RECOMMENDATION ITU-R F.1336-1*

REFERENCE RADIATION PATTERNS OF OMNIDIRECTIONAL, SECTORAL AND OTHER ANTENNAS IN POINT-TO-MULTIPOINT SYSTEMS FOR USE IN SHARING STUDIES IN THE FREQUENCY RANGE FROM 1 GHz TO ABOUT 70 GHz

(Question ITU-R 202/9)

(1997-2000)

The ITU Radiocommunication Assembly,

considering

a) that, for coordination studies and for the assessment of mutual interference between point-to-multipoint (P-MP) radio-relay systems and between stations of such systems and stations of space radiocommunication services sharing the same frequency band, it may be necessary to use reference radiation patterns for radio-relay system antennas;

b) that, for the above studies, radiation patterns based on the level exceeded by a small percentage of the side-lobe peaks may be appropriate;

c) that the side-lobe patterns of antennas of different sizes are strongly influenced by the directivity of the antenna and by the ratio of the antenna diameter to the operating wavelength;

d) that reference radiation patterns are required in situations where information concerning the actual radiation pattern is not available;

e) that, at large angles, the likelihood of local ground reflections must be considered;

f) that the use of antennas with the best available radiation patterns will lead to the most efficient use of the radio-frequency spectrum,

recommends

1 that, in the absence of particular information concerning the radiation pattern of the P-MP radio-relay system antenna involved (see Note 1), the reference radiation pattern as stated below should be used for:

1.1 interference assessment between line-of-sight (LoS) P-MP radio-relay systems;

1.2 coordination studies and interference assessment between P-MP LoS radio-relay stations and other stations of services sharing the same frequency band;

2 that the following reference radiation patterns should be used for frequencies in the range 1 to 3 GHz and, on a provisional basis, for frequencies above 3 GHz to about 70 GHz (see Note 2);

2.1 in cases involving stations that use antennas with omnidirectional pattern in or near the horizontal plane, the following equations should be used for elevation angles that range from 0° to 90° (see Annex 1 and Note 3):

$$G(\theta) = \max\left[G_1(\theta), G_2(\theta)\right] \tag{1a}$$

$$G_1(\theta) = G_0 - 12\left(\frac{\theta}{\theta_3}\right)^2 \tag{1b}$$

$$G_2(\theta) = G_0 - 12 + 10\log\left[\left(\max\left\{\frac{|\theta|}{\theta_3}, 1\right\}\right)^{-1.5} + k\right] \tag{1c}$$

* This Recommendation should be brought to the attention of Radiocommunication Study Groups 4 (WP 4A), 6 (WP 6S), 7 (WP 7B), 8 (WP 8D), and Working Party 4-9S.

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: the maximum gain in or near the horizontal plane (dBi)

θ: absolute value of the elevation angle relative to the angle of maximum gain (degrees)

θ_3: the 3 dB beamwidth in the vertical plane (degrees)

k: parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance,

The relationship between the gain (dBi) and the 3 dB beamwidth in the elevation plane (degrees) is (see Note 4):

$$\theta_3 = 107.6 \times 10^{-0.1\,G_0}$$

2.1.1 in cases involving typical antennas operating in the 1-3 GHz range, the parameter *k* should be 0.7 (see Note 5);

2.1.2 in cases involving antennas with improved side-lobe performance in the 1-3 GHz range, and provisionally for all antennas operating in the 3-70 GHz range, the parameter *k* should be 0;

2.2 that the pattern given in *recommends* 2.1 should be used on a provisional basis for the reference radiation pattern in the elevation plane of sectoral antennas using the value of θ_3 as given in *recommends* 2.2.2;

2.2.1 that the parameter *k* to be used for the reference radiation pattern of sectoral antennas in the elevation plane operating in the 1-70 GHz range should provisionally conform to the values given in *recommends* 2.1.1 and 2.1.2;

2.2.2 in cases involving sectoral antennas with a 3 dB beamwidth in the azimuthal plane less than about 120°, the relationship between the maximum gain in the azimuthal plane and the 3 dB beamwidth in both the azimuthal plane and the elevation plane, on a provisional basis, is (see Note 6 and Annex 3):

$$\theta_3 = \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_s}$$

where φ_s is the 3 dB beamwidth of the sector in the azimuthal plane (degrees) and the other parameters are as defined under *recommends* 2.1;

2.3 in cases involving a low-cost, low-gain antenna in the 1-3 GHz range with circular symmetry about the 3 dB beamwidth and with a gain less than about 20 dBi, the following equations should be used (see Annex 2 and Note 7):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \le \theta < 1.08\,\varphi_3 \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \le \theta < \varphi_1 \\ G_0 - 14 - 32\log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \le \theta < \varphi_2 \\ -8 & \text{for } \varphi_2 \le \theta \le 180° \end{cases}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: the main lobe antenna gain (dBi)

θ: off-axis angle (degrees)

φ_3: the 3 dB beamwidth of the low-gain antenna (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ degrees

$\varphi_1 = 1.9\,\varphi_3$ degrees

$\varphi_2 = \varphi_1 \times 10^{(G_0 - 6)/32}$ degrees.

3 that further studies are required to improve the definition of antenna radiation pattern envelopes. This will ease frequency sharing with other services and improve spectrum utilization;

4 that the following Notes should be regarded as part of this Recommendation:

NOTE 1 – It is essential that every effort be made to utilize the actual antenna pattern in coordination studies and interference assessment.

NOTE 2 – Further study is required to determine if modifications to the reference radiation pattern and the factor –1.5 are required when applied to antennas operating in frequency bands from 3 GHz to about 70 GHz. Further study is also required to develop reference radiation patterns applicable to sectoral antennas in the azimuthal plane. Reference radiation patterns may also need to be developed for other types of antennas which may be used in central stations and subscriber stations of P-MP systems in the frequency range from 1 GHz to about 70 GHz.

NOTE 3 – The reference radiation pattern for omnidirectional antennas given in *recommends* 2.1 primarily applies in situations where the maximum gain in the horizontal plane is between 8 dBi and 13 dBi. Further study is required to establish the full range over which the equations are valid.

NOTE 4 – As discussed in Annex 3, an exponential factor has been replaced to be unity. As a result, the theoretical error introduced by this approximation will be less than 6% for 3 dB beamwidths in the elevation plane less than 45°.

NOTE 5 – The reference radiation pattern for omnidirectional and sectoral antennas given in § A.2.1.2.2.2.3 of RR Resolution 46 (Rev.WRC-97) is:

$$G(\theta) = G_0 - 12(\theta / \varphi_3)^2 \qquad \text{for} \quad 0 \leq \theta < \varphi_3$$

$$G(\theta) = G_0 - 12 - 10 \log (\theta / \varphi_3) \qquad \text{for} \quad \varphi_3 \leq \theta < 90°$$

The parameters are as defined in *recommends* 2.1.

Resolution 46 (Rev.WRC-97) and Annex 1 to RR Appendix S5 indicate that this antenna pattern is provisional and that further study is under way in the ITU-R.

NOTE 6 – Annex 3 derives the gain-beamwidth relationship for omnidirectional and sectoral antennas. The relationship shown in *recommends* 2.2.2 is based on limited measurements of sectoral antennas designed for use in the 25.25-29.5 GHz band. The basis for the constant 31 000 is explained in Annex 3.

NOTE 7 – The reference radiation pattern given in *recommends* 2.3 primarily applies in situations where the maximum gain is less than or equal to 20 dBi and the use of Recommendation ITU-R F.699 produces inadequate results. Further study is required to establish the full range of frequencies and gain over which the equations are valid.

ANNEX 1

Reference radiation pattern for omnidirectional antennas as used in P-MP radio-relay systems

1 Introduction

An omnidirectional antenna is frequently used for transmitting and receiving signals at central stations of P-MP radio-relay systems. Studies involving sharing between these types of radio-relay systems and space service systems in the 2 GHz bands have used the reference radiation pattern described here.

2 Analysis

The reference radiation pattern is based on the following assumptions concerning the omnidirectional antenna:

– that the antenna is an n-element linear array radiating in the broadside mode;

– the elements of the array are assumed to be dipoles;

– the array elements are spaced $3\lambda/4$.

