582	8.400	1.441	26.73	.70	5.150	.78
923.112692	.0079	10.293	29.00	.70	5.000	.80
970.315022	9.009	1.919	25.50	.64	4.940	.67
987.926764	134.6	.257	29.85	.68	4.550	.90
1 780.000000	17506.	.952	196.3	2.00	24.15	5.00

The dry air continuum arises from the non-resonant Debye spectrum of oxygen below 10 GHz and a pressure-induced nitrogen attenuation above 100 GHz.

$$N''_D(f) = f\,p\,\theta^2\left[\frac{6.14\times10^{-5}}{d\left[1+\left(\frac{f}{d}\right)^2\right]}+\frac{1.4\times10^{-12}\,p\,\theta^{1.5}}{1+1.9\times10^{-5}\,f^{1.5}}\right] \tag{8}$$

where *d* is the width parameter for the Debye spectrum:

$$d = 5.6\times10^{-4}(p+e)\theta^{0.8} \tag{9}$$

2 Path attenuation

2.1 Terrestrial paths

For a terrestrial path, or for slightly inclined paths close to the ground, the path attenuation, A, may be calculated as:

$$A = \gamma\, r_0 = (\gamma_o + \gamma_w)\, r_0 \qquad \text{dB} \tag{10}$$

where r_0 is the path length (km).

2.2 Slant paths

Sections 2.2.1 and 2.2.2 provide methods to calculate the Earth-space slant path gaseous attenuation for an ascending path between a location on or near the surface of the Earth and a location above the surface of the Earth or in space using the line-by-line method in Annex 1 for a known temperature, dry air pressure, and water vapour density profile. Section 2.2.3 extends the method to a descending path between a location above the surface of the Earth or in space and a location on or near the surface of the Earth. Sections 2.2.4 and 2.2.5 provide methods to calculate the bending and excess atmospheric path length, respectively, on an Earth-space path.

2.2.1 Non-negative apparent elevation angles

The slant path gaseous attenuation on an ascending path between heights h_1 and h_2 ($h_2 > h_1 \geq 0$ km) is:

$$A_{gas} = \int_{h_1}^{h_2} \frac{\gamma(h)}{\sin \varphi(h)} dh = \int_{h_1}^{h_2} \frac{\gamma(h)}{\sqrt{1-\cos^2 \varphi(h)}} dh \tag{11}$$

where:

$$\cos \varphi(h) = \frac{(R_E+h_1)\, n(h_1)}{(R_E+h)\, n(h)} \cos \varphi_1 \tag{12}$$

$\gamma(h)$ is the specific attenuation at height h, R_E is the average Earth radius (6 371 km), φ_1 is the local apparent elevation angle at height h_1, and $n(h)$ is the refractive index at height h.

While equation (11) can be evaluated by numerical integration[1], the slant path gaseous attenuation is well-approximated by dividing the atmosphere into exponentially increasing layers, determining the specific attenuation (dB/km) of each layer and the path length (km) through each layer, and summing the product of the specific attenuation of each layer and the path length through each layer as shown in equation (13). In the absence of local temperature, dry air pressure, and water vapour partial pressure profiles vs. height (e.g. from radiosonde data), any of the six reference standard atmospheres (i.e. the mean annual global reference atmosphere, the low-latitude reference atmosphere, the mid-latitude summer reference atmosphere, the mid-latitude winter reference atmosphere, the high-latitude summer reference atmosphere, or the high-latitude winter reference atmosphere) given in Recommendation ITU-R P.835 may be used.

$$A_{gas} = \sum_{i=1}^{i_{max}} a_i\, \gamma_i \quad \text{(dB)} \tag{13}$$

where γ_i is the specific attenuation (dB/km) of the i^{th} layer per equation (1), and a_i is the path length (km) through the i^{th} layer.

For a slant path between the surface of the Earth and space and referring to the geometry in Fig. 5, the thickness of the layers increases exponentially from 10 cm at the surface of the Earth to ~1 km at a height of ~100 km to ensure an accurate estimate of the total slant path gaseous attenuation. The thickness of the i^{th} layer, δ_i, is:

$$\delta_i = 0.0001\, e^{\frac{i-1}{100}} \qquad \text{(km)} \tag{14}$$

$h_1 = 0$, and h_i, the height of the bottom of layer i for $i \geq 2$, is:

$$h_i = \sum_{j=1}^{i-1} \delta_j = 0.0001\, \frac{e^{\frac{i-1}{100}}-1}{e^{\frac{1}{100}}-1} \tag{15}$$

If one of the six reference standard atmospheres specified in Recommendation ITU-R P.835 is used, the atmospheric profile is defined for geometric heights up to 100 km, in which case $i_{max} = 922$, $\delta_{922} = 0.999\,66$ km, and $h_{922} = 99.457$ km.

For a slant path between a lower height within the atmosphere, h_{lower}, and an upper height within the atmosphere, h_{upper}, ($0 \text{ km} \leq h_{lower} < h_{upper} \leq 100 \text{ km}$), the slant path attenuation can be calculated by setting r_1 to the radius of the lower height from the centre of the Earth and modifying equations (14) and (15) to approximately preserve the exponentially increasing height progression relative to the surface of the Earth as follows:

a) Calculate i_{lower} and i_{upper}:

$$i_{lower} = \text{floor}\left\{100 \ln\left[10^4\, h_{lower}\left(e^{\frac{1}{100}} - 1\right) + 1\right] + 1\right\} \tag{16a}$$

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[1] Equation (11) can be evaluated using various methods depending on the implementation: e.g. a) the integral function in Matlab, b) the quad function in Octave, c) the quad function in Python, d) several Numerical Recipes functions, and other equivalent methods.

$$i_{upper} = \text{ceiling}\left\{100 \ln\left[10^4\, h_{upper}\left(e^{\frac{1}{100}} - 1\right) + 1\right] + 1\right\} \quad (16b)$$

where floor(x) rounds x down to the next nearest integer, and ceiling(x) rounds x up to the next nearest integer.

b) Replace the lower limit in equation (13) with $i = i_{lower}$ and the upper limit with $i_{upper} - 1$.

c) Replace 0.0001 in equation (14) with m, where:

$$m = \left(\frac{e^{\frac{2}{100}} - e^{\frac{1}{100}}}{e^{\frac{i_{upper}}{100}} - e^{\frac{i_{lower}}{100}}}\right)(h_{upper} - h_{lower}) \quad (16c)$$

d) Replace equation (15) with:

$$h_i = h_{lower} + \sum_{j=i_{lower}}^{i-1} \delta_j = h_{lower} + m\,\frac{e^{\frac{i-1}{100}} - e^{\frac{i_{lower}-1}{100}}}{e^{\frac{1}{100}} - 1},\; i_{lower} \leq\; i \leq i_{upper} \quad (16d)$$

Equations (16a) to (16d) should be used with caution due to possible degraded accuracy for slant paths where $i_{upper} - i_{lower} < 50$ (e.g. paths between two airborne platforms).

a_i is the path length through the i^{th} layer with thickness δ_i, and n_i is the radio refractive index of the i^{th} layer. n_i is a function of the dry air pressure, temperature and water vapour partial pressure of the i^{th} layer using equations (1) and (2) of Recommendation ITU-R P.453. α_i and β_{i+1} are the entry and exit incidence angles at the interface between the i^{th} and $(i+1)^{st}$ layer, r_i is the radius from the centre of the Earth to the beginning of the i^{th} layer, $r_{i+1} = r_i + \delta_i$, and r_1 is the radius from the centre of the Earth to the beginning of the lowest layer, typically the average Earth radius (6 371 km). The refractive index, n_i, and the specific attenuation, γ_i, of the i^{th} layer are their respective values at the midpoint of the i^{th} layer; i.e. at the height $r_i + \delta_i/2$.

The path length a_i is:

$$a_i = -r_i \cos\beta_i + \sqrt{r_i^2 \cos^2\beta_i + 2\, r_i\, \delta_i + \delta_i^2} \quad \text{(km)} \quad (17)$$

and the angle α_i is:

$$\alpha_i = \pi - \cos^{-1}\left(\frac{-a_i^2 - 2\, r_i\, \delta_i - \delta_i^2}{2\, a_i\,(r_i + \delta_i)}\right) \quad (18a)$$

$$= \sin^{-1}\left(\frac{r_i}{r_i + \delta_i}\sin\beta_i\right) \quad (18b)$$

Equation (18a) is deprecated due to degraded accuracy. β_1 is the local zenith angle at or near the surface of the Earth (the complement of the apparent elevation angle, φ; i.e. $\beta_1 = 90° - \varphi$).

