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Recommendation ITU-R F.1336-3

(03/2012)

Reference radiation patterns of omnidirectional, sectoral and other antennas in point-to-multipoint systems for use in sharing studies in the frequency range from 1 GHz to about 70 GHz

F Series

Fixed service

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Note: *This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R F.1336-3*

Reference radiation patterns of omnidirectional, sectoral and other antennas in point-to-multipoint systems for use in sharing studies in the frequency range from 1 GHz to about 70 GHz

(Question ITU-R 242/5)

(1997-2000-2007-2012)

Scope

This Recommendation gives reference models of the peak and average antenna patterns of omnidirectional, sectoral and directional antennas in point-to-multipoint systems to be used in sharing studies in the frequency range 1 GHz to about 70 GHz.

The ITU Radiocommunication Assembly,

considering

a) that, for coordination studies and for the assessment of mutual interference between point-to-multipoint (P-MP) fixed wireless systems (FWSs) and between stations of such systems and stations of space radiocommunication services sharing the same frequency band, it may be necessary to use reference radiation patterns for FWS antennas;

b) that, depending on the sharing scenario, it may be appropriate to consider the peak envelope or average side-lobe patterns in the sharing studies;

c) that it may be appropriate to use the antenna radiation pattern representing average side-lobe levels in the following cases:

– to predict the aggregate interference to a geostationary or non-geostationary satellite from numerous fixed wireless stations;

– to predict the aggregate interference to a fixed wireless station from many geostationary satellites;

– to predict interference to a fixed wireless station from one or more non-geostationary-satellites under continuously varying angles;

– in any other cases where the use of the radiation pattern representing average side-lobe levels is appropriate;

d) that reference radiation patterns may be required in situations where information concerning the actual radiation pattern is not available;

e) that the use of antennas with the best available radiation patterns will lead to the most efficient use of the radio-frequency spectrum;

f) that at large angular distances from the main beam pattern gain may not fully represent the antenna emissions because of local ground reflections,

* This Recommendation should be brought to the attention of Radiocommunication Study Groups 4, 6 and 7.

noting

a) that Recommendations ITU-R F.699 and ITU-R F.1245 give the peak and average reference antenna patterns respectively to be used in coordination studies and interference assessment in cases not referred to in *recommends* 1 to 4 below,

recommends

1 that, in the absence of particular information concerning the radiation pattern of the P-MP FWS antenna involved (see Note 1), the reference radiation pattern as stated below should be used for:

1.1 interference assessment between line-of-sight (LoS) P-MP FWSs;

1.2 coordination studies and interference assessment between P-MP LoS FWSs and other stations of services sharing the same frequency band;

2 that, in the frequency range from 1 GHz to about 70 GHz, the following reference radiation patterns should be used in cases involving stations that use omnidirectional (in azimuth) antennas:

2.1 in the case of peak side-lobe patterns referred to in *considering* b), the following equations should be used for elevation angles that range from –90° to 90° (see Annex 1):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for } 0 \leq |\theta| < \theta_4 \\ G_0 - 12 + 10\log(k+1) & \text{for } \theta_4 \leq |\theta| < \theta_3 \\ G_0 - 12 + 10\log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] & \text{for } \theta_3 \leq |\theta| \leq 90° \end{cases} \quad (1a)$$

with:

$$\theta_3 = 107.6 \times 10^{-0.1\,G_0} \quad (1b)$$

$$\theta_4 = \theta_3\sqrt{1 - \frac{1}{1.2}\log(k+1)} \quad (1c)$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: the maximum gain in the azimuth plane (dBi)

θ: elevation angle relative to the angle of the maximum gain (degrees) ($-90° \leq \theta \leq 90°$)

θ_3: the 3 dB beamwidth in the elevation plane (degrees)

k: parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance (see *recommends* 2.3 and 2.4);

2.2 in the case of average side-lobe patterns referred to in *considering* c), the following equations should be used for elevation angles that range from –90° to 90° (see Annex 1 and Annex 5):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for} \quad 0 \le |\theta| < \theta_3 \\ G_0 - 15 + 10\log(k+1) & \text{for} \quad \theta_3 \le |\theta| < \theta_5 \\ G_0 - 15 + 10\log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] & \text{for} \quad \theta_5 \le |\theta| \le 90^\circ \end{cases} \qquad (1d)$$

with:

$$\theta_5 = \theta_3\sqrt{1.25 - \frac{1}{1.2}\log(k+1)}$$

where θ, θ_3, G_0 and k are defined and expressed in *recommends* 2.1;

2.3 in cases involving typical antennas operating in the 1-3 GHz range, the parameter k should be 0.7;

2.4 in cases involving antennas with improved side-lobe performance in the 1-3 GHz range, and for all antennas operating in the 3-70 GHz range, the parameter k should be 0;

2.5 in cases where the antennas in *recommends* 2.1 through 2.2 operate with a downward electrical tilt, all of the equations in those *recommends* are valid with the definitions of the following variables (see § 3 in Annex 7):

θ_e: elevation angle (degrees) by which the tilted radiation patterns are calculated using equations in *recommends* 2.1 and 2.2

θ_h: elevation angle (degrees) measured from the horizontal plane at the site of the antenna ($-90^\circ \le \theta_h \le 90^\circ$)

β: downward tilt angle, the positive angle (degrees) that the main beam axis is below the horizontal plane at the site of the antenna.

These are interrelated as follows:

$$\theta_e = \frac{90 \cdot (\theta_h + \beta)}{90 + \beta} \qquad \text{for} \quad \theta_h + \beta \ge 0 \qquad (1e)$$

$$\theta_e = \frac{90 \cdot (\theta_h + \beta)}{90 - \beta} \qquad \text{for} \quad \theta_h + \beta < 0$$

Electrically tilted radiation patterns are calculated by using θ_e of equation (1e) instead of θ at the equations in *recommends* 2.1 and 2.2, respectively.

3 that, in the frequency range from 1 GHz to about 70 GHz, the following reference radiation patterns should be used in cases involving stations that use sectoral antennas with a 3 dB beamwidth in the azimuth plane less than about 120° (see Annex 4);

3.1 in the case of peak side-lobe patterns referred to in *considering* b), the following equations should be used for elevation angles that range from −90° to 90° and for azimuth angles that range from –180° to 180°:

$$G(\varphi, \theta) = G_{ref}(x) \qquad (2a1)$$

$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right) \tag{2a2}$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}} \tag{2a3}$$

$$\psi = \arccos(\cos\varphi\cos\theta), \quad 0° \leq \psi \leq 180° \quad \text{(degrees)} \quad \text{(see Note 7)} \tag{2a4}$$

$$x = \psi / \psi_\alpha \tag{2a5}$$

where:

θ: elevation angle relative to the local horizontal plane when the maximum gain is in that plane ($-90° \leq \theta \leq 90°$)

φ: azimuth angle relative to the angle of the maximum gain in the horizontal plane (degrees)

φ_3: the 3 dB beamwidth in the azimuth plane (degrees) (generally equal to the sectoral beamwidth).

Other variables and parameters are as defined in *recommends* 2.1;

3.1.1 in the frequency range from 1 GHz to about 6 GHz (see Annex 6)

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < x_k$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \quad \text{for} \quad x_k \leq x < 4 \tag{2b}$$

$$G_{ref}(x) = G_0 - \lambda_k - 15\log(x) \quad \text{for} \quad x \geq 4$$

with $\lambda_k = 12 - 10\log(1 + 8k)$ and $x_k = \sqrt{1 - 0.36k}$;

3.1.1.1 in cases involving typical antennas the parameter k should be 0.7 (therefore, $\lambda_{k=0.7} = 3.8$ and $x_{k=0.7} = 0.86$) (see Note 2);

3.1.1.2 in cases involving antennas with improved side-lobe performance the parameter k should be 0 (therefore, $\lambda_{k=0} = 12$ and $x_{k=0} = 1$);

3.1.2 in the frequency range from 6 GHz to about 70 GHz:

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < 1 \tag{2c}$$

$$G_{ref}(x) = G_0 - 12 - 15\log(x) \quad \text{for} \quad 1 \leq x$$

3.2 in the case of average side-lobe patterns referred to in *considering* c), for use in a statistical interference assessment, the following equations should be used for elevation angles that range from $-90°$ to $90°$ and for azimuth angles that range from $-180°$ to $180°$ (see Annex 5):

$$G(\varphi, \theta) = G_{ref}(x)$$

3.2.1 in the frequency range from 1 GHz to about 6 GHz (see Annex 6):

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < x_k \tag{2d}$$

$$G_{ref}(x) = G_0 - 15 + 10\log(x^{-1.5} + k) \quad \text{for} \quad x_k \leq x < 4$$

$$G_{ref}(x) = G_0 - \lambda_k - 3 - 15\log(x) \qquad \text{for} \qquad x \geq 4$$

with $\lambda_k = 12 - 10\log(1 + 8k)$ and $x_k = \sqrt{1.25 - 0.36k}$;

3.2.1.1 in cases involving typical antennas the parameter k should be 0.2 (therefore, $\lambda_{k=0.2} = 7.85$ and $x_{k=0.2} = 1.08$) (see Note 2);

3.2.1.2 in cases involving antennas with improved side-lobe performance the parameter k should be 0 (therefore, $\lambda_{k=0} = 12$ and $x_{k=0} = 1.118$);

3.2.2 in the frequency range from 6 GHz to about 70 GHz (see Note 3):

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \qquad 0 \leq x < 1.152 \tag{2e}$$

$$G_{ref}(x) = G_0 - 15 - 15\log(x) \qquad \text{for} \qquad 1.152 \leq x$$

3.3 in cases involving sectoral antennas with a 3 dB beamwidth in the azimuth plane less than about 120°, the relationship between the maximum gain and the 3 dB beamwidth in both the azimuth plane and the elevation plane, on a provisional basis, is (see Annex 3 and Notes 4 and 5):

$$\theta_3 = \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_3} \tag{3}$$

where all parameters are as defined under *recommends* 3.1;

3.4 in cases where the antennas in *recommends* 3.1 through 3.2 operate with a downward mechanical tilt, all of the equations in those *recommends* are valid with the definitions and redefinitions of the following variables (see § 2 in Annex 7):

- θ: elevation angle (degrees) measured from the plane defined by the axis of maximum gain of the antenna and the axis about which the pattern is tilted (θ_3 is also measured from this plane)
- φ: azimuth (degrees) measured from the azimuth of maximum gain in the plane defined by the axis of maximum gain of the antenna and the axis about which the pattern is tilted
- θ_h: elevation angle (degrees) measured from the horizontal plane at the site of the antenna ($-90° \leq \theta_h \leq 90°$)
- φ_h: azimuth angle (degrees) in the horizontal plane at the site of the antenna measured from the azimuth of maximum gain ($-180° \leq \varphi_h \leq 180°$)
- β: downward tilt angle, the positive angle (degrees) that the main beam axis is below the horizontal plane at the site of the antenna.

These are interrelated as follows:

$$\theta = \arcsin(\sin\theta_h \cos\beta + \cos\theta_h \cos\varphi_h \sin\beta), \qquad -90° \leq \theta \leq 90° \tag{2f}$$

$$\varphi = \arccos\left(\frac{(-\sin\theta_h \sin\beta + \cos\theta_h \cos\varphi_h \cos\beta)}{\cos\theta}\right), \quad 0° \leq \varphi \leq 180° \text{ (see Note 1 in Annex 7)} \tag{2g}$$

3.5 in cases where the antennas in *recommends* 3.1 through 3.2 operate with a downward electrical tilt, tilted radiation patterns are also calculated by using θ_e of equation (1e) in *recommends* 2.5 instead of θ at the equations in *recommends* 3.1 and 3.2, respectively;

4 that, in the frequency range from 1 GHz to about 3 GHz, the following reference radiation patterns should be used in cases involving stations that use low-gain antennas with circular symmetry about the 3 dB beamwidth and with a main lobe antenna gain less than about 20 dBi:

4.1 the following equations should be used in the case of peak side-lobe patterns referred to in *considering* b) (see Annex 2 and Note 6):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \leq \theta < 1.08\,\varphi_3 \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \leq \theta < \varphi_1 \\ G_0 - 14 - 32\ \log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \leq \theta < \varphi_2 \\ -8 & \text{for } \varphi_2 \leq \theta \leq 180° \end{cases} \tag{4}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: the main lobe antenna gain (dBi)

θ: off-axis angle (degrees) ($0° \leq \theta \leq 180°$) (see Note 1 in Annex 2)

φ_3: the 3 dB beamwidth of the low-gain antenna (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ (degrees)

$\varphi_1 = 1.9\ \varphi_3$(degrees)

$\varphi_2 = \varphi_1 \times 10^{(G_0 - 6)/32}$ (degrees);

4.2 in the case of average side-lobe patterns referred to in *considering* c), the antenna pattern given in Recommendation ITU-R F.1245 should be used;

5 that the following Notes should be regarded as part of this Recommendation:

NOTE 1 – It is essential that every effort be made to utilize the actual antenna pattern in coordination studies and interference assessment.

NOTE 2 – The different values of parameter *k* in *recommends* 3.1.1.1 and 3.2.1.1 are derived taking into account peak envelopes and average side-lobe levels of a number of typical measured antenna patterns in the 1 to 6 GHz frequency range.

NOTE 3 – Measured results of a specially designed sectoral antenna for use around 20 GHz indicate the possibility of compliance with a more restrictive reference side-lobe radiation pattern. Further studies are required to develop such an optimized pattern.

NOTE 4 – In a case involving an antenna whose 3 dB beamwidth in the elevation plane is already known, it is recommended to use the known θ_3 as an input parameter.

NOTE 5 – As discussed in Annex 3, an exponential factor has been replaced by unity. As a result, the theoretical error introduced by this approximation will be less than 6% for 3 dB beamwidths in the elevation plane less than 45°.

NOTE 6 – The reference radiation pattern given in *recommends* 4.1 primarily applies in situations where the main lobe antenna gain is less than or equal to 20 dBi and the use of Recommendation ITU-R F.699 produces inadequate results. Further study is required to establish the full range of frequencies and gain over which the equations are valid.

NOTE 7 – Within the angle $90° < \psi \leq 180°$, the side-lobe patterns derived from equations (2b) to (2e) may result in conservative gains in particular in the azimuth angles. For cases involving significant interferences from such angles, another approach provided in Annex 8 may be used.

Annex 1

Reference radiation pattern for omnidirectional antennas as used in P-MP fixed wireless systems

1 Introduction

An omnidirectional antenna is frequently used for transmitting and receiving signals at central stations of P-MP fixed wireless systems. Studies involving sharing between these types of fixed-wireless systems and space service systems in the 2 GHz bands have used the reference radiation pattern described here.

2 Analysis

The reference radiation pattern is based on the following assumptions concerning the omnidirectional antenna:

– that the antenna is an *n*-element linear array radiating in the broadside mode;

– the elements of the array are assumed to be dipoles;

– the array elements are spaced $3\lambda/4$.

The 3 dB beamwidth θ_3 of the array in the elevation plane is related to the directivity D by (see Annex 3 for the definition of D):

$$D = 10 \log\left[191.0 \sqrt{0.818 + 1/\theta_3} - 172.4\right] \quad \text{dBi} \tag{5a}$$

Equation (5a) may be solved for θ_3 when the directivity is known:

$$\theta_3 = \frac{1}{\alpha^2 - 0.818} \tag{5b}$$

$$\alpha = \frac{10^{0.1D} + 172.4}{191.0} \tag{5c}$$

The relationship between the 3 dB beamwidth in the elevation plane and the directivity was derived on the assumption that the radiation pattern in the elevation plane was adequately approximated by:

$$f(\theta) = \cos^m(\theta)$$

where m is an arbitrary parameter used to relate the 3 dB beamwidth and the radiation pattern in the elevation plane. Using this approximation, the directivity was obtained by integrating the pattern over the elevation and azimuth planes.

The intensity of the far-field of a linear array is given by:

$$E_T(\theta) = E_e(\theta) \cdot AF(\theta) \tag{6}$$

where:

$E_T(\theta)$: total E-field at an angle of θ normal to the axis of the array

$E_e(\theta)$: *E*-field at an angle of θ normal to the axis of the array caused by a single array element

$AF(\theta)$: array factor at an angle θ normal to the axis of the array.

The normalized E-field of a dipole element is:

$$E_e(\theta) = \cos(\theta) \tag{7}$$

The array factor is:

$$AF_N = \frac{1}{N}\left[\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \tag{8}$$

where:

N: number of elements in the array

$\frac{\psi}{2} = \frac{1}{2}\left[2\pi\frac{d}{\lambda}\sin\theta\right]$

d: spacing of the radiators

λ: wavelength.