The 3 dB beamwidth θ_3 of the array in the elevation plane is related to the directivity D by (see Annex 3 for the definition of D):

$$D = 10 \log\left[191.0\sqrt{0.818 + 1/\theta_3} - 172.4\right] \qquad \text{dBi} \tag{2a}$$

Equation (2a) may be solved for θ_3 when the directivity is known:

$$\theta_3 = \frac{1}{\alpha^2 - 0.818} \tag{2b}$$

$$\alpha = \frac{10^{0.1D} + 172.4}{191.0} \tag{2c}$$

The relationship between the 3 dB beamwidth in the elevation plane and the directivity was derived on the assumption that the radiation pattern in the elevation plane was adequately approximated by:

$$f(\theta) = \cos^m(\theta)$$

where m is an arbitrary parameter used to relate the 3 dB beamwidth and the radiation pattern in the elevation plane. Using this approximation, the directivity was obtained by integrating the pattern over the elevation and azimuth planes.

The intensity of the far-field of a linear array is given by:

$$E_T(\theta) = E_e(\theta) \cdot AF(\theta) \tag{3}$$

where:

$E_T(\theta)$: total E-field at an angle of θ normal to the axis of the array

$E_e(\theta)$: E-field at an angle of θ normal to the axis of the array caused by a single array element

$AF(\theta)$: array factor at an angle θ normal to the axis of the array.

The normalized E-field of a dipole element is:

$$E_e(\theta) = \cos(\theta) \tag{4}$$

The array factor is:

$$AF_N = \frac{1}{N}\left[\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \tag{5}$$

where:

N: number of elements in the array

$$\frac{\psi}{2} = \frac{1}{2}\left[2\pi\frac{d}{\lambda}\sin\theta\right]$$

d: spacing of the radiators

λ: wavelength.

The following procedure has been used to estimate the number of elements N in the array. It is assumed that the maximum gain of the array is identical to the directivity of the array.

– Given the maximum gain of the omnidirectional antenna in the elevation plane, compute the 3 dB beamwidth, θ_3, using equations (2b) and (2c);

– Ignore the small reduction in off-axis gain caused by the dipole element, and note that the array factor, AF_N, evaluates to 0.707 (–3 dB) when $N\frac{\psi}{2} = 1.3916$; and

– N is then determined as the integer value of:

$$N = \left| \frac{2 \times 1.3916}{2\pi \frac{d}{\lambda} \sin\left(\frac{\theta_3}{2}\right)} \right| \tag{6}$$

where $|x|$ means the maximum integer value not exceeding x.

The normalized off-axis discrimination ΔD is given by:

$$\Delta D = 20 \log \left[\left| AF_N \times \cos(\theta) \right| \right] \qquad \text{dB} \tag{7}$$

Equation (7) has been evaluated as a function of the off-axis angle (i.e., the elevation angle) for several values of maximum gain. For values in the range of 8 dBi to 13 dBi, it has been found that the envelope of the radiation pattern in the elevation plane may be adequately approximated by the following equations:

$$G(\theta) = \max\left[G_1(\theta), G_2(\theta)\right] \tag{8a}$$

$$G_1(\theta) = G_0 - 12\left(\frac{\theta}{\theta_3}\right)^2 \qquad \text{dBi} \tag{8b}$$

$$G_2(\theta) = G_0 - 12 + 10 \log\left[\left(\max\left\{\frac{|\theta|}{\theta_3}, 1\right\}\right)^{-1.5} + k\right] \qquad \text{dBi} \tag{8c}$$

k is a parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance.

Figures 1 to 4 compare the reference radiation envelopes with the theoretical antenna patterns generated from equation (8), for gains from 8 dBi to 13 dBi, using a factor of $k = 0$. Figures 5 to 8 compare the reference radiation envelopes with actual measured antenna patterns using a factor of $k = 0$. In Figs. 7 and 8, it can be seen that the side lobes are about 15 dB or more below the level of the main lobe, allowing for a small percentage of side-lobe peaks which might exceed this value. However practical factors such as the use of electrical downtilt, pattern degradations at band-edges and production variations would further increase the side lobes to about 10 dB below the main lobe in actual field installations. The k factor, mentioned above, in equation (8), is intended to characterize this variation in side-lobe levels. Figures 9 and 10 provide a comparison of a 10 dBi and a 13 dBi gain antenna, at 2.4 GHz, with the reference radiation pattern envelope, using $k = 0.5$. A factor of $k = 0.5$ represents side-lobe levels about 15 dB below the main-lobe peak. However, to account for increases in side-lobe levels which may be found in field installations, for typical antennas a factor of $k = 0.7$ should be used, representing side-lobe levels about 13.5 dB below the level of the main lobe. Finally, Figs. 11 and 12 illustrate the effect on elevation patterns of using various values of k.

