

RECOMMENDATION ITU-R P.526-7

Propagation by diffraction

(Question ITU-R 202/3)

(1978-1982-1992-1994-1995-1997-1999-2001)

The ITU Radiocommunication Assembly,

considering

a) that there is a need to provide engineering information for the calculation of field strengths over diffraction paths,

recommends

1 that the methods described in Annex 1 be used for the calculation of field strengths over diffraction paths, which may include a spherical earth surface, or irregular terrain with different kinds of obstacles.

ANNEX 1

1 Introduction

Although diffraction is produced only by the surface of the ground or other obstacles, account must be taken of the mean atmospheric refraction on the transmission path to evaluate the geometrical parameters situated in the vertical plane of the path (angle of diffraction, radius of curvature, height of obstacle). For this purpose, the path profile has to be traced with the appropriate equivalent Earth radius (Recommendation ITU-R P.834). If no other information is available, an equivalent Earth radius of 8 500 km may be taken as a basis.

2 Fresnel ellipsoids and Fresnel zones

In studying radiowave propagation between two points A and B, the intervening space can be subdivided by a family of ellipsoids, known as Fresnel ellipsoids, all having their focal points at A and B such that any point M on one ellipsoid satisfies the relation:

$$AM + MB = AB + n \frac{\lambda}{2} \quad (1)$$

where n is a whole number characterizing the ellipsoid and $n = 1$ corresponds to the first Fresnel ellipsoid, etc., and λ is the wavelength.

As a practical rule, propagation is assumed to occur in line-of-sight, i.e. with negligible diffraction phenomena if there is no obstacle within the first Fresnel ellipsoid.

The radius of an ellipsoid at a point between the transmitter and the receiver is given by the following formula:

$$R_n = \left[\frac{n \lambda d_1 d_2}{d_1 + d_2} \right]^{1/2} \quad (2)$$

or, in practical units:

$$R_n = 550 \left[\frac{n d_1 d_2}{(d_1 + d_2) f} \right]^{1/2} \quad (3)$$

where f is the frequency (MHz) and d_1 and d_2 are the distances (km) between transmitter and receiver at the point where the ellipsoid radius (m) is calculated.

Some problems require consideration of Fresnel zones which are the zones obtained by taking the intersection of a family of ellipsoids by a plane. The zone of order n is the part between the curves obtained from ellipsoids n and $n - 1$, respectively.

3 Diffraction over a spherical earth

The additional transmission loss due to diffraction over a spherical earth can be computed by the classical residue series formula. A computer program GRWAVE, available from the ITU, provides the complete method. A subset of the outputs from this program (for antennas close to the ground and at lower frequencies) is presented in Recommendation ITU-R P.368. At long distances over the horizon, only the first term of this series is important. This first term can be written as the product of a distance term, F , and two height gain terms, G_T and G_R . Sections 3.1 and 3.2 describe how these terms can be obtained either from simple formulae or from nomograms.

It is important to note that:

- the methods described in § 3.1 and 3.2 are limited in validity to transhorizon paths;
- results are more reliable in the deep shadow area well beyond the horizon;
- attenuation in the deep shadow area will, in practice, be limited by the troposcatter mechanism.

3.1 Numerical calculation

3.1.1 Influence of the electrical characteristics of the surface of the Earth

The extent to which the electrical characteristics of the surface of the Earth influence the diffraction loss can be determined by calculating a normalized factor for surface admittance, K , given by the formulae:

in self-consistent units:

$$K_H = \left(\frac{2\pi a_e}{\lambda} \right)^{-1/3} \left[(\epsilon - 1)^2 + (60 \lambda \sigma)^2 \right]^{-1/4} \quad \text{for horizontal polarization} \quad (4)$$

and

$$K_V = K_H \left[\epsilon^2 + (60 \lambda \sigma)^2 \right]^{1/2} \quad \text{for vertical polarization} \quad (5)$$

or, in practical units:

$$K_H = 0.36 (a_e f)^{-1/3} \left[(\epsilon - 1)^2 + (18\,000 \sigma / f)^2 \right]^{-1/4} \quad (4a)$$

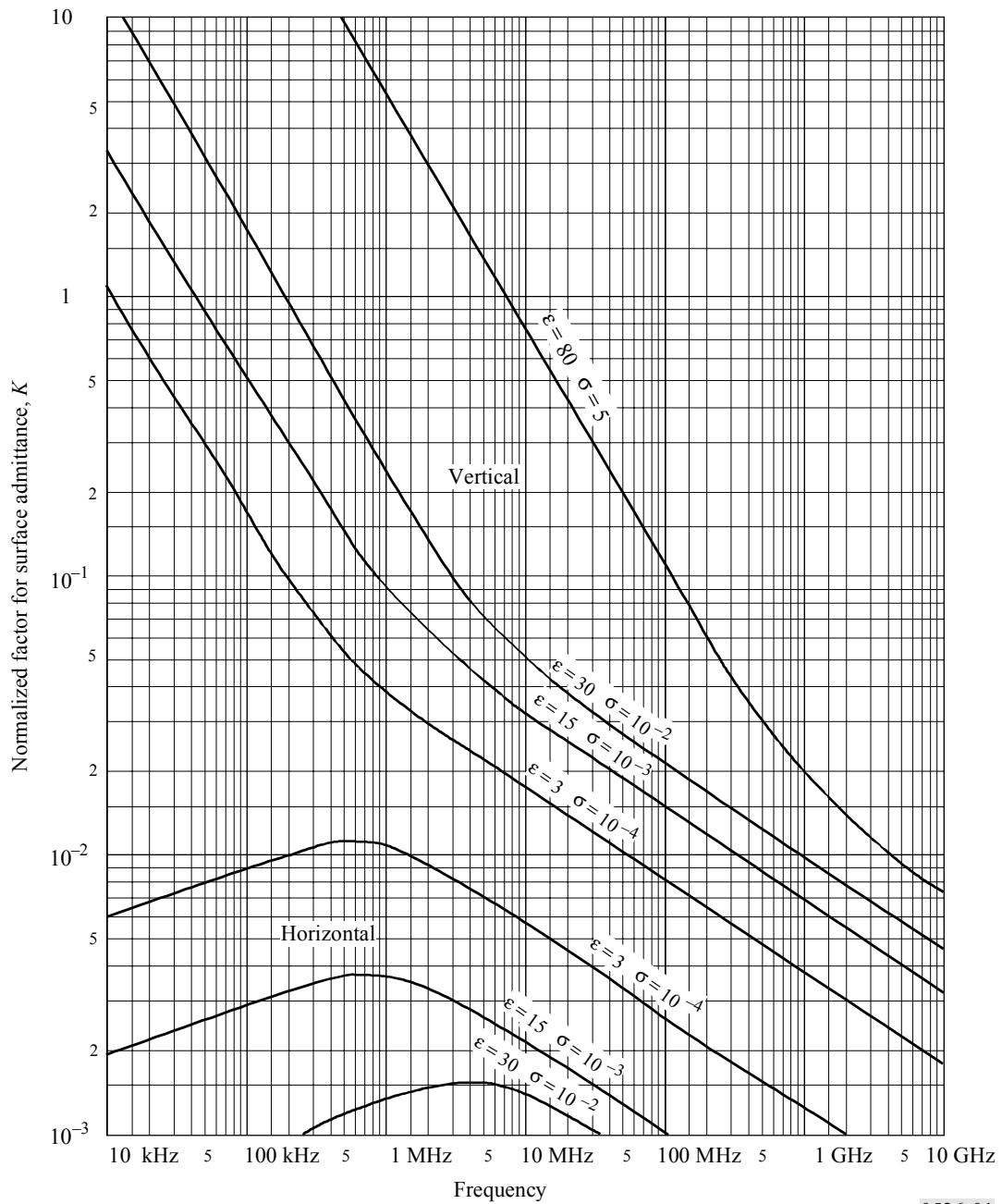
$$K_V = K_H \left[\epsilon^2 + (18\,000 \sigma / f)^2 \right]^{1/2} \quad (5a)$$

where:

- a_e : effective radius of the Earth (km)
- ϵ : effective relative permittivity
- σ : effective conductivity (S/m)
- f : frequency (MHz).

