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International Telecommunication Union
Radiocommunication Sector

Recommendation ITU-R P.1144-12

(08/2023)

P Series: Radiowave propagation

Guide to the application of the propagation methods of Radiocommunication Study Group 3

Foreword

The role of the Radiocommunication Sector is to ensure the rational, equitable, efficient and economical use of the radio-frequency spectrum by all radiocommunication services, including satellite services, and carry out studies without limit of frequency range on the basis of which Recommendations are adopted.

The regulatory and policy functions of the Radiocommunication Sector are performed by World and Regional Radiocommunication Conferences and Radiocommunication Assemblies supported by Study Groups.

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ITU-R policy on IPR is described in the Common Patent Policy for ITU-T/ITU-R/ISO/IEC referenced in Resolution ITU-R 1. Forms to be used for the submission of patent statements and licensing declarations by patent holders are available from http://www.itu.int/ITU-R/go/patents/en where the Guidelines for Implementation of the Common Patent Policy for ITU-T/ITU-R/ISO/IEC and the ITU-R patent information database can also be found.

Series of ITU-R Recommendations

(Also available online at https://www.itu.int/publ/R-REC/en)

Series	Title
BO	Satellite delivery
BR	Recording for production, archival and play-out; film for television
BS	Broadcasting service (sound)
BT	Broadcasting service (television)
F	Fixed service
M	Mobile, radiodetermination, amateur and related satellite services
P	**Radiowave propagation**
RA	Radio astronomy
RS	Remote sensing systems
S	Fixed-satellite service
SA	Space applications and meteorology
SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
TF	Time signals and frequency standards emissions
V	Vocabulary and related subjects

Note*: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R P.1144-12

Guide to the application of the propagation methods of Radiocommunication Study Group 3

(1995-1999-2001-2001-2007-2009-2012-2015-06/2017-12/2017-2019-2021-2023)

Scope

This Recommendation provides a guide to the Recommendations of Radiocommunication Study Group 3, which contain propagation prediction methods. It advises users on the most appropriate methods for particular applications as well as the limits, required input information, and output for each of these methods.

Keywords

Radiowave propagation, prediction methods, digital products, spatial interpolation, height reference system

Glossary

Symbol	Description
n	number of Gaussian quadrature points (nodes)
W_i	Gaussian quadrature weights
X_i	Gaussian quadrature points

Other symbols not listed in the table above are intermediate in nature and have no definition.

The ITU Radiocommunication Assembly,

considering

that there is a need to assist users of the ITU-R Recommendations P Series (developed by Radiocommunication Study Group 3),

recommends

1 that the information contained in Table 1 should be considered for guidance on the application of the various propagation methods contained in the ITU-R Recommendations P Series (developed by Radiocommunication Study Group 3);

2 that the information contained in Table 2 and Annex 1 should be considered for guidance on the use of the various digital maps of geophysical parameters necessary for the application of the propagation methods in *recommends* 1 above.

NOTE – For each of the ITU-R Recommendations in Table 1, there are associated information columns to indicate:

Application: the service(s) or application for which the Recommendation is intended.

Type: the situation to which the Recommendation applies, such as point-to-point, point-to-area, line-of-sight, etc.

Output: the output parameter value produced by the method of the Recommendation, such as basic transmission loss.

Frequency: the applicable frequency range of the Recommendation.

Distance: the applicable distance range of the Recommendation.

% *time*: the applicable time percentage values or range of values of the Recommendation; % time is the percentage of time that the predicted signal is exceeded during an average year.

% *location*: the applicable per cent location range of the Recommendation; % location is the percentage of locations within, say, a square with 100 to 200 m sides that the predicted signal is exceeded.

Terminal height: the applicable terminal antenna height range of the Recommendation.

Input data: a list of parameters used by the method of the Recommendation; the list is ordered by the importance of the parameter and, in some instances, default values may be used.

The information, as shown in Table 1, is already provided in the Recommendations themselves; however, the Table allows users to quickly scan the capabilities (and limitations) of the Recommendations without the requirement to search through the text.