β_{i+1} can be recursively calculated from α_i using Snell's law as follows:

$$\beta_{i+1} = \sin^{-1}\left(\frac{n_i}{n_{i+1}}\sin\alpha_i\right) \quad (19a)$$

Alternatively, β_i can be calculated directly without calculating α_i using Snell's law in polar coordinates as follows:

$$\beta_i = \sin^{-1}\left(\frac{n_1\, r_1}{n_i\, r_i}\sin\beta_1\right) \quad (19b)$$

and similarly, α_i can be calculated as follows:

$$\alpha_i = \sin^{-1}\left(\frac{n_1 r_1}{n_i\, r_{i+1}}\sin\beta_1\right) \quad (19c)$$

In the Earth-to-space direction, equations (19a) or (19b) and (19c) may be invalid at initial apparent elevation angles < 1° (i.e. initial apparent zenith angle, $\beta_1, > 89^o$) when the radio refractivity gradient dN/dh is less than −157 N-units/km, which may occur when radiosonde data from certain regions of the world susceptible to ducting are used as the atmospheric profile. In these cases, the radio wave is reflected by the atmosphere and follows the curvature of the Earth (i.e. travels in ducts), and the argument of the inverse sine function in equations (19a) or (19b) and (19c) is greater than 1. Equations (19a), (19b), and (19c) are valid for all non-negative apparent elevation angles when any of the six reference standard atmospheres in Recommendation ITU-R P.835 are used as input, since these reference atmospheres do not have refractivity gradients characteristic of ducting.

Figure 4 shows the zenith attenuation calculated at 1 GHz intervals for the mean annual global reference standard atmosphere in Recommendation ITU-R P.835. The "Standard" atmosphere is the mean annual global reference atmosphere with ρ_o = 7.5 g/m^3, and the "Dry" atmosphere is the mean annual global reference atmosphere with ρ_o = 0 g/m^3.

2.2.2 Negative apparent elevation angles

Equation (13) assumes the height increases between the earth station and space. However, for negative apparent elevation angles from an elevated earth station, the height decreases along the propagation path between the earth station and the minimum grazing height and then increases along the propagation path between the minimum grazing height and space. This is shown in Fig. 3 for an earth station at height h_1 with an apparent elevation angle of $90^o - \beta_1$.

From Snell's law in polar coordinates:

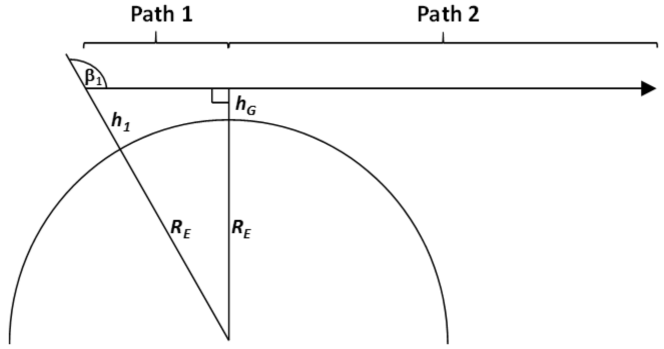
$$n(h_G)(R_E + h_G) = n(h_1)\,(R_E + h_1)\sin\beta_1 \quad (20)$$

in which case the grazing height, h_G, can be determined by iteratively solving equation (20). The refractive index, $n(h)$, can be determined from equations (1) and (2) of Recommendation ITU-R P.453 for the specific atmospheric profile of interest, typically one of the reference profiles in Recommendation ITU-R P.835.

The net gaseous attenuation is the sum of the gaseous attenuations for Path 1 and Path 2. Path 1 is the gaseous attenuation between a virtual earth station at height h_G km and the actual earth station at a height of h_1 km at an apparent elevation angle of 0°, and Path 2 is the gaseous attenuation between a virtual earth station at height h_G km and the maximum atmospheric height (typically 100 km) at an apparent elevation angle of 0°.

FIGURE 3

Grazing height geometry



2.2.3 Space-Earth Earth-space reciprocity

For a path between a space station and an earth station, where the apparent elevation angle, φ_s, at the space station is negative, and the apparent elevation angle at the earth station is φ_e, the apparent elevation angles are related by:

$$\varphi_s = -\cos^{-1}\left(\frac{r_e\, n_e}{r_s\, n_s}\cos\varphi_e\right) \tag{21a}$$

and

$$\varphi_e = \cos^{-1}\left(\frac{r_s\, n_s}{r_e\, n_e}\cos\varphi_s\right) \tag{21b}$$

where n_e is the refractive index at the earth station height, r_e is the radius from the centre of the Earth to the earth station ($r_e \geq R_E$), n_s is the refractive index at the space station height, and r_s is the radius from the centre of the Earth to the space station ($r_s > r_e$). If the height of the space station is greater than 100 km above the surface of the Earth, then n_s = 1.

Since propagation through the atmosphere is reciprocal, the gaseous attenuation for a space-Earth path, where the apparent elevation angle at the space station is φ_s, is identical to the gaseous attenuation for the reciprocal Earth-space path, where the apparent elevation angle at the earth station is φ_e. As a result, the gaseous attenuation for a descending space-Earth path can be calculated as the gaseous attenuation for the corresponding ascending Earth-space path. If $\frac{r_s\, n_s}{r_e\, n_e}\cos\varphi_s > 1$, then the space-Earth path does not intercept the Earth.

2.2.4 Atmospheric bending

The total atmospheric bending, $Bending$, along the Earth-space path is:

$$Bending = \sum_{i=1}^{i_{max}-1}(\beta_{i+1} - \alpha_i) \tag{22a}$$

$$= \sum_{i=1}^{i_{max}-1}\left[\sin^{-1}\left(\frac{n_1\, r_1}{n_{i+1}\, r_{i+1}}\sin\beta_1\right) - \sin^{-1}\left(\frac{n_1\, r_1}{n_i\, r_{i+1}}\sin\beta_1\right)\right] \tag{22b}$$

where a positive value of bending means the ray bends toward the Earth. Equation (9) of Recomendation ITU-R P.834 is an approximation to equations (22a) and (22b) for the mean annual global reference atmosphere.

2.2.5 Excess atmospheric path length

Since the tropospheric refractive index is greater than 1, the effective atmospheric path length exceeds the geometrical path length, in which case the excess atmospheric path length, ΔL, is:

$$\Delta L = \sum_{i=1}^{i_{max}} a_i(n_i - 1) \quad \text{(km)} \tag{23}$$

The term excess atmospheric path length is synonymous with the term excess radio path length in Recommendation ITU-R P.834; and a method of predicting the excess radio path length as a function of location, day of the year, and apparent elevation angle is given in § 6 of Recommendation ITU-R P.834.

FIGURE 4

Zenith attenuation due to atmospheric gases, calculated at 1 GHz intervals, including line centres