Typical values of K are shown in Fig. 1.

FIGURE 1
Calculation of K



If K is less than 0.001, the electrical characteristics of the Earth are not important. For values of K greater than 0.001, the appropriate formulae given below should be used.

3.1.2 Diffraction field strength formulae

The diffraction field strength, E , relative to the free-space field strength, E_0 , is given by the formula:

$$20 \log \frac{E}{E_0} = F(X) + G(Y_1) + G(Y_2) \quad \text{dB} \quad (6)$$

where X is the normalized length of the path between the antennas at normalized heights Y_1 and Y_2 (and where $20 \log \frac{E}{E_0}$ is generally negative).

In self-consistent units:

$$X = \beta \left(\frac{\pi}{\lambda a_e^2} \right)^{1/3} d \quad (7)$$

$$Y = 2\beta \left(\frac{\pi^2}{\lambda^2 a_e} \right)^{1/3} h \quad (8)$$

or, in practical units:

$$X = 2.2 \beta f^{1/3} a_e^{-2/3} d \quad (7a)$$

$$Y = 9.6 \times 10^{-3} \beta f^{2/3} a_e^{-1/3} h \quad (8a)$$

where:

d : path length (km)

a_e : equivalent Earth's radius (km)

h : antenna height (m)

f : frequency (MHz).

β is a parameter allowing for the type of ground and for polarization. It is related to K by the following semi-empirical formula:

$$\beta = \frac{1 + 1.6 K^2 + 0.75 K^4}{1 + 4.5 K^2 + 1.35 K^4} \quad (9)$$

For horizontal polarization at all frequencies, and for vertical polarization above 20 MHz over land or 300 MHz over sea, β may be taken as equal to 1.

For vertical polarization below 20 MHz over land or 300 MHz over sea, β must be calculated as a function of K . However, it is then possible to disregard ϵ and write:

$$K^2 \approx 6.89 \frac{\sigma}{k^{2/3} f^{5/3}} \quad (9a)$$

where σ is expressed in S/m, f (MHz) and k is the multiplying factor of the Earth's radius.

The distance term is given by the formula:

$$F(X) = 11 + 10 \log(X) - 17.6 X \quad (10)$$

The height gain term, $G(Y)$ is given by the following formulae:

$$G(Y) \cong 17,6 (Y - 1,1)^{1/2} - 5 \log(Y - 1,1) - 8 \quad \text{for } Y > 2 \quad (11)$$

For $Y < 2$ the value of $G(Y)$ is a function of the value of K computed in § 3.1.1:

$$G(Y) \cong 20 \log(Y + 0.1 Y^3) \quad \text{para } 10 K < Y < 2 \quad (11a)$$

$$G(Y) \cong 2 + 20 \log K + 9 \log(Y/K) [\log(Y/K) + 1] \quad \text{para } K/10 < Y < 10 K \quad (11b)$$

$$G(Y) \cong 2 + 20 \log K \quad \text{para } Y < K/10 \quad (11c)$$

3.2 Calculation by nomograms

Under the same approximation condition (the first term of the residue series is dominant), the calculation may also be made using the following formula:

$$20 \log \frac{E}{E_0} = F(d) + H(h_1) + H(h_2) \quad \text{dB} \quad (12)$$

where:

E : received field strength

E_0 : field strength in free space at the same distance

d : distance between the extremities of the path

h_1 and h_2 : heights of the antennas above the spherical earth.

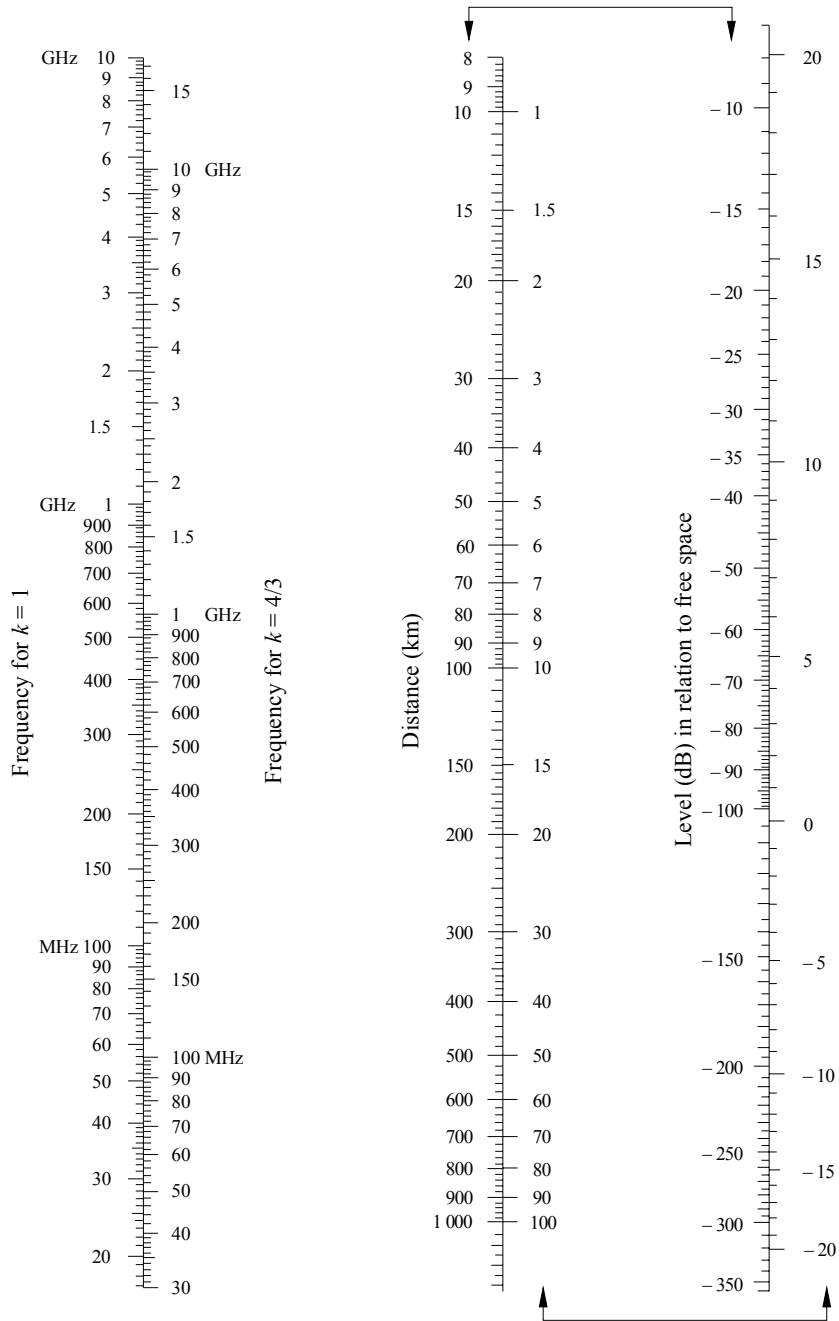
The function F (influence of the distance) and H (height-gain) are given by the nomograms in Figs. 2, 3, 4 and 5.

These nomograms (Figs. 2 to 5) give directly the received level relative to free space, for $k = 1$ and $k = 4/3$, and for frequencies greater than approximately 30 MHz. k is the effective Earth radius factor, defined in Recommendation ITU-R P.310. However, the received level for other values of k may be calculated by using the frequency scale for $k = 1$, but replacing the frequency in question by a hypothetical frequency equal to f/k^2 for Figs. 2 and 4 and f/\sqrt{k} , for Figs. 3 and 5.

Very close to the ground the field strength is practically independent of the height. This phenomenon is particularly important for vertical polarization over the sea. For this reason Fig. 5 includes a heavy black vertical line AB. If the straight line should intersect this heavy line AB, the real height should be replaced by a larger value, so that the straight line just touches the top of the limit line at A.

NOTE 1 – Attenuation relative to free space is given by the negative of the values given by equation (12). If equation (12) gives a value above the free-space field, the method is invalid.