TABLE 1

ITU-R radiowave propagation prediction methods

Method	Title	Application	Type	Output	Frequency	Distance	% time	% location	Terminal height	Input data
Rec. ITU-R P.368	Ground-wave propagation curves for frequencies between 10 kHz and 30 MHz	All services	Point-to-point	Field strength	10 kHz to 30 MHz	1 to 10 000 km	Not applicable	Not applicable	Ground-based	Frequency Ground conductivity
Rec. ITU-R P.452	Prediction procedure for the evaluation of interference between stations on the surface of the Earth at frequencies above about 0.1 GHz	Services employing stations on the surface of the Earth; interference	Point-to-point	Basic transmission loss	100 MHz to 50 GHz	Not specified but up to and beyond the radio horizon	0.001 to 50 Average year and worst month	Not applicable	No limits specified, within the surface layer of the atmosphere. (Not suitable for aeronautical applications)	Path profile data Frequency Percentage time Tx antenna height Rx antenna height Latitude and longitude of Tx Latitude and longitude of Rx Meteorological data Polarization
Rec. ITU-R P.528	Propagation curves for aeronautical mobile and radionavigation services using the VHF, UHF and SHF bands	Aeronautical mobile	Point-to-area	Basic transmission loss	100 MHz to 30 GHz	Not specified but up to and beyond the radio horizon For aeronautical applications 0 km great circle distance does not mean 0 km path length.	1 to 99	Not applicable	Terminal heights: 1.5 - 20 000 m	Distance Tx height Frequency Rx height Percentage time Polarization
Rec. ITU-R P.530	Propagation data and prediction methods required for the design of terrestrial line-of-sight systems	Line-of-sight fixed links	Point-to-point line-of-sight	Propagation loss Diversity improvement (clear air conditions) XPD[(2)] Outage Error performance	Approximately 150 MHz to 100 GHz	Up to 200 km if line-of-sight	All percentages of time in clear-air conditions; 1 to 0.001 in precipitation conditions And worst month for attenuation	Not applicable	High enough to ensure specified path clearance	Distance Tx height Frequency Rx height Percentage time Path obstruction data Climate data Terrain information

TABLE 1 (*continued*)

Method	Title	Application	Type	Output	Frequency	Distance	% time	% location	Terminal height	Input data
Rec. ITU-R P.533	Method for the prediction of the performance of HF circuits	Broadcasting Fixed Mobile	Point-to-point	Basic MUF Sky-wave field strength Available receiver power Signal-to-noise ratio LUF Circuit reliability	2 to 30 MHz	0 to 40 000 km	All percentages	Not applicable	Not applicable	Latitude and longitude of Tx Latitude and longitude of Rx Sunspot number Month Time(s) of day Frequencies Tx power Tx antenna type Rx antenna type
Rec. ITU-R P.534	Method for calculating sporadic-E field strength	Fixed Mobile Broadcasting	Point-to-point via sporadic E	Field strength	30 to 100 MHz	0 to 4 000 km	0.1 to 50	Not applicable	Not applicable	Distance Frequency
Rec. ITU-R P.617	Propagation prediction techniques and data required for the design of trans-horizon radio-relay systems	Trans-horizon fixed links	Point-to-point	Basic transmission loss	> 30 MHz	100 to 1 000 km	0.001 to 99.999	Not applicable	No limits specified within the surface layer of the atmosphere. (Not suitable for aeronautical applications)	Frequency Tx antenna gain Rx antenna gain Path geometry
Rec. ITU-R P.618	Propagation data and prediction methods required for the design of Earth-space telecommunication systems	Satellite	Point-to-point	Propagation loss Diversity gain and (for precipitation condition) XPD(2)	1 to 55 GHz	Any practical orbit height	0.001-5 for rain attenuation; 0.001-50 for total attenuation, 0.001-1 for XPD(2) Also worst month for attenuation	Not applicable	No limit	Meteorological data Frequency Elevation angle Height of earth station Separation and angle between earth station sites (for diversity gain) Antenna diameter and efficiency (for scintillation) Polarization angle (for XPD(2))
Rec. ITU-R P.619	Propagation Data Required for the Evaluation of Interference Between Stations in Space and those on the Surface of the Earth	Satellite	Point-to-point	Basic transmission loss for single-entry interference Clear-air basic transmission loss for multiple-entry interference	0.1 to 100 GHz	Any practical orbit height	0.001 to 50	Not applicable	No limit	Frequency Earth-station elevation angle Angular path separation Path length Gaseous attenuation Scintillation “gain” Maximum allowed attenuation of the wanted signal

TABLE 1 (*continued*)

Method	Title	Application	Type	Output	Frequency	Distance	% time	% location	Terminal height	Input data
Rec. ITU-R P.620	Propagation data required for the evaluation of coordination distances in the frequency range 100 MHz to 105 GHz	Earth station frequency coordination	Coordination distance	Distance of which the required propagation loss is achieved	100 MHz to 105 GHz	Up to 1 200 km	0.001 to 50	Not applicable	No limits specified within the surface layer of the atmosphere. (Not suitable for aeronautical applications)	Minimum basic transmission loss Frequency Percentage of time Earth-station elevation angle
Rec. ITU-R P.678	Characterization of the variability of propagation phenomena and estimation of the risk associated with propagation margin	Rain rate models Satellite	Point-to-point	Variability of propagation phenomena	12 to 50 GHz	Any practical orbit height	0.01-2 for rainfall rate and rain attenuation along slant paths	Not applicable	No limit	Probability of exceedance
Rec. ITU-R P.679	Propagation data required for the design of broadcasting-satellite systems	Broadcast satellite	Point-to-area	Excess basic transmission loss Effect of local environment	0.5 to 5.1 GHz	Any practical orbit height	Not applicable	No limits specified	No limits specified	Frequency Elevation angle Features of local environment
Rec. ITU-R P.680	Propagation data required for the design of Earth-space maritime mobile telecommunication systems	Maritime mobile satellite	Point-to-point	Sea-surface fading Fade duration Interference (adjacent satellite)	0.8-8 GHz	Any practical orbit height	To 0.001% via Rice-Nakagami distribution Limit of 0.01% for interference[(1)]	Not applicable	No limit	Frequency Elevation angle Maximum antenna boresight gain
Rec. ITU-R P.681	Propagation data required for the design of Earth-space land mobile telecommunication systems	Land mobile satellite	Point-to-point	Path fading Fade duration Non-fade duration	0.8 to 20 GHz	Any practical orbit height	Not applicable Percentage of distance travelled 1 to 80%[(1)]	Not applicable	No limit	Frequency Elevation angle Percentage of distance travelled Approximate level of optical shadowing
Rec. ITU-R P.682	Propagation data required for the design of Earth-space aeronautical mobile telecommunication systems	Aeronautical mobile satellite	Point-to-point	Sea-surface fading Multipath from ground and aircraft during landing	1 to 2 GHz (sea-surface fading) 1 to 3 GHz (multipath from ground)	Any practical orbit height	To 0.001% via Rice-Nakagami distribution[(1)]	Not applicable	No limit for sea-surface fading Up to 1 km for ground reflection during landing	Frequency Elevation angle Polarization Maximum antenna boresight gain Antenna height