FIGURE 1

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 10 dBi, k = 0

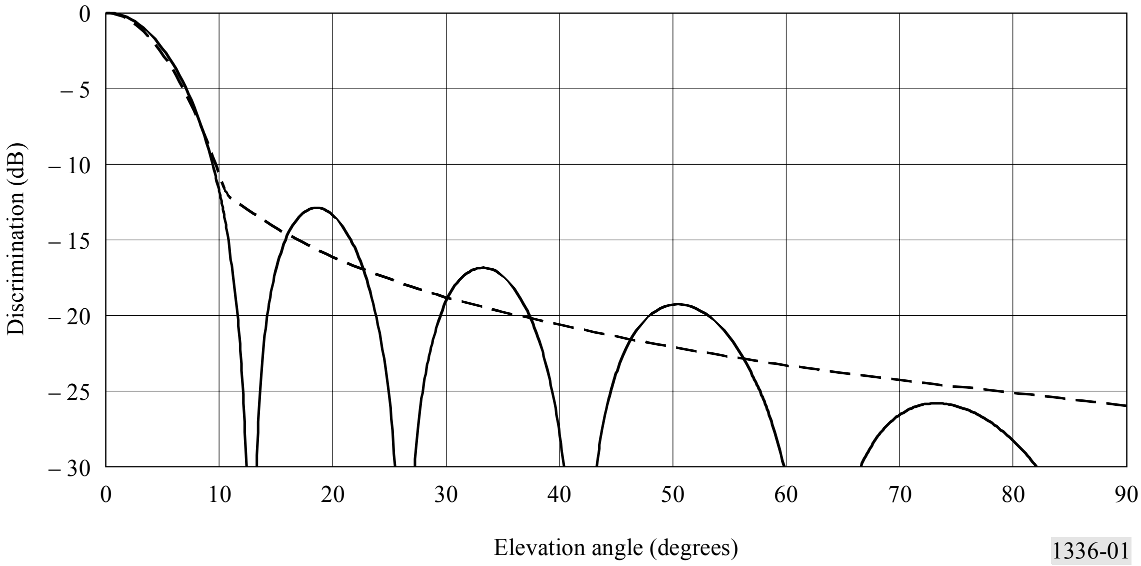

FIGURE 2

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 11 dBi, k = 0

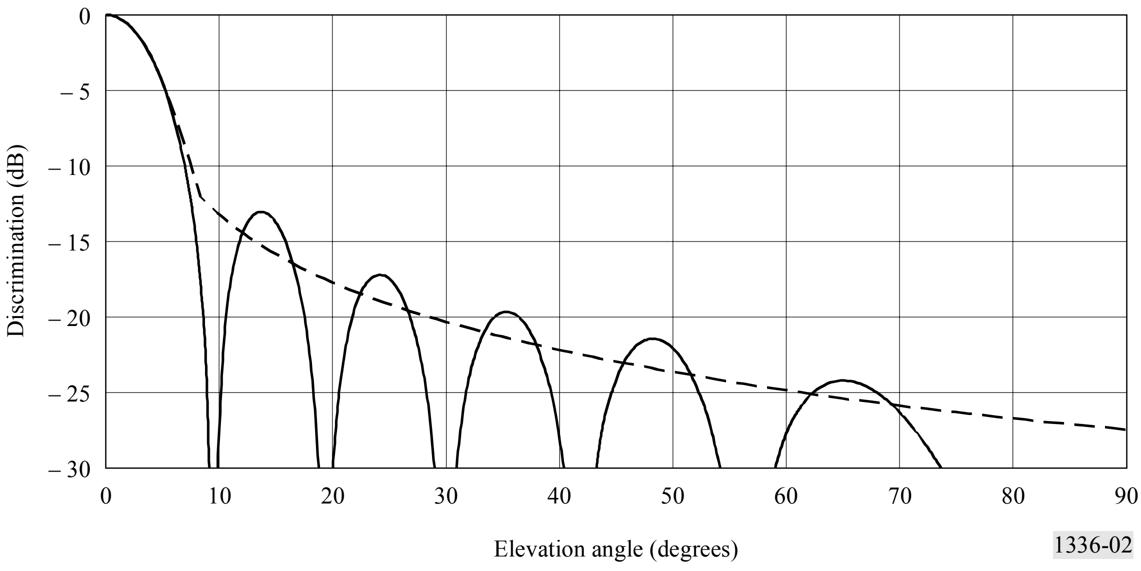

FIGURE 3

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 12 dBi, k = 0

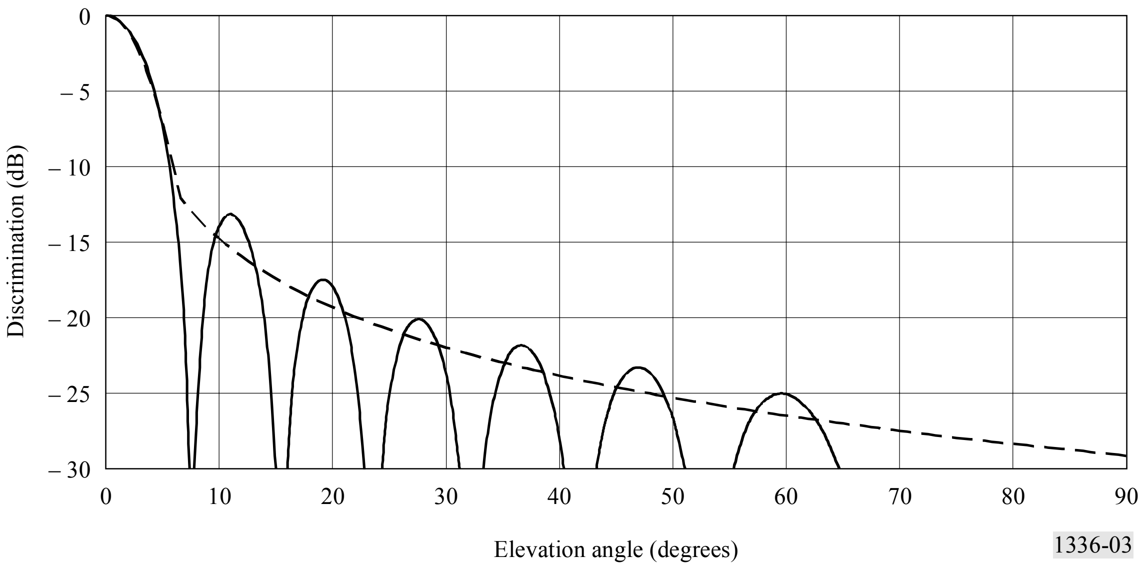

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FIGURE 4

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 13 dBi, k = 0

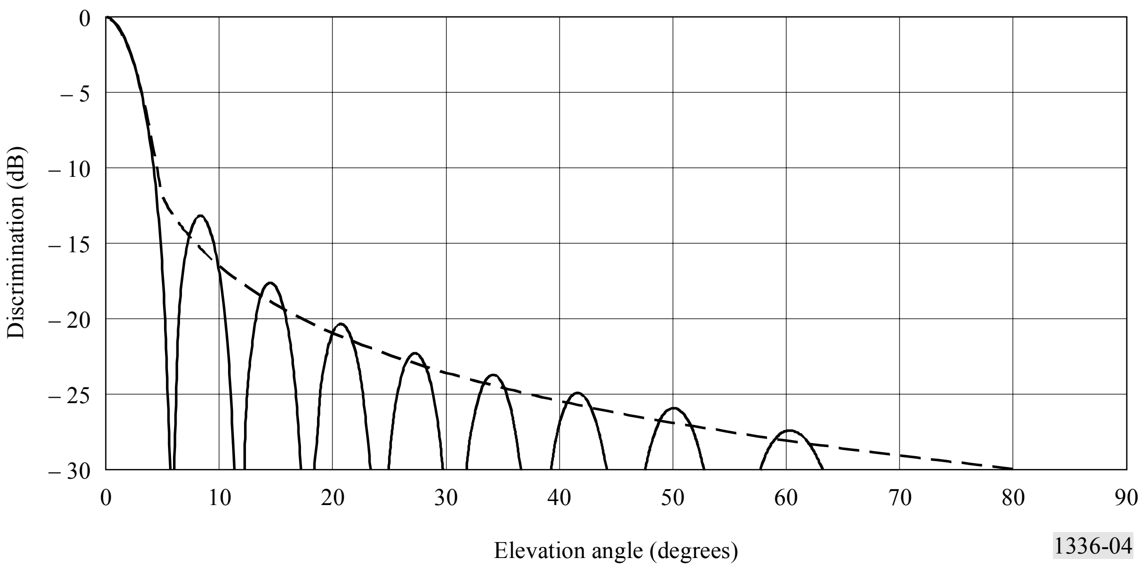

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FIGURE 5

Comparison of measured pattern and reference radiation pattern envelope for an omnidirectional antenna with 11 dBi gain and operating in the band 928-944 MHz, *k* = 0

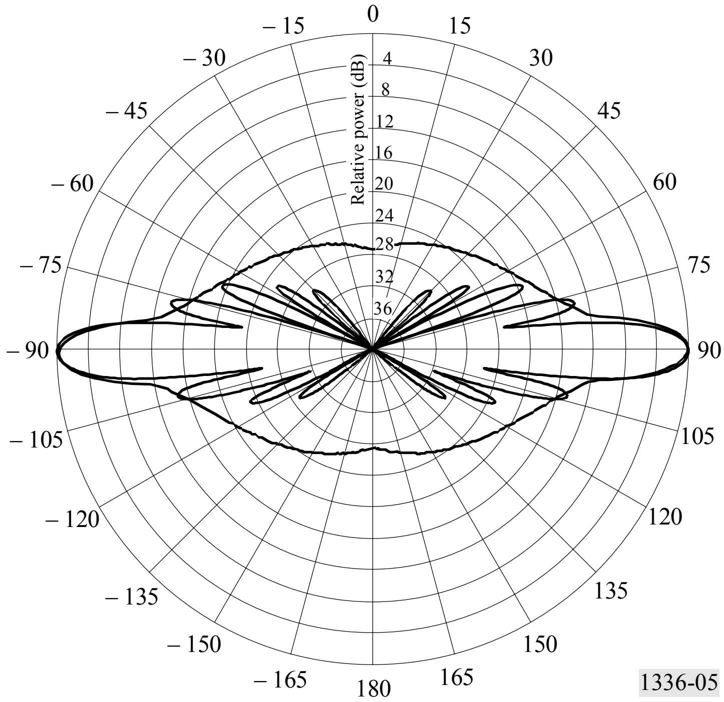

FIGURE 6

Comparison of measured pattern and the reference radiation pattern envelope for an omnidirectional antenna with 8 dBi gain and operating in the band 1 850-1 990 MHz, *k* = 0