The following procedure has been used to estimate the number of elements *N* in the array. It is assumed that the maximum gain of the array is identical to the directivity of the array.

– Given the maximum gain of the omnidirectional antenna in the elevation plane, compute the 3 dB beamwidth, θ_3, using equations (5b) and (5c);

– Ignore the small reduction in off-axis gain caused by the dipole element, and note that the array factor, AF_N, evaluates to 0.707 (–3 dB) when $N\frac{\psi}{2} = 1.396$; and

– *N* is then determined as the integer value of:

$$N = \left|\frac{2 \times 1.3916}{2\pi\frac{d}{\lambda}\sin\left(\frac{\theta_3}{2}\right)}\right| \tag{9}$$

where $|x|$ means the maximum integer value not exceeding *x*.

The normalized off-axis discrimination ΔD is given by:

$$\Delta D = 20\log\left[\left|AF_N \times \cos(\theta)\right|\right] \qquad \text{dB} \tag{10}$$

Equation (10) has been evaluated as a function of the off-axis angle (i.e., the elevation angle) for several values of maximum gain. For values in the range of 8 dBi to 13 dBi, it has been found that the envelope of the radiation pattern in the elevation plane may be adequately approximated by the following equations:

$$G(\theta) = \max\left[G_1(\theta), G_2(\theta)\right] \tag{11a}$$

$$G_1(\theta) = G_0 - 12\left(\frac{\theta}{\theta_3}\right)^2 \qquad \text{dBi} \qquad (11b)$$

$$G_2(\theta) = G_0 - 12 + 10\log\left[\left(\max\left\{\frac{|\theta|}{\theta_3}, 1\right\}\right)^{-1.5} + k\right] \qquad \text{dBi} \qquad (11c)$$

k is a parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance.

Figures 1 to 4 compare the reference radiation envelopes with the theoretical antenna patterns generated from equation (11), for gains from 8 dBi to 13 dBi, using a factor of $k = 0$. Figures 5 to 8 compare the reference radiation envelopes with actual measured antenna patterns using a factor of $k = 0$. In Figs 7 and 8, it can be seen that the side lobes are about 15 dB or more below the level of the main lobe, allowing for a small percentage of side-lobe peaks which might exceed this value. However practical factors such as the use of electrical downtilt, pattern degradations at band-edges and production variations would further increase the side lobes to about 10 dB below the main lobe in actual field installations. The k factor, mentioned above, in equation (11), is intended to characterize this variation in side-lobe levels. Figures 9 and 10 provide a comparison of a 10 dBi and a 13 dBi gain antenna, at 2.4 GHz, with the reference radiation pattern envelope, using $k = 0.5$. A factor of $k = 0.5$ represents side-lobe levels about 15 dB below the main-lobe peak. However, to account for increases in side-lobe levels which may be found in field installations, for typical antennas a factor of $k = 0.7$ should be used, representing side-lobe levels about 13.5 dB below the level of the main lobe. Finally, Figs 11 and 12 illustrate the effect on elevation patterns of using various values of k.

FIGURE 1

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern
G_0 = 10 dBi, k = 0

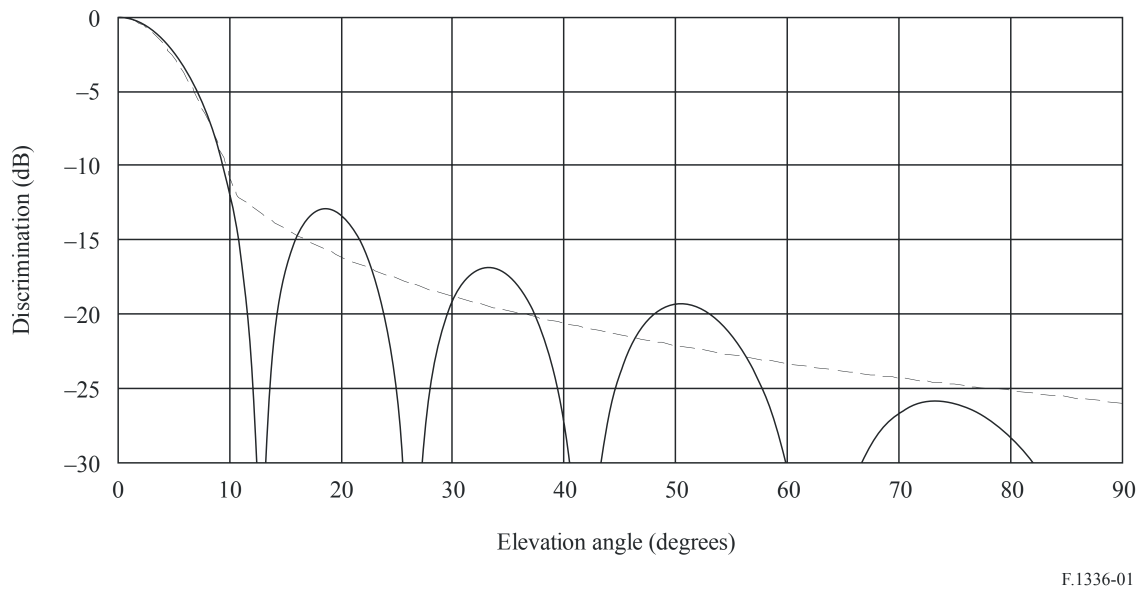

FIGURE 2

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern
G_0 = 11 dBi, k = 0

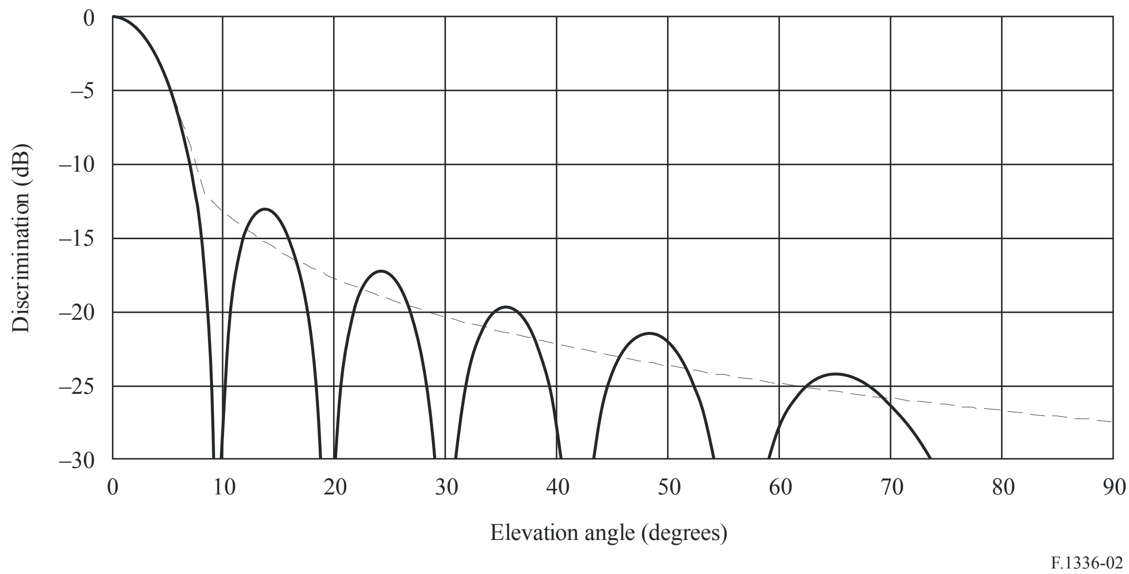

FIGURE 3

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern
G_0 = 12 dBi, k = 0

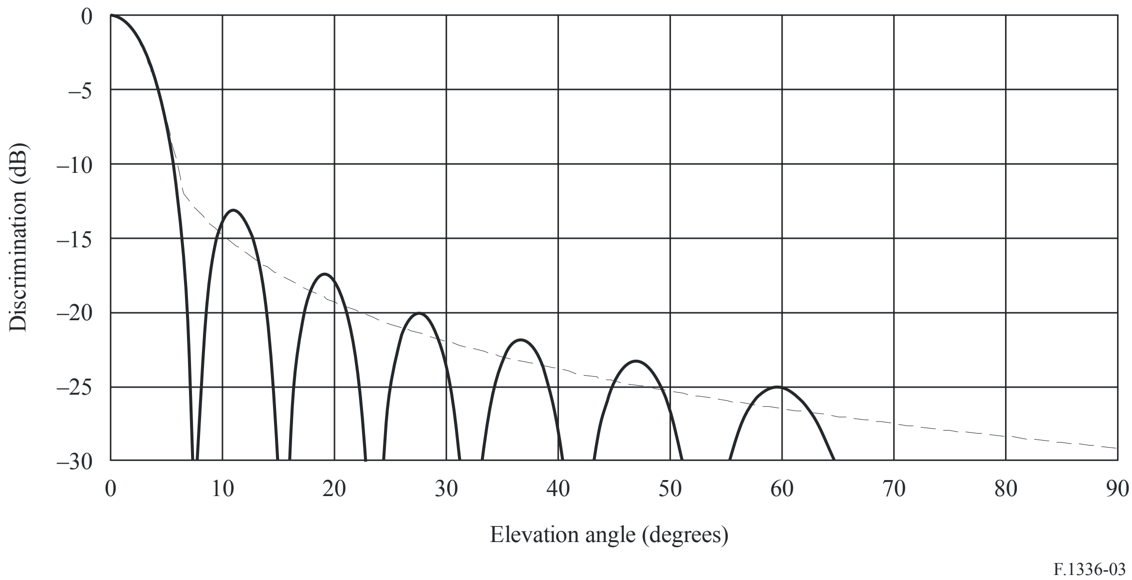

FIGURE 4

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern
G_0 = 13 dBi, k = 0

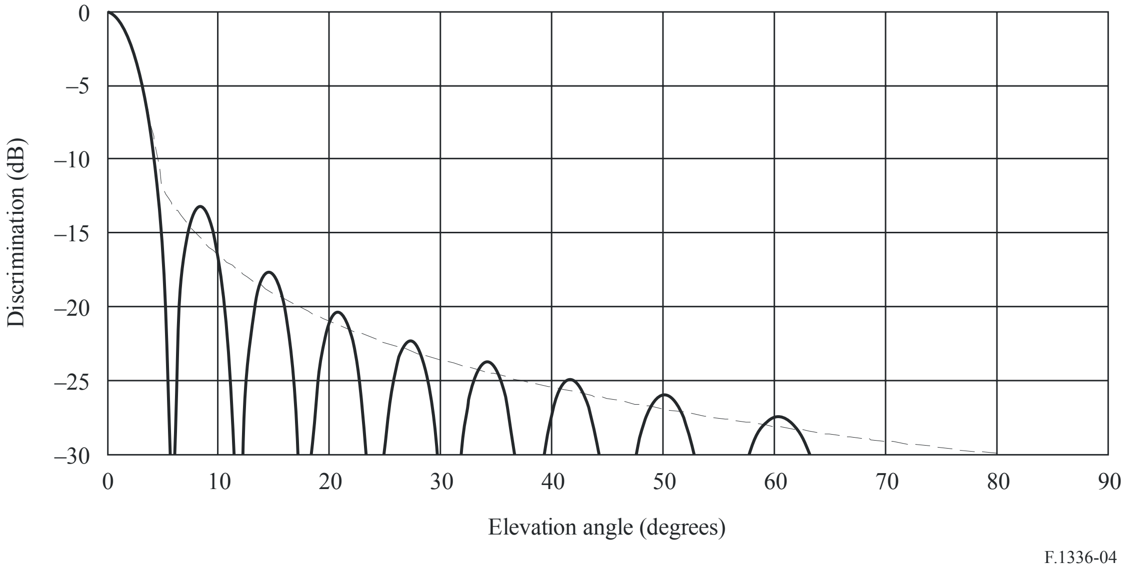

FIGURE 5

Comparison of measured pattern and reference radiation pattern envelope for an omnidirectional antenna with 11 dBi gain and operating in the band 928-944 MHz, k = 0

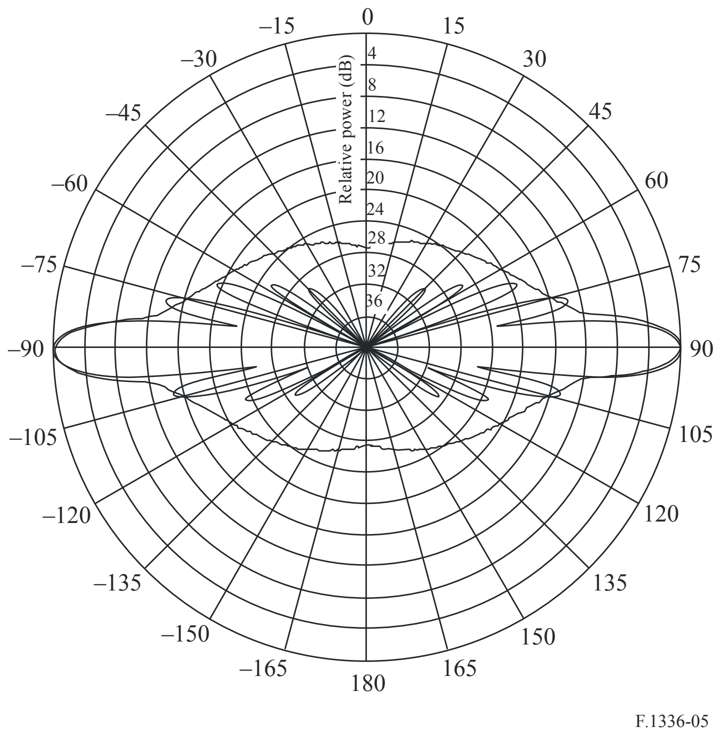

FIGURE 6

Comparison of measured pattern and the reference radiation pattern envelope for an omnidirectional antenna with 8 dBi gain and operating in the band 1 850-1 990 MHz, *k* = 0

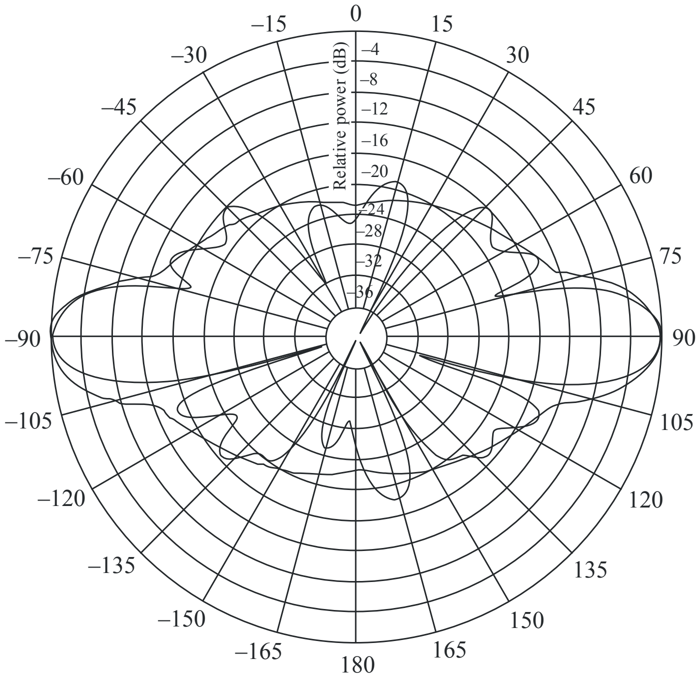

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FIGURE 7

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 10 dBi gain and operating in the 1.4 GHz band

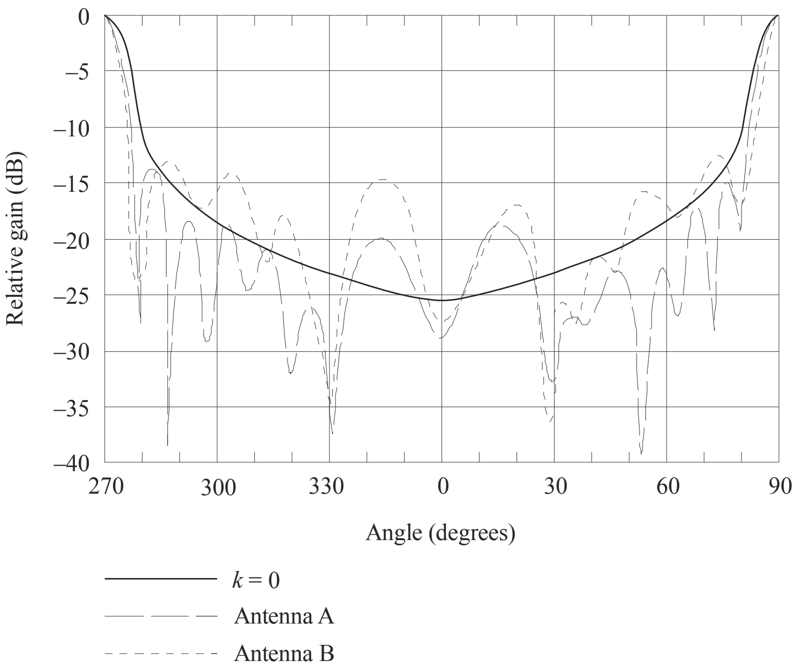

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FIGURE 8

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 13 dBi gain and operating in the 1.4 GHz band