TABLE 1 (*continued*)

Method	Title	Application	Type	Output	Frequency	Distance	% time	% location	Terminal height	Input data
Rec. ITU-R P.684	Prediction of field strength at frequencies below about 150 kHz	Fixed Mobile	Point-to-point Point-to-area	Sky-wave field strength	30 to 150 kHz	0 to 16 000 km	50	Not applicable	Not applicable	Latitude and longitude of Tx Latitude and longitude of Rx Distance Tx power Frequency Ground constants Season Sunspot number Hour of day
Rec. ITU-R P.843	Communication by meteor-burst propagation	Fixed Mobile Broadcasting	Point-to-point via meteor-burst	Received power Burst rate	30 to 100 MHz	100 to 1 000 km	0 to 5	Not applicable	Not applicable	Frequency Distance Tx power Antenna gains
Rec. ITU-R P.1147	Prediction of sky-wave field strength at frequencies between about 150 and 1 700 kHz	Broadcasting	Point-to-area	Sky-wave field strength	0.15 to 1.7 MHz	50 to 12 000 km	1, 10, 50	Not applicable	Not applicable	Latitude and longitude of Tx Latitude and longitude of Rx Distance Sunspot number Tx power Frequency
Rec. ITU-R P.1238	Propagation data and prediction methods for the planning of indoor radiocommunication systems and radio local area networks in the frequency range 300 MHz to 100 GHz	Mobile RLAN	In-building propagation methods	Basic transmission loss Delay spread	300 MHz to 450 GHz	Within buildings	Not applicable	Not applicable	Base: about 2-3 m Mobile: about 0.5-3 m	Frequency Distance Floor and wall factors
Rec. ITU-R P.1410	Propagation data and prediction methods required for the design of terrestrial broadband radio access systems operating in a frequency range from 3 to 60 GHz	Broadband radio access	Point-to-area	Coverage Temporal coverage reduction due to rain	3 to 60 GHz	0-5 km	0.001 to 1 (for calculating reduction in coverage due to rain)	Up to 100	No limit; 0-300 m (typical)	Frequency Cell size Terminal heights Building height statistical parameters

TABLE 1 (*continued*)

Method	**Title**	**Application**	Type	**Output**	**Frequency**	**Distance**	**% time**	**% location**	**Terminal height**	**Input data**
Rec. ITU-R P.1411	Propagation data and prediction methods for the planning of short-range outdoor radiocommunication systems and radio local area networks in the frequency range 300 MHz to 100 GHz	Mobile	Short-path propagation methods	Basic transmission loss Delay spread	300 MHz to 100 GHz	< 1 km	Not applicable	Not applicable	Base: about 4-50 m Mobile: about 0.5-3 m	Frequency Distance Street dimensions Structure heights
Rec. ITU-R P.1546	Method for point-to-area predictions for terrestrial services in the frequency range 30 MHz to 4 000 MHz	Terrestrial services	Point-to-area	Field strength	30 to 4 000 MHz	1 to 1 000 km	1 to 50	1 to 99	*Tx/base:* effective height from less than 0 m to 3 000 m *Rx/mobile:* ≥ 1 m	Terrain height and ground cover (optional) Path classification Distance Tx antenna height Frequency Percentage time Rx antenna height Terrain clearance angle Percentage locations Refractivity gradient
Rec. ITU-R P.1622	Prediction methods required for the design of Earth-space systems operating between 20 THz and 375 THz	Satellite optical links	Point-to-point	Absorption loss Scattering loss Background noise Amplitude scintillation Angle of arrival Beam wander Beam spreading	20 to 375 THz	Far-field Earth-to-space optical links	Not applicable	Not applicable	No limit	Wavelength Terminal height Elevation angle Turbulence structure parameter
Rec. ITU-R P.1623	Prediction method of fade dynamics on Earth-space paths	Satellite	Point-to-point	Fade duration, fade slope	10 to 50 GHz	Any practical orbit height	Not applicable	Not applicable	No limit	Frequency Elevation angle Attenuation threshold Filter bandwidth
Rec. ITU-R P.1812	A path-specific propagation prediction method for point-to-area terrestrial services in the frequency range 30 MHz to 6 000 MHz	Terrestrial services	Point-to-area	Field strength	30 MHz to 6 000 MHz	Not specified but up to and beyond the radio horizon	1 to 50	1 to 99	No limits specified, within the surface layer of the atmosphere. (Not suitable for aeronautical applications)	Path profile data Frequency Percentage time Tx antenna height Rx antenna height Latitude and longitude of Tx Latitude and longitude of Rx Meteorological data Polarization