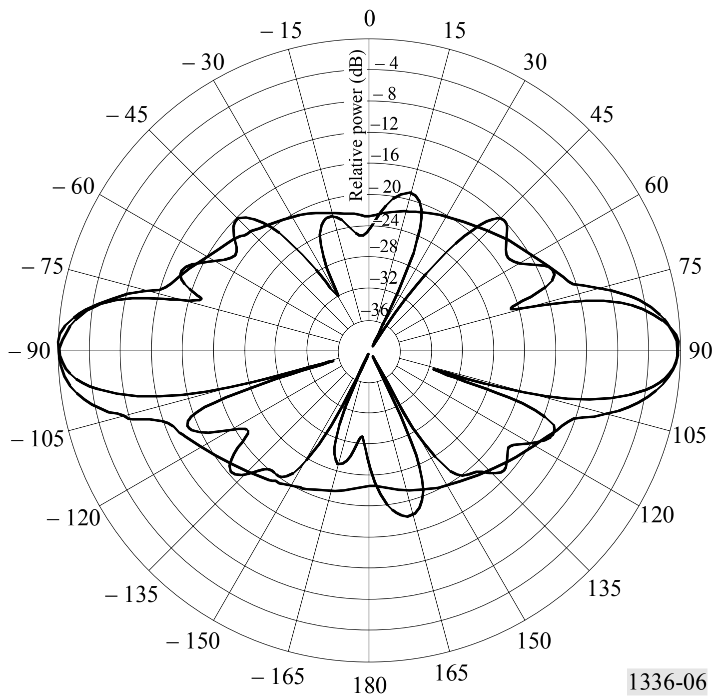

FIGURE 7

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 10 dBi gain and operating in the 1.4 GHz band

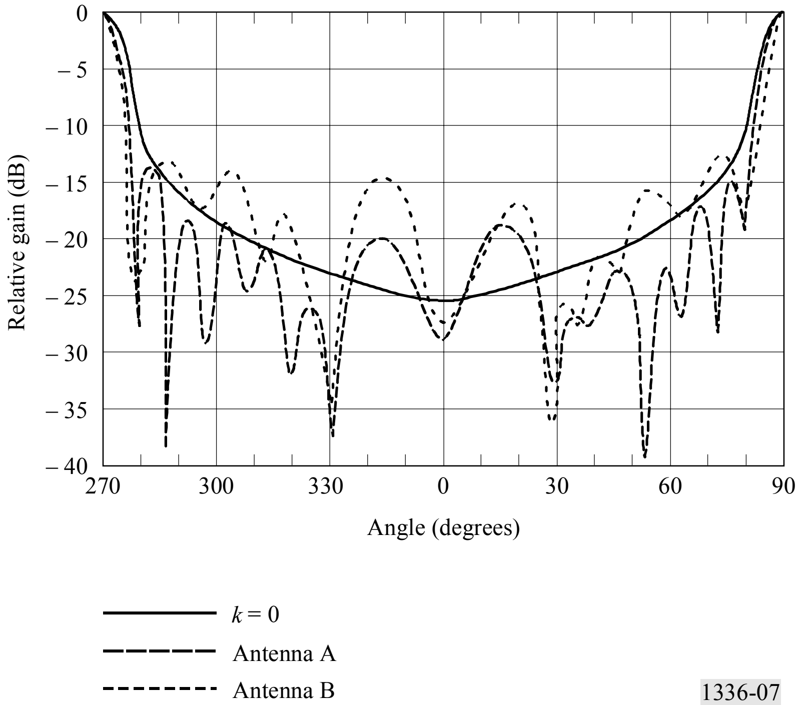

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FIGURE 8

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 13 dBi gain and operating in the 1.4 GHz band

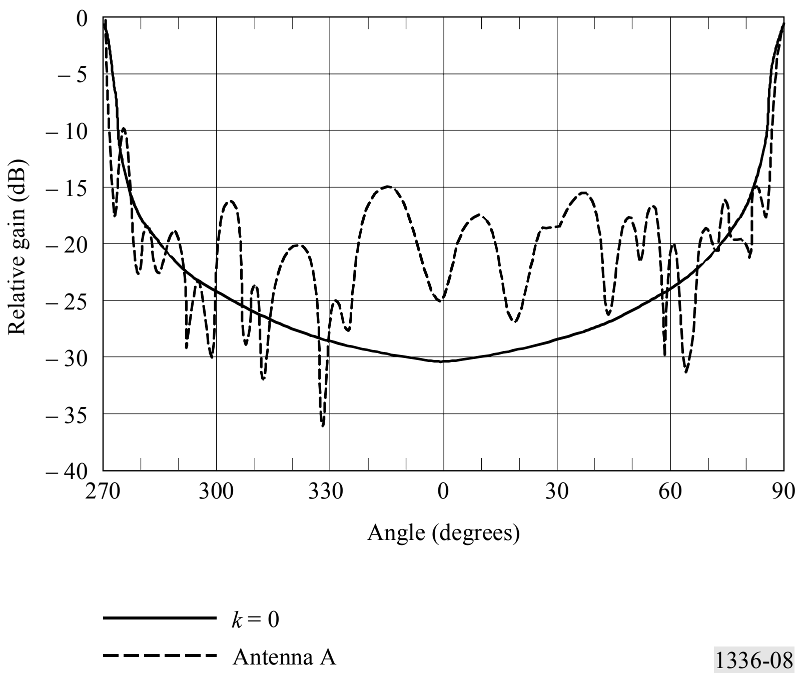

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FIGURE 9

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 10 dBi gain and operating in the 2.4 GHz band

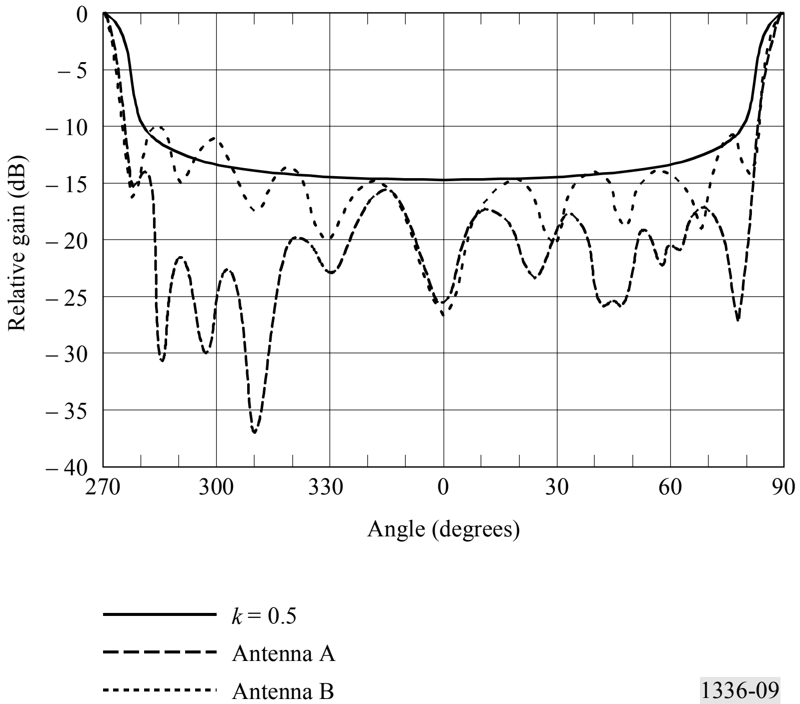


FIGURE 10

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 13 dBi gain and operating in the 2.4 GHz band