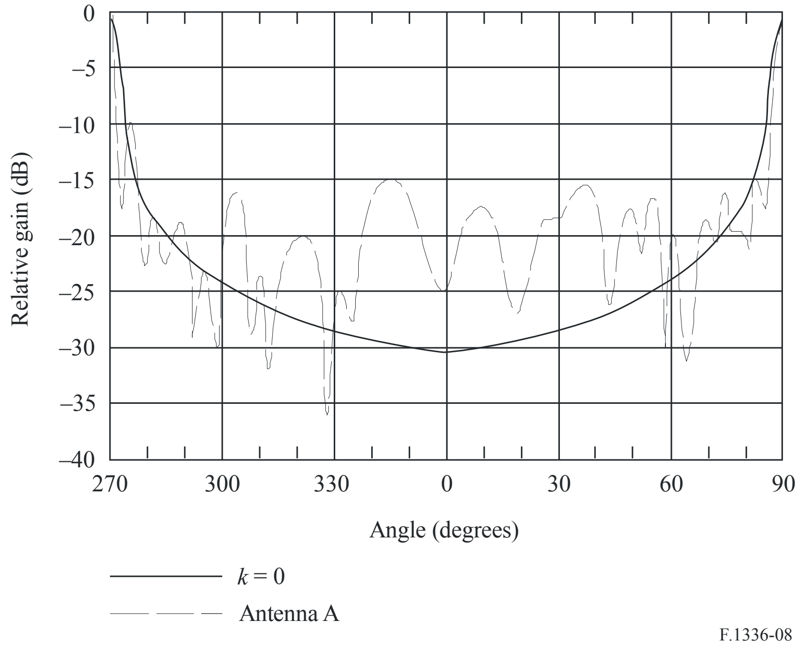


FIGURE 9

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 10 dBi gain and operating in the 2.4 GHz band

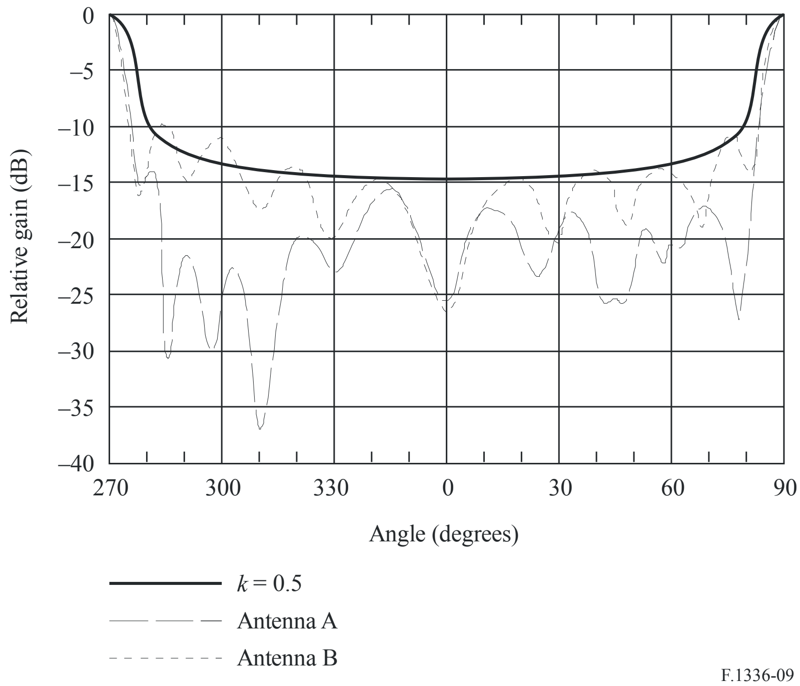


FIGURE 10

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 13 dBi gain and operating in the 2.4 GHz band

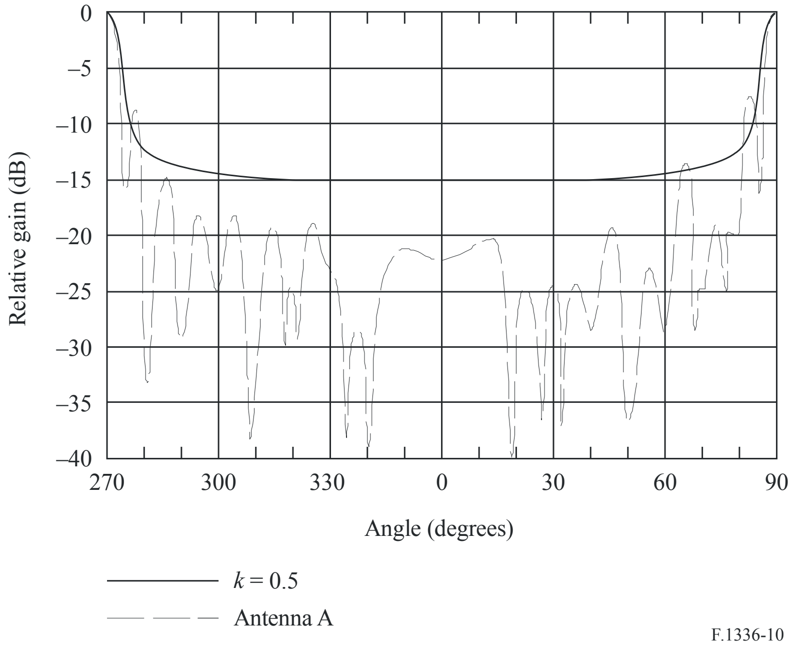


FIGURE 11

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 10 dBi gain

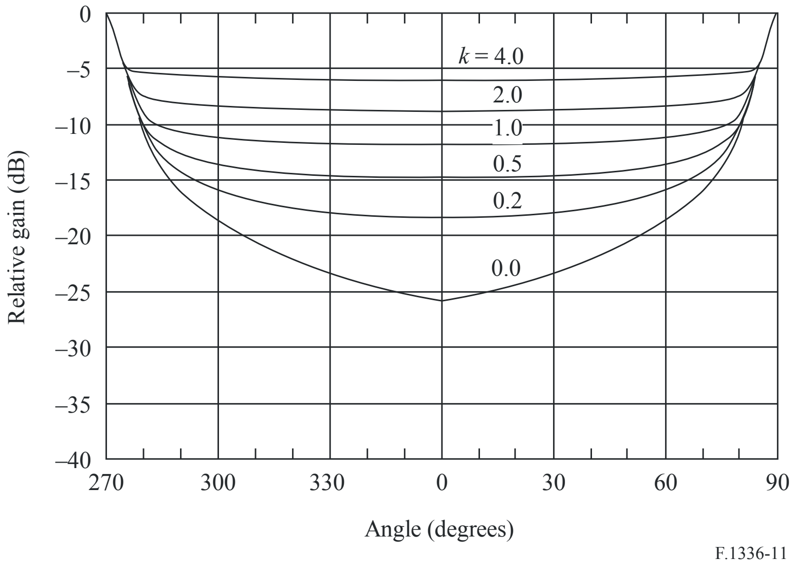
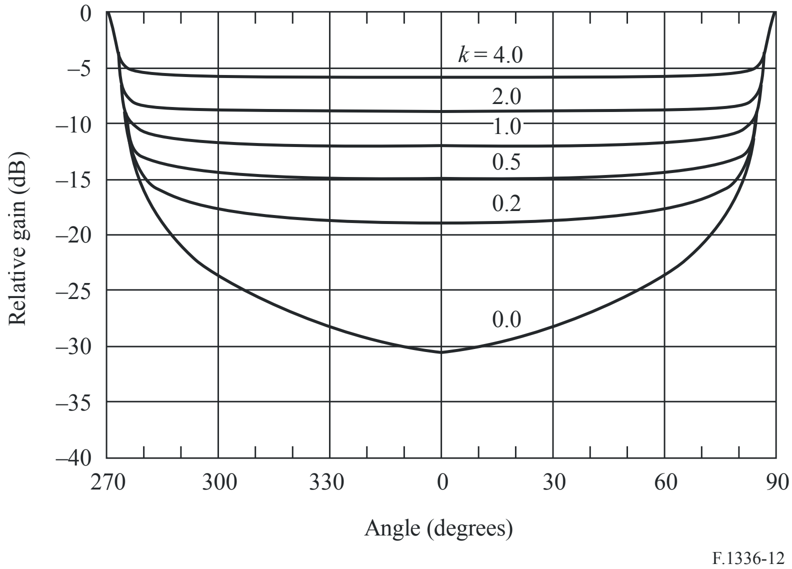


FIGURE 12

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 13 dBi gain



3 Summary, conclusions and further analyses

A reference radiation pattern has been presented for omnidirectional antennas exhibiting a gain between 8 dBi and 13 dBi. The reference radiation pattern has been derived on the basis of theoretical considerations of the radiation pattern of a collinear array of dipoles. The proposed pattern has been shown to adequately represent the theoretical patterns and measured patterns over the range from 8 dBi to 13 dBi. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate especially with regard to antennas operating in frequency bands above 3 GHz.

Annex 2

Reference radiation pattern for low-gain circularly symmetric subscriber antennas as used in P-MP fixed wireless systems in the 1-3 GHz range

1 Introduction

An antenna with relatively low gain is frequently used for transmitting and receiving signals at the out-stations or in sectors of central stations of P-MP fixed wireless systems. These antennas may exhibit a gain of the order of 20 dBi or less. It has been found that using the reference radiation pattern given in Recommendation ITU-R F.699 for these relatively low-gain antennas will result in an overestimate of the gain for relatively large off-axis angles. As a consequence, the amount of interference caused to other systems and the amount of interference received from other systems at relatively large off-axis angles will likely be substantially overestimated if the pattern of Recommendation ITU-R F.699 is used.

2 Analysis

The reference radiation pattern for a subscriber antenna is based on the following assumptions:

– that the directivity of the antenna is less than about 20 dBi;

– that the antenna pattern exhibits circularly symmetric about the main lobe;

– that the main-lobe gain is equal to the directivity.

The proposed reference radiation pattern is given by:

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \le \theta < 1.08\,\varphi_3 \quad (12) \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \le \theta < \varphi_1 \quad (13) \\ G_0 - 14 - 32\log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \le \theta < \varphi_2 \quad (14) \\ -8 & \text{for } \varphi_2 \le \theta \le 180° \quad (15) \end{cases}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: maximum on-axis gain (dBi)

θ: off-axis angle (degrees) ($0° \le \theta \le 180°$) (see Note 1)

φ_3: the 3 dB beamwidth (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ degrees

φ_1 $= 1.9\,\varphi_3$ degrees

φ_2 $= \varphi_1 \times 10^{(G_0 - 6)/32}$ degrees.

NOTE 1 – The angle θ in this Annex 2 is defined differently from other parts in this Recommendation.

3 Summary and conclusions

A reference radiation pattern has been presented for low-gain subscriber antennas exhibiting a gain of less than or equal to 20 dBi. The reference radiation pattern has been derived on the basis of limited data on the radiation patterns of flat plate array antennas considered for use in a local access P-MP system operating in the 2 GHz bands. The proposed pattern has been shown to more accurately represent the actual pattern than the pattern given in Recommendation ITU-R F.699. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate and to compare the reference radiation pattern to measured patterns.

Annex 3

Relationship between gain and beamwidth for omnidirectional and sectoral antennas

1 Introduction

The purpose of this Annex is to derive the relationship between the gain of omnidirectional and sectoral antennas and their beamwidth in the azimuthal and elevation planes. Section 2 is an analysis of the directivity of omnidirectional and sectoral antennas assuming two different radiation intensity functions in the azimuthal plane. For both cases, the radiation intensity in the elevation plane was assumed to be an exponential function. Section 3 provides a comparison between the gain-beamwidth results obtained using the methods of Section 2 and results contained in the previous version of this Recommendation for omnidirectional antennas. Section 4 summarizes the results, proposes a provisional equation for gain-beamwidth for omnidirectional and sectoral antennas, and suggests areas for further study.

2 Analysis

The far-field pattern of the sectoral antenna in the elevation plane is assumed to conform to an exponential function, whereas the far-field pattern in the azimuth plane is assumed to conform to either a rectangular function or an exponential function. With these assumptions, the directivity, D, of the sectoral antenna may be derived from the following formulation in (spherical coordinates):

$$D = \frac{U_M}{U_0} \tag{16}$$

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} F(\varphi)\, F(\theta) \cos(\theta)\, d\theta\, d\varphi \tag{17}$$

where:

- U_M: maximum radiation intensity
- U_0: radiation intensity of an isotropic source
- φ: angle in the azimuthal plane
- θ: angle in the elevation plane
- $F(\varphi)$: radiation intensity in the azimuthal plane
- $F(\theta)$: radiation intensity in the elevation plane.

The directivity of omnidirectional and sector antennas is evaluated in the following sub-sections assuming the radiation intensity in the azimuthal plane is either a rectangular function or an exponential function.

2.1 Rectangular sectoral radiation intensity

Rectangular sectoral radiation intensity function, $F(\varphi)$, is assumed to be:

$$F(\varphi) = U\left(\frac{\varphi_s}{2} - |\varphi|\right) \tag{18}$$

where:

φ_s : beamwidth of the sector,

$$U(x) = 1 \quad \text{for} \quad x \geq 0$$
$$U(x) = 0 \quad \text{for} \quad x < 0 \tag{19}$$

For either rectangular or exponential sectoral radiation intensity functions, it is assumed that the radiation intensity in the elevation plane is given by:

$$F(\theta) = \mathrm{e}^{-a^2\theta^2} \tag{20}$$

where:

$$a^2 = -\ln(0.5) \times \left(\frac{2}{\theta_3}\right)^2 = \frac{2.773}{\theta_3^2} \tag{21}$$

θ_3 : 3 dB beamwidth of the antenna in the elevation plane (degrees).

Substituting equations (18) and (20) into equation (17) results in:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} U\left(\frac{\varphi_s}{2} - |\varphi|\right) \mathrm{d}\varphi \int_{-\pi/2}^{\pi/2} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta \tag{22}$$

This double integral may be solved as the product of two independent integrals. The integral over φ is evaluated in a straightforward way. However, evaluating the integral over θ is somewhat more difficult. The integral over θ could be evaluated numerically with the results either tabulated or a polynomial fitted to the data. However, it is noted that if the limits of integration are changed to ±∞, the integral over θ is given in closed-form by:

$$\int_{-\pi/2}^{\pi/2} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta \approx \int_{-\infty}^{\infty} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta = \frac{1}{a}\sqrt{\pi}\, \mathrm{e}^{-1/4a^2} \tag{23}$$

This is a rather simple and flexible formulation that, depending on its accuracy, could be quite useful in evaluating the directivity of sector antennas as well as omnidirectional antennas.

The accuracy with which the infinite integral approximates the finite integral has been evaluated. The finite integral, i.e., the integral on the left-hand side of equation (23), has been evaluated for several values of 3 dB beamwidth using the 24 point Gaussian Quadrature method and compared with the value obtained using the formula corresponding to the infinite integral on the right-hand side of equation (23). (Actually, because of its symmetry, the finite integral has been numerically evaluated over the range 0 to π/2 and the result doubled.) The results for a range of example values of the 3 dB beamwidth in the elevation plane are shown in Table 1. The Table shows that for a 3 dB beamwidth of 45°, the difference between the values produced by the finite integral and the infinite integral approximation is less than 0.03%. At 25° and below, the error is essentially zero. Equation (22) is now readily evaluated:

$$U_0 = \frac{\varphi_s \theta_3}{4\pi} \sqrt{\frac{\pi}{2.773}} \times e^{\frac{\theta_3^2}{11.09}} \qquad (24)$$

TABLE 1

Relative accuracy of the infinite integral in equation (23) in the evaluation of the average radiation intensity

3 dB beamwidth in the elevation plane (degrees)	Finite integral	Infinite integral	Relative error (%)
45	1.116449558	1.116116449	0.0298
25	0.67747088	0.67747088	0.0000
20	0.549744213	0.549744213	0.0000
15	0.416896869	0.416896869	0.0000
10	0.280137168	0.280137168	0.0000
5	0.140734555	0.140734558	0.0000

From equations (18) and (20), $U_M = 1$. Substituting these values and equation (24) into equation (16) yields the directivity of a sector antenna given the beamwidth in the elevation and azimuthal planes:

$$D = \frac{11.805}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{11.09}} \qquad (25)$$

where the angles are given in radians. When the angles are expressed in degrees, equation (25) becomes:

$$D = \frac{38\,750}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36\,400}} \qquad (26)$$

Note that for an omnidirectional antenna, equation (26) reduces to:

$$D = \frac{107.64}{\theta_3} e^{\frac{\theta_3^2}{36\,400}} \qquad (27a)$$

If it is assumed that the radiation efficiency is 100% and that the antenna losses are negligible, then the gain and the directivity of the omnidirectional antenna are identical. Additionally, for omnidirectional antennas with a 3 dB beamwidth less than about 45°, the relationship between the gain and the 3 dB beamwidth in the elevation plane may be simplified by setting the exponential factor equal to unity. The resulting error is less than 6%.

$$G_0 \approx \frac{107.64}{\theta_3} \qquad (27b)$$

2.2 Exponential sectoral radiation intensity

The second case considered for the sectoral radiation intensity is that of an exponential function. Specifically:

$$F(\varphi) = \mathrm{e}^{-b^2\varphi^2} \tag{28}$$

where:

$$b^2 = -\ln(0.5) \times \left(\frac{2}{\varphi_s}\right)^2 \tag{29}$$

and φ_s is the 3 dB beamwidth of the sector.