TABLE 1 (*end*)

Method	Title	Application	Type	Output	Frequency	Distance	% time	% location	Terminal height	Input data
Rec. ITU-R P.1814	Prediction methods required for the design of terrestrial free-space optical links	Terrestrial optical links	Point-to-point	Absorption loss Scattering loss Background noise Amplitude scintillation Beam spreading	20 to 375 THz	No limit	Not applicable	Not applicable	No limit	Wavelength Visibility (in fog) Path length Turbulence structure parameter
Rec. ITU-R P.1853	Tropospheric attenuation time series synthesis	Terrestrial satellite	Point-to-point	Rain attenuation for terrestrial paths Total attenuation and tropospheric scintillation for Earth-space paths	4 to 40 GHz for terrestrial paths 4 to 55 GHz for Earth-space paths	Between 2 and 60 km for terrestrial paths GEO satellite	Not applicable	Not applicable	No limit	Meteorological data Frequency Elevation angle Height of earth station Separation and angle between earth station sites (for diversity gain) Antenna diameter and efficiency (for scintillation)
Rec. ITU-R P.2001	A general purpose wide-range terrestrial propagation model in the frequency range 30 MHz to 50 GHz	Terrestrial services	Point-to-point	Basic transmission loss	30 MHz to 50 GHz	3 to 1 000 km	0.001 to 99.999	Not applicable	<8000 m above m.s.l. but near the ground, within the troposphere	Path profile data Frequency Percentage time Tx antenna height, gain and azimuthal direction Rx antenna height, gain and azimuthal direction Latitude and longitude of Tx Latitude and longitude of Rx Polarization
Rec. ITU-R P.2041	Prediction of path attenuation on links between an airborne platform and Space and between an airborne platform and the surface of the Earth	Airborne	Point-to-point	Total attenuation	1 to 55 GHz	Any height	0.001 to 50	Not applicable	Between the surface of the Earth and space	Meteorological data Frequency Elevation angle Availability Height of airborne platform Antenna diameter and efficiency (for scintillation)

(1) Time percentage of outage; for service availability, subtract value from 100.

(2) XPD: Cross-polarization discrimination.

TABLE 2

ITU-R digital products for radiowave propagation predictions methods

Recommendation ITU-R	Description	Grid resolution	Spatial interpolation required (see Annex 1)	Interpolation in probability	Interpolation of the variable	Comments
P.452	Median annual ΔN Median annual N_0	1.5° × 1.5°	Bi-linear	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.453	Annual and monthly probability distributions of the wet term of the refractivity at the surface (Nwet)	0.75° × 0.75°	Bi-linear	Logarithmic	Not applicable	Refer to the associated Readme file for the applicable file names(2)
	• Refractivity gradient in the lowest 65 m of the atmosphere (N-units/km) • Refractivity gradient in the lowest 1 km of the atmosphere (N-units/km) • Percentage of time for which refractivity gradient in the lowest 100 m < −100 N-unit/km	0.75° × 0.75°	Bi-linear	Not defined	Not applicable	Refer to the associated Readme file for the applicable file names(2)
	Surface duct data	1.5° × 1.5°	Bi-linear	Not defined	Not applicable	Refer to the Software for ionospheric and tropospheric propagation and radio noise web page
	Elevated duct data	1.5° × 1.5°	Bi-linear	Not defined	Not applicable	Refer to the Software for ionospheric and tropospheric propagation and radio noise web page
P.530	*LogK* logarithm of *K*%, the geoclimatic factor for the average worst month	0.25° × 0.25°	Bi-linear	Not defined	Linear	See the Recommendation for the application and use of these data sets.
	dN_{75} empirical prediction of 0.1% of the average worst month refractivity increase with height over the lowest 75 m of the atmosphere	0.25° × 0.25°	Bi-linear	Not defined	Linear	See the Recommendation for the application and use of these data sets.
P.534	foEs exceeded for annual percentage times	1.5° × 1.5°	Bi-linear	Linear	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.617	Median annual ΔN Median annual N_0	1.5° × 1.5°	Bi-linear	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.678	Map of the climatic ratio	0.5° × 0.5°	Bi-linear	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)