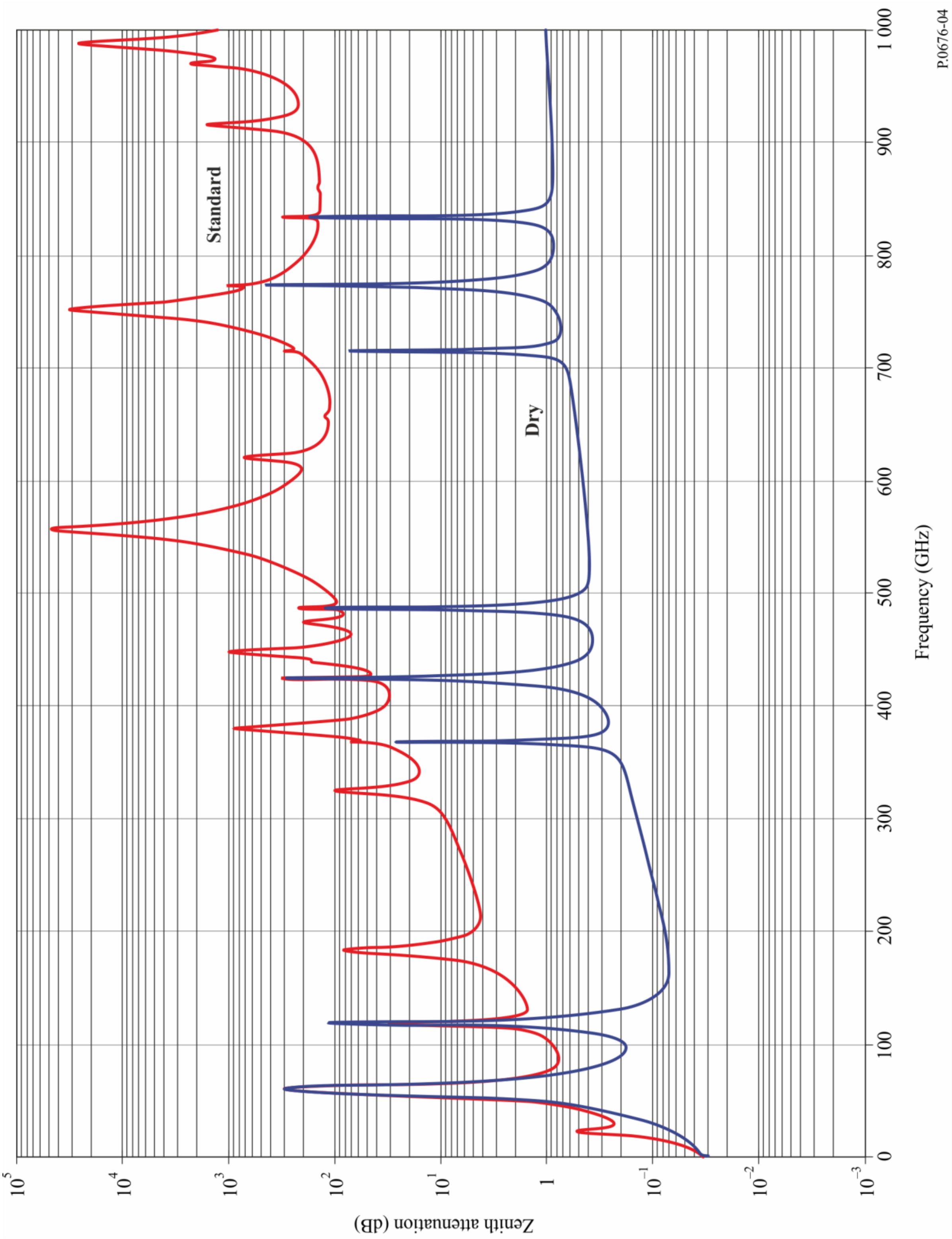
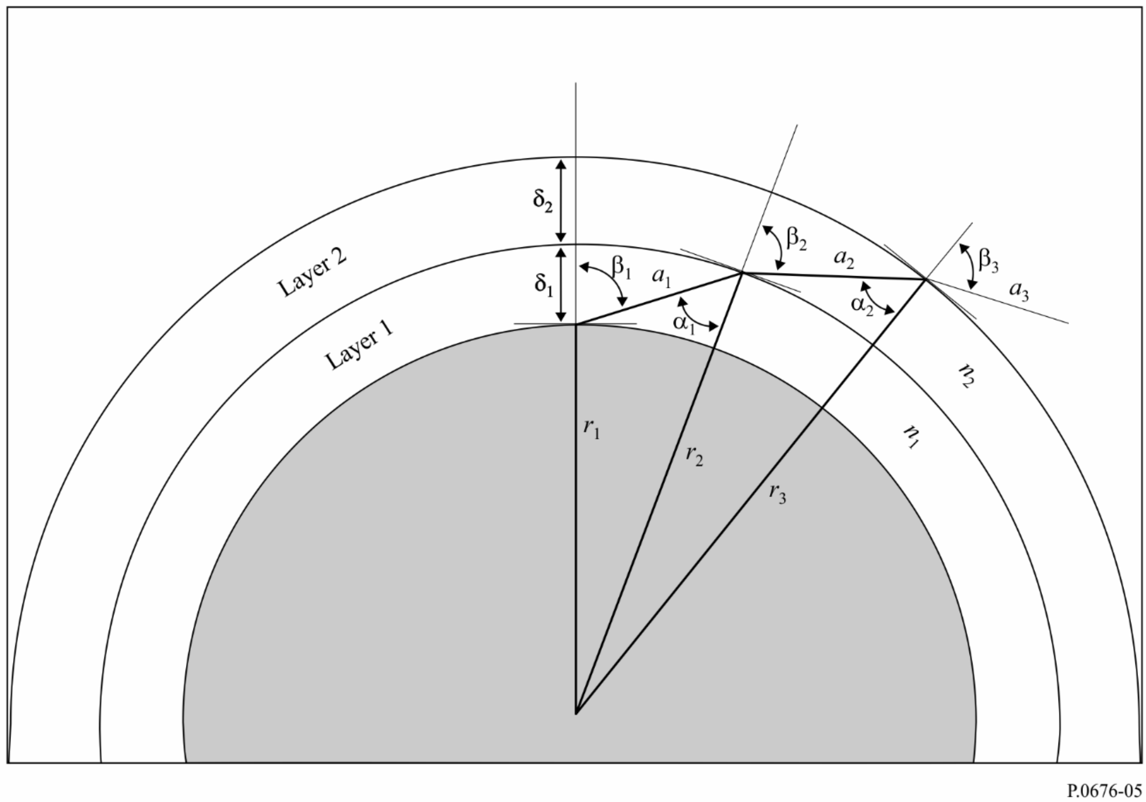


FIGURE 5

Path through the atmosphere



3 Dispersive effects

In addition to the attenuation described in the previous paragraph, which is based on the imaginary part of the frequency-dependent complex refractivity, oxygen and water vapour introduce dispersion, which is based on the real part of the frequency-dependent complex refractivity. This effect is described in terms of phase dispersion vs. frequency (deg/km) or group delay vs. frequency (ps/km), and, similar to attenuation, dispersion can be calculated for slant paths.

Similar to equation (1), the specific gaseous phase dispersion, φ, is given by:

$$\varphi = \varphi_o + \varphi_w = -1.2008 f \left(N'_{Oxygen}(f) + N'_{WaterVapour}(f) \right) \qquad \text{(deg/km)} \qquad (24)$$

where φ_o is the specific phase dispersion (deg/km) due to dry air, φ_w is the specific phase dispersion due to water vapour; f is the frequency (GHz); and $N'_{Oxygen}(f)$ and $N'_{WaterVapour}(f)$ are the real parts of the frequency-dependent complex refractivities:

$$N'_{Oxygen}(f) = \sum_{i\,(Oxygen)} S_i F'_i + N'_D(f) \qquad (25a)$$

$$N'_{Water\,Vapour}(f) = \sum_{i\,(Water\,Vapour)} S_i F'_i \qquad (25b)$$

where:

S_i is the strength of the i^{th} oxygen or water vapour line from equation (3), F'_i is the real part of the oxygen or water vapour line shape factor:

$$F'_i = \frac{f}{f_i}\left[\frac{(f_i - f) + \delta\Delta f}{(f_i - f)^2 + \Delta f^2} - \frac{(f_i + f) + \delta\Delta f}{(f_i + f)^2 + \Delta f^2}\right] \qquad (25c)$$

and the summations extend over all spectral lines in Tables 1 and 2.

$N'_D(f)$ is the real-part of the dry continuum due to pressure-induced nitrogen absorption:

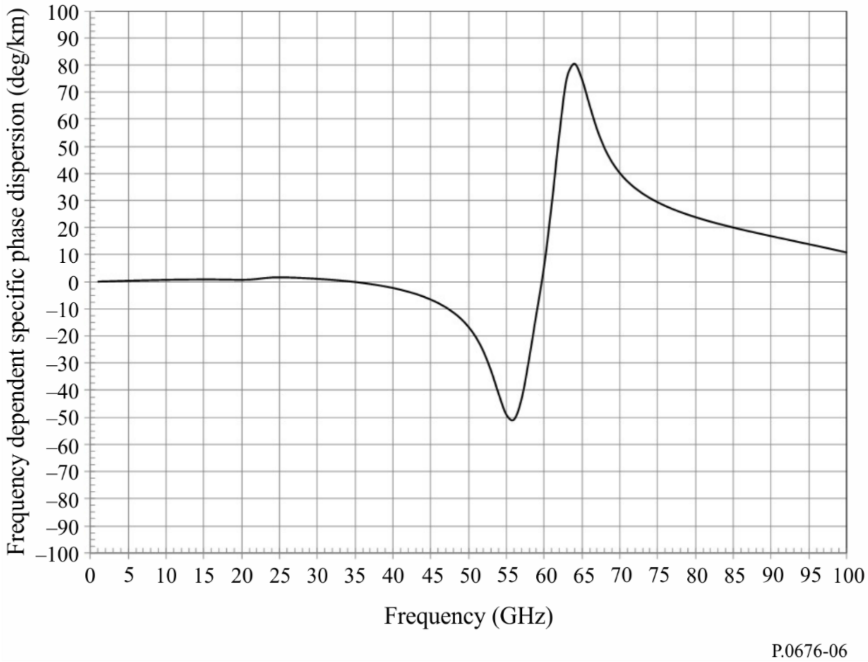
$$N_D'(f) = \frac{-6.14 \times 10^{-5} p\, \theta^2 f^2}{f^2 + d^2} \tag{25d}$$

Δf is defined in equation (6b), Δf δ is defined in equation (7), and d is defined in equation (9).