Substituting equations (20) and (28) into equation (14), changing the limits of integration so that the finite integrals become infinite integrals, integrating and then substituting the result into equation (16) yields the following approximation:

$$D = \frac{11.09}{\varphi_s \theta_3}\,\mathrm{e}^{\frac{\theta_3^2}{11.09}} \tag{30}$$

where the angles are as defined previously and are expressed in radians. Converting the angles to degrees transforms equation (30) into:

$$D = \frac{36\,400}{\varphi_s \theta_3}\,\mathrm{e}^{\frac{\theta_3^2}{36\,400}} \tag{31}$$

Comparing equations (26) and (31), it is seen that the difference between the directivity computed using either of the equations is less than 0.3 dB.

The results given by equation (31) should be compared to a number of measured patterns to determine the inherent effect of the radiation efficiency of the antenna and other losses on the coefficient. At this time, only two sets of measurements are available for sectoral antennas designed to operate in the 25.25 GHz to 29.5 GHz band. Measured patterns in the azimuthal and elevation planes are given, respectively, in Figs 13 and 14 for one set of antennas and Figs 15 and 16, respectively, for the second set. From Figs 13 and 14, the 3 dB beamwidth in the azimuthal plane is 90° and the 3 dB beamwidth in the elevation plane is 2.5°. From equation (31), the directivity is 22.1 dB. This is to be compared with a measured gain of 20.5-21.4 dBi for the antenna over the range 25.5-29.5 GHz. Assuming the gain G_0 of the antenna in the band around 28 GHz is 0.7 dB less than its directivity, and the exponential factor is replaced by unity which introduces an increasing error with increasing beamwidth. The error reaches 6% at 45°. A larger beamwidth leads to a larger error. Based on these considerations, the semi-empirical relationship between the gain and the beamwidth of a sectoral antenna is given by:

$$G_0 \approx \frac{31\,000}{\varphi_s \theta_3} \tag{32a}$$

Similarly, from Figs 15 and 16, the semi-empirical relationship between the gain and the beamwidth of that sectoral antenna is:

$$G_0 \approx \frac{34\,000}{\varphi_s \theta_3} \tag{32b}$$

FIGURE 13

Measured pattern in the azimuthal plane of a 90° sector antenna. Pattern measured over the band 27.5 GHz to 29.5 GHz. The band drawn cross marks on the left side of the Figure correspond to valutes obtained from equation (28) (when expressed in dB) for an assumed 3 dB beamwidth of 90° in the azimuthal plane

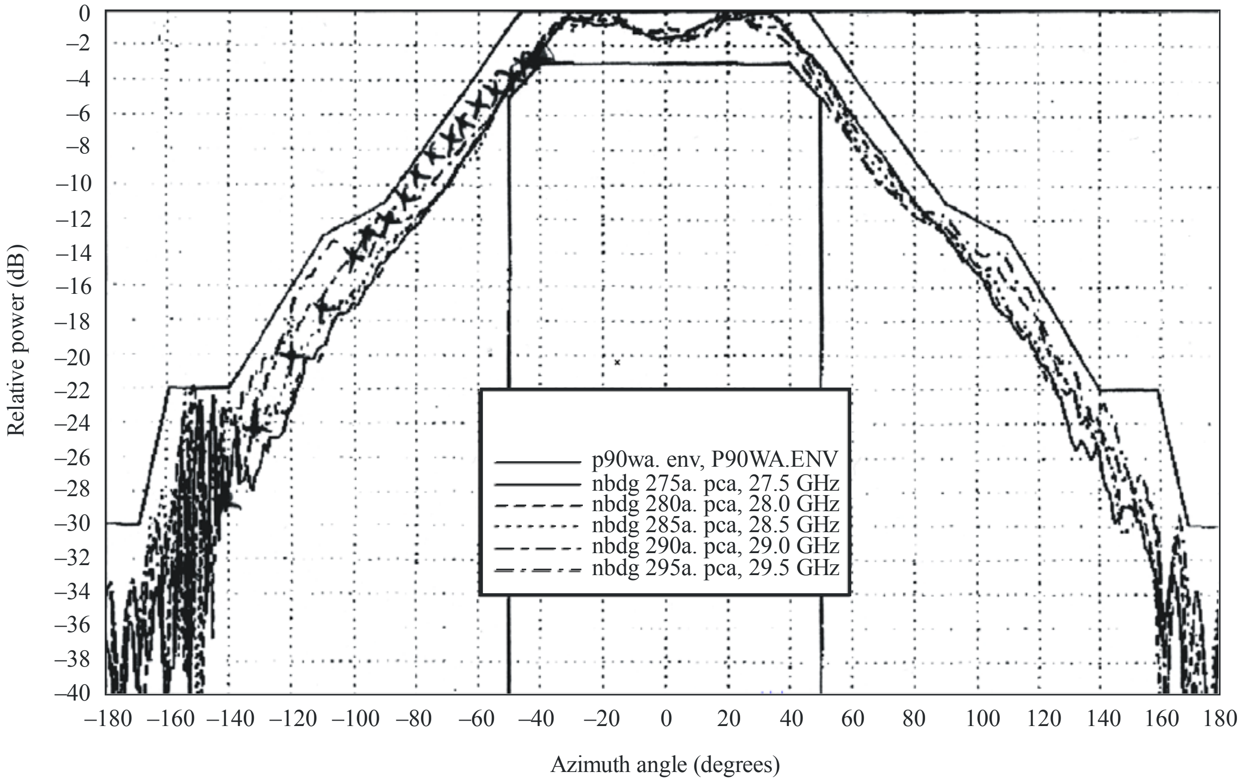


FIGURE 14

Measured pattern in the azimuthal plane of a 90° sector antenna.
Pattern measured over the band 27.5 GHz to 29.5 GHz

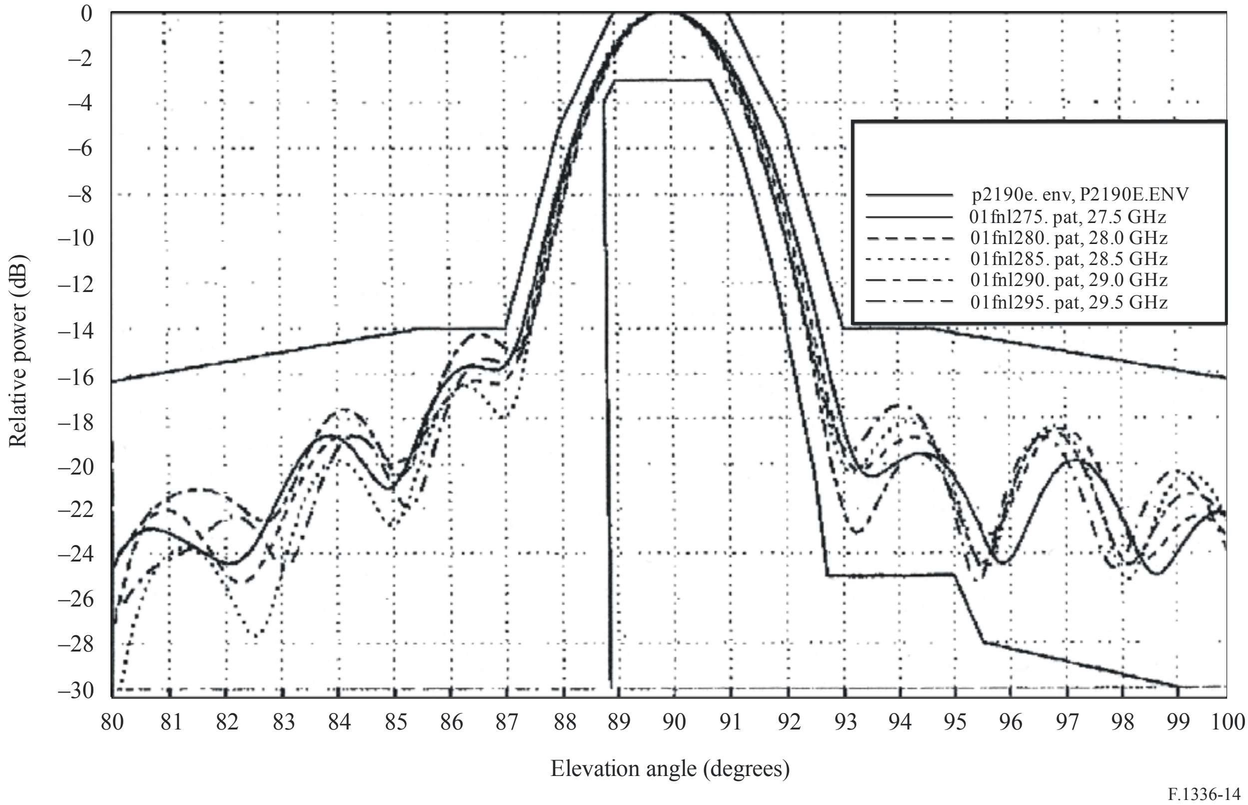


FIGURE 15

Azimuth pattern of typical 90° sectoral antenna (V-polarization)
15 dBi half-value angle: 90° (horn type antenna at 26 GHz)

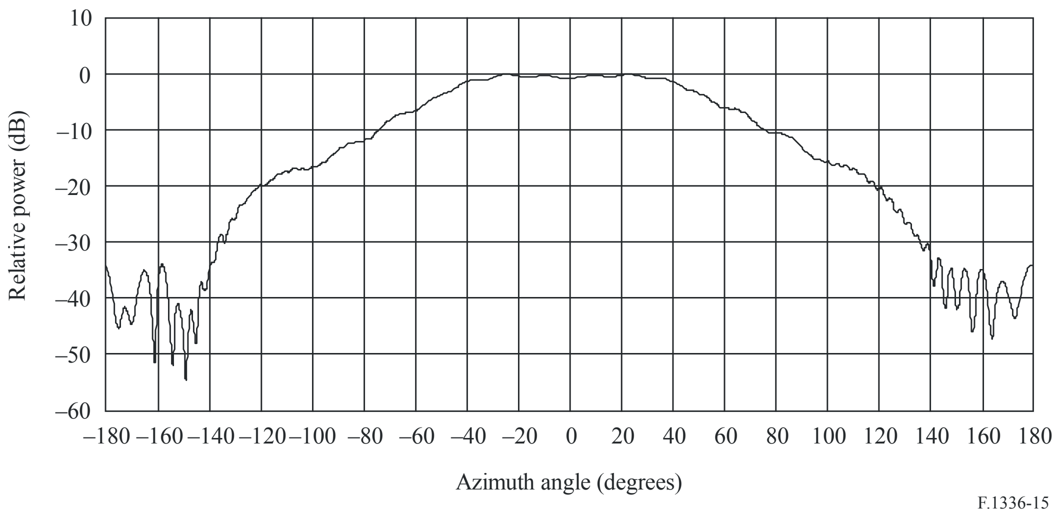
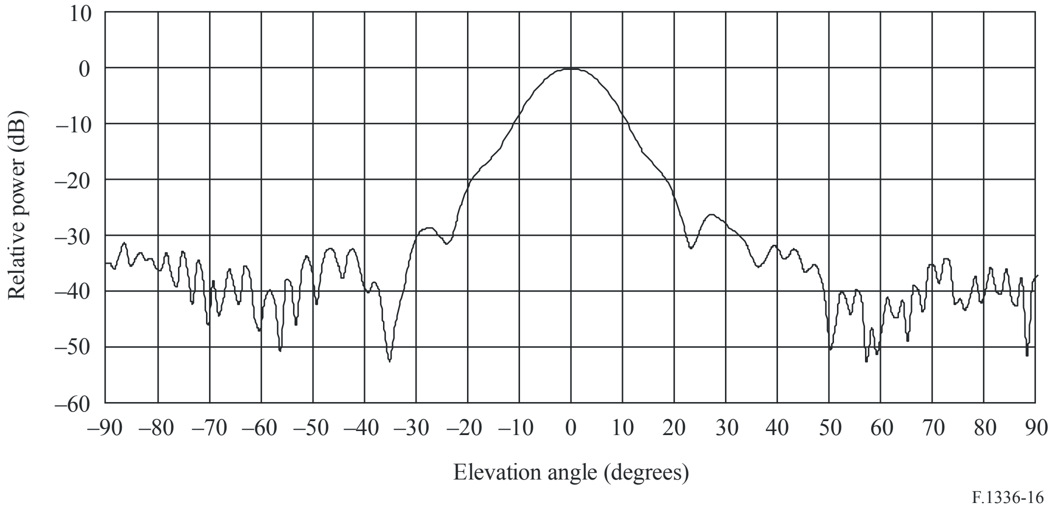


FIGURE 16

Elevation pattern of typical 90° sectoral antenna (V-polarization)
15 dBi half-value angle: 12° (horn type antenna at 26 GHz)



3 Comparison with previous results for omnidirectional antennas

The purpose of this section is to compare the results obtained for an omnidirectional antenna given by equation (27) with previous results reported in and summarized in Annex 1 of this Recommendation.

The radiation intensity in the elevation plane used in for an omnidirectional antenna was of the form:

$$F(\theta) = \cos^{2N} \theta \tag{33}$$

Substituting equation (33) into equation (17), and assuming that $F(\varphi) = 1$, yields:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos^{2N}(\theta) \cos(\theta)\, d\theta\, d\varphi \tag{34}$$

This double integral evaluates to:

$$U_0 = \frac{(2N)!!}{(2N+1)!!} \tag{35}$$

where $(2N)!!$ is the double factorial defined as $(2 \cdot 4 \cdot 6 ... (2N))$, and $(2N + 1)!!$ is also a double factorial defined as $(1 \cdot 3 \cdot 5 ... (2N + 1))$.

Thus, the directivity becomes:

$$D = \frac{(2N+1)!!}{(2N)!!} \tag{36}$$

The 3 dB beamwidth in the elevation plane is given by:

$$\theta_3 = 2 \cos^{-1}\left(0.5^{1/2N}\right) \tag{37}$$

A comparison between the directivity computed using the assumptions and methods embodied in equation (27) and those used in the derivation of equations (36) and (37) is given in Table 2. It is shown that results obtained using equation (27) compare favourably with the results using equations (36) and (37). In all cases equation (27) slightly underestimates the directivity obtained using equations (36) and (37). The relative error (%) of the estimates, when expressed in dB, is greatest for a 3 dB beamwidth in the elevation plane of 65°, amounting to –2.27%. The error (dB) for this case, expressed in dB, is –0.062 dB. For 3 dB beamwidth angles less than 65°, the relative error (%) and the error (dB), are monotonically decreasing functions as the 3 dB beamwidth decreases. For a 16° 3 dB beamwidth, the relative error (%) is about –0.01% and the error (dB) is less than about –0.0085 dB. An evaluation similar to that shown in Table 2 for values of 2*N* up to 10 000 (corresponds to a 3 dB beamwidth of 1.35° and a directivity of 19.02 dB) confirms that the results of the two approaches converge.

