RECOMMENDATION ITU-R P.1623-1*

Prediction method of fade dynamics on Earth-space paths

(Question ITU-R 201/3)

(2003-2005)

The ITU Radiocommunication Assembly,

considering

a) that for a variety of radiocommunication services, design objectives have to be met that require information on the dynamics of outage events;

b) that for the evaluation of parameters associated with the risk of failure to provide a certain quality and reliability of service, the probability of occurrence of fades of a certain duration must be known;

c) that for the evaluation of internal parameters inside a fade mitigation technique (FMT) control loop (used to improve the quality and reliability of service), the probability of occurrence of fade slope corresponding to a given attenuation threshold must be known;

d) that there is a need to provide engineering information for the calculation of fade duration, interfade duration and fade slope statistics,

recommends

1 that the methods described in Annex 1 § 2.2 be used for the calculation of the statistics of fade duration due to the combined effects of attenuation (gases, clouds and rain) and scintillation on Earth-space paths;

2 that the methods described in Annex 1 § 3.2 be used for the calculation of the statistics of fade slope due to attenuation on Earth-space paths.

Annex 1

1 Introduction

In the design of a variety of telecommunication systems, the dynamic characteristics of fading due to atmospheric propagation are of concern to optimize system capacity and meet quality and reliability criteria. Examples are fixed networks that include a space segment and systems that apply fade mitigation or resource sharing techniques.

Several temporal scales can be defined, and it is useful to have information on fade slope, fade duration and interfade duration statistics for a given attenuation level (Fig. 1).

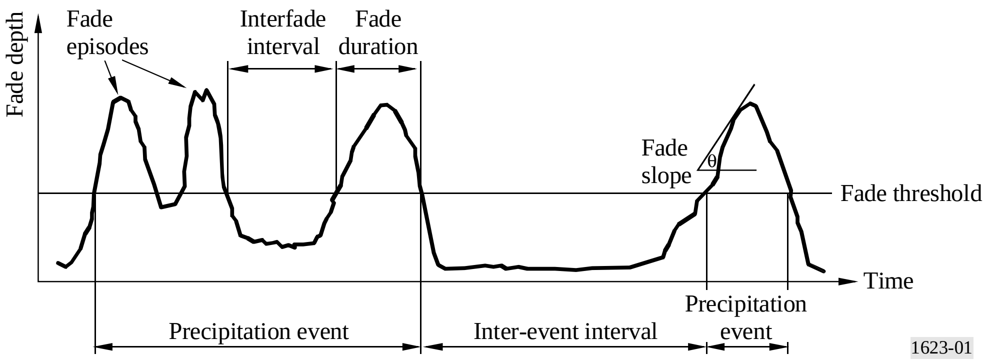
Fade duration is defined as the time interval between two crossings above the same attenuation threshold whereas interfade duration is defined as the time interval between two crossings below the same attenuation threshold. Fade slope is defined as the rate of change of attenuation with time.

---

* *Note by the BR Secretariat* – Equation (20) was amended editorially in English in September 2008.

FIGURE 1

Features characterizing the dynamics of fade events



Of particular interest in the context of availability criteria is the distinction between fades of shorter and longer duration than 10 s. Knowledge of the distribution of fade duration as a function of fade depth is also a prerequisite for the application of risk concepts in the provision of telecommunication services.

In addition, information about the expected fade slope is essential to assess the required minimum tracking rate of a fade mitigation system.

2 Fade and interfade duration

2.1 Requirements for fade duration information

Fade duration is an important parameter to be taken into account in system design for several reasons:

– system outage and unavailability: fade duration statistics provide information on number and duration of outages and system unavailability due to propagation on a given link and service;

– sharing of the system resource: it is important from the operator's point-of-view to have an insight into the statistical duration of an event in order to assign the resource for other users;

– FMT: fade duration is of concern to define statistical duration for the system to stay in a compensation configuration before coming back to its nominal mode;

– system coding and modulation: fade duration is a key element in the process of choosing forward error correction codes and best modulation schemes; for satellite communication systems, the propagation channel does not produce independent errors but blocks of errors. Fade duration impacts directly on the choice of the coding scheme (size of the coding word in block codes, interleaving in concatenated codes, etc.).