The frequency dependent specific phase dispersion is shown in Fig. 6 for a standard atmosphere (p = 1 013.25 hPa, ρ = 7.5 g/m^3, T = 15ºC).

FIGURE 6

The frequency dependent specific phase dispersion for a standard atmosphere (p = 1 013.25 hPa, ρ = 7.5 g/m^3, T = 15ºC)



4 Downwelling and upwelling microwave brightness temperature

Microwave brightness temperature is defined as the noise temperature at the output of a lossless antenna due to the incident atmospheric brightness. Noise power spectral density, $S(f)$, and noise temperature, $T(f)$, are related by $S(f) = k\, T(f)$, where k is Boltzmann's constant. The downwelling space-to-Earth microwave brightness temperature looking up and the upwelling Earth-to-space microwave brightness temperature looking down can be calculated similar to equation (13). Layer 1 is typically at the surface of the Earth, and layer k is at the top of the atmosphere (typically 100 km). The aggregate microwave brightness temperature is the sum of the microwave brightness temperatures of each atmospheric layer multiplied by the loss between that atmospheric layer and the observation point. It is assumed that the atmosphere is in local thermodynamic equilibrium and scattering is negligible.

In the following paragraphs, $T_B(f_{GHz}, T_j)$ is the microwave brightness temperature of the jth layer defined by:

$$T_B(f_{GHz}, T_j) = 0.048\, f_{GHz} \left[\frac{1}{\exp\left(\frac{0.048 f_{GHz}}{T_j}\right) - 1} \right] \text{ (K)} \tag{26}$$

where $\boldsymbol{T_j}$ is the physical temperature of the j^th^ layer. $T_B(f_{GHz}, T_j)$ can be well-approximated by T_j for $f_{GHz} < 0.42\ T_j$; γ_j is the specific attenuation (dB/km) of the j^{th} layer given in equation (1), and a_j is the path length (km) through the j^{th} layer given in equation (17).

The difference between the physical temperature, T, and the microwave brightness temperature of a blackbody source, T_B, is shown in Fig. 7. For a given frequency, f_{GHz}, $T - T_B \rightarrow 0.024 f_{GHz}$ as the physical temperature, T, increases.

4.1 Downwelling microwave brightness temperature

If the profiles of physical temperature, pressure and water vapour along the path are known, the downwelling microwave brightness temperature, which is the sum of: a) the cosmic microwave brightness temperature attenuated by the atmospheric attenuation and b) the downwelling atmospheric microwave brightness temperature, can be calculated as follows:

$$T_{downwelling} = T_B(f_{GHz}, 2.73)\ 10^{-\left(\frac{\sum_{j=1}^{k} a_j \gamma_j}{10}\right)}$$

$$+\sum_{j=1}^{k} T_B(f_{GHz}, T_j)\left(10^{\frac{a_j \gamma_j}{10}} - 1\right) 10^{-\left(\frac{\sum_{i=1}^{j} a_i \gamma_i}{10}\right)} \text{ (K)} \qquad (27)$$

However, it may be more convenient to implement the net microwave brightness temperature as a recursion using the following recursive method:

Step 1: Set $T_{B,Downwelling} = 0.048\left[\frac{f_{GHz}}{\exp\left(\frac{0.048 f_{GHz}}{2.73}\right) - 1}\right]$ (27a)

Repeat Steps 2 through 5 for $j = k$ to $j = 1$ decrementing j by 1 at each iteration:

Step 2: Set $T_{B,Downwelling,last} = T_{B,Downwelling}$ (27b)

Step 3: Set $T_B = 0.048\left[\frac{f_{GHz}}{\exp\left(\frac{0.048 f_{GHz}}{T_j}\right) - 1}\right]$ (27c)

Step 4: Set $L_j = 10^{\frac{-a_j \gamma_j}{10}}$ (27d)

Step 5: Set $T_{B,Downwelling} = \left[T_{B,Downwelling,last}\ L_j + \left(1 - L_j\right) T_B\right]$ (27e)

where 2.73 K is the exoatmospheric cosmic microwave background blackbody temperature.

The downwelling microwave brightness temperature for a zenith path and standard atmosphere is shown in Fig. 8.

If the profiles are not known, the method in § 3 of Annex 1 of Recommendation ITU-R P.618 can be used to estimate the downwelling microwave brightness temperature, including other effects from the total atmospheric attenuation.

Recommendation ITU-R P.372 can be used to determine the earth station system noise temperature from the brightness temperatures.

4.2 Upwelling microwave brightness temperature

The net upwelling microwave brightness temperature, which is the sum of: a) the upwelling atmospheric microwave brightness temperature, b) the downwelling atmospheric microwave brightness temperature reflected by the Earth's surface attenuated by the net atmospheric attenuation, and c) upwelling microwave brightness temperature of the Earth's surface attenuated by the atmospheric attenuation, can be calculated as follows:

$$T_{B,upwelling} = (\epsilon T_B(f_{GHz}, T_{Earth}) + \rho\, T_{downwelling}) \times 10^{-\left(\frac{\sum_{j=1}^{k} a_j \gamma_j}{10}\right)}$$

$$+ \sum_{j=1}^{k} T_B(f_{GHz}, T_j) \left(10^{\frac{a_j \gamma_j}{10}} - 1\right) 10^{-\left(\frac{\sum_{i=j}^{k} a_i \gamma_i}{10}\right)} \quad \text{(K)} \qquad (28)$$

However, it may be more convenient to implement the net microwave brightness temperature as a recursion using the following recursive method:

Step 1: Set $T_{B,Upwelling} = \epsilon\ 0.048 \left[\frac{f_{GHz}}{\exp\left(\frac{0.048 f_{GHz}}{T_{Earth}}\right)} - 1\right] + \rho\ T_{B,downwelling}$ (28a)

Repeat Steps 2 through 5 for $j = 1$ to $j = k$ incrementing j by 1 after each iteration:

Step 2: Set $T_{B,Upwelling,last} = T_{B,Upwelling}$ (28b)

Step 3: Set $T_B = 0.048 \left[\frac{f_{GHz}}{\exp\left(\frac{0.048 f_{GHz}}{T_j}\right)} - 1\right]$ (28c)

Step 4: Set $L_j = 10^{\frac{-a_j \gamma_j}{10}}$ (28d)

Step 5: Set $T_{B,Upwelling} = \left[T_{B,Upwelling,last}\ L_j + \left(1 - L_j\right) T_B\right]$ (28e)

where:

ϵ : emissivity of the Earth's surface

ρ : reflectivity of the Earth's surface

$\rho = 1 - \epsilon$.

In the absence of local data or other guidance, a value of ϵ of 0.95 can be used.

The upwelling microwave brightness temperature for a zenith path and the standard (i.e., the mean annual global reference atmosphere) is shown in Fig. 9, where ϵ = 0.95, ρ = 0.05, and T_{Earth} = 290 K.