TABLE 2

Comparison of the directivity of omnidirectional antennas computed using equation (27a) with the directivity computed using equations (36) and (37)

2*N*	**θ_3 (degrees) (equation (37))**	**Directivity (dB) (equation (36))**	**Directivity (dB) (equation (27a))**	**Relative error (%)**	**Error (dB)**
2	90.0000	1.7609	1.7437	–0.98	–0.0172
4	65.5302	2.7300	2.6677	–2.28	–0.0623
6	54.0272	3.3995	3.3419	–1.69	–0.0576
8	47.0161	3.9110	3.8610	–1.28	–0.0500
10	42.1747	4.3249	4.2814	–1.01	–0.0435
12	38.5746	4.6726	4.6343	–0.82	–0.0383
14	35.7624	4.9722	4.9381	–0.69	–0.0341
16	33.4873	5.2355	5.2047	–0.59	–0.0307
18	31.5975	5.4703	5.4423	–0.51	–0.0280
20	29.9953	5.6822	5.6565	–0.45	–0.0256
22	28.6145	5.8752	5.8516	–0.40	–0.0237
24	27.4083	6.0525	6.0305	–0.36	–0.0220
26	26.3428	6.2164	6.1959	–0.33	–0.0205
28	25.3927	6.3688	6.3496	–0.30	–0.0192
30	24.5384	6.5112	6.4931	–0.28	–0.0181
32	23.7649	6.6449	6.6278	–0.26	–0.0171
34	23.0603	6.7708	6.7545	–0.24	–0.0162
36	22.4148	6.8897	6.8743	–0.22	–0.0154
38	21.8206	7.0026	6.9879	–0.21	–0.0147
40	21.2714	7.1098	7.0958	–0.20	–0.0140
42	20.7616	7.2120	7.1986	–0.19	–0.0134
44	20.2868	7.3096	7.2967	–0.18	–0.0129

TABLE 2 (*end*)

2*N*	θ_3 (degrees) (equation (37))	Directivity (dB) (equation (36))	Directivity (dB) (equation (27a))	Relative error (%)	Error (dB)
46	19.8431	7.4030	7.3906	–0.17	–0.0124
48	19.4274	7.4925	7.4806	–0.16	–0.0119
50	19.0367	7.5785	7.5671	–0.15	–0.0115
52	18.6687	7.6613	7.6502	–0.14	–0.0111
54	18.3212	7.7410	7.7302	–0.14	–0.0107
56	17.9924	7.8178	7.8075	–0.13	–0.0104
58	17.6808	7.8921	7.8820	–0.13	–0.0100
60	17.3847	7.9638	7.9541	–0.12	–0.0097
62	17.1031	8.0333	8.0239	–0.12	–0.0094
64	16.8347	8.1007	8.0915	–0.11	–0.0092
66	16.5786	8.1660	8.1571	–0.11	–0.0089
68	16.3338	8.2294	8.2207	–0.11	–0.0087
70	16.0996	8.2910	8.2825	–0.10	–0.0085
72	15.8751	8.3509	8.3426	–0.10	–0.0083
74	15.6598	8.4092	8.4011	–0.10	–0.0081

4 Summary and conclusions

Equations have been developed that permit easy calculation of the directivity and the relationship between the beamwidth and gain of omnidirectional and sectoral antennas as used in P-MP radio-relay systems. It is proposed to use the following equations to determine the directivity of sectoral antennas:

$$D = \frac{k}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36\,400}} \tag{38}$$

where:

$$k = 38\,750 \qquad \text{for} \quad \varphi_s > 120° \tag{39}$$

$$k = 36\,400 \qquad \text{for} \quad \varphi_s \leq 120°$$

and φ_s = 3 dB beamwidth of the sectoral antenna in the azimuthal plane (degrees) for an assumed exponential radiation intensity in azimuth and θ_3 is the 3 dB beamwidth of the sectoral antenna in the elevation plane (degrees).

For omnidirectional antennas, it is proposed to use the following simplified equation to determine the 3 dB beamwidth in the elevation plane given the gain in dBi (see equation (27b)):

$$\theta_3 \approx 107.6 \times 10^{-0.1\,G_0}$$

It is proposed to use, on a provisional basis, the following semi-empirical equation relating the gain of a sectoral antenna (dBi) to the 3 dB beamwidths in the elevation plane and the azimuthal plane, where the sector is on the order of 120° or less and the 3 dB beamwidth in the elevation plane is less than about 45° (see equation (32a)):

$$\theta_3 \approx \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_s}$$

Further study is required to determine how to handle the transition region implicit in equation (39), and to determine the accuracy of these approximations as they apply to measured patterns of sectoral and omnidirectional antennas designed for use in P-MP radio-relay systems for bands in the range from 1 GHz to about 70 GHz.

Annex 4

Procedure for determining the gain of a sectoral antenna at an arbitrary off-axis angle specified by an azimuth angle and an elevation angle referenced to the boresight of the antenna

1 Analysis

The basic geometry for determining the gain of a sectoral antenna at an arbitrary off-axis angle is shown in Fig. 17. It is assumed that the antenna is located at the centre of the spherical coordinate system; the direction of maximum radiation is along the x-axis; the x-y plane is the local horizontal plane; the elevation plane contains the z-axis; and, u_0 is a unit vector whose direction is used to determine the gain of the sectoral antenna. In analysing sectoral antennas in particular, it is important to observe the range of validity of the azimuth and elevation angles:

$$-180° \leq \varphi \leq +180°$$

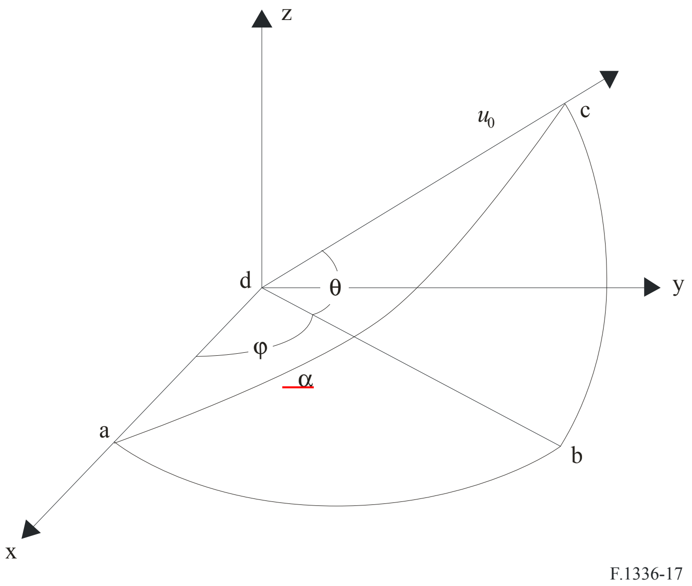
$$-90° \leq \theta \leq +90°$$

Also observe that the range of validity of the angle α is

$$-90° \leq \alpha \leq +90°$$

FIGURE 17

Determining the off-boresight angle given the azimuth and elevation angle of interest



The two fundamental assumptions regarding this procedure are that:

– the –3 dB gain contour of the far-field pattern when plotted in two-dimensions as a function of the azimuth and elevation angles will be an ellipse as shown in Fig. 18; and

– the gain of the sectoral antenna at an arbitrary off-axis angle is a function of the 3 dB beamwidth and the beamwidth of the antenna when measured in the plane containing the x-axis and the unit vector u_0 (see Fig. 17).

Given the 3 dB beamwidth (degrees) of the sectoral antenna in the azimuth and elevation planes, φ_3 and θ_3, the numerical value of the boresight gain is given, on a provisional basis, by (see *recommends* 3.3 and equation (32a)).

$$G_0 = \frac{31\,000}{\varphi_3 \theta_3} \tag{40}$$

The first step in evaluating the gain of the sectoral antenna at an arbitrary off-axis angle, φ and θ, is to determine the value of α. Referring to Fig. 17 and recognizing that abc is a right-spherical triangle, α is given by:

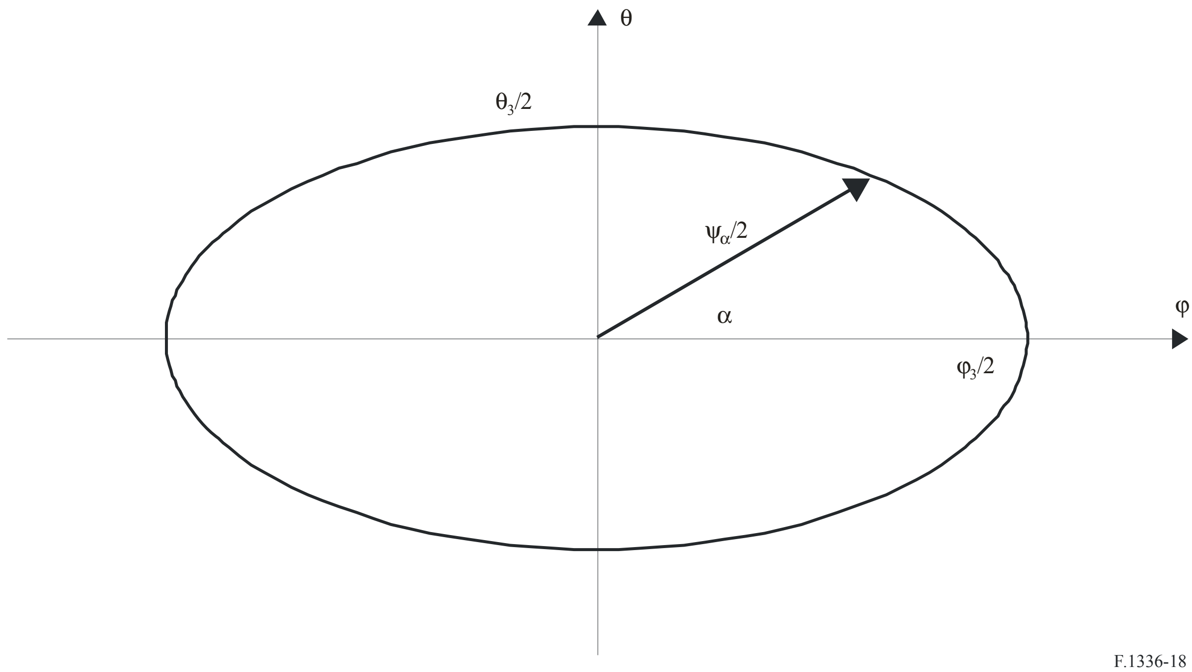
$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right), \quad -90° \leq \alpha \leq +90° \tag{41a}$$

and the off-axis angle in the plane adc is given by:

$$\psi = \arccos(\cos\varphi \cos\theta), \quad 0° \leq \psi \leq 180° \tag{41b}$$

FIGURE 18

Determination of the 3 dB beamwidth of an elliptical beam at an arbitrary inclination angle α



Given that the beam is elliptical, the 3 dB beamwidth of the sectoral antenna in the plane adc in Fig. 17 is determined from:

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}} \tag{42}$$

The gain of the sectoral antenna at this arbitrary off-axis angle may be determined, on a provisional basis, using the reference radiation pattern given in *recommends* 3.1 and 3.2 of this Recommendation.

2 Conclusion

A procedure has been given to evaluate the gain of a sectoral antenna at an arbitrary off-axis angle as referenced to the direction of the maximum gain of the antenna. The importance of observing the range of validity of the azimuth and elevation angles in modelling the radiation pattern of a sectoral antenna has been emphasized. Further study is required to demonstrate the range of gain and beamwidths in the azimuth and elevation planes over which the reference gain representation used here (equations (2), (3) and (40)) is valid for sectoral antennas. Administrations are requested to submit measured patterns of sectoral antennas in order that this determination may be made (see also Annex 8).

Annex 5

Mathematical model of generic average radiation patterns of omnidirectional and sectoral antennas for P-MP FWSs for use in statistical interference assessment

1 Introduction

The main text of this Recommendation (in *recommends* 2.2 and 3.2) gives reference radiation patterns, representing average side-lobe levels for both omnidirectional (in azimuth) and sectoral antennas, which can be applied in the case of multiple interference entries or time-varying interference entries.

On the other hand, for use in spatial statistical analysis of the interference, e.g. from a few GSO satellite systems into a large number of interfered-with FWS, a mathematical model is required for generic radiation patterns as given in the later sections in this Annex.

It should be noted that these mathematical models based on the sinusoidal functions, when applied in multiple entry interference calculations, may lead to biased results unless the interference sources are distributed over a large range of azimuth/elevation angles. Therefore, use of these patterns is recommended only in the case stated above.

2 Mathematical model for omnidirectional antennas

In case of spatial analysis of the interference from one or a few GSO satellite systems into a large number of FS stations, the following average side-lobe patterns should be used for elevation angles that range from −90° to 90° (see Annex 1):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for} \quad 0 \le |\theta| < \theta_4 \\ G_0 - 12 + 10\log(k+1) + F(\theta) & \text{for} \quad \theta_4 \le |\theta| < \theta_3 \\ G_0 - 12 + 10\log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] + F(\theta) & \text{for} \quad \theta_3 \le |\theta| \le 90° \end{cases} \tag{43a}$$

with:

$$F(\theta) = 10\log\left(0.9\sin^2\left(\frac{3\pi\theta}{4\theta_3}\right) + 0.1\right) \tag{43b}$$

where θ, θ_3, θ_4, G_0 and k are defined and expressed in *recommends* 2.1 in the main text.

NOTE 1 – In cases involving typical antennas operating in the 1-3 GHz range, the parameter k should be 0.7.

NOTE 2 – In cases involving antennas with improved side-lobe performance in the 1-3 GHz range, and for all antennas operating in the 3-70 GHz range, the parameter k should be 0.

3 Mathematical model for sectoral antennas

In case of spatial analysis of the interference from one or a few GSO satellite systems into a large number of FS stations, the following average side-lobe patterns should be used for elevation angles that range from −90° to 90° and for azimuth angles from –180° to 180°:

$$G(\varphi, \theta) = G_{ref}(x) \tag{44}$$

where:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \leq x < 1.396$$

$$G_{ref}(x) = G_0 - 12 - 15\log(x) + F_{ref}(x) \qquad \text{for} \quad 1.396 \leq x$$

$$F_{ref}(x) = 10\log(0.9\sin^2(0.75\pi x) + 0.1)$$

$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right)$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}}$$

$$\psi = \arccos(\cos\varphi \cdot \cos\theta), \qquad 0° \leq \psi \leq 180° \ \text{(degrees)}$$

$$x = \psi / \psi_\alpha$$

where all variables and parameters are as defined in *recommends* 3.1 in the main text (also, see Note 7 of the *recommends*).

NOTE 1 – In cases involving sectoral antennas with a 3 dB beamwidth in the azimuth plane less than about 120°, the relationship between the maximum gain and the 3 dB beamwidth in both the azimuth plane and the elevation plane, on a provisional basis, is (see Annex 3):

$$\theta_3 = \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_3}$$

where all parameters are as defined in *recommends* 3.1 in the main text.

Annex 6

Rationale used to develop equations for sectoral peak and average antennas between 1 GHz and about 6 GHz

1 Development of the equations for sectoral antennas between 1 GHz and 6 GHz

1.1 Rationale of the development

In order to appropriately model measured antenna pattern data at frequencies around 2 GHz a *k* parameter was introduced to account for side-lobe level performance into the equations used for sectoral antennas, similarly to the equations used for omnidirectional antennas.

It was found that a sectoral antenna pattern with a *k* parameter greater than zero agrees with measured antenna patterns whose first side-lobe levels are not well-estimated when using equations for omnidirectional antennas.

Equation (45) applies for peak sectoral antenna patterns:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \le x < 1$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \qquad \text{for} \quad 1 \le x \qquad (45)$$

The definition of all parameters is the same as in the main text of this Recommendation (*recommends* 3.1).

Note that there is a small discontinuity at $x = 1$ in equation (45).

When $k = 0.7$, for example, the lower formula becomes $G_{ref}(x) = G_0 - 9.7$ while the upper one remains as $G_{ref}(x) = G_0 - 12$ (about 2 dB difference). This discontinuity becomes smaller for smaller values of k.

In order to define more precisely the breakpoint between these two equations, it is found that with small approximations (k<<1 and x_k the breakpoint close to 1):

$$G_{ref}(x_k) = G_0 - 12 + 10\ \log({x_k}^{-1.5} + k) = G_0 - 12x_k^2$$

$$\Rightarrow -12x_k^2 = -12 + \frac{10}{\ln(10)}\ln({x_0}^{-1.5} + k) \approx -12 + \frac{10}{\ln(10)}k$$

$$\Rightarrow x_k \approx \sqrt{1 - \frac{5k}{6\ln 10}}$$

Thus, the breakpoint "1" can be replaced by a floating breakpoint x_k.

In that case, equation (45) becomes:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \le x < x_k$$

$$(46)$$

$$G_{ref}(x) = G_0 - 12 + 10\ \log(x^{-1.5} + k) \qquad \text{for} \quad x_k \le x$$

with:

$$x_k = \sqrt{1 - 0.36k}$$

1.2 Determination of the domain for which the equations are valid

It was determined, from evaluating measured antenna patterns, that different equations are needed for antennas operating from 1 GHz to about 6 GHz.