TABLE 2 (*continued*)

Recommendation ITU-R	Description	Grid resolution	Spatial interpolation required (see Annex 1)	Interpolation in probability	Interpolation of the variable	Comments
P.834	• Harmonic coefficients of excess path length along Earth-space paths • Harmonic coefficients of the hydrostatic and wet mapping functions	1.5° × 1.5° 5° × 5°	Bi-linear Not Required	Not defined	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.835	Experimental Data of Atmospheric Vertical Profiles (Annex 2)	353 Locations	Not required	Not applicable	Not applicable	Refer to the Software for ionospheric and tropospheric propagation and radio noise web page
	Weather Prediction Data of Atmospheric Vertical Profiles (Annex 3)	1.5° × 1.5°	Not specified	Not applicable	Not applicable	Refer to the Software for ionospheric and tropospheric propagation and radio noise web page
P.836	Total columnar water vapour exceedance probability (%) (IWVC)	1.125° × 1.125°	Bi-linear(1)	Logarithmic	Linear	Refer to the associated Readme file for the applicable file names(2)
	Surface water vapour density exceedance probability (%) (Rho)	1.125° × 1.125°	Bi-linear(1)	Logarithmic	Linear	Refer to the associated Readme file for the applicable file names(2)
	Water vapour scale height	1.125° × 1.125°	Bi-linear(1)	Logarithmic	Linear	Refer to the associated Readme file for the applicable file names(2)
	Topographic altitude (a.m.s.l.) (km)	0.5° × 0.5°	Bi-cubic	Not applicable	Not applicable	Refer to the Recommendation
P.837	Monthly mean total rainfall (mm) $R_{0.01}$ (mm/h)	0.25° × 0.25° 0.125° × 0.125°	Bi-linear Bi-linear	Not applicable Not applicable	Not applicable Not applicable	Refer to the associated readme file for the applicable filenames(2)
	Conversion of rain rate statistics at different integration times (Annex 2)	Not applicable	Not required	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.839	Mean annual 0°C isotherm height (km)	1.5° × 1.5°	Bi-linear	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.840	Annual and monthly statistics (mean, standard deviation and CCDF) statistics of integrated cloud liquid water content Approximation of annual integrated cloud liquid water content by a log-normal distribution	0.25° × 0.25°	Bi-linear	Logarithmic	Linear	Refer to the associated Readme file for the applicable file names(2)

TABLE 2 (*continued*)

Recommendation ITU-R	**Description**	**Grid resolution**	**Spatial interpolation required (see Annex 1)**	**Interpolation in probability**	**Interpolation of the variable**	**Comments**
P.1510	Mean monthly and annual surface temperature	0.75° × 0.75°	Bi-linear	Not applicable	Not applicable	Refer to the associated readme file for the applicable filenames(2)
P.1511	Topographic altitude (a.m.s.l.) (m)	0.08333° × 0.083 33°	Bi-cubic	Not applicable	Not applicable	This Recommendation contains definitions of latitude, longitude, and height in P-Series Recommendations. Refer to the associated Readme file for the applicable file names(2)
	Earth gravitational model 2008 (EGM2008) (m)	0.08333° × 0.08333°	Bi-cubic	Not applicable	Not applicable	Refer to the associated readme file for the applicable filenames(2)
P.1812	Median annual ΔN Median annual N_0	1.5° × 1.5°	Bi-linear	Not applicable	Not applicable	Refer to the associated Readme file for the applicable file names(2)
P.1853	Annual mean surface pressure Annual mean water vapour density	0.75° × 0.75°	Bi-linear	Not applicable	Not applicable	WV_Annual.txt P_Annual.txt
P.2001	Surface level refractivity and gradient in the lowest 1 km of the atmosphere	Multiple	Bi-linear	Not applicable	Linear	Refer to the associated Readme file for the applicable file names(2)
P.2001 and P.534	Critical frequency for sporadic-*E* (F_0E_s)	1.5° × 1.5°	Bi-linear	Linear	Linear	FoEs50.txt FoEs10.txt FoEs01.txt FoEs0.1.txt
P.2145	Annual and monthly statistics (mean, standard deviation and CCDF) of surface pressure Annual and monthly statistics (mean, standard deviation and CCDF) of surface temperature Annual and monthly statistics (mean, standard deviation and CCDF) of surface water vapour density Annual and monthly statistics (mean, standard deviation and CCDF) of integrated water vapour content Approximation of annual integrated water vapour content by a Weibull distribution	0.25° × 0.25°	Bi-linear	Logarithmic	Linear	Refer to the Recommendation

TABLE 2 (*end*)