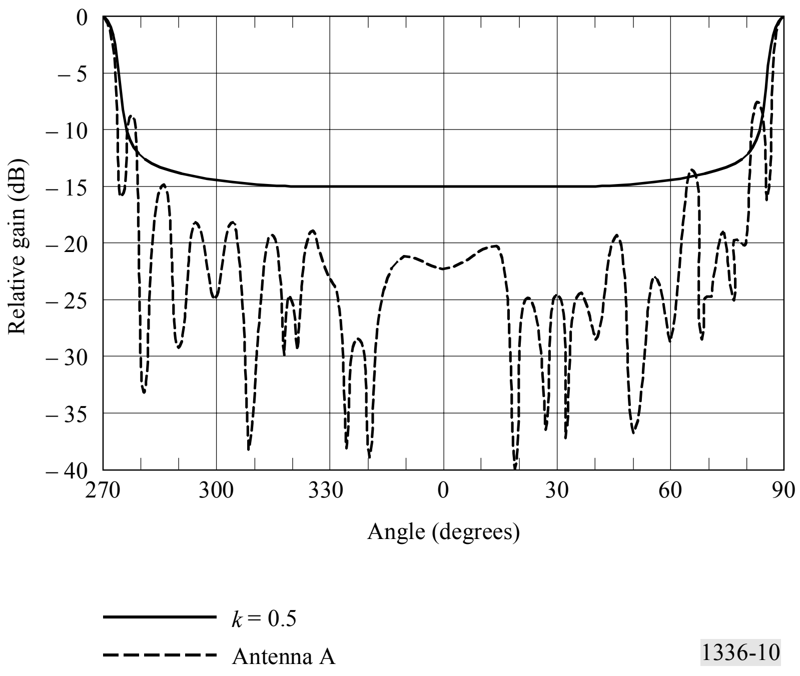


FIGURE 11

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 10 dBi gain

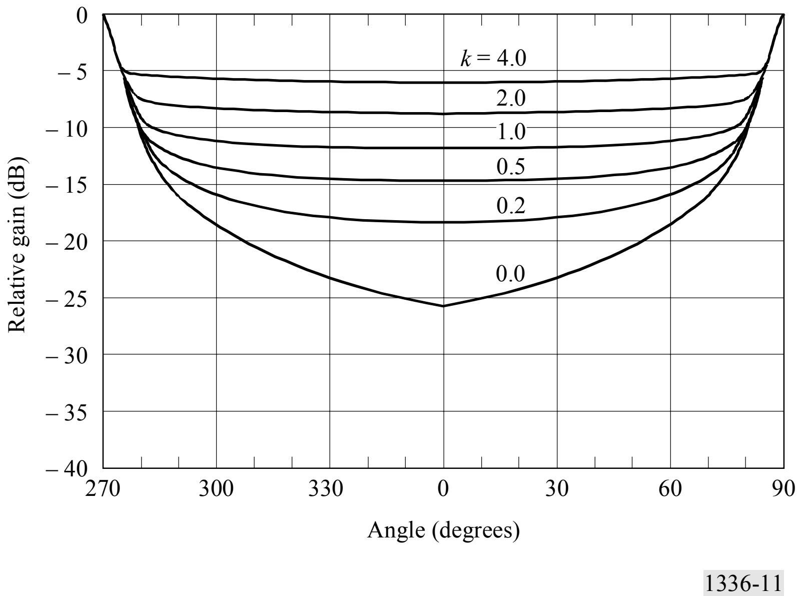
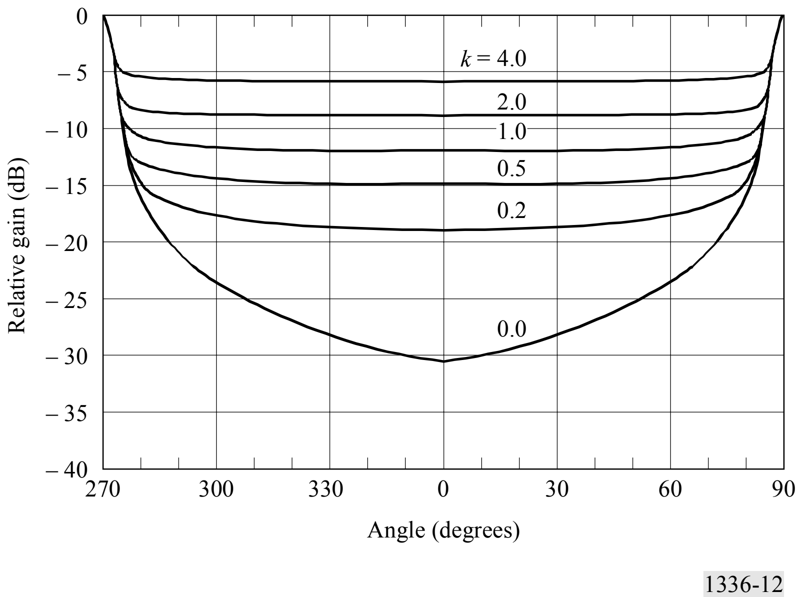


FIGURE 12

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 13 dBi gain



3 Summary, conclusions and further analyses

A reference radiation pattern has been presented for omnidirectional antennas exhibiting a gain between 8 dBi and 13 dBi. The reference radiation pattern has been derived on the basis of theoretical considerations of the radiation pattern of a collinear array of dipoles. The proposed pattern has been shown to adequately represent the theoretical patterns and measured patterns over the range from 8 dBi to 13 dBi. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate especially with regard to antennas operating in frequency bands above 3 GHz.

ANNEX 2

Reference radiation pattern for low-gain circularly symmetric subscriber antennas as used in P-MP radio-relay systems in the 1-3 GHz range

1 Introduction

An antenna with relatively low gain is frequently used for transmitting and receiving signals at the out-stations or in sectors of central stations of P-MP radio-relay systems. These antennas may exhibit a gain of the order of 20 dBi or less. It has been found that using the reference radiation pattern given in Recommendation ITU-R F.699 for these relatively low-gain antennas will result in an overestimate of the gain for relatively large off-axis angles. As a consequence, the amount of interference caused to other systems and the amount of interference received from other systems at relatively large off-axis angles will likely be substantially overestimated if the pattern of Recommendation ITU-R F.699 is used.

2 Analysis

The reference radiation pattern for a subscriber antenna is based on the following assumptions:

– that the directivity of the antenna is less than about 20 dBi;

– that the antenna pattern exhibits circularly symmetric about the main lobe;

– that the main-lobe gain is equal to the directivity.

The proposed reference radiation pattern is given by:

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \le \theta < 1.08\,\varphi_3 \quad (9) \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \le \theta < \varphi_1 \quad (10) \\ G_0 - 14 - 32\log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \le \theta < \varphi_2 \quad (11) \\ -8 & \text{for } \varphi_2 \le \theta \le 180° \quad (12) \end{cases}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: maximum on-axis gain (dBi)

θ: off-axis angle (degrees)

φ_3: the 3 dB beamwidth (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ degrees

$\varphi_1 = 1.9\ \varphi_3$ degrees

$\varphi_2 = \varphi_1 \times 10^{(G_0 - 6)/32}$ degrees.

3 Summary and conclusions

A reference radiation pattern has been presented for low-gain subscriber antennas exhibiting a gain of less than or equal to 20 dBi. The reference radiation pattern has been derived on the basis of limited data on the radiation patterns of flat plate array antennas considered for use in a local access P-MP system operating in the 2 GHz bands. The proposed pattern has been shown to more accurately represent the actual pattern than the pattern given in Recommendation ITU-R F.699. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate and to compare the reference radiation pattern to measured patterns.

ANNEX 3

Relationship between gain and beamwidth for omnidirectional and sectoral antennas

1 Introduction

The purpose of this Annex is to derive the relationship between the gain of omnidirectional and sectoral antennas and their beamwidth in the azimuthal and elevation planes. Section 2 is an analysis of the directivity of omnidirectional and sectoral antennas assuming two different radiation intensity functions in the azimuthal plane. For both cases, the radiation intensity in the elevation plane was assumed to be an exponential function. Section 3 provides a comparison between the gain-beamwidth results obtained using the methods of Section 2 and results contained in the previous version of this Recommendation for omnidirectional antennas. Section 4 summarizes the results, proposes a provisional equation for gain-beamwidth for omnidirectional and sectoral antennas, and suggests areas for further study.

2 Analysis

The far-field pattern of the sectoral antenna in the elevation plane is assumed to conform to an exponential function, whereas the far-field pattern in the azimuth plane is assumed to conform to either a rectangular function or an exponential function. With these assumptions, the directivity, D, of the sectoral antenna may be derived from the following formulation in (spherical coordinates):

$$D = \frac{U_M}{U_0} \tag{13}$$

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} F(\varphi)\, F(\theta) \cos(\theta)\, \mathrm{d}\theta\, \mathrm{d}\varphi \tag{14}$$

where:

U_M: maximum radiation intensity

U_0: radiation intensity of an isotropic source

φ: angle in the azimuthal plane

θ: angle in the elevation plane

$F(\varphi)$: radiation intensity in the azimuthal plane

$F(\theta)$: radiation intensity in the elevation plane.

The directivity of omnidirectional and sector antennas is evaluated in the following sub-sections assuming the radiation intensity in the azimuthal plane is either a rectangular function or an exponential function.