2.2 Fade duration prediction method

Fade duration can be described by two different cumulative distribution functions:

1 $P(d > D|a > A)$, the probability of occurrence of fades of duration d longer than D (s), given that the attenuation a is greater than A (dB). This probability can be estimated from the ratio of the number of fades of duration longer than D to the total number of fades observed, given that the threshold A is exceeded.

2 $F(d > D|a > A)$, the cumulative exceedance probability, or, equivalently, the total fraction (between 0 and 1) of fade time due to fades of duration d longer than D (s), given that the attenuation a is greater than A (dB). This probability can be estimated from the ratio of the total fading time due to fades of duration longer than D given that the threshold A is exceeded, to the total exceedance time of the threshold.

For a given reference period, the number of fades of duration longer than D is estimated by multiplying the probability of occurrence $P(d > D|a > A)$ by the total number of fades exceeding the threshold, $N_{tot}(A)$. Likewise, an estimate of the total exceedance time due to fade events of duration longer than D is obtained by multiplying the fraction of time $F(d > D|a > A)$ by the total time that the threshold is exceeded, $T_{tot}(A)$.

The two-segment model presented here consists of a log-normal distribution function for long fades and a power-law function for short fades. The boundary between short and long fades is given by the threshold duration D_t calculated in the model. The power-law model is valid for fade durations longer than 1 s. Fades of shorter duration do not contribute significantly to total outage time.

The following provides estimates of the parameters required for the model and finally defines the two-segment model for both distribution functions, i.e. the occurrence probability P and the exceedance probability (or fraction of time) F.

The model is expected to be valid for durations longer than 1 s.

The following parameters are required as input to the model:

f: frequency (GHz): 10-50 GHz

φ: elevation angle (degrees): 5-60°

A: attenuation threshold (dB).

The step-by-step calculation of the fade duration distribution is as follows:

Step 1: Calculate the mean duration D_0 of the log-normal distribution of the fraction of fading time due to fades of long duration, given that the attenuation is greater than A, as:

$$D_0 = 80\, \varphi^{-0.4}\, f^{1.4}\, A^{-0.39} \quad \text{s} \tag{1}$$

Step 2: Calculate the standard deviation σ of the lognormal distribution of the fraction of fading time due to fades of long duration as:

$$\sigma = 1.85\, f^{-0.05}\, A^{-0.027} \tag{2}$$

Step 3: Calculate the exponent γ of the power-law distribution of the fraction of fading time due to fades of short duration as:

$$\gamma = 0.055\, f^{0.65}\, A^{-0.003} \tag{3}$$

Step 4: Calculate the boundary between short and long fade durations, D_t, as:

$$D_t = D_0\, e^{p_1 \sigma^2 + p_2 \sigma - 0.39} \quad \text{s} \tag{4}$$

where:

$$p_1 = 0.885\gamma - 0.814 \tag{5}$$

$$p_2 = -1.05\gamma^2 + 2.23\gamma - 1.61 \tag{6}$$

Step 5: Calculate the mean duration D_2 of the log-normal distribution of the probability of occurrence of fading events of long duration as:

$$D_2 = D_0 \cdot e^{-\sigma^2} \qquad \text{s} \tag{7}$$

Step 6: Calculate the fraction of time k due to fades of duration less than D_t as:

$$k = \left[1 + \frac{\sqrt{D_0 D_2}\,(1-\gamma)\,Q\left(\dfrac{\ln(D_t) - \ln(D_0)}{\sigma}\right)}{D_t \gamma\, Q\left(\dfrac{\ln(D_t) - \ln(D_2)}{\sigma}\right)} \right]^{-1} \tag{8}$$

where:

Q: standard cumulative distribution function for a normally distributed variable:

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\frac{1}{2}x^2}\, dx \tag{9}$$

Step 7: Calculate the probability of occurrence of fade events of duration d longer than D given that attenuation a is greater than A as:

For $1 \le D \le D_t$ $$P(d > D | a > A) = D^{-\gamma} \tag{10}$$

For $D > D_t$ $$P(d > D | a > A) = D_t^{-\gamma} \cdot \frac{Q\left(\dfrac{\ln(D) - \ln(D_2)}{\sigma}\right)}{Q\left(\dfrac{\ln(D_t) - \ln(D_2)}{\sigma}\right)} \tag{11}$$

Step 8: Calculate the cumulative probability of exceedance, i.e. the total fraction of fade time due to fades of duration d longer than D:

For $1 \le D \le D_t$ $$F(d > D | a > A) = \left[1 - k \left(\frac{D}{D_t} \right)^{1-\gamma} \right] \tag{12}$$

For $D > D_t$ $$F(d > D | a > A) = (1-k) \cdot \frac{Q\left(\dfrac{\ln(D) - \ln(D_0)}{\sigma}\right)}{Q\left(\dfrac{\ln(D_t) - \ln(D_0)}{\sigma}\right)} \tag{13}$$

Step 9: If required, the total number of fades of duration d longer than D for a given threshold A can be calculated from:

$$N(D, A) = P(d > D | a > A) \times N_{tot}(A) \tag{14}$$

Similarly, the total fading time due to fades of duration d longer than D for the threshold A is:

$$T(d > D|a > A) = F(d > D|a > A) \times T_{tot}(A) \quad \text{s} \tag{15}$$

for the reference period of interest, where $T_{tot}(A)$ is the total time the threshold A is exceeded and $N_{tot}(A)$ is the total number of fades exceeding the minimum duration of 1 s. These parameters can be obtained in the following way:

$T_{tot}(A)$ should be obtained from local data. If this long-term statistic is not available, an estimate can be calculated from Recommendation ITU-R P.618. In this case the procedure consists in calculating the CDF of total attenuation, deriving the percentage of time the considered attenuation threshold A is exceeded and then the associated total exceedance time $T_{tot}(A)$ for the reference period considered.

Once $T_{tot}(A)$ has been obtained, $N_{tot}(A)$ can be calculated as:

$$N_{tot}(A) = T_{tot}(A) \cdot \frac{k}{\gamma} \cdot \frac{1 - \gamma}{D_t^{1-\gamma}} \tag{16}$$

The above method was tested against the Radiocommunication Study Group 3 fade duration data bank for frequencies between 11 and 50 GHz and for elevation angles between 6° and 60°. The arithmetic mean of the logarithmic error (ratio of the predicted to the measured fade duration at the same probability level) was found to be 30% for fade durations shorter than 10 s and between –25% and –80% for fade durations longer than 10 s. As far as the standard deviation is concerned, it was found to range between 80% and 150%, demonstrating the high natural variability of this parameter.

2.3 Interfade duration

Apart from fade duration statistics, it is also useful to characterize the time interval between two fades, called interfade duration. Once the level of the received signal has just crossed back over the margin threshold after an outage event, it is essential from the operator's point-of-view to know statistically the duration before another outage event.

Experimental results indicate that interfade duration statistics may be log-normally distributed; however, short duration interfade intervals resulting from tropospheric scintillation are expected to follow a power-law form as found with short-term fade duration statistics.

3 Fade slope

3.1 Requirements for fade slope information

It is important to be able to quantify fade slope for satellite communication systems for which Fade Mitigation Techniques may be implemented. The knowledge of the fade slope of the received signal is useful either to design a control loop that can follow signal variations, or to allow a better short term prediction of the propagation conditions. In both cases, the relevant information is the slope of the slowly varying component of the signal which involves filtering out scintillation and rapid variations of rain attenuation.

3.2 Fade slope prediction method

The fade slope probability distribution depends on climatic parameters, drop size distribution and therefore on the type of rain. The horizontal wind velocity perpendicular to the path is another climatic parameter of influence, determining the speed at which the horizontal rain profile passes across the propagation path. Also, the expected fade slope at a given attenuation level is likely to decrease as path length increases due to the smoothing effect of summing different rain contributions, and, therefore, will increase as elevation angle increases on Earth-space paths.