FIGURE 7

Difference between the physical and microwave brightness temperatures of a blackbody source

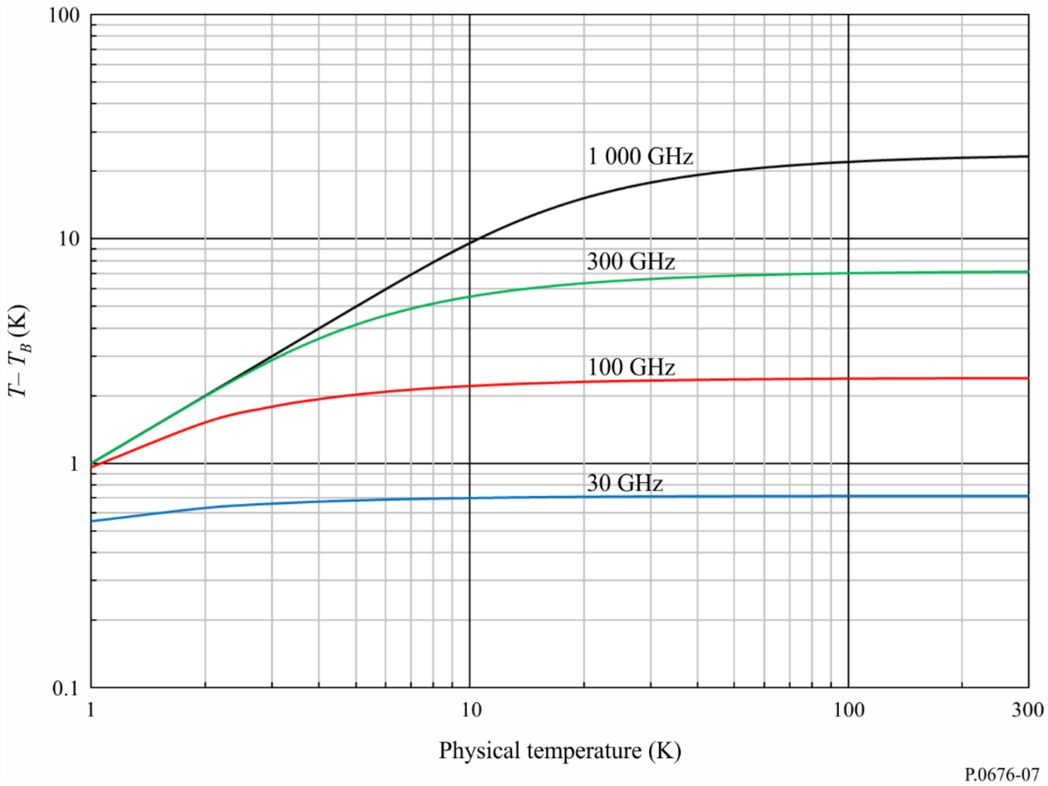


FIGURE 8

Zenith downwelling microwave brightness temperature for a standard atmosphere (1 GHz centres)

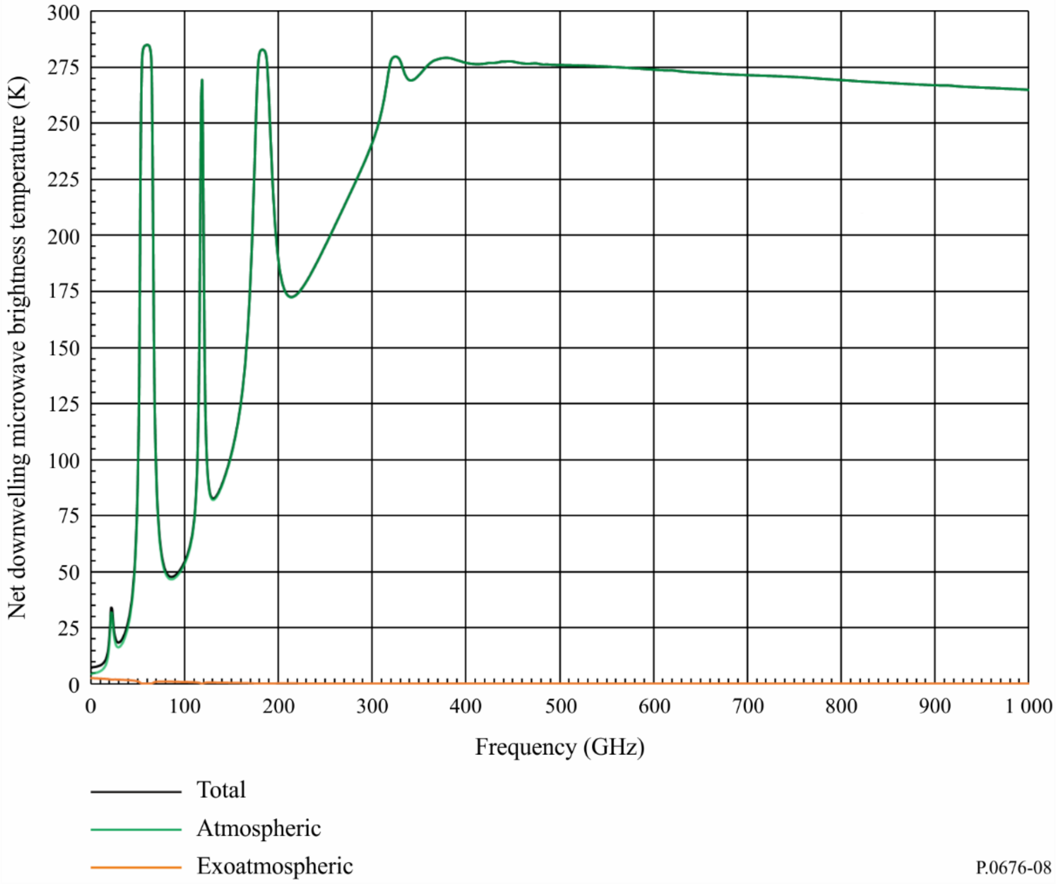


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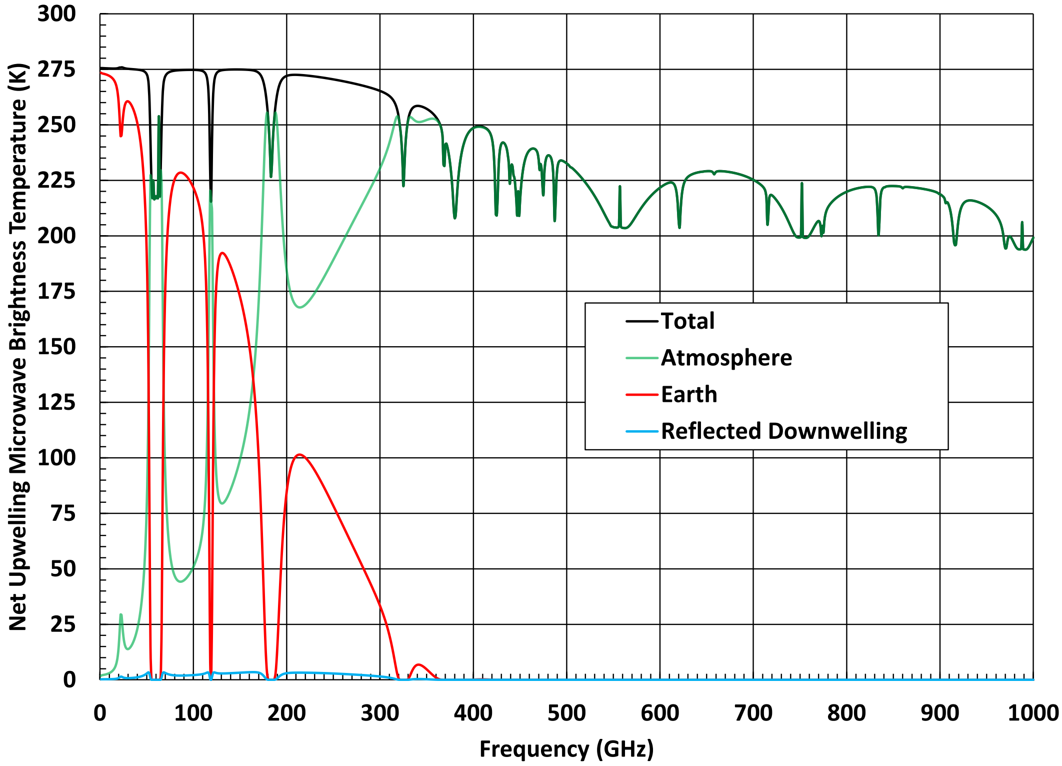
FIGURE 9

Zenith upwelling microwave brightness temperature for a standard atmosphere (1 GHz centres)



5 Slant path attenuation using vertical atmospheric profiles

The slant path gaseous attenuation for any specific profile in Annex 3 of Recommendation ITU-R P.835 can be calculated using the procedure in § 2.2 of Annex 1 noting the following:

1 Convert the water vapour density, ρ, to water vapour partial pressure, *e*, using equation (4).

2 Convert the total air pressure ($p_{tot} = p_{dry} + e$) to dry air pressure, p_{dry}, by subtracting the water vapour partial pressure, *e*.

3 Calculate the total attenuation using equation (13) where the exponential layer thicknesses are defined in equation (14).

4 If the height of the surface of the Earth above mean sea level is not available from local data, an estimate can be obtained from Recommendation ITU-R P.1511.