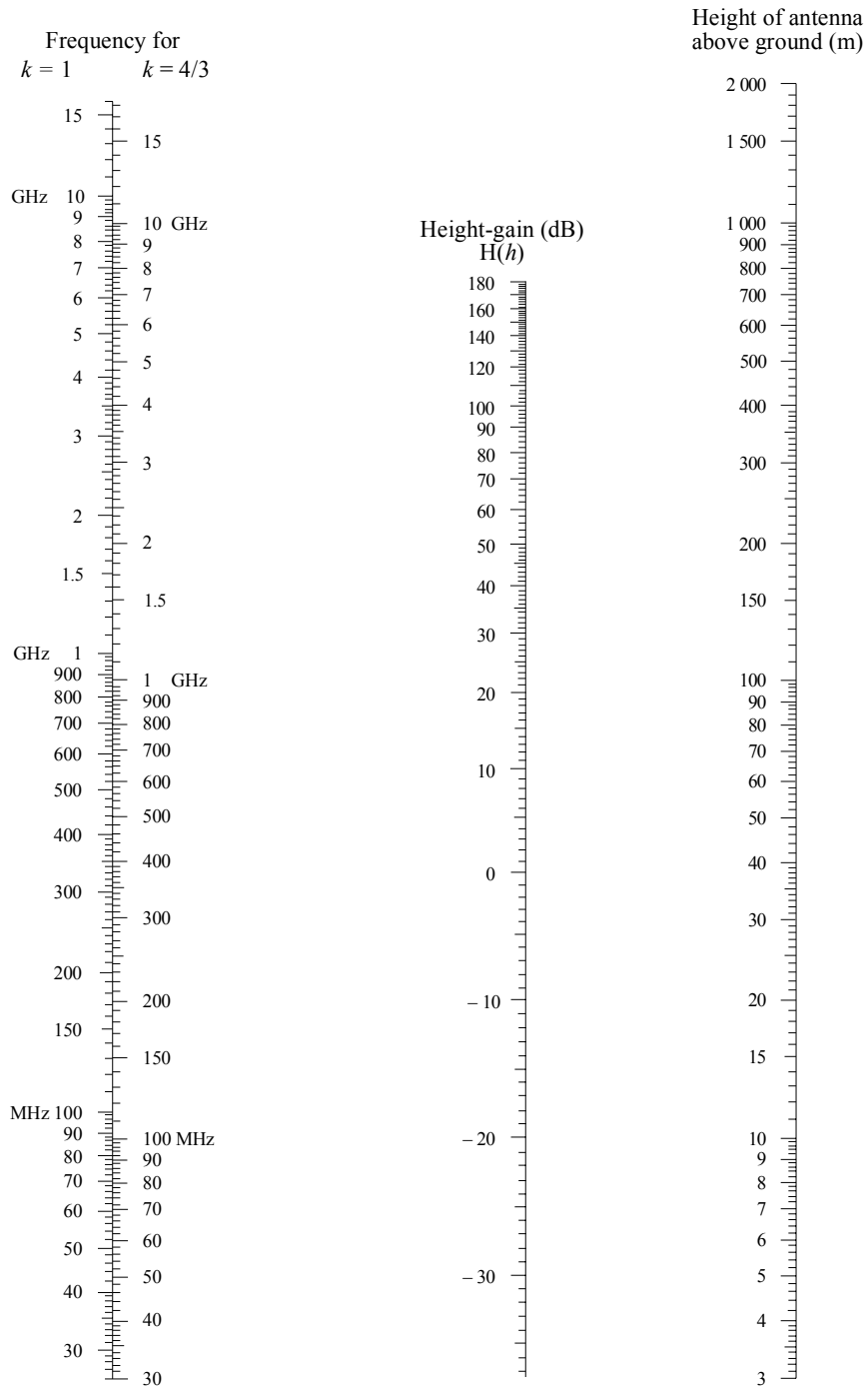
FIGURE 2
Diffraction by a spherical earth – effect of distance



Horizontal polarization over land and sea
Vertical polarization over land

(The scales joined by arrows should be used together)

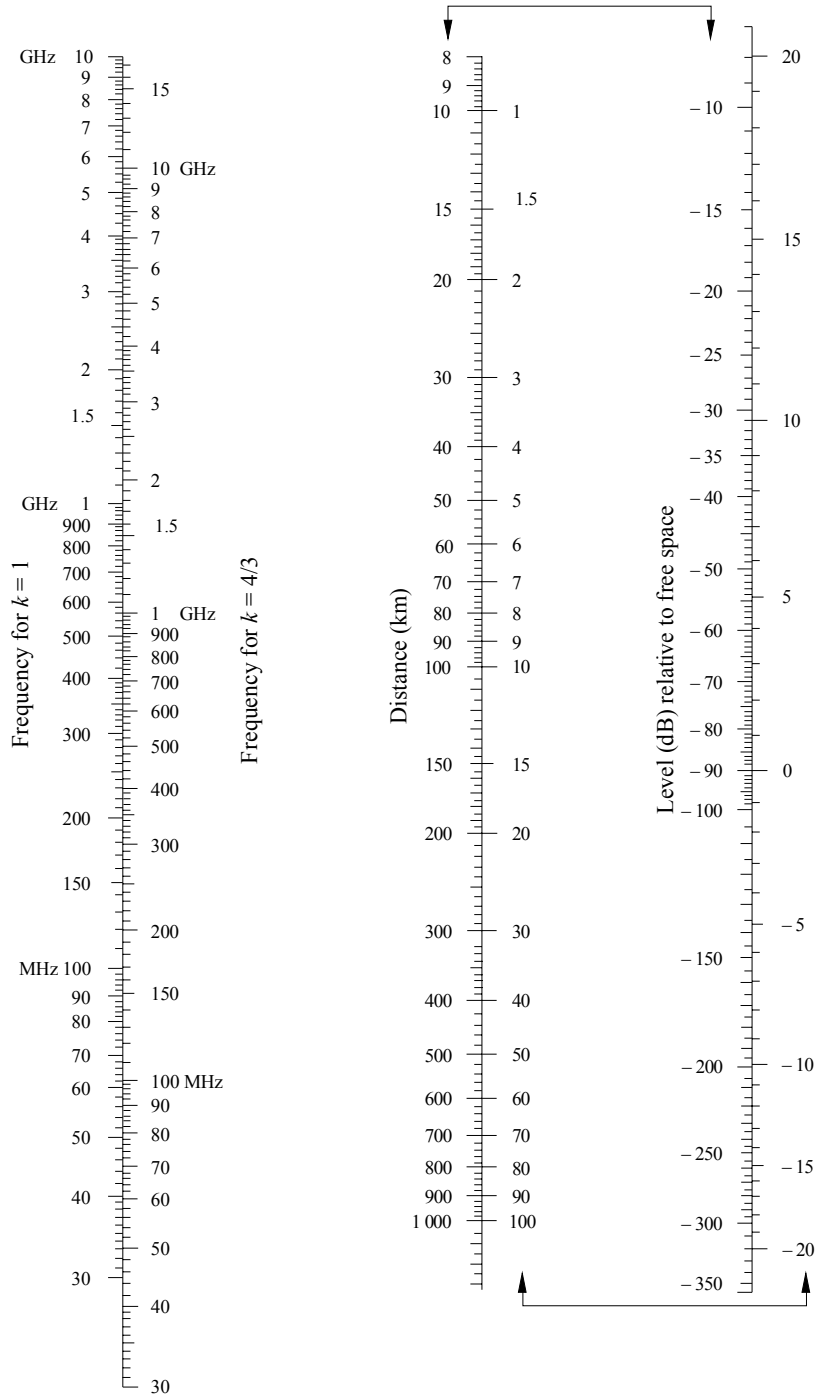
FIGURE 3
 Diffraction by a spherical earth – height-gain