2.1 Rectangular sectoral radiation intensity

Rectangular sectoral radiation intensity function, $F(\varphi)$, is assumed to be:

$$F(\varphi) = U\left(\frac{\varphi_s}{2} - |\varphi|\right) \tag{15}$$

where:

φ_s: beamwidth of the sector,

$$\begin{aligned} U(x) &= 1 \quad \text{for} \quad x \geq 0 \\ U(x) &= 0 \quad \text{for} \quad x < 0 \end{aligned} \tag{16}$$

For either rectangular or exponential sectoral radiation intensity functions, it is assumed that the radiation intensity in the elevation plane is given by:

$$F(\theta) = e^{-a^2\theta^2} \tag{17}$$

where:

$$a^2 = -\ln(0.5) \times \left(\frac{2}{\theta_3}\right)^2 = \frac{2.773}{\theta_3^2} \tag{18}$$

θ_3 : 3 dB beamwidth of the antenna in the elevation plane (degrees).

Substituting equations (15) and (17) into equation (14) results in:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} U\left(\frac{\varphi_s}{2} - |\varphi|\right) d\varphi \int_{-\pi/2}^{\pi/2} e^{-a^2\theta^2} \cos(\theta)\, d\theta \tag{19}$$

This double integral may be solved as the product of two independent integrals. The integral over φ is evaluated in a straightforward way. However, evaluating the integral over θ is somewhat more difficult. The integral over θ could be evaluated numerically with the results either tabulated or a polynomial fitted to the data. However, it is noted that if the limits of integration are changed to $\pm\infty$, the integral over θ is given in closed-form by:

$$\int_{-\pi/2}^{\pi/2} e^{-a^2\theta^2} \cos(\theta)\, d\theta \approx \int_{-\infty}^{\infty} e^{-a^2\theta^2} \cos(\theta)\, d\theta = \frac{1}{a}\sqrt{\pi}\, e^{-1/4a^2} \tag{20}$$

This is a rather simple and flexible formulation that, depending on its accuracy, could be quite useful in evaluating the directivity of sector antennas as well as omnidirectional antennas.

The accuracy with which the infinite integral approximates the finite integral has been evaluated. The finite integral, i.e., the integral on the left-hand side of equation (20), has been evaluated for several values of 3 dB beamwidth using the 24 point Gaussian Quadrature method and compared with the value obtained using the formula corresponding to the infinite integral on the right-hand side of equation (20). (Actually, because of its symmetry, the finite integral has been numerically evaluated over the range 0 to $\pi/2$ and the result doubled.) The results for a range of example values of the 3 dB beamwidth in the elevation plane are shown in Table 1. The Table shows that for a 3 dB beamwidth of 45°, the difference between the values produced by the finite integral and the infinite integral approximation is less than 0.03%. At 25° and below, the error is essentially zero. Equation (19) is now readily evaluated:

$$U_0 = \frac{\varphi_s \theta_3}{4\pi} \sqrt{\frac{\pi}{2.773}} \times e^{\frac{\theta_3^2}{11.09}} \tag{21}$$

TABLE 1

Relative accuracy of the infinite integral in equation (20) in the evaluation of the average radiation intensity

3 dB beamwidth in the elevation plane (degrees)	Finite integral	Infinite integral	Relative error (%)
45	1.116449558	1.116116449	0.0298
25	0.67747088	0.67747088	0.0000
20	0.549744213	0.549744213	0.0000
15	0.416896869	0.416896869	0.0000
10	0.280137168	0.280137168	0.0000
5	0.140734555	0.140734558	0.0000

From equations (15) and (17), $U_M = 1$. Substituting these values and equation (21) into equation (13) yields the directivity of a sector antenna given the beamwidth in the elevation and azimuthal planes:

$$D = \frac{11.805}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{11.09}} \tag{22}$$

where the angles are given in radians. When the angles are expressed in degrees, equation (22) becomes:

$$D = \frac{38750}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36400}} \tag{23}$$

Note that for an omnidirectional antenna, equation (23) reduces to:

$$D = \frac{107.64}{\theta_3} e^{\frac{\theta_3^2}{36400}} \tag{24a}$$

If it is assumed that the radiation efficiency is 100% and that the antenna losses are negligible, then the gain and the directivity of the omnidirectional antenna are identical. Additionally, for omnidirectional antennas with a 3 dB beamwidth less than about 45°, the relationship between the gain and the 3 dB beamwidth in the elevation plane may be simplified by setting the exponential factor equal to unity. The resulting error is less than 6%.

$$G_0 \approx \frac{107.64}{\theta_3} \tag{24b}$$

2.2 Exponential sectoral radiation intensity

The second case considered for the sectoral radiation intensity is that of an exponential function. Specifically:

$$F(\varphi) = e^{-b^2 \varphi^2} \tag{25}$$

where:

$$b^2 = -\ln(0.5) \times \left(\frac{2}{\varphi_s} \right)^2 \tag{26}$$

and φ_s is the 3 dB beamwidth of the sector.

Substituting equations (17) and (25) into equation (14), changing the limits of integration so that the finite integrals become infinite integrals, integrating and then substituting the result into equation (13) yields the following approximation:

$$D = \frac{11.09}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{11.09}} \tag{27}$$

where the angles are as defined previously and are expressed in radians. Converting the angles to degrees transforms equation (27) into:

$$D = \frac{36400}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36400}} \tag{28}$$

Comparing equations (23) and (28), it is seen that the difference between the directivity computed using either of the equations is less than 0.3 dB.

The results given by equation (28) should be compared to a number of measured patterns to determine the inherent effect of the radiation efficiency of the antenna and other losses on the coefficient. At this time, only two sets of measurements are available for sectoral antennas designed to operate in the 25.25 GHz to 29.5 GHz band. Measured patterns in the azimuthal and elevation planes are given, respectively, in Figs. 13 and 14 for one set of antennas and Figs. 15 and 16, respectively, for the second set. From Figs. 13 and 14, the 3 dB beamwidth in the azimuthal plane is 90° and the 3 dB beamwidth in the elevation plane is 2.5°. From equation (28), the directivity is 22.1 dB. This is to be compared with a measured gain of 20.5-21.4 dBi for the antenna over the range 25.5-29.5 GHz. Assuming the gain G_0 of the antenna in the band around 28 GHz is 0.7 dB less than its directivity, and the exponential factor is replaced by unity which introduces an increasing error with increasing beamwidth. The error reaches 6% at 45°. A larger beamwidth leads to a larger error. Based on these considerations, the semi-empirical relationship between the gain and the beamwidth of a sectoral antenna is given by:

$$G_0 \approx \frac{31\,000}{\varphi_s \theta_3} \tag{29a}$$

Similarly, from Figs. 15 and 16, the semi-empirical relationship between the gain and the beamwidth of that sectoral antenna is:

$$G_0 \approx \frac{34\,000}{\varphi_s \theta_3} \tag{29b}$$

FIGURE 13

Measured pattern in the azimutal plane of a 90° sector antenna. Pattern measured over the band 27.5 GHz to 29.5 GHz. The hand drawn cross marks on the left side of the Figure correspond to values obtained from equation (25) (when expressed in dB) for an assumed 3 dB beamwidth of 90° in the azimuthal plane