1.3 Studies on a value of a *k* parameter

Results of the analysis with respect to the k parameter and measured antenna patterns are summarized in Table 3, which also shows a trend of side-lobe performance improvement over a 10-year period.

1.4 Impact of the *k* parameter on sectoral antenna pattern

It is shown in general, that a peak antenna pattern with a typical value of k = 0.7 (see § 2 of this Annex) is appropriate in most cases (typical side-lobe case).

Note that k = 0 allows these antenna patterns to fit with sectoral antenna patterns with improved side-lobe performance.

TABLE 3

Value of parameter *k* in this Recommendation

Recommendation ITU-R F.1336			**1997**	**2000**	***2006 (used in this Recommendation)***
Omnidirectional antenna	Typical side lobe	1-3 GHz	k = 1.5	k = 0.7	k = 0.7
		3-70 GHz	k = 1.5	k = 0	k = 0
	Improved side lobe	1-70 GHz	k = 0	k = 0	k = 0
Sectoral antenna	Typical side lobe	1-3 GHz	–	k = 0.7	Peak: k = 0.7 Average: k = 0.2 (Note 1)
		3-6 GHz	–	k = 0	
		6-70 GHz	–	k = 0	k = 0
	Improved side lobe	1-70 GHz	–	k = 0	k = 0

NOTE 1 – For sectoral antennas, equations (46) are used.

Equation (47) should be used for peak sectoral antenna patterns:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \qquad 0 \le x < x_k$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \qquad \text{for} \qquad x_k \le x < 4 \qquad (47)$$

$$G_{ref}(x) = G_0 - \lambda_k - 15\log(x) \qquad \text{for} \qquad x \ge 4$$

with $\lambda_k = 12 - 10\ \log(1 + 8k)$ and $x_k = \sqrt{1 - 0.36k}$

and

x_k: breakpoint which ensures continuity between the main lobe and the first side lobes

λ_k: needed attenuation factor below the antenna gain which ensures continuity between side lobes and back lobes for x = 4.

Equation (47) is used in *recommends* 3.1.1 and 3.2.1.

2 Consideration on parameter *k* for sectoral antennas in the 1-6 GHz range

In order to evaluate an appropriate value for *k,* the total difference was calculated between the reference pattern and measured antenna patterns provided by several countries for both fixed and mobile applications. These measured patterns provided the gain for many values of elevation angle.

For the peak pattern, the experimental data were compared directly with equation (47) with a *k* factor equal to 0.7.

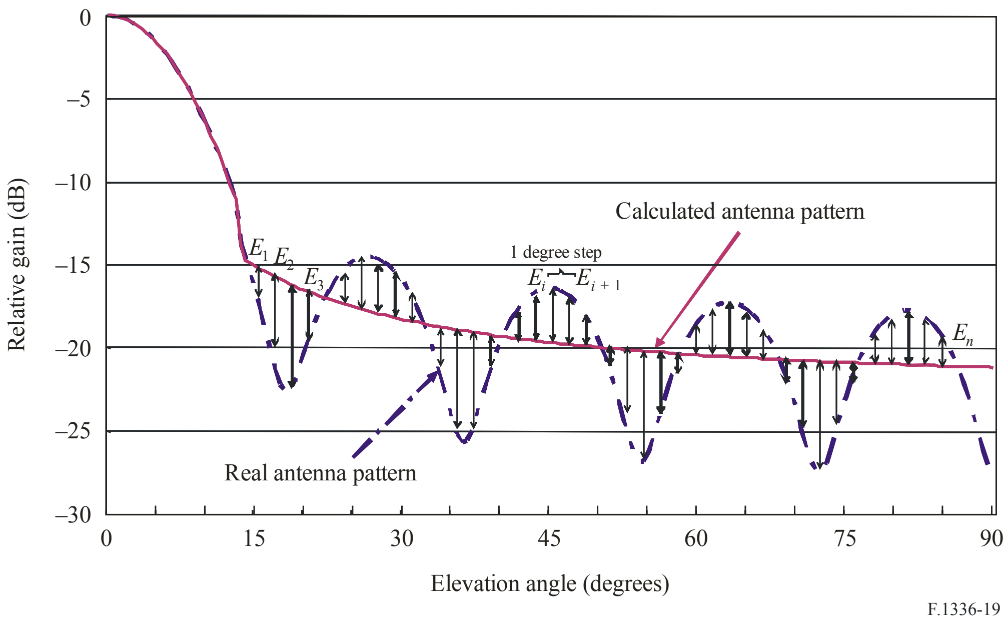
For the average pattern, the calculation was conducted only for the side-lobe range (not for the main-lobe portion); differences between the computed and real antenna patterns were sampled every one degree to determine the "Total error".

Total error is defined as below. E_i is calculated for a real value and not for a dB value.

$$\text{Total error} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} E_i^2}$$

FIGURE 19

Total error calculation to evaluate *k* parameter for the average pattern



The total error was calculated for each pattern using several *k* parameters between 0 and 0.3. The results are shown in Fig. 19. One could regard the value of the *k* parameter that provides the minimum total error as the optimum value. Based on this analysis, the value *k* = 0.2 should be used for average antenna patterns.

Another important factor to consider is "Sigma value" which is defined by the total power integration over the range of angles.

The basic idea is that:

– for peak envelope patterns, Sigma value should be in the range 2-4 dB;

– for average side-lobe patterns, Sigma value should be in the range 0-1 dB.

Calculation results of the Sigma values for the equations recommended for representative examples of typical antennas are given in Table 4.

For the peak envelope patterns, the Sigma value for *k* = 0.7 is within the permissible level. In addition, *k* = 0.2 will be a possible value for the average side-lobe patterns.

FIGURE 20

Example of evaluation of an optimum *k* value for the average pattern

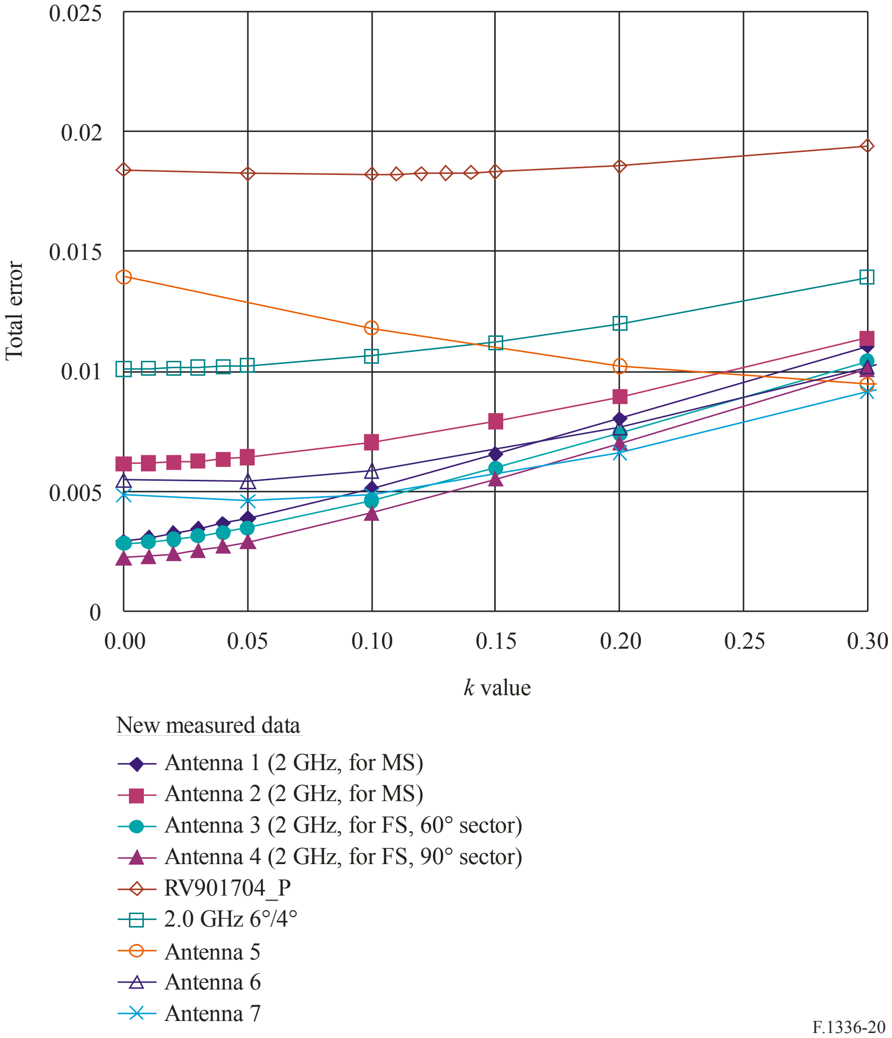


TABLE 4

Calculation results of the Sigma values

	Pattern	Equations	*k* parameter	Sigma value	
				16 dBi, 60° sector	16 dBi, 120° sector
Typical antennas in 1-6 GHz range	Peak envelope	*recommends* 3.1.1	$k = 0.7$	3.8 dB	2.55 dB
	Average side lobe	*recommends* 3.2.1	$k = 0.2$	0.8 dB	0.12 dB
			$k = 0.4$	1.43 dB	0.57 dB
			$k = 0.6$	1.93 dB	0.97 dB

Annex 7

Procedure for determining the radiation pattern of an antenna at an arbitrary off-axis angle when the boresight of the antenna is mechanically or electrically tilted downward

1 Introduction

This Annex presents methods to account for the radiation pattern of a sectoral antenna when tilted downwardly by either mechanical or electrical means. The analysis of the mechanical means is presented in § 2 and the electrical means in § 3.

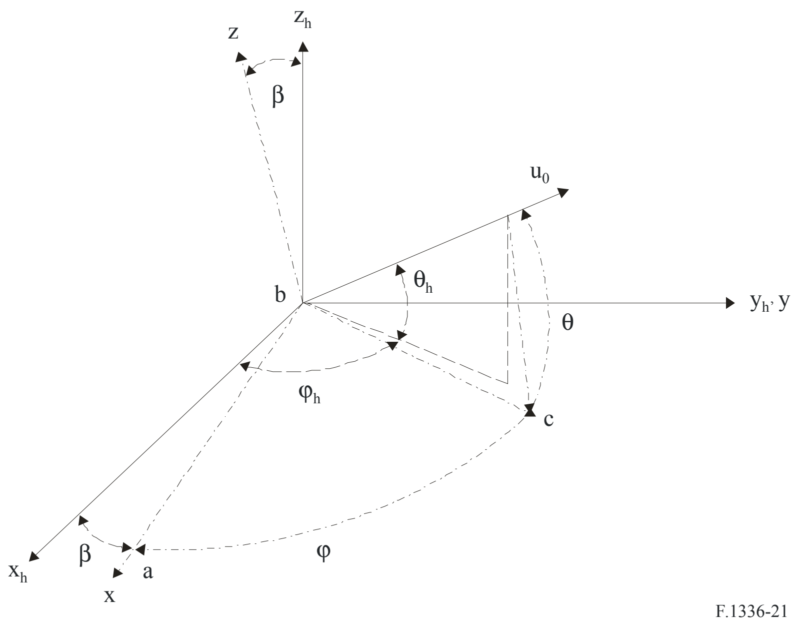
2 Analysis of mechanical tilt

The basic geometry for determining the gain of a sectoral antenna at an arbitrary off-axis angle is shown in Fig. 21. It is assumed that the antenna is located at the centre of the spherical coordinate system; the direction of maximum radiation is along the x-axis. If the antenna is tilted downward, it becomes necessary to distinguish between the antenna-based coordinates (θ, φ) and the coordinates referenced to the horizontal plane (θ_h, φ_h). The relationship between these coordinate systems is best determined by considering the rectangular coordinate systems attached to them.

If the antenna is down-tilted to a specified tilt angle by rotating the coordinate system about the y-axis, the x-y plane contains the main beam axis of the sectoral antenna, and this plane intersects the local horizontal plane along the y-axis. The tilt angle β is defined as the positive angle (degrees) that the main beam axis is below the horizontal plane at the site of the antenna.

FIGURE 21

Right-handed coordinate systems used to account for the radiation pattern of a tilted sectoral antenna



In a rectangular coordinate system located at the antenna, with its x-axis in the vertical plane containing the maximum gain of the antenna, the coordinates of the unit vector are given as follows:

$$z_h = \sin\theta_h$$
$$x_h = \cos\theta_h \cos\varphi_h \qquad (48)$$
$$y_h = \cos\theta_h \sin\varphi_h$$

Note that this is a non-standard spherical coordinate system in that the elevation is measured in the range from –90 to +90 degrees. This is the same convention that was used in *recommends* in the main text and in the previous annexes.

Consider the rectangular coordinate system of Fig. 21, which contains the main beam axis of the antenna and is rotated downward about the *y*-axis by an angle of β degrees. The unit vector in this system has the coordinates x, y, and z given by:

$$z = z_h \cos\beta + x_h \sin\beta$$
$$x = -z_h \sin\beta + x_h \cos\beta \qquad (49)$$
$$y = y_h$$

In the corresponding spherical coordinate system referenced to the plane defined by the main beam axis and the y-axis, the spherical angles are related to the coordinates x, y and z by sinθ = z and tanφ = y/x. The determination of the value of φ, which lies between -180 and +180 degrees, is given by the arctan(y/x) with possible corrections depending on the algebraic sign of x and y. Alternatively, making use of the fact that the sum of the squares of x, y and z is unity, it can be shown that cosφ = x/cosθ over a restricted range of values of φ. Substituting equations (48) into (49) and then substituting the resultant values of z and x for the relationships z = sinθ and x = cosθcosφ, the following expressions for the values of the spherical coordinates are obtained (see Note 1):

$$\theta = \arcsin(z) = \arcsin(\sin\theta_h \cos\beta + \cos\theta_h \cos\varphi_h \sin\beta), \qquad -90° \le \theta \le 90°$$
$$\varphi = \arccos\left(\frac{x}{\cos\theta}\right) = \arccos\left(\frac{(-\sin\theta_h \sin\beta + \cos\theta_h \cos\varphi_h \cos\beta)}{\cos\theta}\right), \qquad 0° \le \varphi \le 180° \qquad (50)$$

NOTE 1 – The range of the function "arccos" is from 0° to 180°. However, this does not limit the applicability of the methodology because the antenna patterns used exhibit mirror symmetry with respect to the x-z plane and the x-y plane.

The equations in *recommends* 3.4 come from equation (50).

3 Application of the radiation pattern equations in *recommends* 2.5 and 3.5 to electrical tilt antennas

In the case of the electrical tilt, the radiation pattern equations should be theoretically a function of the tilt angle β, which depends on the phase shift amount of the flux radiated from the vertically placed antenna elements. However, taking into account that β is actually a small value in general (e.g. within 15°), the following assumption could be applied for simplification.

Since the tilted radiation gains at the zenith and the nadir have to remain the same values respectively regardless of the tilt angle β (see Fig. 22), the actual radiation pattern, compared to the pattern before tilting, slightly expands or contracts above the maximum gain axis or below that axis, respectively, as shown in the solid line pattern in Fig. 22.

This radiation pattern's gains (illustrated by the solid line) could be approximated by those of another pattern (illustrated by the broken line in Fig. 22) using a parameter conversion. This broken line pattern is derived from an ideal uniform elevation angle shift of β for the original pattern calculated from the equations in *recommends* 2.1, 2.2, 3.1 and 3.2 in the respective cases.

Thus, the electrically tilted radiation patterns are derived using the parameter conversion in the equations in recommends (in 2.1, 2.2, 3.1 and 3.2) as follows:

The elevation angle θ from the maximum gain axis can be described as:

$$\theta = \theta_h + \beta \qquad (51)$$

where,

θ_h: elevation angle (degrees) measured from the horizontal plane at the site of the antenna for the tilted radiation pattern ($-90° \leq \theta_h \leq 90°$)

β: electrical tilt angle as defined in § 2 of this Annex or *recommends* 2.5 and 3.4.