Horizontal polarization – land and sea
 Vertical polarization – land

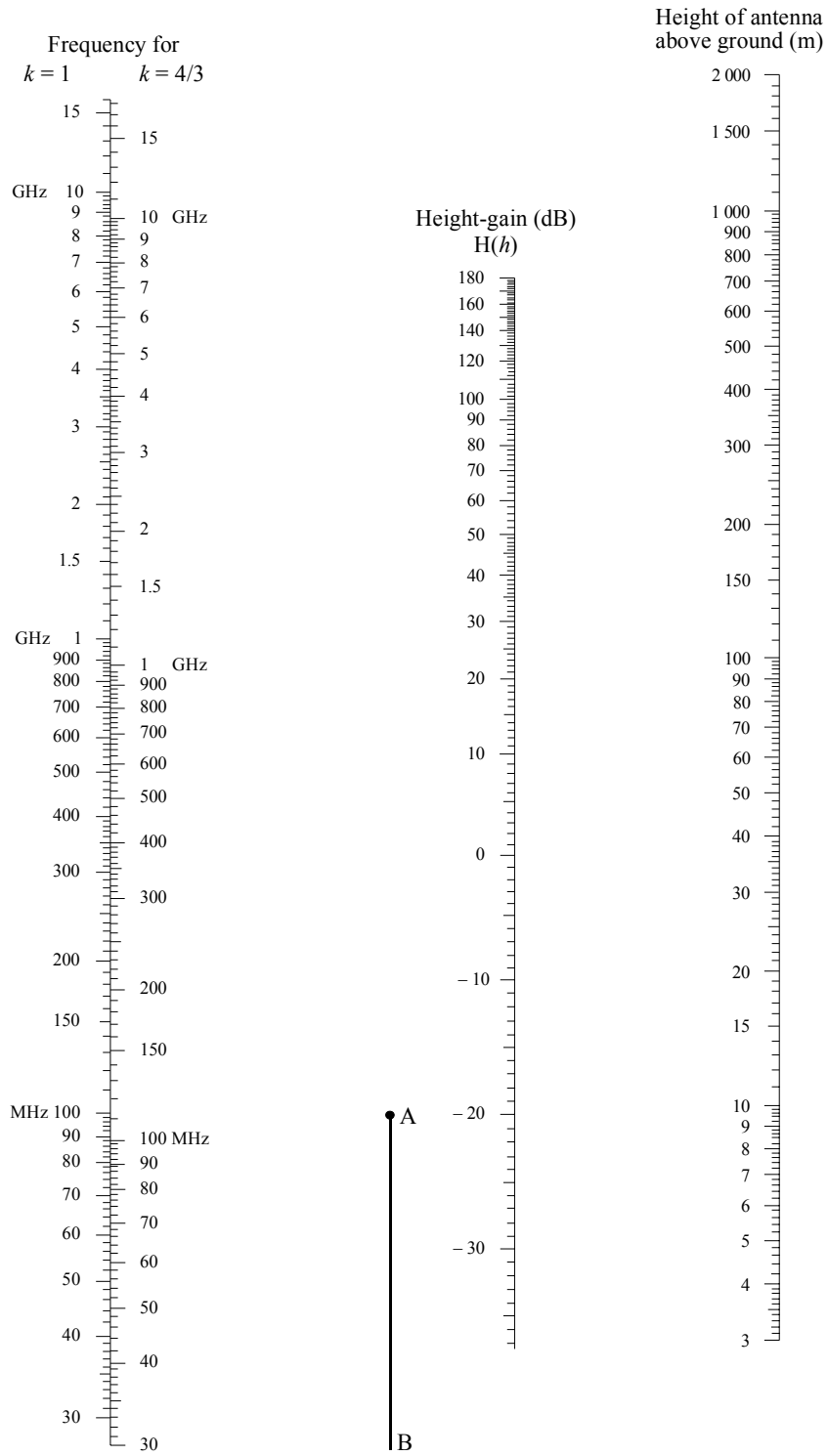
FIGURE 4

Diffraction by a spherical earth – effect of distance



Vertical polarization over sea
 (The scales joined by arrows should be used together)

FIGURE 5
 Diffraction by a spherical earth – height-gain



Vertical polarization – sea

4 Diffraction over obstacles and irregular terrain

Many propagation paths encounter one obstacle or several separate obstacles and it is useful to estimate the losses caused by such obstacles. To make such calculations it is necessary to idealize the form of the obstacles, either assuming a knife-edge of negligible thickness or a thick smooth obstacle with a well-defined radius of curvature at the top. Real obstacles have, of course, more complex forms, so that the indications provided in this Recommendation should be regarded only as an approximation.

In those cases where the direct path between the terminals is much shorter than the diffraction path, it is necessary to calculate the additional transmission loss due to the longer path.

The data given below apply when the wavelength is fairly small in relation to the size of the obstacles, i.e., mainly to VHF and shorter waves ($f > 30$ MHz).

4.1 Single knife-edge obstacle

In this extremely idealized case (Figs. 6a) and 6b)), all the geometrical parameters are combined together in a single dimensionless parameter normally denoted by v which may assume a variety of equivalent forms according to the geometrical parameters selected:

$$v = h \sqrt{\frac{2}{\lambda} \left(\frac{1}{d_1} + \frac{1}{d_2} \right)} \quad (13)$$

$$v = \theta \sqrt{\frac{2}{\lambda \left(\frac{1}{d_1} + \frac{1}{d_2} \right)}} \quad (14)$$

$$v = \sqrt{\frac{2 h \theta}{\lambda}} \quad (v \text{ has the sign of } h \text{ y } \theta) \quad (15)$$

$$v = \sqrt{\frac{2 d}{\lambda} \cdot \alpha_1 \alpha_2} \quad (v \text{ has the sign of } \alpha_1 \text{ and } \alpha_2) \quad (16)$$

where:

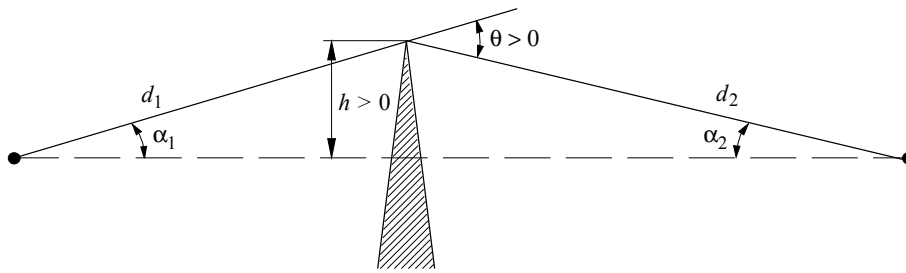
- h : height of the top of the obstacle above the straight line joining the two ends of the path. If the height is below this line, h is negative
- d_1 and d_2 : distances of the two ends of the path from the top of the obstacle
- d : length of the path
- θ : angle of diffraction (rad); its sign is the same as that of h . The angle θ is assumed to be less than about 0.2 rad, or roughly 12°
- α_1 and α_2 : angles between the top of the obstacle and one end as seen from the other end. α_1 and α_2 are of the sign of h in the above equations.

NOTE 1 – In equations (13) to (16) inclusive h , d , d_1 , d_2 and λ should be in self-consistent units.

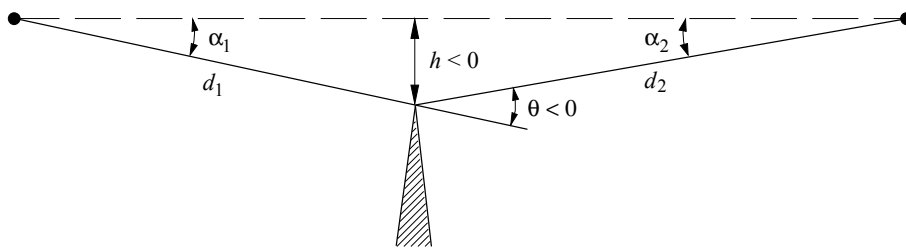
FIGURE 6

Geometrical elements

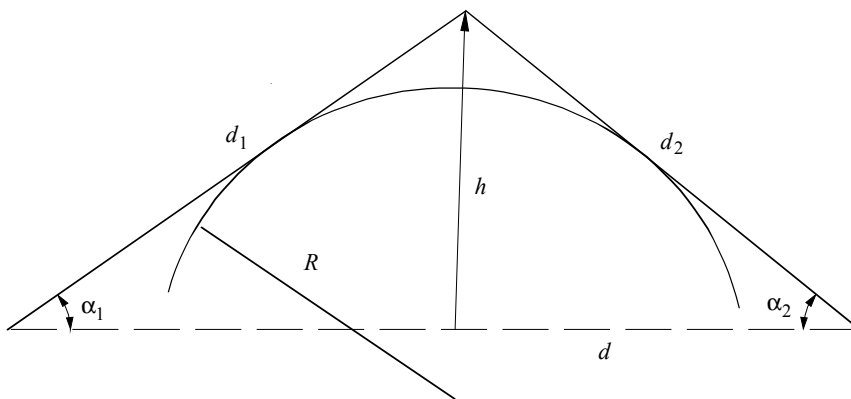
(For definitions of θ , α_1 , α_2 , d , d_1 , d_2 and R , see § 4.1 and 4.3)



a)



b)