5 The summation in equation (13) should be from the height of the surface of the Earth above mean sea level to the maximum height in the data set.

6 The 32 levels in each profile should be interpolated and extrapolated (to the surface of the Earth, if required) per the exponential layer thicknesses defined in equation (14) assuming:

a) A linear relation between the logarithm of pressure and height.

b) A linear relation between temperature and height.

c) A linear relation between the logarithm of water vapour density and height.

If needed, equations (24a) to (24c) in Annex 1 of Recommendation ITU-R P.834 (and the associated maps) can be used for interpolation and extrapolation of these profiles.

7 The elevation angle at or near the surface of the Earth is the apparent rather than the free-space elevation angle. For free-space elevation angles less than or equal to 10 degrees, the apparent elevation angle can be calculated from the free-space elevation angle using equation (13) of Recommendation ITU-R P.834.

8 The estimated slant path gaseous attenuation at any latitude and longitude between grid points can be estimated by bilinear interpolation of the corresponding estimates of slant path gaseous attenuation at the surrounding grid points using the procedure in Annex 1 of Recommendation ITU-R P.1144. The slant path gaseous attenuation at each surrounding grid point should be from the height of the surface of the Earth above mean sea level at the latitude and longitude of interest to the maximum height in each profile.

Annex 2

Approximate estimation of gaseous attenuation in the frequency range 1-350 GHz

This Annex contains simplified algorithms for approximate estimates of gaseous attenuation for the limited frequency range from 1 GHz to 350 GHz, path elevation angles of 5 degrees and above, a limited range of meteorological conditions, and a limited variety of geometrical configurations.

1 Specific attenuation

The specific attenuation attributable to oxygen, γ_o (dB/km), and the specific attenuation attributable to water vapour, γ_w (dB/km), are identical to γ_o and γ_w in equation (1). The specific attenuation for moist air attributable to oxygen, γ_o (dB/km), and the specific attenuation for moist air attributable to water vapour, γ_w (dB/km), used in these simplified methods are identical to γ_o and γ_w in equation (1).

The dry pressure, p, temperature, T, and water vapour density, ρ, are values at the surface of the Earth. If local data is not available, the mean annual global reference atmosphere given in Recommendation ITU-R P.835 can be used to determine p, T and ρ .

Figure 10 shows the dry air (Dry), water vapour only with a density of 7.5 g/m^3 (water vapour), and total (Total) specific attenuation from 1 to 350 GHz at sea-level for the mean annual global reference atmosphere given in Recommendation ITU-R P.835. Moreover, values for ρ at the surface of the Earth can be found in Recommendation ITU-R P.836.

2 Path attenuation

2.1 Terrestrial paths

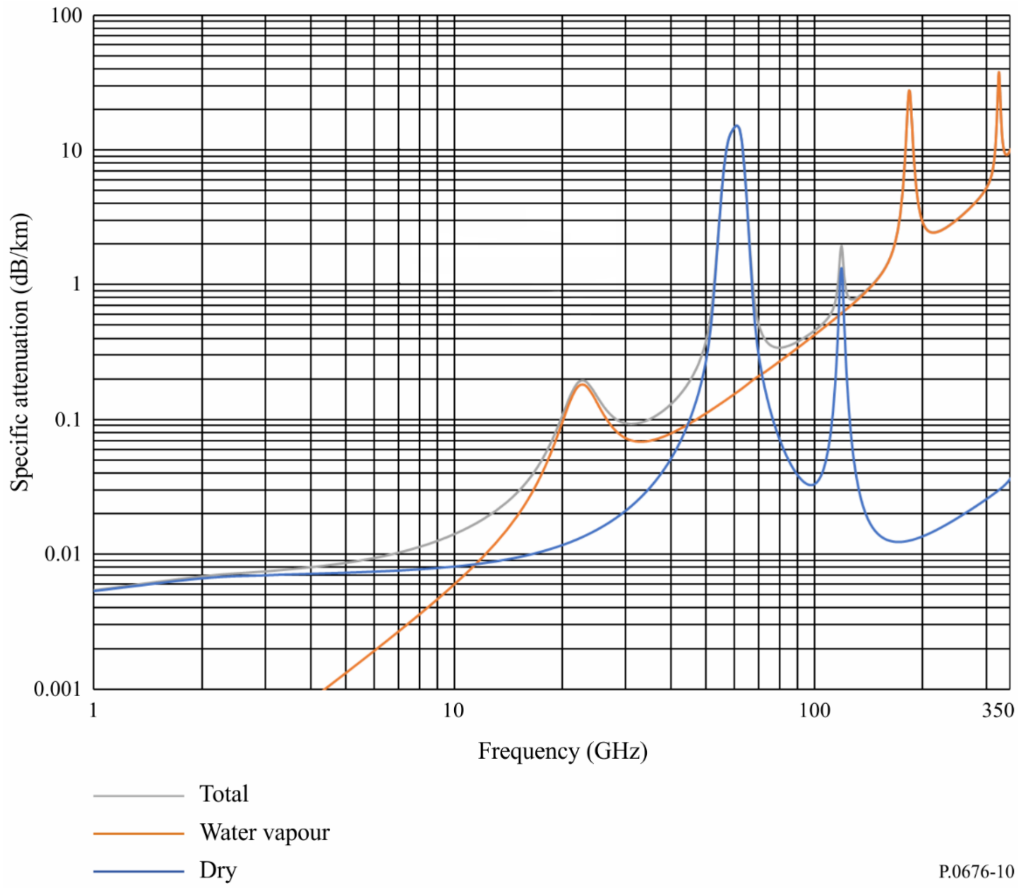
For a horizontal path, or for slightly inclined paths close to the ground, the path attenuation, A, may be calculated as:

$$A = \gamma r_0 = (\gamma_o + \gamma_w) r_0 \quad \text{dB} \tag{29}$$

where r_0 is the path length (km).

FIGURE 10

Specific attenuation due to atmospheric gases
(Pressure = 1 013.25 hPa; Temperature = 15°C; Water Vapour Density = 7.5 g/m^3)



2.2 Slant paths

This section contains algorithms for estimating the total gaseous attenuation for slant paths through the Earth's atmosphere by defining oxygen and water vapour equivalent heights by which the oxygen and water vapour specific attenuations are multiplied to estimate the corresponding zenith oxygen and water vapour attenuations. The oxygen and water vapour specific attenuations are calculated at the pressure, temperature, and water vapour density at the Earth station's altitude using the method in section 1 of Annex 1; and the oxygen and water vapour equivalent heights are calculated at the pressure, temperature, and water vapour density at the surface of the Earth. The concept of equivalent height assumes an exponential decay in atmospheric specific attenuation versus altitude. These algorithms can be used to estimate the total slant path gaseous attenuation for frequencies outside of 0.5 GHz of spectral line centres for earth station altitudes up to 10 km above the surface of the Earth. For frequencies within 0.5 GHz of spectroscopic line centres at any earth station altitude, the line-by-line method in Annex 1 should be used. The expressions below were derived from the reference atmospheric profiles in Annex 1 of Recommendation ITU-R P.835 and are accurate to within 10% for these specific atmospheric profiles. The accuracy of these algorithms at a specific location and time can be assessed by comparing the attenuation estimated by these algorithms with the attenuation calculated using the method in Annex 1 for representative atmospheric profiles in Annex 2 and Annex 3 of Recommendation ITU-R P.835, or radiosonde data.

The equivalent height attributable to the oxygen component of gaseous attenuation is given by:

$$h_o = \frac{6.1A}{1+0.17r_p^{-1.1}}\left(1+t_1+t_2+t_3\right) \tag{30}$$

where:

$$t_1 = \frac{5.1040}{\left(1+0.066r_p^{-2.3}\right)}\exp\left(-\left(\frac{f-59.7}{2.87+12.4\times\exp\left(-7.9r_p\right)}\right)^2\right) \tag{31}$$

$$t_2 = \sum_{i=1}^{7}\frac{c_i\exp\left(2.12r_p\right)}{\left(f-f_i\right)^2+0.025\exp\left(2.2r_p\right)} \tag{32}$$

$$t_3 = \frac{0.0114f}{1+0.14r_p^{-2.6}}\,\frac{15.02f^2-1353f+5.333\times10^4}{f^3-151.3f^2+9629f-6803} \tag{33}$$

$$A = 0.7832+0.00709(T-273.15) \tag{34}$$

where f_i and c_i vs. i are shown in Table 3.