In order to apply the reference radiation pattern equations in *recommends* 2.1, 2.2, 3.1 and 3.2 to the electrically tilt antennas, based on the above assumption, a compression /extension ratio R_{CE} is introduced. The compression /extension ratio R_{CE} can be defined as:

$$R_{CE} = \frac{90}{90 \pm \beta}. \qquad (52)$$

Elevation angles θ_e, by which the tilted radiation gains are calculated using equations in *recommends* 2.1, 2.2, 3.1, and 3.2, can be expressed as follows:

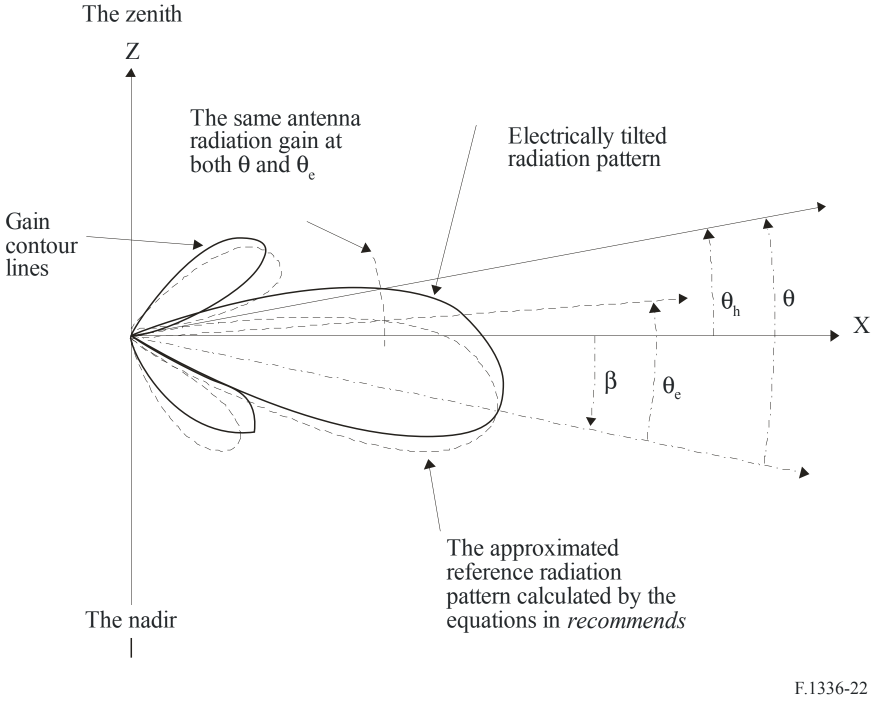
$$\theta_e = \theta \cdot R_{CE} = \frac{90 \cdot \theta}{90 + \beta} = \frac{90 \cdot (\theta_h + \beta)}{90 + \beta} \qquad \text{for} \quad \theta_h + \beta \geq 0$$

$$\theta_e = \theta \cdot R_{CE} = \frac{90 \cdot \theta}{90 - \beta} = \frac{90 \cdot (\theta_h + \beta)}{90 - \beta} \qquad \text{for} \quad \theta_h + \beta < 0 \qquad (53)$$

The electrically tilted radiation patterns are calculated by using θ_e of equations of (53) instead of θ in the equations in *recommends* 3.1 and 3.2 for sectoral antennas and also in *recommends* 2.1 and 2.2 for omnidirectional antennas.

FIGURE 22

Approximation of the reference radiation pattern for an electrically tilted antenna



Annex 8

An approach to improve the calculated side-lobe patterns defined in *recommends* 3.1 in the main part

Scope of Annex

This Annex considers an alternative approach to improve the sectoral antenna reference radiation patterns specified in *recommends* 3 in the main text of this Recommendation. The equations presented in this Annex have been derived from the practical analysis based on the measured data of the sectoral antennas and may be considered as a provisional solution to compensate the inconsistency, which may be observed between the recommended reference patterns and those in actual antennas in the off-axis region (in the direction opposite to the main lobe) in particular in the azimuth plane. The content of this Annex may be further reviewed with a view to development of more appropriate and comprehensive reference patterns for the sectoral antennas.

1 Introduction

The reference radiation patterns specified for sectoral antennas in *recommends* 3, which are defined by a series of the equations (2a1-2a5, 2b, 2c, 2d and 2e), are based on the mathematical model elaborated in Annex 4.

On one hand, these specified patterns in *recommends* 3.1 (for the peak side-lobe pattern) and 3.2 (for the average side-lobe pattern) have played an important role in many cases of interference evaluation after the latest version of this Recommendation was approved.

However, on the other hand, it has been reported that the reference patterns do not present good approximation to the measured antenna patterns in particular in the azimuth angle outside about ±90° (more specifically, depending on the 3 dB beamwidth φ_3, from $\pm\varphi_3 \cdot x_k$ to ±180°).

Through comparisons with the measured antenna patterns, solutions for the above inconsistency are considered in the later sections including the applicability of the mathematical models in Annex 4.

2 Points for consideration

The calculated side-lobe patterns using equation (2b) for peak side-lobe do not well fit to the measured patterns in particular in the angles outside the angle corresponding to x_k of the azimuth plane, as indicated in Figs 24 to 27 in this Annex (see Curves 1 and 3 in these Figures).

Due to a difference between the 3 dB beamwidth values, i.e. φ_3 and θ_3, in the azimuth and the elevation planes, the calculated patterns based on these values result in different gains at the cross point of $(\varphi, \theta) = (\pm 180, 0)$, although the gain values in the both planes should be theoretically equal at this cross point.

All the calculated patterns, specifically in the azimuth plane outside angle corresponding x_k, are quite different from that of the measured patterns, and the calculated values represent 10 dB or higher gains than the measured patterns at the azimuth angle of ±180°.

It is therefore noted that, as a cause of such inconsistency, the basic mathematical model and the associated assumptions (as illustrated in Figs 17 and 18 in Annex 4), which is adopted in the algorithm deriving the sectoral antenna patterns, may not applicable to the entire 3-dimension angles.

For the elevation plane, the calculated patterns based on the current algorithm represent fairly good approximation to the measured patterns.

Taking into account the above points, other algorithms to overcome the inconsistency between the calculated and the measured patterns are provided in § 3 as a provisional solution.

3 An approach to improve the applicability of the side-lobe patterns calculated from equations defined in *recommends* 3.1 in the main part

3.1 Consideration

In the angle range where ψ is greater than 90°, it is proposed to modify the 3 dB beamwidth values, φ_3 and θ_3, to variable parameters φ_{3m} and θ_{3m} so as to gradually get to a single value $\phi_{3(180)}$ at the cross point (±180, 0) since the inconsistency at this point is caused by the difference between φ_3 and θ_3.

As a possible value of $\phi_{3(180)}$, the existing constant θ_3 could be adopted assuming that there is no more discrimination at the cross point between elevation and azimuth planes, and it is the simplest selection as far as we consider the cross point being included in the elevation plane.

Therefore,

$$\phi_{3(180)} = \theta_3 \text{ (see Note 1)} \tag{54}$$

NOTE 1 – When a front-to-back ratio (FBR) of the reference antenna is available, it may also be possible to adopt $\phi_{3(180)}$ as follows:

$$\phi_{3(180)} = \frac{180}{10^{(FBR-\lambda k)/15}} \tag{55}$$

Since the difference of the patterns starts from the angle corresponding to x_k for the frequency range 1-6 GHz (or the angle corresponding to x_k=1 for the frequency range 6-70 GHz), the azimuth angle at this point φ_{th} is expressed as follows (see equation (2a5)):

$$\varphi_{th} = \varphi_3 \cdot x_k \qquad \text{(for 1-6 GHz)} \tag{56a}$$

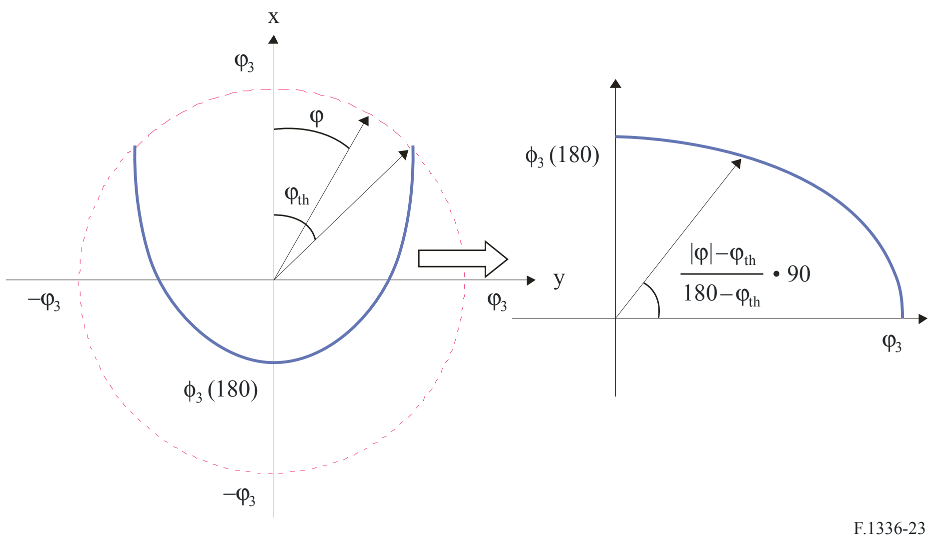
$$\varphi_{th} = \varphi_3 \qquad \text{(for 6-70 GHz)} \tag{56b}$$

The newly defined 3 dB beamwidth variable φ_{3m} gradually changes from φ_3 at $\pm\varphi_{th}$ to $\phi_{3(180)}$ at the azimuth angle of ±180°. Given that the changing locus is a part of ellipse, the difference between azimuth angles of $|\varphi|$ and $\varphi_{th}$is compressed by the factor of $90/(180 - \varphi_{th})$ as shown in Fig. 23. Then φ_{3m} is generally expressed by the following equation:

$$\varphi_{3m} = \frac{1}{\sqrt{\left(\dfrac{\cos\left(\dfrac{|\varphi| - \varphi_{th}}{180 - \varphi_{th}} \cdot 90\right)}{\varphi_3}\right)^2 + \left(\dfrac{\sin\left(\dfrac{|\varphi| - \varphi_{th}}{180 - \varphi_{th}} \cdot 90\right)}{\phi_{3(180)}}\right)^2}} \qquad \text{for} \quad \varphi_{th} < |\varphi| \le 180° \tag{57}$$

FIGURE 23

Determining the compression factor for the ellipse equation



Since the value of φ_{3m} in the range $\varphi_{th} < \varphi \le 90°$ is described as equation (57), a consequential modification to equation (2a3) in *recommends* 3.1 is required as follows:

$$\psi_\alpha = \frac{1}{\sqrt{\left(\dfrac{\cos\alpha}{\varphi_{3m}}\right)^2 + \left(\dfrac{\sin\alpha}{\theta_3}\right)^2}} \qquad \text{for} \quad 0° \le \psi \le 90° \tag{58}$$

where:

$$\varphi_{3m} = \varphi_3 \qquad \text{for} \quad 0° \leq \psi \leq \varphi_{th}$$

Furthermore, within the angle ψ between 90° and 180° in the elevation plane (in this case $\theta = 180 - \psi$), the following new variable θ_{3m} is defined which gradually changes from θ_3 at 90° to $\phi_{3(180)}$ at 180°. Given that the changing locus is a part of ellipse, θ_{3m} is generally expressed by the following equation (it is noted that, in the case of $\phi_{3(180)} = \theta_3$, θ_{3m} is a constant value θ_3):

$$\theta_{3m} = \frac{1}{\sqrt{\left(\frac{\cos\theta}{\phi_{3(180)}}\right)^2 + \left(\frac{\sin\theta}{\theta_3}\right)^2}} \qquad \text{for} \quad 90° < \psi \leq 180° \tag{59}$$

In the same manner, taking into account equation (A8-4), in the range ψ greater than 90°, the value of ψ_α is not dependent on α but on θ and is represented by the following equation:

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\theta}{\varphi_{3m}}\right)^2 + \left(\frac{\sin\theta}{\theta_3}\right)^2}} \qquad \text{for} \quad 90° < \psi \leq 180° \tag{60}$$

3.2 Alternative equations used in *recommends* 3.1 and 3.2 of the main part

Taking into account the modified parameters considered in the previous section, the text and the equations in *recommends* 3.1 in the main part of this Recommendation could be rewritten as follows.

3.1 in the case of peak side-lobe patterns referred to in *considering* b), the following equations should be used for elevation angles that range from –90° to 90° and for azimuth angles that range from –180° to 180°:

$$G(\varphi,\theta) = G_{ref}(x) \tag{2a1}$$

$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right), \qquad -90° \leq \alpha \leq +90° \tag{2a2}$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_{3m}}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}} \qquad \text{for} \quad 0° \leq \psi \leq 90° \tag{2a3}$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\theta}{\varphi_{3m}}\right)^2 + \left(\frac{\sin\theta}{\theta_3}\right)^2}} \qquad \text{for} \quad 90° < \psi \leq 180°$$

$$\psi = \arccos(\cos\varphi \cdot \cos\theta), \qquad 0° \leq \psi \leq 180° \tag{2a4}$$

$$x = \psi / \psi_\alpha \tag{2a5}$$

where:

θ: elevation angle relative to the local horizontal plane when the maximum gain is in that plane ($-90° \leq \theta \leq 90°$)

φ: azimuth angle relative to the angle of the maximum gain in the horizontal plane (degrees)

φ_3: the 3 dB beamwidth in the azimuth plane (degrees) (generally equal to the sectoral beamwidth)

φ_{3m}: the equivalent 3 dB beamwidth in the azimuth plane for an adjustment of horizontal gains (degrees)

$$\varphi_{3m} = \varphi_3 \qquad \text{for} \quad 0° \leq |\varphi| \leq \varphi_{th} \tag{2a6}$$

$$\varphi_{3m} = \frac{1}{\sqrt{\left(\dfrac{\cos\left(\dfrac{|\varphi| - \varphi_{th}}{180 - \varphi_{th}} \cdot 90\right)}{\varphi_3}\right)^2 + \left(\dfrac{\sin\left(\dfrac{|\varphi| - \varphi_{th}}{180 - \varphi_{th}} \cdot 90\right)}{\theta_3}\right)^2}} \qquad \text{for} \quad \varphi_{th} < |\varphi| \leq 180° \tag{2a7}$$

where:

φ_{th}: the boundary azimuth angle (degrees) (See equations (56a) and (56b)).

It is noted that, in the case of using an available FBR, θ_3 in equation (2a7) is replaced with the value calculated by equation (55).

3.1.1 in the frequency range from 1 GHz to about 6 GHz:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \leq x < x_k$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \qquad \text{for} \quad x_k \leq x < 4 \tag{2b}$$

$$G_{ref}(x) = G_0 - \lambda_k - 15\log(x) \qquad \text{for} \quad x \geq 4$$

with $\lambda_k = 12 - 10\log(1 + 8k)$ and $x_k = \sqrt{1 - 0.36k}$;

3.1.1.1 in cases involving typical antennas the parameter k should be 0.7 (therefore, $\lambda_{k\,=\,0.7} = 3.8$ and $x_{k\,=\,0.7} = 0.86$) (see Note 2);

3.1.1.2 in cases involving antennas with improved side-lobe performance the parameter k should be 0 (therefore, $\lambda_{k\,=\,0} = 12$ and $x_{k\,=\,0} = 1$);

3.1.2 in the frequency range from 6 GHz to about 70 GHz:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \leq x < 1 \tag{2c}$$

$$G_{ref}(x) = G_0 - 12 - 15\log(x) \qquad \text{for} \quad 1 \leq x$$

The alternative approach in this Annex using the modified parameter ψ_α as defined in equation (2a3) can naturally be applied to the equations (2d) and (2e) used in *recommends* 3.2 in the case of average side-lobe patterns, although there is no apparent change to these equations.

4 Comparison between the measured patterns and calculated patterns

Comparison has been made in several bands between the measured side-robe patterns, the calculated ones by using above-mentioned equations (Section 3 in this Annex), and the same equations in *recommends* 3.1 in the main part of this Recommendation.

The detailed characteristics of the four sets of the measured sectoral antenna patterns are shown in Table 5.

TABLE 5

The measured sectoral antenna pattern characteristics

No.	φ_3 (°)	θ_3 (°)	β (°) (Electrical tilt)	Measured frequency *f* (GHz)
1	62	5.5	6	2.045
2	89	5.5	6	2.045
3	69	6.5	10	2.61
4	87	10.5	0	25.3

For other parameters, the following values are used in the calculation:

G_0 = 0

k = 0.7 (for 1-6 GHz)

k: not defined (for 6-70 GHz).

The four sets of the measured antenna data and both the calculated peak side-lobe patterns using the above parameters are shown in Figs 24 through 27.

As for the azimuth plane, each figure gives the following three patterns from the top:

– Curve 1: pattern using the equations in *recommends* 3.1;

– Curve 2: pattern using the modified equations in § 3 in this Annex;

– Curve 3: the measured pattern.

For the elevation plane, since the Curves 1 and 2 represent the same result, the only 2 patterns are depicted in each Figure.

In some actual antennas, e.g. the measured example in Fig. 27, maximum gain compression technique (about 3 dB) is applied in order to expand the 3 dB beamwidth in the azimuth plane. This may cause a bit difference from the calculated patterns near the first side-lobe in the elevation plane and also near the main lobe in the azimuth one, however it would not become a significant problem in the application of the proposed approach.