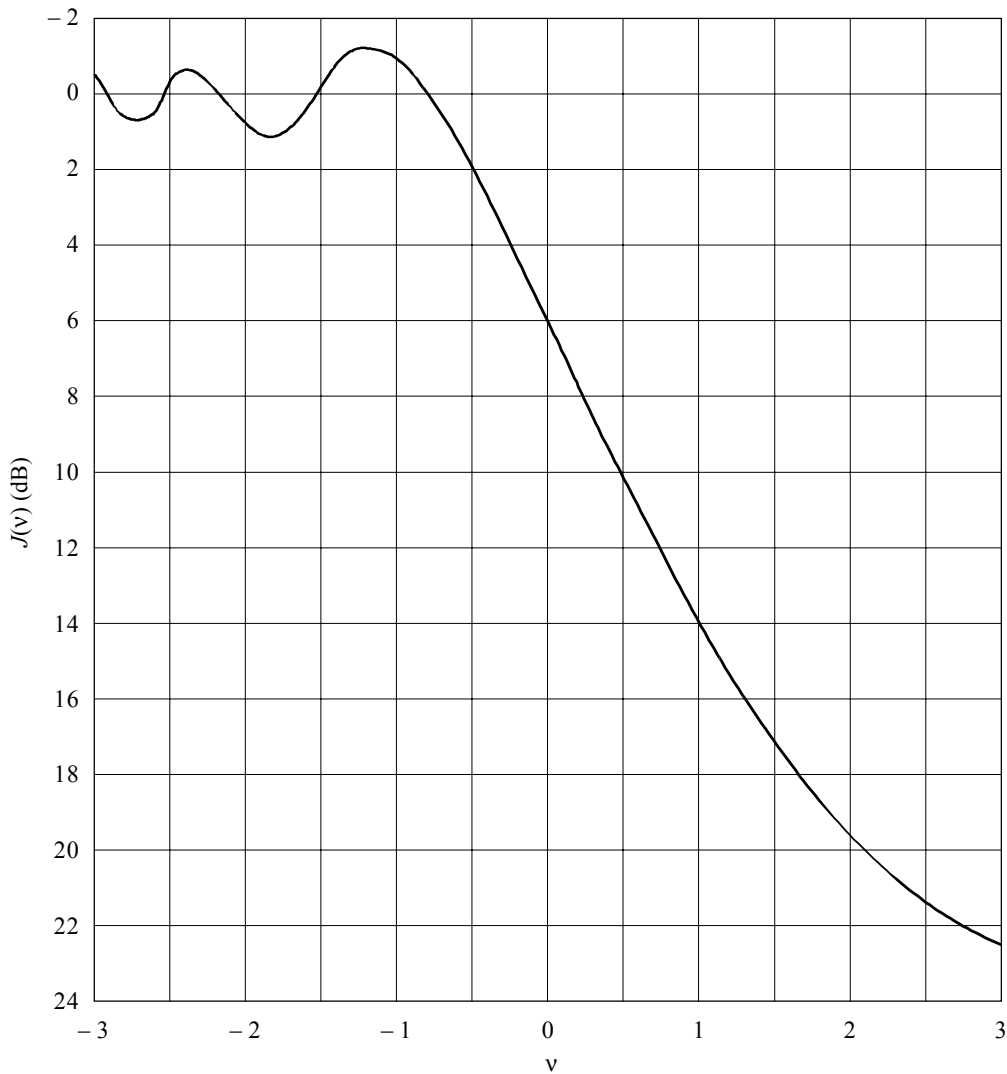


c)

Figure 7 gives, as a function of v , the loss (dB) caused by the presence of the obstacle. For v greater than -0.7 an approximate value can be obtained from the expression:

$$J(v) = 6.9 + 20 \log \left(\sqrt{(v - 0.1)^2 + 1} + v - 0.1 \right) \quad \text{dB} \quad (17)$$

FIGURE 7
Knife-edge diffraction loss



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4.2 Finite-width screen

Interference suppression for a receiving site (e.g. a small earth station) may be obtained by an artificial screen of finite width transverse to the direction of propagation. For this case the field in the shadow of the screen may be calculated by considering three knife-edges, i.e. the top and the two sides of the screen. Constructive and destructive interference of the three independent contributions will result in rapid fluctuations of the field strength over distances of the order of a wavelength. The following simplified model provides estimates for the average and minimum

diffraction loss as a function of location. It consists of adding the amplitudes of the individual contributions for an estimate of the minimum diffraction loss and a power addition to obtain an estimate of the average diffraction loss. The model has been tested against accurate calculations using the uniform theory of diffraction (UTD) and high-precision measurements.

Step 1: Calculate the geometrical parameter v for each of the three knife-edges (top, left side and right side) using any of equations (13) to (16).

Step 2: Calculate the loss factor $j(v) = 10^{J(v)/20}$ associated with each edge from equation (17).

Step 3: Calculate minimum diffraction loss J_{min} from:

$$J_{min}(v) = -20 \log \left[\frac{1}{j_1(v)} + \frac{1}{j_2(v)} + \frac{1}{j_3(v)} \right] \quad \text{dB} \quad (18)$$

or, alternatively,

Step 4: Calculate average diffraction loss J_{av} from:

$$J_{av}(v) = -10 \log \left[\frac{1}{j_1^2(v)} + \frac{1}{j_2^2(v)} + \frac{1}{j_3^2(v)} \right] \quad \text{dB} \quad (19)$$

4.3 Single rounded obstacle

The geometry of a rounded obstacle of radius R is illustrated in Fig. 6c). Note that the distances d_1 and d_2 , and the height h above the baseline, are all measured to the vertex where the projected rays intersect above the obstacle. The diffraction loss for this geometry may be calculated as:

$$A = J(v) + T(m,n) \quad \text{dB} \quad (20)$$

where:

- a) $J(v)$ is the Fresnel-Kirchoff loss due to an equivalent knife-edge placed with its peak at the vertex point. The dimensionless parameter v may be evaluated from any of equations (13) to (16) inclusive. For example, in practical units equation (13) may be written:

$$v = 0.0316 h \left[\frac{2(d_1 + d_2)}{\lambda d_1 d_2} \right]^{1/2} \quad (21)$$

where h and λ are in metres, and d_1 and d_2 are in kilometres.

$J(v)$ may be obtained from Fig. 7 or from equation (17). Note that for an obstruction to line-of-sight propagation, v is positive and equation (17) is valid.

- b) $T(m,n)$ is the additional attenuation due to the curvature of the obstacle:

$$T(m,n) = k m^b \quad (22a)$$

where:

$$k = 8.2 + 12.0 n \quad (22b)$$

$$b = 0.73 + 0.27 [1 - \exp(-1.43 n)] \quad (22c)$$

and

$$m = R \left[\frac{d_1 + d_2}{d_1 d_2} \right] \left/ \left[\frac{\pi R}{\lambda} \right]^{1/3} \right. \quad (23)$$