TABLE 3

Parameters f_i and c_i

i	**c*i***	**f_i (GHz)**
1	0.1597	118.750334
2	0.1066	368.498246
3	0.1325	424.763020
4	0.1242	487.249273
5	0.0938	715.392902
6	0.1448	773.839490
7	0.1374	834.145546

with the constraint that:

$$h_o \le 10.7r_p^{0.3} \quad \text{when} \quad f < 70\text{ GHz} \tag{35a}$$

T is the temperature at the Earth's surface in K, ρ is the water vapour density at the Earth's surface in g/m^3, $e = \frac{\rho T}{216.7}$ hPa, and $r_p = (p+e)/1013.25$.

The equivalent height attributable to the water vapour component of gaseous attenuation is:

$$h_w = A + B\times\sum_{i=1}^{14}\frac{a_i\sigma_w}{\left(f-f_i\right)^2+b_i\sigma_w} \tag{35b}$$

where a_i, b_i and f_i vs. i are shown in Table 4, and:

$$A = 1.9298-0.04166(T-273.15)+0.0517\rho \tag{36}$$

$$B = 1.1674-0.00622(T-273.15)+0.0063\rho \tag{37}$$

$$\sigma_w = \frac{1.013}{1 + \exp[-8.6\,(r_p - 0.57)]} \tag{38}$$

TABLE 4

Parameters f_i, a_i, and b_i

i	f_i (GHz)	a_i	b_i
1	22.235080	1.52	2.56
2	183.310087	7.62	10.2
3	325.152888	1.56	2.70
4	380.197353	4.15	5.70
5	439.150807	0.20	0.91
6	448.001085	1.63	2.46
7	474.689092	0.76	2.22
8	488.490108	0.26	2.49
9	556.935985	7.81	10.0
10	620.70087	1.25	2.35
11	752.033113	16.2	20.0
12	916.171582	1.47	2.58
13	970.315022	1.36	2.44
14	987.926764	1.60	1.86

T is the temperature at the Earth's surface in K, ρ is the water vapour density at the Earth's surface in g/m^3, $e = \frac{\rho T}{216.7}$ hPa, and $r_p = (p + e)/1013.25$.

The zenith attenuation between 50 to 70 GHz is a complicated function of frequency, as shown in Fig. 12, and the above expressions for equivalent height can provide only an approximate estimate, in general, of the levels of attenuation likely to be encountered in this frequency range. For greater accuracy, the procedure in Annex 1 should be used.

The total zenith attenuation is then:

$$A = \gamma_o\, h_o + \gamma_w\, h_w \qquad \text{(dB)} \tag{39}$$

Figure 11 shows the total zenith attenuation at sea level (Total), as well as the attenuation due to dry air (Dry) and water vapour (Water Vapour), using the mean annual global reference atmosphere given in Recommendation ITU-R P.835.

2.2.1 Elevation angles between 5° and 90°

2.2.1.1 Earth-space paths

For an elevation angle, φ, between 5° and 90°, the path attenuation is obtained using the cosecant law, as follows:

For path attenuation based on surface meteorological data:

$$A = \frac{A_o + A_w}{\sin\varphi} \qquad \text{dB} \tag{40}$$

where $A_o = h_o \gamma_o$ and $A_w = h_w \gamma_w$

and for path attenuation based on integrated water vapour content:

$$A = \frac{A_o + A_w}{\sin \varphi} \qquad \text{dB} \tag{41}$$

where $A_o = h_o \gamma_o$, and A_w is given in § 2.3.

2.2.1.2 Inclined paths

To determine the attenuation values on an inclined path between a station situated at altitude h_1 and another at a higher altitude h_2, where both altitudes are less than 10 km above mean sea level, the values h_o and h_w in equation (39) must be replaced by the following h_o' and h_w' values:

$$h_o' = h_o \left[e^{-h_1/h_o} - e^{-h_2/h_o} \right] \qquad \text{km} \tag{42}$$

$$h_w' = h_w \left[e^{-h_1/h_w} - e^{-h_2/h_w} \right] \qquad \text{km} \tag{43}$$

it being understood that the value ρ of the water vapour density used in equation (1) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \times \exp(h_1/2) \tag{44}$$

where ρ_1 is the value corresponding to altitude h_1 of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).

Equations (42) and (43) use different normalizations for the dry air and water vapour equivalent heights. While the mean air pressure referred to sea level can be considered constant around the world (equal to 1 013.25 hPa), the water vapour density not only has a wide range of climatic variability but is measured at the surface (i.e. at the height of the ground station). For values of surface water vapour density, see Recommendation ITU-R P.836.

2.2.2 Elevation angles between 0 and 5 degrees

2.2.2.1 Earth-space paths

In this case, Annex 1 of this Recommendation should be used. Annex 1 should also be used for elevations less than zero.

2.2.2.2 Inclined paths

The attenuation on an inclined path between a station situated at altitude h_1 and a higher altitude h_2 (where both altitudes are less than 10 km above mean sea level), can be determined from the following:

$$\begin{aligned} A = {} & \gamma_o \sqrt{h_o} \left[\frac{\sqrt{R_e + h_1} \cdot F(x_1)\, e^{-h_1/h_o}}{\cos \varphi_1} - \frac{\sqrt{R_e + h_2} \cdot F(x_2)\, e^{-h_2/h_o}}{\cos \varphi_2} \right] \\ & + \gamma_w \sqrt{h_w} \left[\frac{\sqrt{R_e + h_1} \cdot F(x_1')\, e^{-h_1/h_w}}{\cos \varphi_1} - \frac{\sqrt{R_e + h_2} \cdot F(x_2')\, e^{-h_2/h_w}}{\cos \varphi_2} \right] \qquad \text{dB} \end{aligned} \tag{45}$$

where:

R_e : effective Earth radius including refraction, given in Recommendation ITU-R P.834, expressed in km (a value of 8 500 km is generally acceptable for the immediate vicinity of the Earth's surface)

φ_1 : elevation angle at altitude h_1

F : function defined by:

$$F(x) = \frac{1}{0.661\,x + 0.339\sqrt{x^2 + 5.51}} \tag{46}$$

$$\varphi_2 = \arccos\left(\frac{R_e + h_1}{R_e + h_2}\cos\varphi_1\right) \tag{47a}$$

$$x_i = \tan\varphi_i\sqrt{\frac{R_e + h_i}{h_o}} \qquad \text{for } i = 1, 2 \tag{47b}$$

$$x_i' = \tan\varphi_i\sqrt{\frac{R_e + h_i}{h_w}} \qquad \text{for } i = 1, 2 \tag{47c}$$

it being understood that the value ρ of the water vapour density used in equation (1) is the hypothetical value at sea level calculated as follows:

$$\rho = \rho_1 \cdot \exp\left(h_1 / 2\right) \tag{48}$$

where ρ_1 is the value corresponding to altitude h_1 of the station in question, and the equivalent height of water vapour density is assumed as 2 km (see Recommendation ITU-R P.835).

FIGURE 11

Total, dry air and water vapour zenith attenuation from sea level

(Pressure = 1 013.25 hPa; Temperature = 15ºC; Water Vapour Density = 7.5 g/m³)