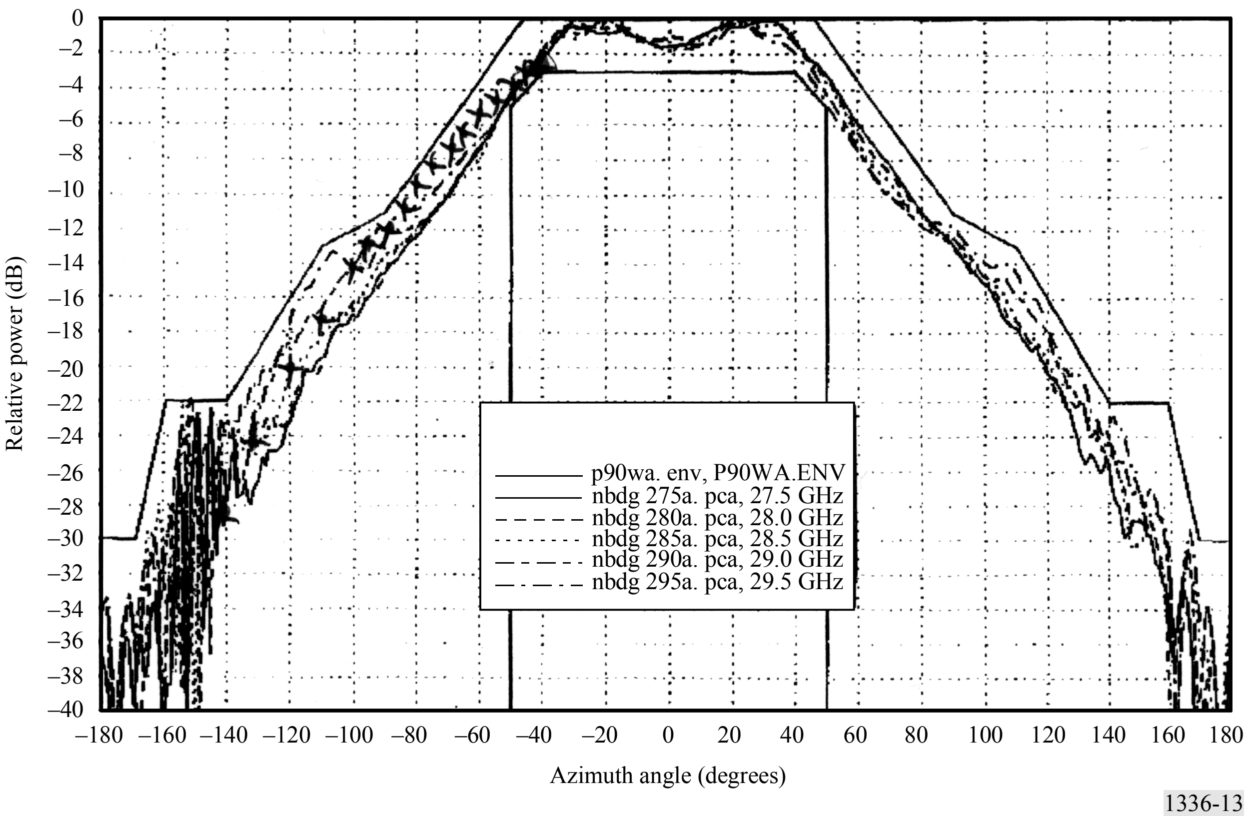
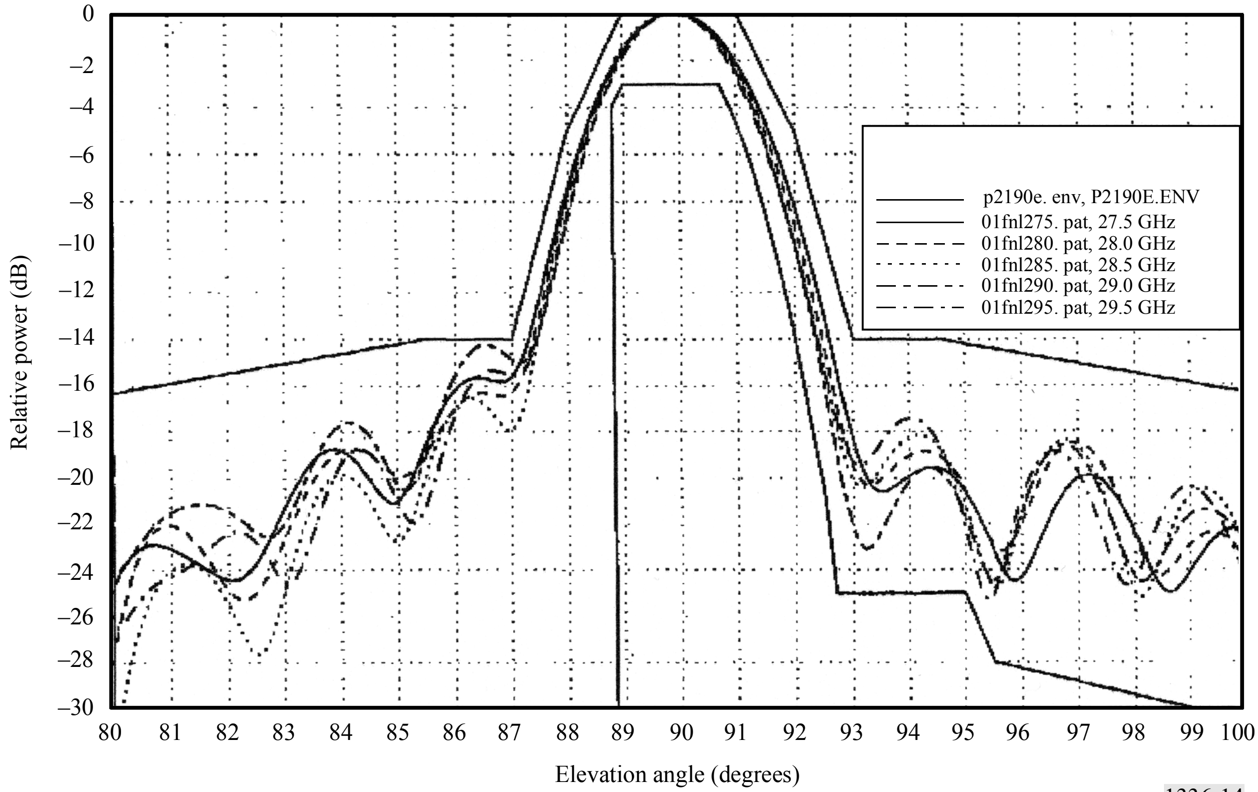


FIGURE 14

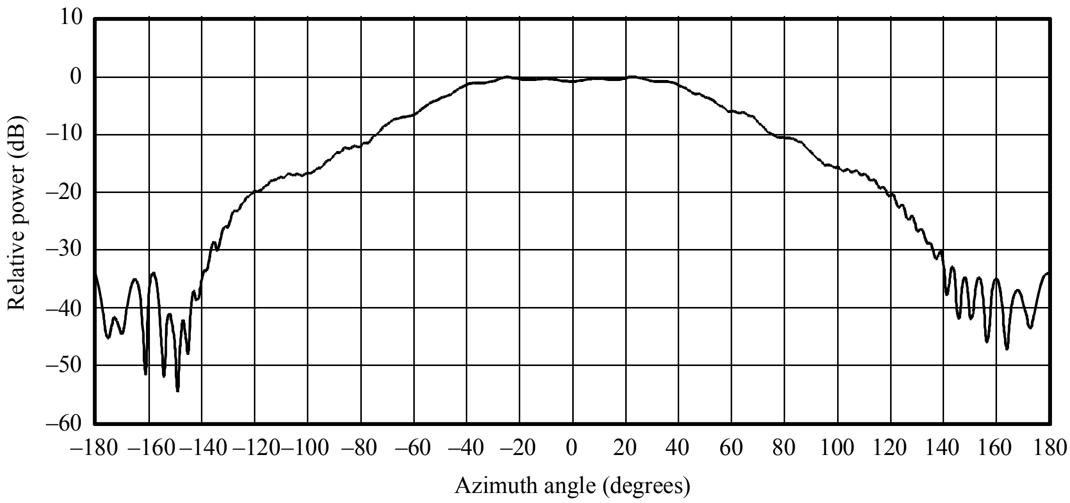
Measured pattern in the azimutal plane of a 90° sector antenna. Pattern measured over the band 27.5 GHz to 29.5 GHz