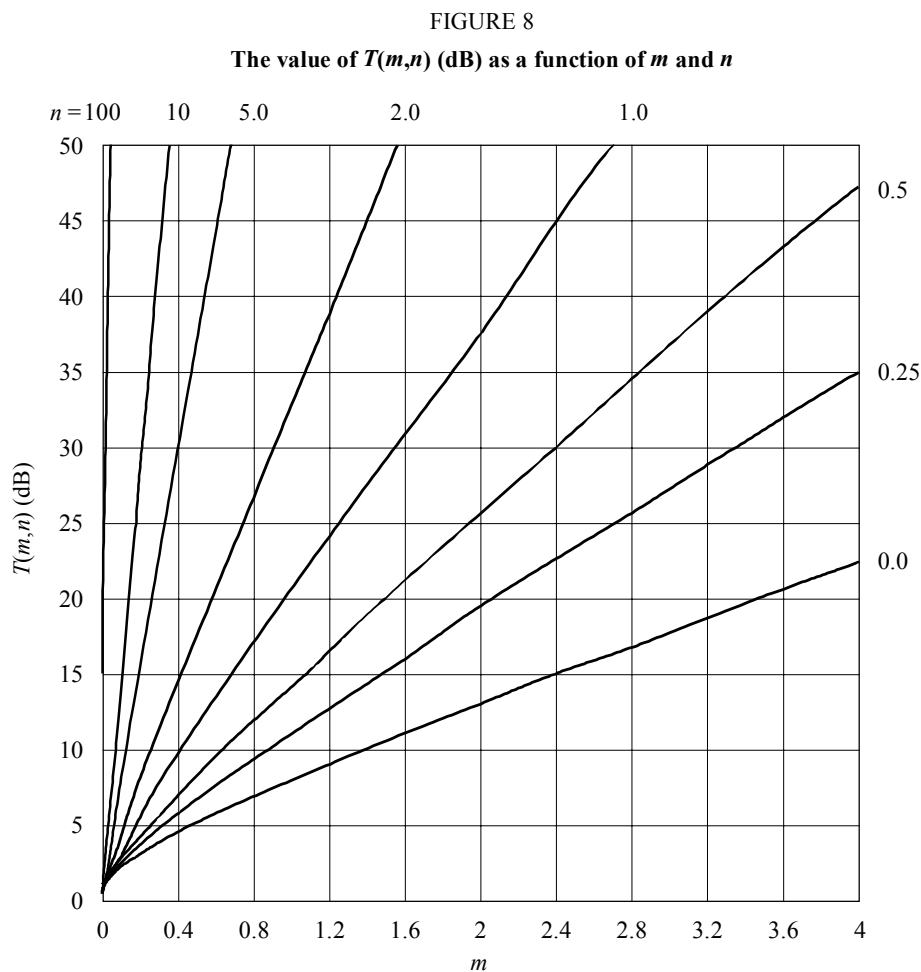
$$n = h \left[\frac{\pi R}{\lambda} \right]^{2/3} \left/ R \right. \quad (24)$$

and R , d_1 , d_2 , h and λ are in self-consistent units.

$T(m,n)$ can also be derived from Fig. 8.

Note that as R tends to zero, m , and hence $T(m,n)$, also tend to zero. Thus equation (20) reduces to knife-edge diffraction for a cylinder of zero radius.

It should be noted that the cylinder model is intended for typical terrain obstructions. It is not suitable for trans-horizon paths over water, or over very flat terrain, when the method of § 3 should be used.



4.4 Double isolated edges

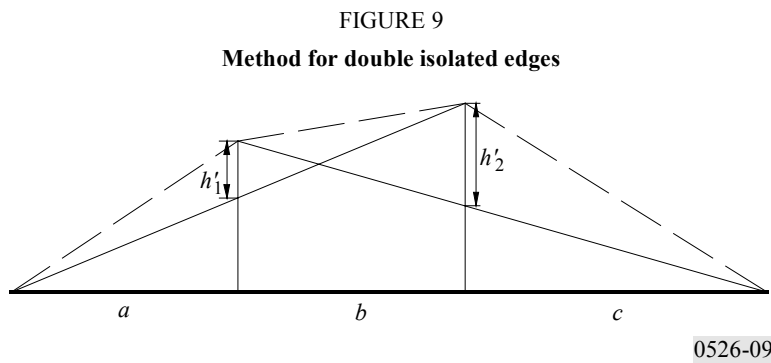
This method consists of applying single knife-edge diffraction theory successively to the two obstacles, with the top of the first obstacle acting as a source for diffraction over the second obstacle (see Fig. 9). The first diffraction path, defined by the distances a and b and the height h'_1 , gives a loss L_1 (dB). The second diffraction path, defined by the distances b and c and the height h'_2 , gives a loss L_2 (dB). L_1 and L_2 are calculated using formulae of § 4.1. A correction term L_c (dB) must be added to take into account the separation b between the edges. L_c may be estimated by the following formula:

$$L_c = 10 \log \left[\frac{(a + b) (b + c)}{b (a + b + c)} \right] \tag{25}$$

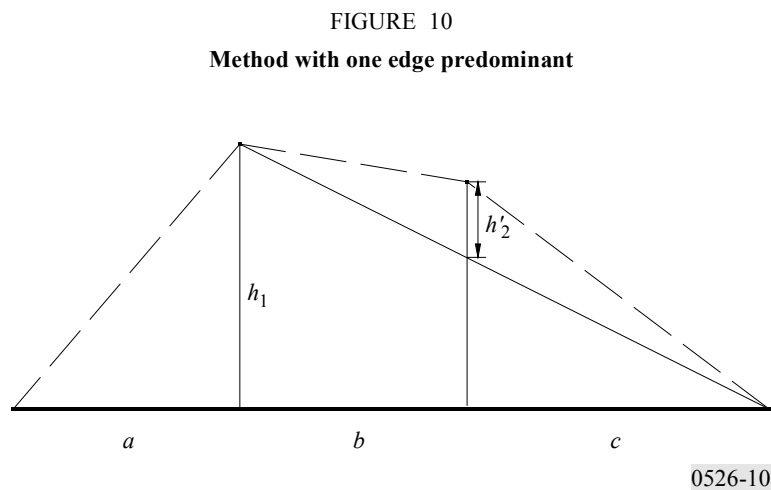
which is valid when each of L_1 and L_2 exceeds about 15 dB. The total diffraction loss is then given by:

$$L = L_1 + L_2 + L_c \tag{26}$$

The above method is particularly useful when the two edges give similar losses.



If one edge is predominant (see Fig. 10), the first diffraction path is defined by the distances a and $b + c$ and the height h_1 . The second diffraction path is defined by the distances b and c and the height h'_2 . The losses corresponding to these two paths are added, without addition of a third term.



The same method may be applied to the case of rounded obstacles using § 4.3.

In cases where the diffracting obstacle may be clearly identified as a flat-roofed building a single knife-edge approximation is not sufficient. It is necessary to calculate the phasor sum of two components: one undergoing a double knife-edge diffraction and the other subject to an additional reflection from the roof surface. It has been shown that, where the reflectivity of the roof surface and any difference in height between the roof surface and the side walls are not accurately known, then a double knife-edge model produces a good prediction of the diffracted field strength, ignoring the reflected component.

4.5 General method for one or more obstacles

The following method is recommended for the diffraction loss over irregular terrain which forms one or more obstacles to line-of-sight propagation. The calculation takes Earth curvature into account via the concept of an effective Earth radius (see Recommendation ITU-R P.452, § 4.3). This method is suitable in cases where a single general procedure is required for terrestrial paths over land or sea and for both line-of-sight and transhorizon.

A profile of the radio path should be available consisting of a set of samples of ground height above sea level ordered at intervals along the path, the first and last being the heights of the transmitter and receiver above sea level, and a corresponding set of horizontal distances from the transmitter. Each height and distance pair are referred to as a profile point and given an index, with indices incrementing from one end of the path to the other. Although it is not essential to the method, in the following description it is assumed that indices increment from the transmitter to the receiver. It is preferable but not essential for the profile samples to be equally spaced horizontally.