Furthermore, the measured fade slope is influenced by dynamic parameters, or time constants, of the receiving system. A receiver with a longer integrating time reduces the instantaneous fade change and spreads it over a longer period of time.

The predicted distribution of fade slope is dependent on attenuation level $A(t)$ and on the time interval length Δt. Also, the distribution depends on the 3 dB cut-off frequency of the low-pass filter which is used to remove tropospheric scintillation and rapid variations of rain attenuation from the signal. Experimental results show that a 3 dB cut-off frequency of 0.02 Hz allows scintillation and rapid variations of rain attenuation to be filtered out adequately. If scintillation and rapid variations of rain attenuation are not filtered out, the signal will exhibit stronger fluctuations and the model will only predict those due to rain attenuation. In that case, the cut-off frequency required as an input is the sampling frequency.

In the model, the fade slope ζ at a certain point in time is defined from the filtered data as:

$$\zeta(t) = \frac{A\left(t + \frac{1}{2}\Delta t\right) - A\left(t - \frac{1}{2}\Delta t\right)}{\Delta t} \qquad \text{dB/s} \qquad (17)$$

The model is valid for the following ranges of parameters:

– frequencies from 10 to 30 GHz

– elevation angles from 10° to 50°.

The following parameters are required as input to the model:

A: attenuation level (dB): 0-20 dB

f_B: 3 dB cut-off frequency of the low pass filter (Hz): 0.001-1 Hz

Δt: time interval length over which fade slope is calculated (s): 2-200 s.

The step-by-step calculation of the fade slope distribution is as follows:

Step 1: Calculate the function F which gives the dependence on the time interval length Δt and the 3 dB cut-off frequency of the low pass filter f_B:

$$F(f_B, \Delta t) = \sqrt{\frac{2\pi^2}{\left(1/f_B^b + (2\Delta t)^b\right)^{1/b}}} \qquad (18)$$

with $b = 2.3$.

Step 2: Calculate the standard deviation σ_ζ of the conditional fade slope at a given attenuation level as:

$$\sigma_\zeta = s\, F(f_B, \Delta t) A \qquad \text{dB/s} \qquad (19)$$

where s is a parameter which depends on climate and elevation angle; an overall average value in Europe and the United States of America, at elevations between 10° and 50°, is $s = 0.01$.

Step 3a: Calculate $p(\zeta|A)$ the conditional probability (probability density function) that the fade slope is equal to ζ for a given attenuation value, A:

$$p(\zeta \mid A) = \frac{2}{\pi\sigma_\zeta\left(1 + (\zeta/\sigma_\zeta)^2\right)^2} \qquad (20)$$

Step 3b: If required, calculate $p(\zeta|A)$, the conditional probability (complementary cumulative distribution function) that the fade slope ζ is exceeded for a given attenuation value, A:

$$P\,(\zeta|A) = \frac{1}{2} - \frac{(\zeta/\sigma_\zeta)}{\pi(1+(\zeta/\sigma_\zeta)^2)} - \frac{\arctan(\zeta/\sigma_\zeta)}{\pi} \qquad (21)$$

or calculate $p(\zeta|A)$, the conditional probability that the absolute value of the fade slope ζ is exceeded for a given attenuation value, A:

$$P(|\zeta|\,|\,A) = \int_{-\infty}^{-\zeta} p(x\,|\,A)\,\mathrm{d}x + \int_{\zeta}^{\infty} p(x\,|\,A)\,\mathrm{d}x = 1 - \frac{2(|\zeta|/\sigma_\zeta)}{\pi(1+(|\zeta|/\sigma_\zeta)^2)} - \frac{2\arctan(|\zeta|/\sigma_\zeta)}{\pi} \qquad (22)$$

The model given in equation (22) was tested against data between 12.5 GHz and 50 GHz. These results showed a good match to the shape of the fade slope cumulative distribution function as well as its variation with attenuation threshold A, interval length Δt and 3 dB cut-off frequency of the low pass filter f_B.