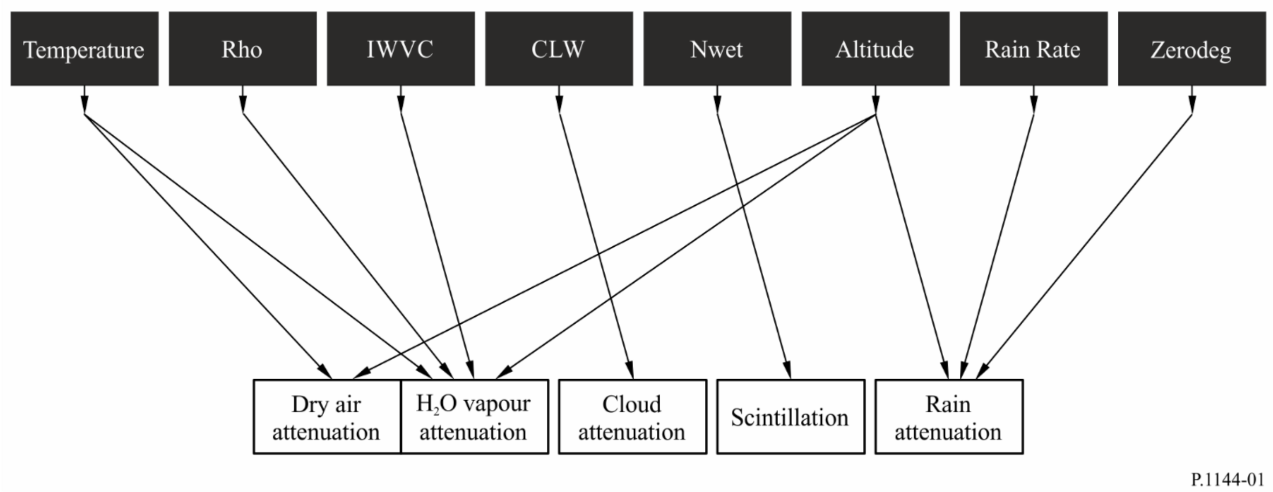
Recommendation ITU-R	**Description**	**Grid resolution**	**Spatial interpolation required (see Annex 1)**	**Interpolation in probability**	**Interpolation of the variable**	**Comments**
P.2148	Annual statistics of wind speed at a height of 10 m above the surface of the Earth	0.25° × 0.25°	Bi-linear	Logarithmic	Linear	Refer to the Recommendation

(1) The variables at the surrounding grid points are scaled to the desired altitude prior to spatial interpolation per the scaling procedure in the Recommendation.

(2) The readme file is contained within the Zip (Components) file on the web page associated with the Recommendation.

For easy reference, Fig. 1 shows the relationship between the geophysical maps (black boxes) and propagation effects (white boxes).

FIGURE 1



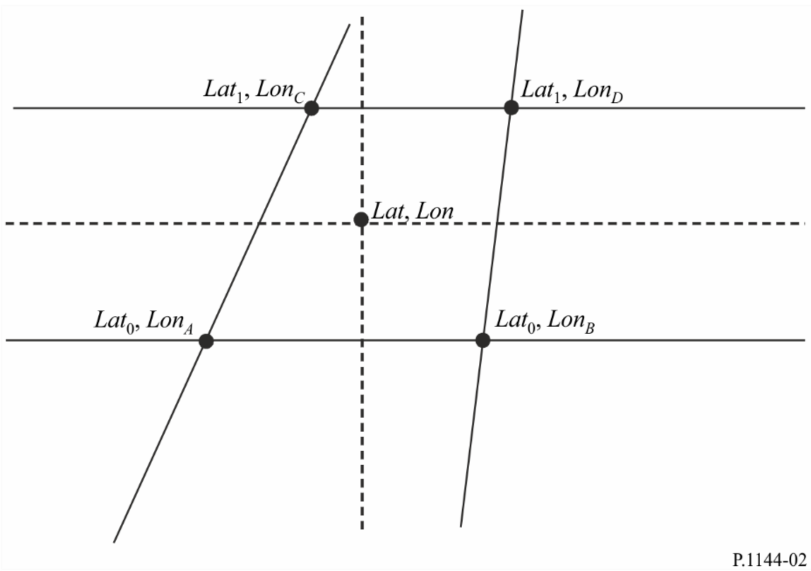
Annex 1

1a Bi-linear interpolation on a trapezoidal grid

Given: Values of X at four surrounding points:(Lat_1, Lon_C), (Lat_1, Lon_D), (Lat_0, Lon_A), and (Lat_0, Lon_B); i.e. $X(Lat_1, Lon_C)$, $X(Lat_1, Lon_D)$, $X(Lat_0, Lon_A)$, and $X(Lat_0, Lon_B)$.

Problem: Determine the value $X(Lat, Lon)$ at an intervening point (Lat, Lon) using bi-linear interpolation.