The method is based on a procedure which is used from 1 to 3 times depending on the path profile. The procedure consists of finding the point within a given section of the profile with the highest value of the geometrical parameter v as described in § 4.1. The section of the profile to be considered is defined from point index a to point index b ($a < b$). If $a + 1 = b$ there is no intermediate point and the diffraction loss for the section of the path being considered is zero. Otherwise the construction is applied by evaluating v_n ($a < n < b$) and selecting the point with the highest value of v . The value of v for the n -th profile point is given by:

$$v_n = h\sqrt{2d_{ab}/\lambda d_{an}d_{nb}} \quad (27)$$

where:

$$h = h_n + [d_{an} d_{nb}/2 r_e] - [(h_a d_{nb} + h_b d_{an})/d_{ab}] \quad (27a)$$

h_a, h_b, h_n : vertical heights as shown in Fig. 11

d_{an}, d_{nb}, d_{ab} : horizontal distances as shown in Fig. 11

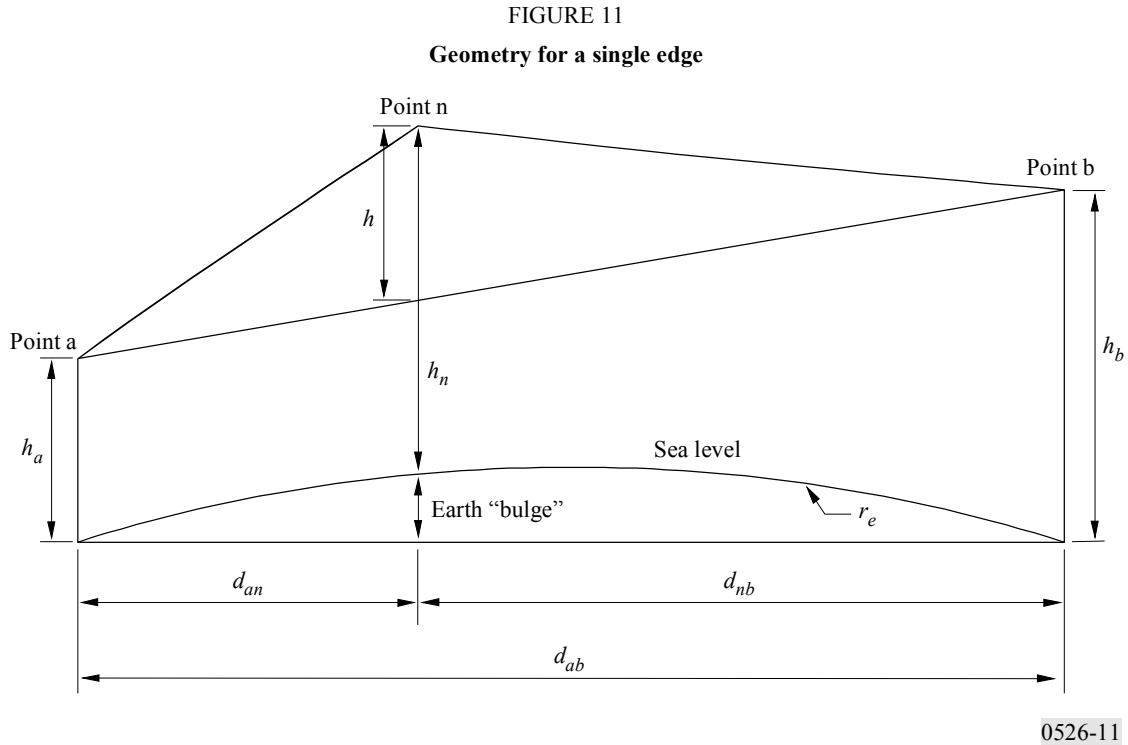
r_e : effective Earth radius

λ : wavelength

and all h, d, r_e and λ are in self-consistent units.

The diffraction loss is then given as the knife-edge loss $J(v)$ according to equation (17) for $v > -0.78$, and is otherwise zero.

Note that equation (27) is derived directly from equation (13). The geometry of equation (27a) is illustrated in Fig. 11. The second term in equation (27a) is a good approximation to the additional height at point n due to Earth curvature.



The above procedure is first applied to the entire profile from transmitter to receiver. The point with the highest value of v is termed the principal edge, p , and the corresponding loss is $J(v_p)$.

If $v_p > -0.78$ the procedure is applied twice more:

- from the transmitter to point p to obtain v_t and hence $J(v_t)$;
- from point p to the receiver to obtain v_r and hence $J(v_r)$.

The excess diffraction loss for the path is then given by:

$$L = J(v_p) + T [J(v_t) + J(v_r) + C] \quad \text{for} \quad v_p > -0.78 \quad (28a)$$

$$L = 0 \quad \text{for} \quad v_p \leq -0.78 \quad (28b)$$

where:

C : empirical correction

$$C = 10.0 + 0.04D \quad (29)$$

D : total path length (km)

and

$$T = 1.0 - \exp [-J(v_p)/6.0] \quad (30)$$

Note that the above procedure, for transhorizon paths, is based on the Deygout method limited to a maximum of 3 edges. For line-of-sight paths it differs from the Deygout construction in that two secondary edges are still used in cases where the principal edge results in a non-zero diffraction loss.

Where this method is used to predict diffraction loss for different values of effective Earth radius over the same path profile, it is recommended that the principal edge, and if they exist the auxiliary edges on either side, are first found for median effective Earth radius. These edges should then be used when calculating diffraction losses for other values of effective Earth radius, without repeating the procedure for locating these points. This avoids the possibility, which may occur in a few cases, of a discontinuity in predicted diffraction loss as a function of effective Earth radius due to different edges being selected.

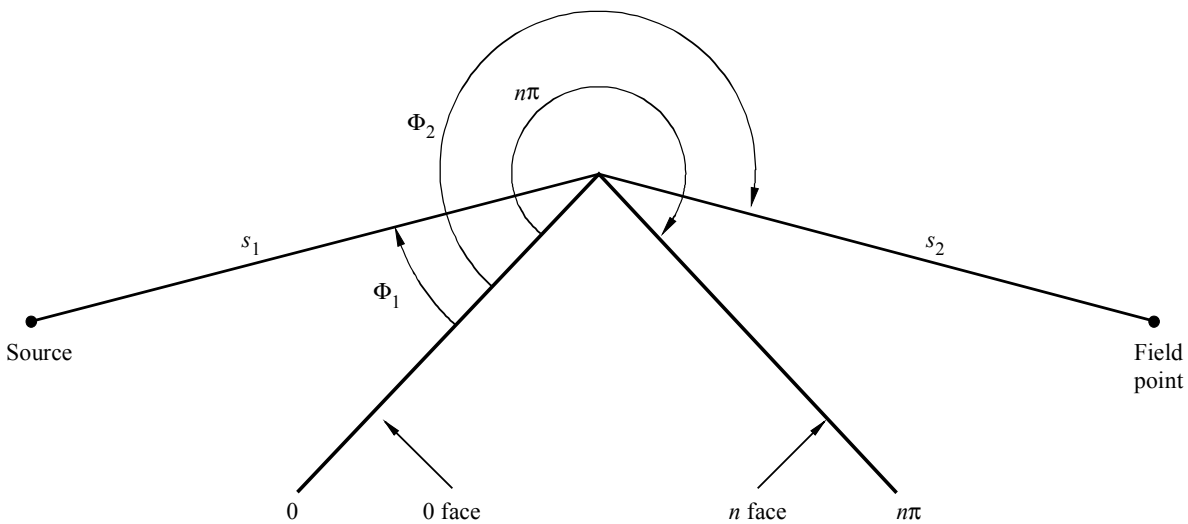
4.6 Finitely conducting wedge obstacle

The method described below can be used to predict the diffraction loss due to a finitely conducting wedge. Suitable applications are for diffraction around the corner of a building or over the ridge of a roof, or where terrain can be characterized by a wedge-shaped hill. The method requires the conductivity and relative dielectric constant of the obstructing wedge, and assumes that no transmission occurs through the wedge material.

The method is based on the Uniform Theory of Diffraction (UTD). It takes account of diffraction in both the shadow and line-of-sight region, and a method is provided for a smooth transition between these regions.

The geometry of a finitely conducting wedge-shaped obstacle is illustrated in Fig. 12.

FIGURE 12
Geometry for application of UTD wedge diffraction



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The UTD formulation for the electric field at the field point, specializing to two dimensions, is

$$e_{UTD} = e_0 \frac{\exp(-jks_1)}{s_1} D_{\parallel}^{\dagger} \cdot \sqrt{\frac{s_1}{s_2(s_1 + s_2)}} \cdot \exp(-jks_2) \quad (31)$$

where:

- e_{UTD} : electric field at the field point
- e_0 : relative source amplitude
- s_1 : distance from source point to diffracting edge
- s_2 : distance from diffracting edge to field point
- k : wave number $2\pi/\lambda$
- D_{\parallel}^{\perp} : diffraction coefficient depending on the polarization (parallel or perpendicular to the plane of incidence) of the incident field on the edge

and s_1, s_2 and λ are in self-consistent units.