1336-14

FIGURE 15

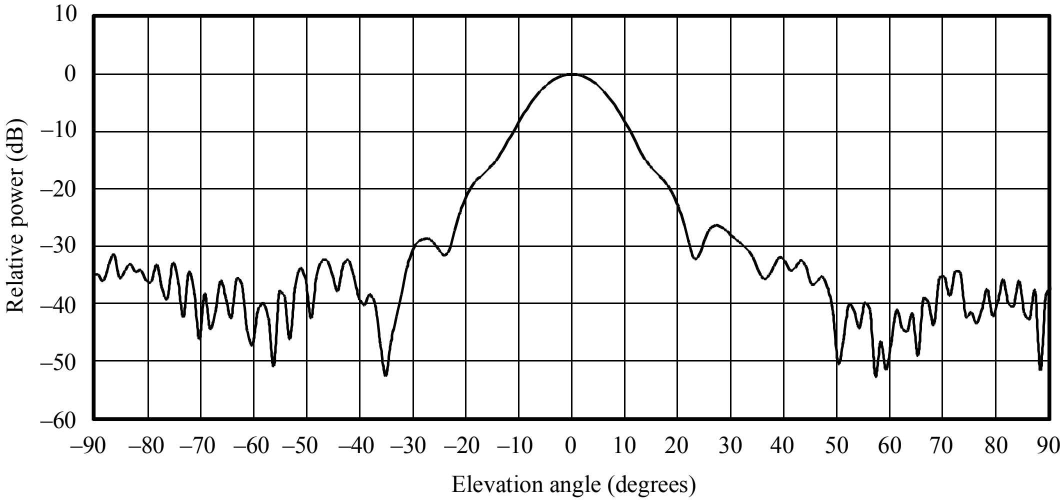
Azimuth pattern of typical 90° sectoral antenna (V-polarization) 15 dBi half-value angle: 90° (horn type antenna at 26 GHz)



1336-15

FIGURE 16

Elevation pattern of typical 90° sectoral antenna (V-polarization)
15 dBi half-value angle: 12° (horn type antenna at 26 GHz)



1336-16

3 Comparison with previous results for omnidirectional antennas

The purpose of this section is to compare the results obtained for an omnidirectional antenna given by equation (24) with previous results reported in and summarized in Annex 1 of this Recommendation.

The radiation intensity in the elevation plane used in for an omnidirectional antenna was of the form:

$$F(\theta) = \cos^{2N} \theta \tag{30}$$

Substituting equation (30) into equation (14), and assuming that $F(\varphi) = 1$, yields:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos^{2N}(\theta) \cos(\theta)\, d\theta\, d\varphi \tag{31}$$

This double integral evaluates to:

$$U_0 = \frac{(2N)!!}{(2N+1)!!} \tag{32}$$

where $(2N)!!$ is the double factorial defined as $(2 \cdot 4 \cdot 6 ... (2N))$, and $(2N + 1)!!$ is also a double factorial defined as $(1 \cdot 3 \cdot 5 ... (2N + 1))$.

Thus, the directivity becomes:

$$D = \frac{(2N+1)!!}{(2N)!!} \tag{33}$$

The 3 dB beamwidth in the elevation plane is given by:

$$\theta_3 = 2 \cos^{-1}\left(0.5^{1/2N}\right) \tag{34}$$

A comparison between the directivity computed using the assumptions and methods embodied in equation (24) and those used in the derivation of equations (33) and (34) is given in Table 2. It is shown that results obtained using equation (24) compare favourably with the results using equations (33) and (34). In all cases equation (24) slightly underestimates the directivity obtained using equations (33) and (34). The relative error (%) of the estimates, when expressed in dB, is greatest for a 3 dB beamwidth in the elevation plane of 65°, amounting to –2.27%. The error (dB) for this case, expressed in dB, is –0.062 dB. For 3 dB beamwidth angles less than 65°, the relative error (%) and the error (dB), are monotonically decreasing functions as the 3 dB beamwidth decreases. For a 16° 3 dB beamwidth, the relative error (%) is about –0.01% and the error (dB) is less than about –0.0085 dB. An evaluation similar to that shown in Table 2 for values of $2N$ up to 10 000 (corresponds to a 3 dB beamwidth of 1.35° and a directivity of 19.02 dB) confirms that the results of the two approaches converge.

TABLE 2

Comparison of the directivity of omnidirectional antennas computed using equation (24a) with the directivity computed using equations (33) and (34)

$2N$	θ_3 (degrees) (equation (34))	Directivity (dB) (equation (33))	Directivity (dB) (equation (24a))	Relative error (%)	Error (dB)
2	90.0000	1.7609	1.7437	–0.98	–0.0172
4	65.5302	2.7300	2.6677	–2.28	–0.0623
6	54.0272	3.3995	3.3419	–1.69	–0.0576
8	47.0161	3.9110	3.8610	–1.28	–0.0500
10	42.1747	4.3249	4.2814	–1.01	–0.0435
12	38.5746	4.6726	4.6343	–0.82	–0.0383
14	35.7624	4.9722	4.9381	–0.69	–0.0341
16	33.4873	5.2355	5.2047	–0.59	–0.0307
18	31.5975	5.4703	5.4423	–0.51	–0.0280
20	29.9953	5.6822	5.6565	–0.45	–0.0256
22	28.6145	5.8752	5.8516	–0.40	–0.0237
24	27.4083	6.0525	6.0305	–0.36	–0.0220
26	26.3428	6.2164	6.1959	–0.33	–0.0205
28	25.3927	6.3688	6.3496	–0.30	–0.0192
30	24.5384	6.5112	6.4931	–0.28	–0.0181
32	23.7649	6.6449	6.6278	–0.26	–0.0171
34	23.0603	6.7708	6.7545	–0.24	–0.0162
36	22.4148	6.8897	6.8743	–0.22	–0.0154
38	21.8206	7.0026	6.9879	–0.21	–0.0147
40	21.2714	7.1098	7.0958	–0.20	–0.0140
42	20.7616	7.2120	7.1986	–0.19	–0.0134
44	20.2868	7.3096	7.2967	–0.18	–0.0129
46	19.8431	7.4030	7.3906	–0.17	–0.0124
48	19.4274	7.4925	7.4806	–0.16	–0.0119
50	19.0367	7.5785	7.5671	–0.15	–0.0115
52	18.6687	7.6613	7.6502	–0.14	–0.0111
54	18.3212	7.7410	7.7302	–0.14	–0.0107
56	17.9924	7.8178	7.8075	–0.13	–0.0104
58	17.6808	7.8921	7.8820	–0.13	–0.0100
60	17.3847	7.9638	7.9541	–0.12	–0.0097
62	17.1031	8.0333	8.0239	–0.12	–0.0094
64	16.8347	8.1007	8.0915	–0.11	–0.0092
66	16.5786	8.1660	8.1571	–0.11	–0.0089
68	16.3338	8.2294	8.2207	–0.11	–0.0087
70	16.0996	8.2910	8.2825	–0.10	–0.0085
72	15.8751	8.3509	8.3426	–0.10	–0.0083
74	15.6598	8.4092	8.4011	–0.10	–0.0081

4 Summary and conclusions

Equations have been developed that permit easy calculation of the directivity and the relationship between the beamwidth and gain of omnidirectional and sectoral antennas as used in P-MP radio-relay systems. It is proposed to use the following equations to determine the directivity of sectoral antennas:

$$D = \frac{k}{\varphi_s \theta_3} \mathrm{e}^{\frac{\theta_3^2}{36\,400}} \tag{35}$$

where:

$$\begin{aligned} k &= 38\,750 \qquad \text{for} \quad \varphi_s > 120° \\ k &= 36\,400 \qquad \text{for} \quad \varphi_s \le 120° \end{aligned} \tag{36}$$

and φ_s = 3 dB beamwidth of the sectoral antenna in the azimuthal plane (degrees) for an assumed exponential radiation intensity in azimuth and θ_3 is the 3 dB beamwidth of the sectoral antenna in the elevation plane (degrees).

For omnidirectional antennas, it is proposed to use the following simplified equation to determine the 3 dB beamwidth in the elevation plane given the gain in dBi (see equation (24b)):

$$\theta_3 \approx 107.6 \times 10^{-0.1\,G_0}$$

It is proposed to use, on a provisional basis, the following semi-empirical equation relating the gain of a sectoral antenna (dBi) to the 3 dB beamwidths in the elevation plane and the azimuthal plane, where the sector is on the order of 120° or less and the 3 dB beamwidth in the elevation plane is less than about 45° (see equation (29a)):

$$\theta_3 \approx \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_s}$$

Further study is required to determine how to handle the transition region implicit in equation (36), and to determine the accuracy of these approximations as they apply to measured patterns of sectoral and omnidirectional antennas designed for use in P-MP radio-relay systems for bands in the range from 1 GHz to about 70 GHz.