FIGURE 2



Solution: Define two auxiliary variables, t and s:

$$t = \frac{Lat - Lat_0}{Lat_1 - Lat_0}$$

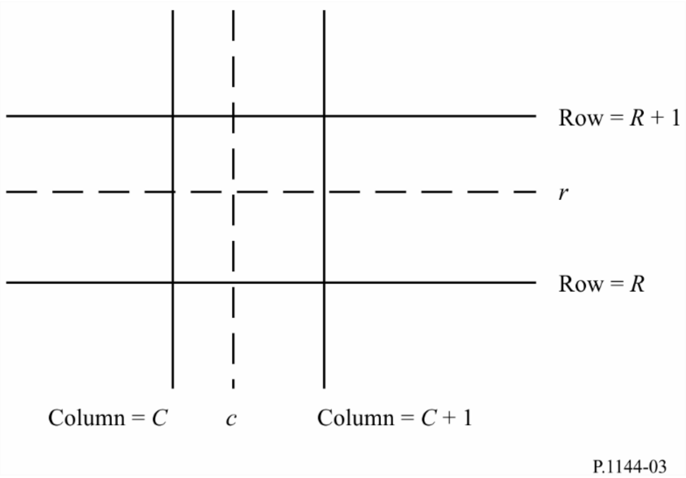
$$s = \frac{Lon - Lon_A + t\,(Lon_A - Lon_C)}{Lon_B - Lon_A + t\,(Lon_A - Lon_C + Lon_D - Lon_B)}$$

and calculate:

$$\begin{aligned} X(Lat, Lon) = {} & (1-s)\,(1-t)\,X(Lat_0, Lon_A) \\ & +(1-s)\,t\,X(Lat_1, Lon_C) \\ & +s\,(1-t)\,X(Lat_0, Lon_B) \\ & +t\,s\,X(Lat_1, Lon_D) \end{aligned}$$

1b Bi-linear interpolation on a square grid

FIGURE 3



Given: Values of I at four surrounding grid points: $I(R,C)$, $I(R,C+1)$, $I(R+1,C)$, and $I(R+1,C+1)$, where R, $R+1$, C, and $C+1$ are integer row and column numbers.

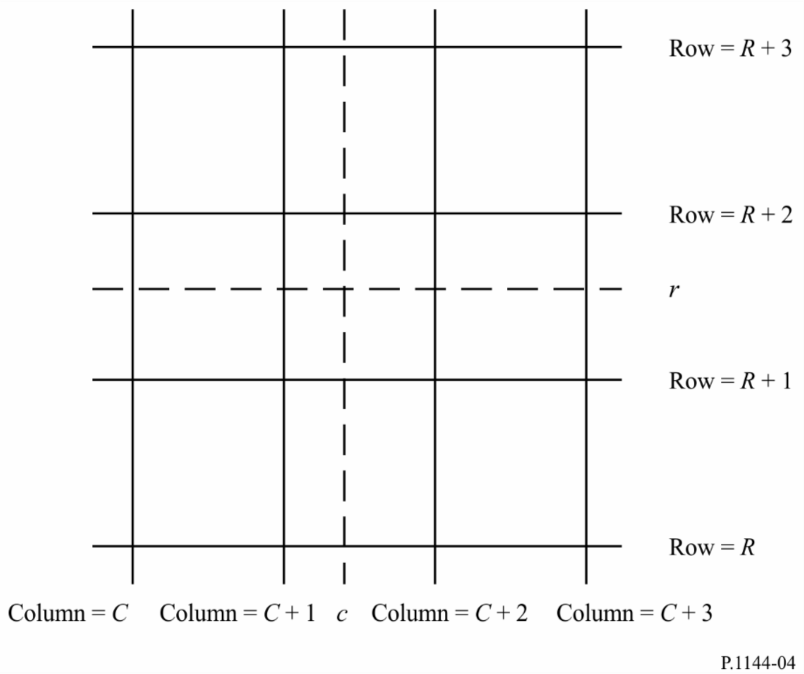
Problem: Determine $I(r,c)$, where r is a fractional row number between R and $R+1$ and c is a fractional column number between C and $C+1$, using bi-linear interpolation.

Solution: Calculate:

$$\begin{aligned} I(r,c) = {} & I(R,C)\,[(R+1-r)(C+1-c)] \\ & + I(R+1,C)\,[(r-R)(C+1-c)] \\ & + I(R,C+1)\,[(R+1-r)(c-C)] \\ & + I(R+1,C+1)\,[(r-R)(c-C)] \end{aligned}$$

2 Bi-cubic interpolation

FIGURE 4



Given: Values of I at 16 surrounding grid points:

$I(R,C)$, $I(R,C+1)$, $I(R,C+2)$, $I(R,C+3)$,

$I(R+1,C)$, $I(R+1,C+1)$, $I(R+1,C+2)$, $I(R+1,C+3)$,

$I(R+2,C)$, $I(R+2,C+1)$, $I(R+2,C+2)$, $I(R+2,C+3)$,

$I(R+3,C)$, $I(R+3,C+1)$, $I(R+3,C+2)$, $I(R+3,C+3)$

where R, $R + 1$, etc.; and C, $C + 1$, etc. are integers.

Problem: Calculate $I(r,c)$, where r is a fractional row number between $R + 1$ and $R + 2$ and c is a fractional column number between $C + 1$ and $C + 2$, using bi-cubic interpolation.