The diffraction coefficient for a finitely conducting wedge is given as

$$D_{\parallel}^{\perp} = \frac{-\exp(-j\pi/4)}{2n\sqrt{2\pi k}} \left\{ \begin{array}{l} \cot\left(\frac{\pi + (\Phi_2 - \Phi_1)}{2n}\right) \cdot F(kLa^+(\Phi_2 - \Phi_1)) \\ + \cot\left(\frac{\pi - (\Phi_2 - \Phi_1)}{2n}\right) \cdot F(kLa^-(\Phi_2 - \Phi_1)) \\ + R_0^{\perp} \cdot \cot\left(\frac{\pi - (\Phi_2 + \Phi_1)}{2n}\right) \cdot F(kLa^-(\Phi_2 + \Phi_1)) \\ + R_n^{\perp} \cdot \cot\left(\frac{\pi + (\Phi_2 + \Phi_1)}{2n}\right) \cdot F(kLa^+(\Phi_2 + \Phi_1)) \end{array} \right\} \quad (32)$$

where:

- Φ_1 : incidence angle, measured from incidence face (0 face)
- Φ_2 : diffraction angle, measured from incidence face (0 face)
- n : external wedge angle as a multiple of π radians (actual angle = $n\pi$ (rad))
- $j = \sqrt{-1}$

and where $F(x)$ is a Fresnel integral:

$$F(x) = 2j\sqrt{x} \cdot \exp(jx) \cdot \int_{\sqrt{x}}^{\infty} \exp(-jt^2) dt \quad (33)$$

$$\int_{\sqrt{x}}^{\infty} \exp(-jt^2) dt = \sqrt{\frac{\pi}{8}}(1 - j) - \int_0^{\sqrt{x}} \exp(-jt^2) dt \quad (34)$$

The integral may be calculated by numerical integration.

Alternatively a useful approximation is given by:

$$\int_{\sqrt{x}}^{\infty} \exp(-jt^2) dt = \sqrt{\frac{\pi}{2}} A(x) \quad (35)$$

where:

$$A(x) = \left\{ \begin{array}{ll} \frac{1-j}{2} - \exp(-jx) \sqrt{\frac{x}{4}} \sum_{n=0}^{11} \left[(a_n + jb_n) \left(\frac{x}{4}\right)^n \right] & \text{si } x < 4 \\ -\exp(-jx) \sqrt{\frac{4}{x}} \sum_{n=0}^{11} \left[(c_n + jd_n) \left(\frac{4}{x}\right)^n \right] & \text{otherwise} \end{array} \right\} \quad (36)$$

and the coefficients a, b, c, d have the values:

$a_0 = +1.595769140$	$b_0 = -0.000000033$	$c_0 = +0.000000000$	$d_0 = +0.199471140$
$a_1 = -0.000001702$	$b_1 = +4.255387524$	$c_1 = -0.024933975$	$d_1 = +0.000000023$
$a_2 = -6.808568854$	$b_2 = -0.000092810$	$c_2 = +0.000003936$	$d_2 = -0.009351341$
$a_3 = -0.000576361$	$b_3 = -7.780020400$	$c_3 = +0.005770956$	$d_3 = +0.000023006$
$a_4 = +6.920691902$	$b_4 = -0.009520895$	$c_4 = +0.000689892$	$d_4 = +0.004851466$
$a_5 = -0.016898657$	$b_5 = +5.075161298$	$c_5 = -0.009497136$	$d_5 = +0.001903218$
$a_6 = -3.050485660$	$b_6 = -0.138341947$	$c_6 = +0.011948809$	$d_6 = -0.017122914$
$a_7 = -0.075752419$	$b_7 = -1.363729124$	$c_7 = -0.006748873$	$d_7 = +0.029064067$
$a_8 = +0.850663781$	$b_8 = -0.403349276$	$c_8 = +0.000246420$	$d_8 = -0.027928955$
$a_9 = -0.025639041$	$b_9 = +0.702222016$	$c_9 = +0.002102967$	$d_9 = +0.016497308$
$a_{10} = -0.150230960$	$b_{10} = -0.216195929$	$c_{10} = -0.001217930$	$d_{10} = -0.005598515$
$a_{11} = +0.034404779$	$b_{11} = +0.019547031$	$c_{11} = +0.000233939$	$d_{11} = +0.000838386$

$$L = \frac{s_2 \cdot s_1}{s_2 + s_1} \quad (37)$$

$$a^\pm(\beta) = 2 \cos^2 \left(\frac{2n\pi N^\pm - \beta}{2} \right) \quad (38)$$

where:

$$\beta = \Phi_2 \pm \Phi_1 \quad (39)$$

In equation (38), N^\pm are the integers which most nearly satisfy the equation.

$$N^\pm = \frac{\beta \pm \pi}{2n\pi} \quad (40)$$

R_0^\perp, R_n^\perp are the reflection coefficients for either perpendicular or parallel polarization given by:

$$R^\perp = \frac{\sin(\Phi) - \sqrt{\eta - \cos(\Phi)^2}}{\sin(\Phi) + \sqrt{\eta - \cos(\Phi)^2}} \quad (41)$$

$$R^\parallel = \frac{b \cdot \sin(\Phi) - \sqrt{\eta - \cos(\Phi)^2}}{b \cdot \sin(\Phi) + \sqrt{\eta - \cos(\Phi)^2}} \quad (42)$$

where:

$$\Phi = \Phi_1 \text{ for } R_0 \text{ and } \Phi = (n\pi - \Phi_2) \text{ for } R_n$$

$$\eta = \epsilon_r - j \times 18 \times 10^9 \sigma / f$$

ϵ_r : relative dielectric constant of the wedge material

σ : conductivity of the wedge material (S/m)

f : frequency (Hz).

Note that if necessary the two faces of the wedge may have different electrical properties.

At shadow and reflection boundaries one of the cotangent functions in equation (32) becomes singular.

However D^{\dagger} remains finite, and can be readily evaluated. The term containing the singular cotangent function is given for small ε as:

$$\cot\left(\frac{\pi \pm \beta}{2n}\right) \cdot F(kLa^{\pm}(\beta)) \cong n \cdot \left[\sqrt{2\pi kL} \cdot \text{sign}(\varepsilon) - 2kL\varepsilon \cdot \exp(j\pi/4) \right] \cdot \exp(j\pi/4) \quad (43)$$

with ε defined by:

$$\varepsilon = \pi + \beta - 2\pi nN^+ \quad \text{for} \quad \beta = \Phi_2 + \Phi_1 \quad (44)$$

$$\varepsilon = \pi - \beta + 2\pi nN^- \quad \text{for} \quad \beta = \Phi_2 - \Phi_1 \quad (45)$$

The resulting diffraction coefficient will be continuous at shadow and reflection boundaries, provided that the same reflection coefficient is used when calculating reflected rays.

The field e_{LD} due to the diffracted ray, plus the line-of-sight ray for $(\Phi_2 - \Phi_1) < \pi$, is given by:

$$e_{LD} = \begin{cases} e_{UTD} + \frac{\exp(-jks)}{s} & \text{for} \quad \Phi_2 < \Phi_1 + \pi \\ e_{UTD} & \text{for} \quad \Phi_2 \geq \Phi_1 + \pi \end{cases} \quad (46)$$

where:

s : straight-line distance between the source and field points.

Note that at $(\Phi_2 - \Phi_1) = \pi$ the 2nd cotangent term in equation (32) will become singular, and that the alternative approximation given by equation (43) must be used.

The field strength at the field point (dB) relative to the field which would exist at the field point in the absence of the wedge-shaped obstruction (i.e., dB relative to free space) is given by setting e_0 to unity in equation (31) and calculating:

$$E_{UTD} = 20 \log \left(\left| \frac{s \cdot e_{UTD}}{\exp(-jks)} \right| \right) \quad (47)$$

where:

s : straight-line distance between the source and field points.

Note that, for $n = 2$ and zero reflection coefficients, this should give the same results as the knife edge diffraction loss curve shown in Fig. 7.

A MathCAD version of the UTD formulation is available from the Radiocommunication Bureau.