FIGURE 24

Comparison between the measured patterns and the calculated peak side-lobe patterns ($\varphi_3 = 62°$, $\theta_3 = 5.5°$, $\beta = 6°$, $f = 2.045$ GHz)

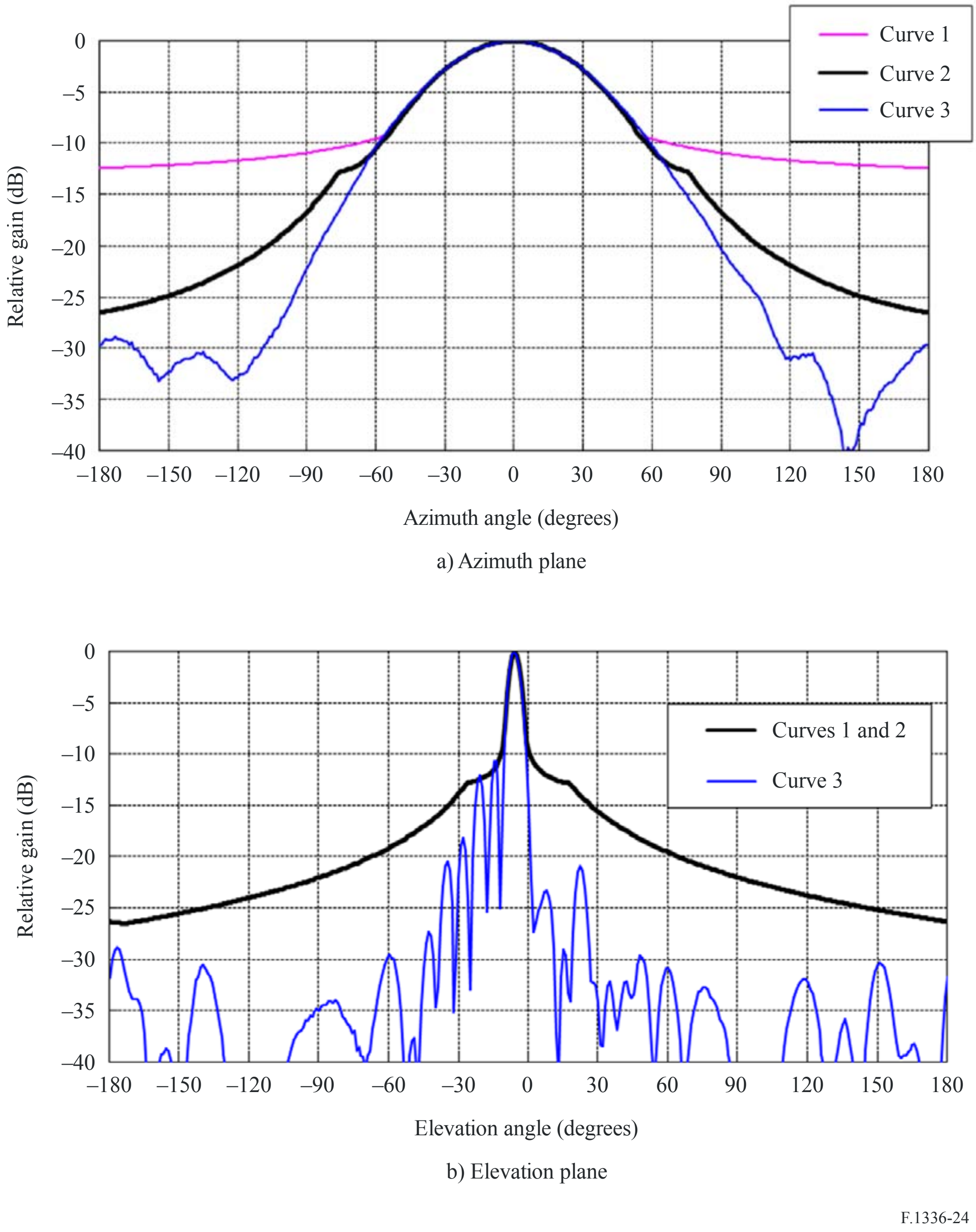
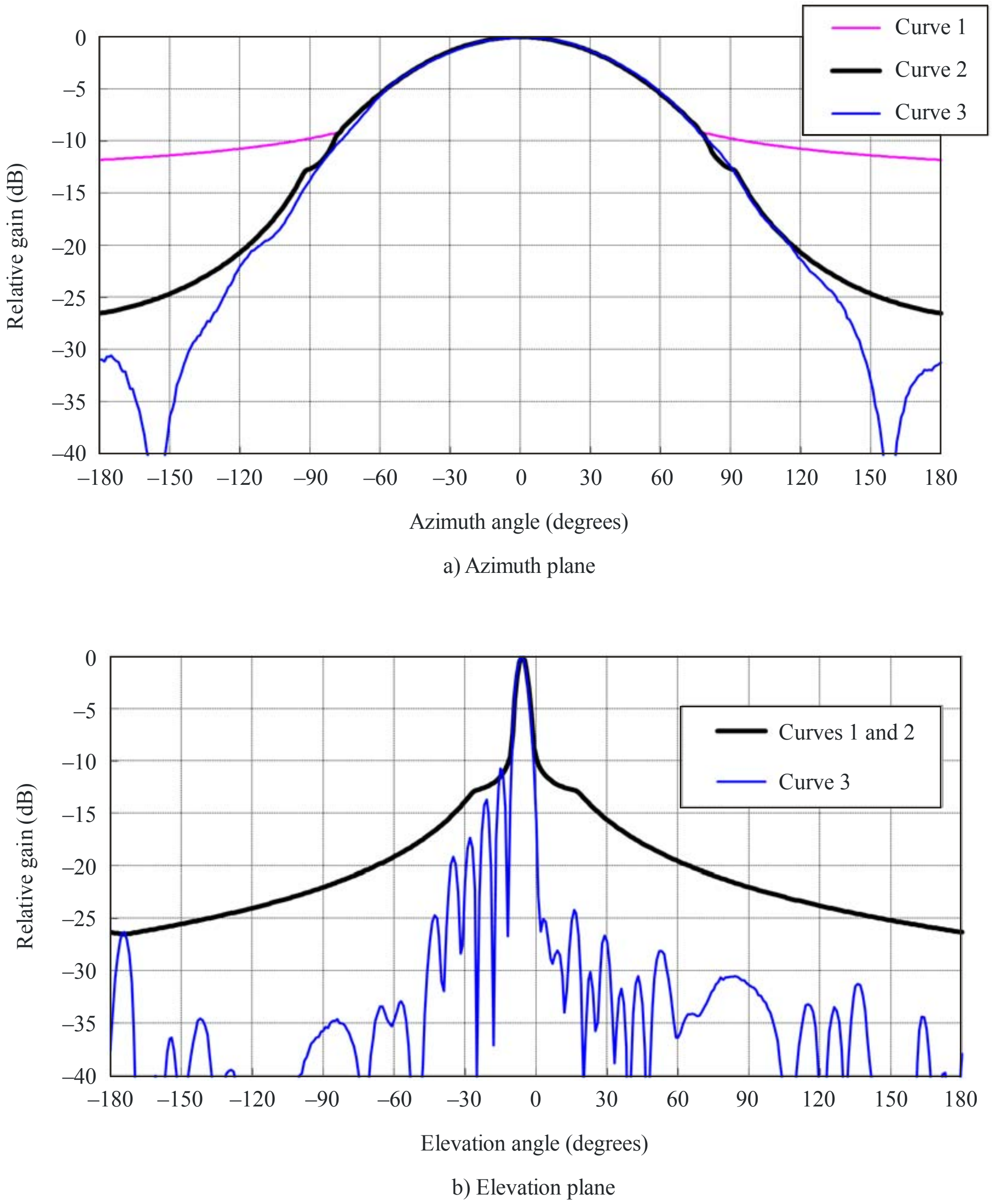


FIGURE 25

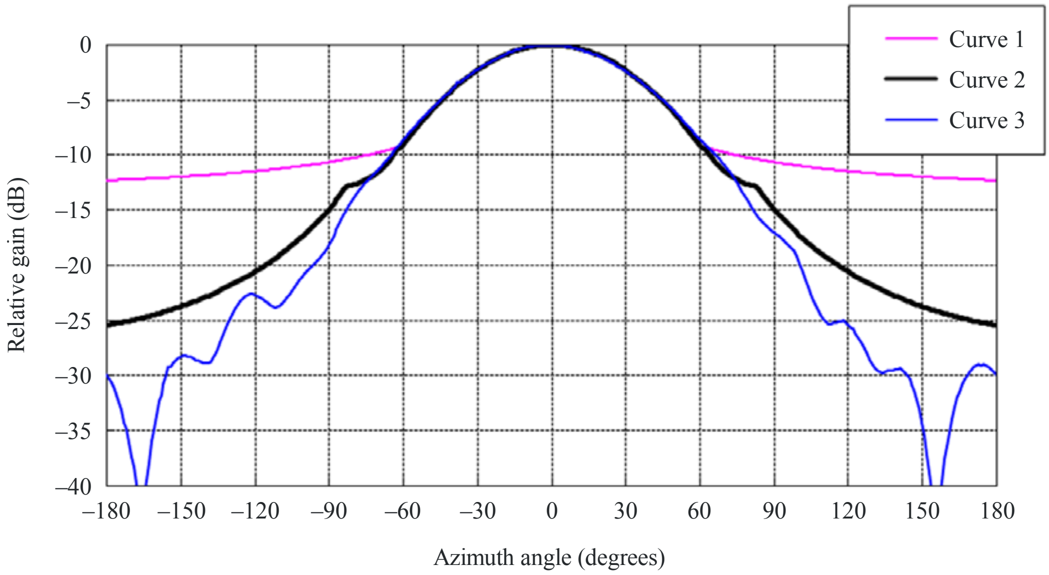
Comparison between the measured patterns and the calculated peak side-lobe patterns
($\varphi_3 = 89°$, $\theta_3 = 5.5°$, $\beta = 6°$, $f = 2.045$ GHz)



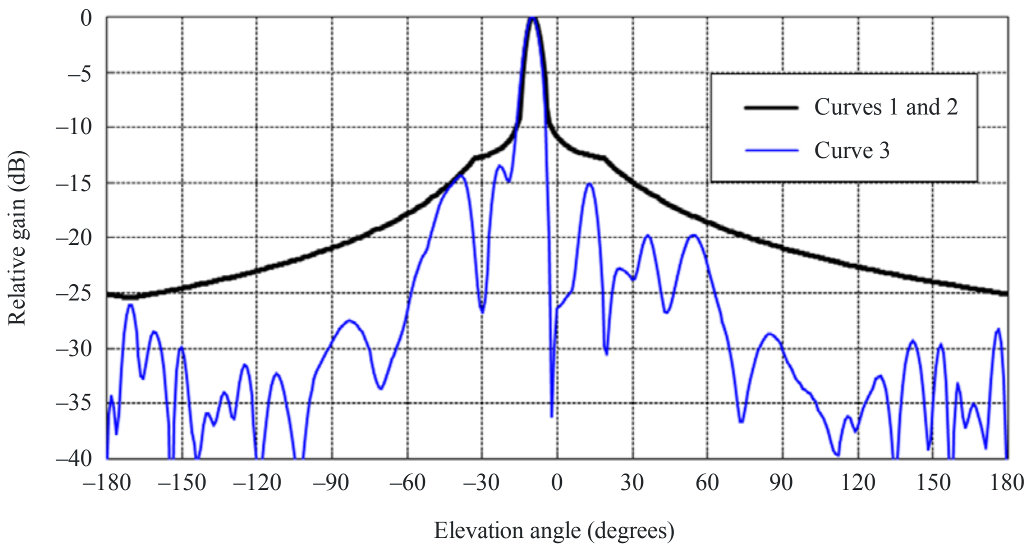
F.1336-25

FIGURE 26

Comparison between the measured patterns and the calculated peak side-lobe patterns
(φ_3 = 69°, θ_3 = 6.5°, β = 10°, *f* = 2.61 GHz)



a) Azimuth plane

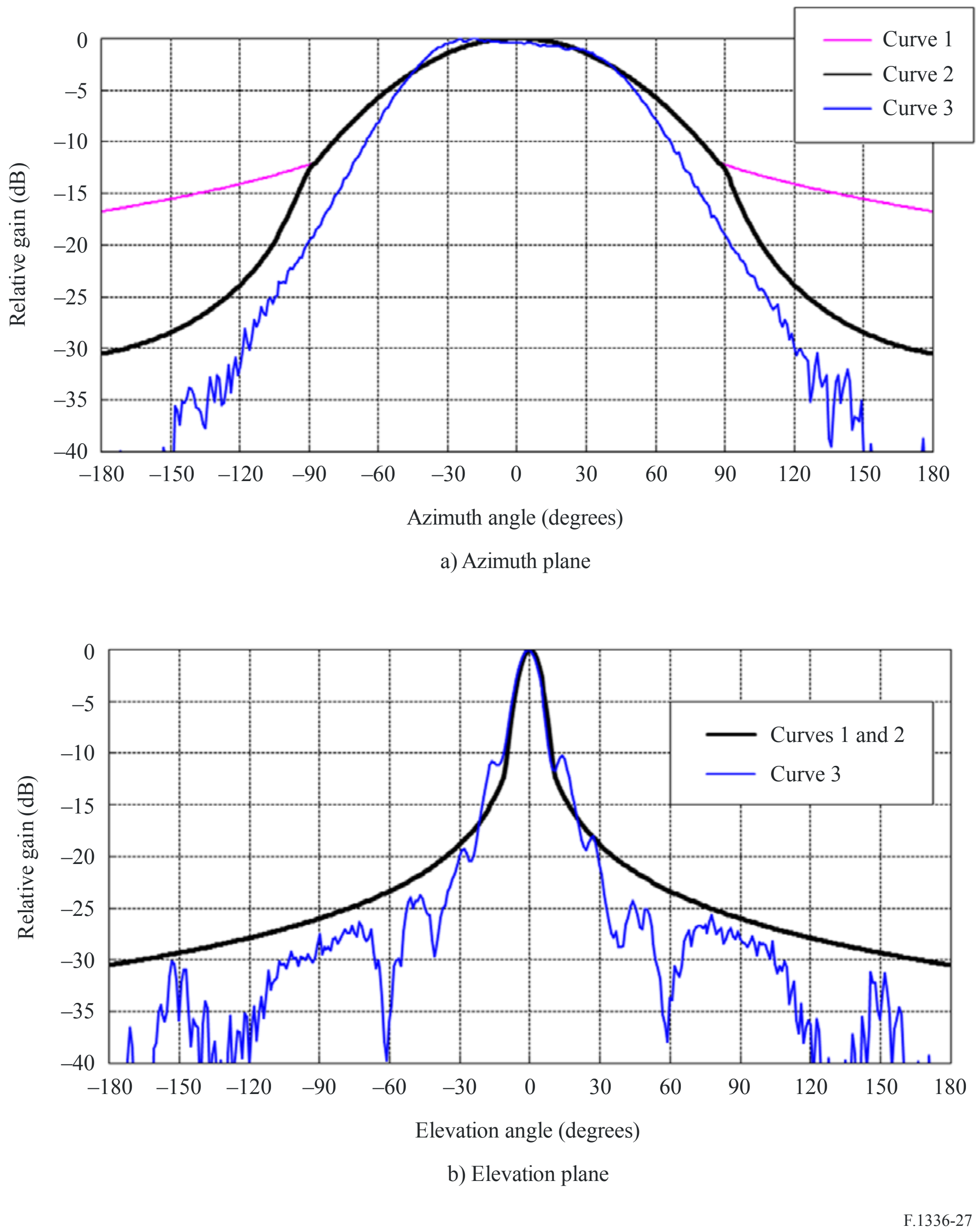


b) Elevation plane

F.1336-26

FIGURE 27

Comparison between the measured patterns and the calculated peak side-lobe patterns
(φ_3 = 87°, θ_3 = 10°, β = 0°, *f* = 25.3 GHz)



5 Summary

As shown in § 4, it has been demonstrated that the approach considered in § 3 would provide much improved side-lobe patterns for sectoral antennas in the azimuth plane outside the angle of φ_{th}. These proposed changes will not significantly affect the elevation angle where the inconsistency with the measured data is rather small.

It should also be noted that the new equations proposed in this Annex are still based on the mathematical model in Annex 4 and the imperfection of the original algorithm in *recommends* 3.1 and 3.2 are due to the applicability of this model to the entire 3-dimension angles. Therefore, further study may be required to establish a more appropriate model applicable to all the angles in both elevation and azimuth planes.

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