Solution:

Step 1: For each row, X, where $X = \{R, R+1, R+2, R+3\}$, compute the interpolated value at the desired fractional column c as:

$$RI(X,c) = \sum_{j=C}^{C+3} I(X,j)\,K(c-j)$$

where:

$$K(\delta) = \begin{cases} (a+2)|\delta|^3 - (a+3)|\delta|^2 + 1 & \text{for} \quad 0 \le |\delta| \le 1 \\ a|\delta|^3 - 5a|\delta|^2 + 8a|\delta| - 4a & \text{for} \quad 1 \le |\delta| \le 2 \\ 0 & \text{for} \quad 2 \le |\delta| \end{cases}$$

and

$$a = -0.5$$

Step 2: Calculate $I(r,c)$ by interpolating the one-dimensional interpolations, $RI(R,c)$, $RI(R+1,c)$, $RI(R+2,c)$, and $RI(R+3,c)$ in the same manner as the row interpolations.

3 Gaussian quadrature integration

Gaussian quadrature integration is accurately approximate to a definite integral if the integrand, $f(x)$, is well-approximated by a polynomial of degree $2n$-1 or less over the integration interval. The value of n should be selected based on the desired approximation accuracy.

3.1 Single integral

A single integral can be well-approximated by Gaussian quadrature integration noting that:

$$\int_a^b f(x)dx = \int_{-1}^1 \left(\frac{b-a}{2}\right) f\left(\frac{a+b}{2} + \frac{b-a}{2}t\right) dt$$

$$\approx \sum_{i=1}^{n} W_i' f(X_i')$$

where:

$$W_i' = \left(\frac{b-a}{2}\right) W_i$$

$$X_i' = \frac{a+b}{2} + \frac{b-a}{2} X_i$$

3.2 Double integral

A double integral can be well-approximated by Gaussian quadrature integration noting that:

$$\int_a^b \int_c^d f(x,y)\, dx\, dy = \int_{-1}^1 \int_{-1}^1 \left(\frac{b-a}{2}\right)\left(\frac{d-c}{2}\right) f\left(\frac{a+b}{2} + \frac{b-a}{2}s, \frac{c+d}{2} + \frac{d-c}{2}t\right) ds\, dt$$

$$\approx \sum_{i=1}^{n}\sum_{j=1}^{n} W_i'\, Y_j'\, f(X_i', Z_j{}')$$

where:

$$W_i' = \left(\frac{b-a}{2}\right) W_i$$

$$X_i' = \frac{a+b}{2} + \frac{b-a}{2} X_i$$

$$Y_j' = \left(\frac{d-c}{2}\right) W_j$$

$$Z_j' = \frac{c+d}{2} + \frac{d-c}{2} X_j$$

3.3 Algorithm to calculate Gaussian quadrature points (nodes) and weights

This algorithm calculates the points (nodes), X_i, and weights, W_i, for $i = 1, 2, \ldots, n$, where n is the number of Gaussian quadrature points (nodes). The variable eps is the accuracy of the machine's floating-point system[1]. On machines that support IEEE floating point arithmetic, eps is approximately 2.2204e-16 for double precision. The function $floor(x)$ rounds x to the nearest integer less than or equal to x.

Step 1: Calculate $m = floor(\frac{n+1}{2})$

Repeat Steps 2 to 13 for $i = 1$ to m

Step 2: Calculate $X_i = \cos\left(\frac{4i-1}{4n+2}\pi\right)$

Step 3: Calculate $Pm1 = 1$ and $P = X_i$

Repeat Steps 4 and 5 for $j = 2$ to n

Step 4: Calculate $Pm2 = Pm1$ and $Pm1 = P$

Step 5: Calculate $P = \left(2 - \frac{1}{j}\right) X_i\, Pm1 - \left(1 - \frac{1}{j}\right) Pm2$

Step 6: Calculate $P' = \frac{n \cdot (X_i \cdot P - Pm1)}{X_i^2 - 1}$

Step 7: Calculate $\Delta = \frac{P}{P'}$

Step 8: Calculate $X_i = X_i - \Delta$

Step 9: If $|\Delta| > eps$ then go to Step 3, otherwise go to Step 10

Step 10: Calculate $Xlast = X_i + \Delta$

Step 11: Calculate $Pm1' = \frac{(n-1) \cdot Xlast \cdot Pm1 - Pm2)}{(Xlast^2 - 1)}$

Step 12: Calculate $Pm1 = Pm1 - \Delta \cdot Pm1'$

Step 13: Calculate $W_i = \frac{2\,(1 - X_i^2)}{(n \cdot Pm1)^2}$

Repeat Step 14 for $i = m + 1$ to n

Step 14: Calculate $X_i = -X_{floor\left(\frac{n}{2}\right)+m+1-i}$ and $W_i = W_{floor\left(\frac{n}{2}\right)+m+1-i}$

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[1] Example values of X_i, the Gaussian quadrature points, and W_i, the Gaussian quadrature weights, are provided in a supplemental product on the ITU-R Study Group 3 website on digital products.