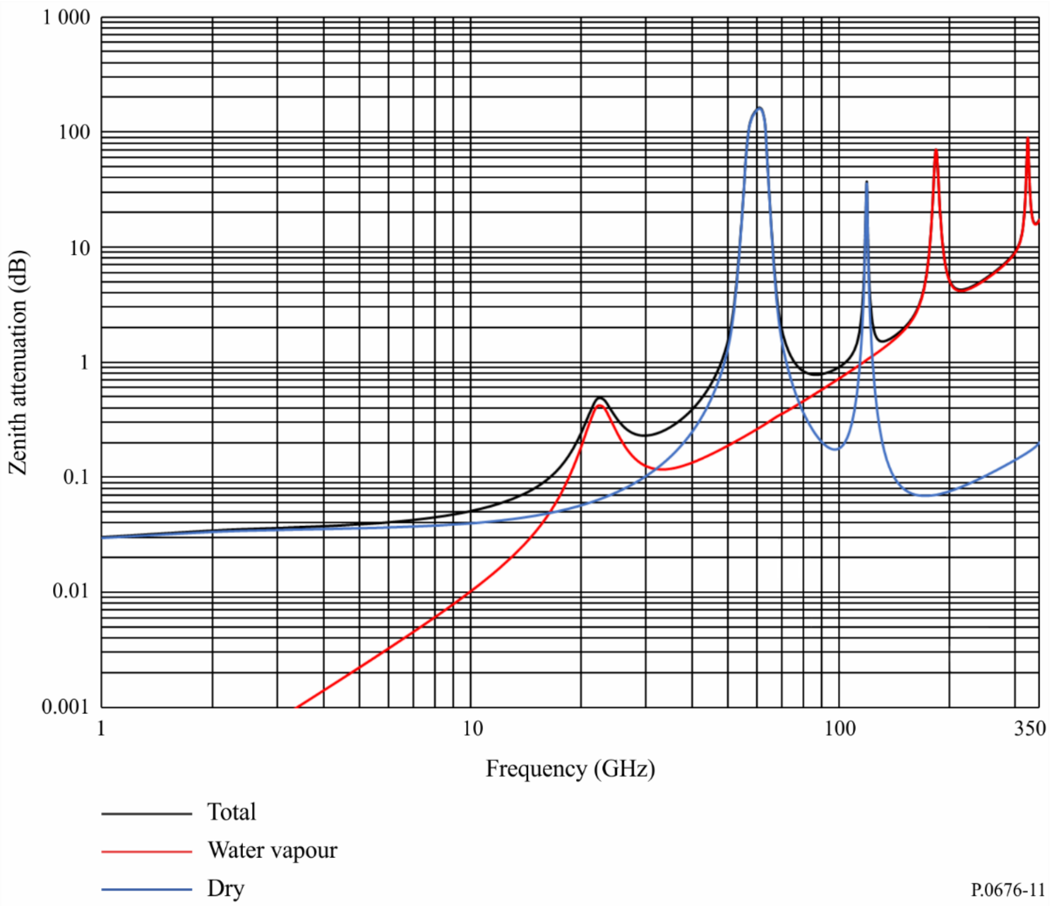
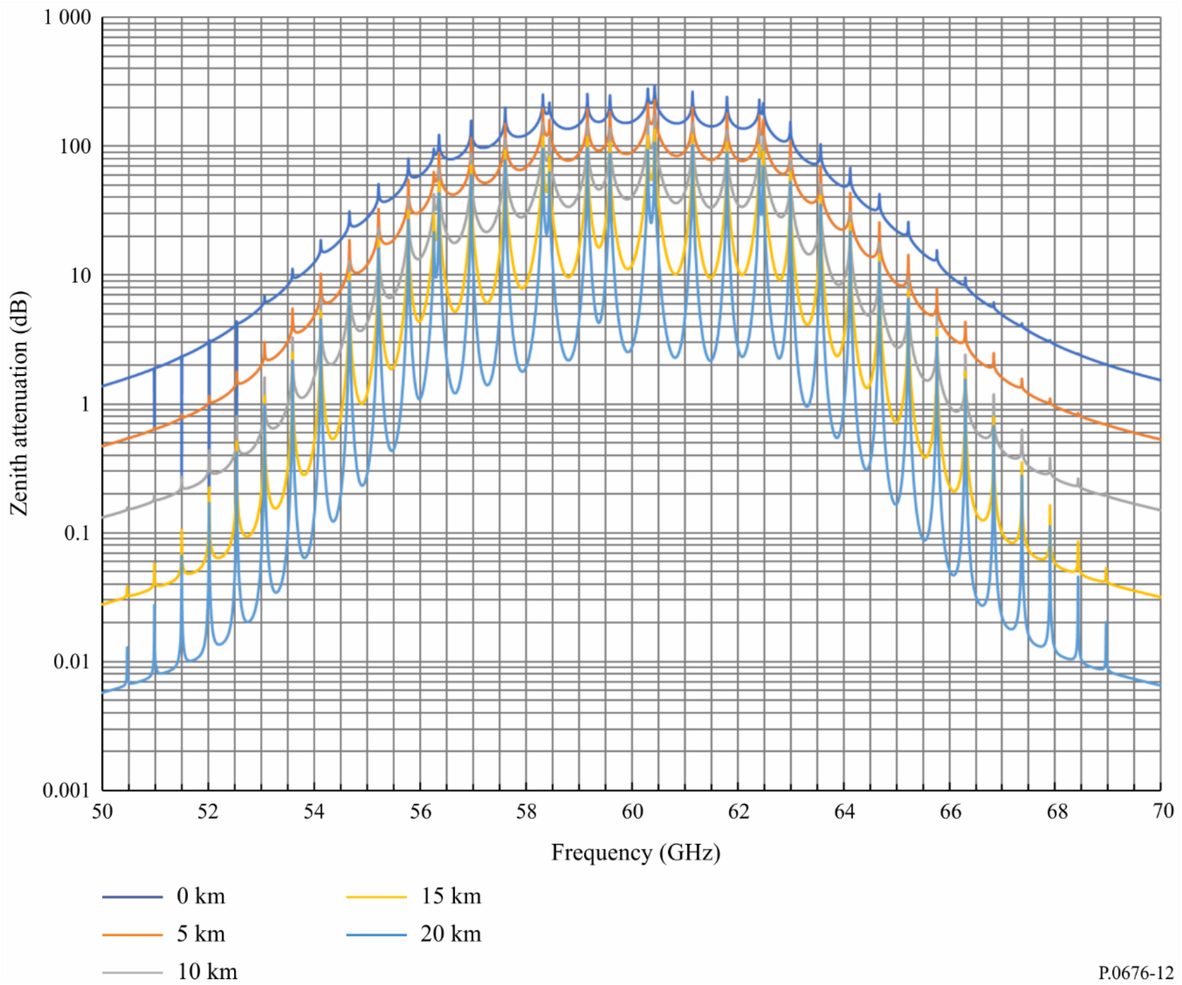


FIGURE 12

Zenith oxygen attenuation from the altitudes indicated, calculated at intervals of 10 MHz, including line centres (0 km, 5 km, 10 km, 15 km and 20 km)



Values for ρ_1 at the surface can be found in Recommendation ITU-R P.836.

The different formulation for dry air and water vapour is explained in § 2.2.2.2.

2.3 Zenith path water vapour attenuation

The above method for calculating slant path attenuation relies on knowledge of the water vapour density at the surface of the Earth. If the integrated water vapour content, V_t, is known, the total water vapour attenuation can be estimated as follows:

$$A_w = \begin{cases} \dfrac{0.0176\, V_t \gamma_w(f, p_{ref}, \rho_{v,ref}, t_{ref})}{\gamma_w(f_{ref}, p_{ref}, \rho_{v,ref}, t_{ref})}, & 1\text{ GHz} \le f \le 20\text{ GHz} \\ \dfrac{0.0176\, V_t \gamma_w(f, p_{ref}, \rho_{v,ref}, t_{ref})}{\gamma_w(f_{ref}, p_{ref}, \rho_{v,ref}, t_{ref})}(ah^b + 1), & 20\text{ GHz} < f \le 350\text{ GHz} \end{cases} \quad \text{dB} \qquad (49)$$

where:

$$a = 0.2048\, exp\left[-\left(\frac{f-22.43}{3.097}\right)^2\right] + 0.2326\, exp\left[-\left(\frac{f-183.5}{4.096}\right)^2\right] + 0.2073\, exp\left[-\left(\frac{f-325}{3.651}\right)^2\right] - 0.1113 \qquad (50)$$

$$b = 8.741\ x\ 10^4 \exp(-0.587f) + 312.2f^{-2.38} + 0.723 \tag{51}$$

$$h = \begin{cases} 0 & h_s < 0 \text{ km} \\ h_s & 0 \text{ km} \le h_s \le 4 \text{ km} \\ 4 & h_s > 4 \text{ km} \end{cases} \quad \text{(km)} \tag{52}$$

$$\rho_{v,ref} = \frac{V_t}{2.38} \ (\text{g/m}^3) \tag{53}$$

$$t_{ref} = 14 \ln\left(0.22 \frac{V_t}{2.38}\right) + 3 \ (°\text{C}) \tag{54}$$

and

f : frequency (GHz)

f_{ref} : 20.6 (GHz)

p_{ref} = 845 (hPa)

V_t: integrated water vapour content from: a) local radiosonde or radiometric data or b) at the required percentage of time (kg/m^2 or mm) obtained from the digital maps in Recommendation ITU-R P.836 (kg/m^2 or mm)

$\gamma_W(f, p, \rho, t)$: specific attenuation as a function of frequency, pressure, water vapour density, and temperature calculated from the water vapour component of equation (1) (dB/km)

h_s: earth station height above mean sea level (a.m.s.l) (km).