International Telecommunication Union

ITU-R

Radiocommunication Sector of ITU

Recommendation ITU-R F.1245-3

(01/2019)

Mathematical model of average and related radiation patterns for point-to-point fixed wireless system antennas for use in interference assessment in the frequency range from 1 GHz to 86 GHz

F Series

Fixed service

International Telecommunication Union

Foreword

The role of the Radiocommunication Sector is to ensure the rational, equitable, efficient and economical use of the radio-frequency spectrum by all radiocommunication services, including satellite services, and carry out studies without limit of frequency range on the basis of which Recommendations are adopted.

The regulatory and policy functions of the Radiocommunication Sector are performed by World and Regional Radiocommunication Conferences and Radiocommunication Assemblies supported by Study Groups.

Policy on Intellectual Property Right (IPR)

ITU-R policy on IPR is described in the Common Patent Policy for ITU-T/ITU-R/ISO/IEC referenced in Resolution ITU-R 1. Forms to be used for the submission of patent statements and licensing declarations by patent holders are available from http://www.itu.int/ITU-R/go/patents/en where the Guidelines for Implementation of the Common Patent Policy for ITU-T/ITU-R/ISO/IEC and the ITU-R patent information database can also be found.

Series of ITU-R Recommendations

(Also available online at http://www.itu.int/publ/R-REC/en)

Series	Title
BO	Satellite delivery
BR	Recording for production, archival and play-out; film for television
BS	Broadcasting service (sound)
BT	Broadcasting service (television)
F	**Fixed service**
M	Mobile, radiodetermination, amateur and related satellite services
P	Radiowave propagation
RA	Radio astronomy
RS	Remote sensing systems
S	Fixed-satellite service
SA	Space applications and meteorology
SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
TF	Time signals and frequency standards emissions
V	Vocabulary and related subjects

Note*: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

Electronic Publication
Geneva, 2019

RECOMMENDATION ITU-R F.1245-3*

Mathematical model of average and related radiation patterns for point-to-point fixed wireless system antennas for use in interference assessment in the frequency range from 1 GHz to 86 GHz

(Question ITU-R 110-3/5)

(1997-2000-2012-2019)

Scope

This Recommendation provides average and related reference radiation patterns for point-to-point fixed wireless system (FWS) antennas in the frequency range from 1 GHz to 86 GHz. The analysis in this Recommendation may be used in interference assessments when particular information concerning the FWS antenna is not available.

Keywords

Antenna, azimuth and elevation beamwidths, cross polarization, fixed service, frequency sharing, radio-relay station, reference radiation pattern, side-lobe envelope, statistical interference analyses

Abbreviations/Glossary

FWS Fixed wireless system

Related ITU Recommendations

Recommendation ITU-R F.699 – Reference radiation patterns for fixed wireless system antennas for use in coordination studies and interference assessment in the frequency range from 100 MHz to 86 GHz

Recommendation ITU-R F.1336 – Reference radiation patterns of omnidirectional, sectoral and other antennas for the fixed and mobile service for use in sharing studies in the frequency range from 400 MHz to about 70 GHz

The ITU Radiocommunication Assembly,

considering

a) that the reference radiation pattern of point-to-point fixed wireless system (FWS) antennas stated in Recommendation ITU-R F.699 provides the peak envelope of side-lobe patterns;

b) that if the peak envelope radiation pattern is used in the assessment of the aggregate interference consisting of many interference entries, the predicted interference will result in values that are greater than values that would be experienced in practice;

c) that, therefore, it is necessary to use the antenna radiation pattern representing average side-lobe levels in the following cases:

– to predict the aggregate interference to a geostationary or non-geostationary satellite from numerous radio-relay stations;

– to predict the aggregate interference to a radio-relay station from many geostationary satellites;

* This Recommendation should be brought to the attention of Radiocommunication Study Groups 4 and 7.

– to predict interference to a radio-relay station from one or more non-geostationary satellites under the continuously variable angle which should be averaged;

– in any other cases where the use of the radiation pattern representing average side-lobe levels is appropriate;

d) that a simple mathematical formula is preferable to the radiation pattern representing average side-lobe levels;

e) that a mathematical model is also required for generalized radiation patterns of antennas for statistical interference analyses involving a few interference entries such as from geostationary satellites into systems in the fixed service,

recommends

1 that, in the absence of particular information concerning the radiation pattern of the FWS antenna involved, the mathematical model of the average radiation pattern as stated below should be used for the applications referred to in *considering c)*;

2 that the following mathematical model of the average radiation pattern should be used for frequencies in the range 1-86 GHz;

2.1 in cases where the ratio between the antenna diameter and the wavelength is greater than 100 ($D/\lambda > 100$), the following equation should be used (see Notes 1 and 7):

2.1.1 for frequencies in the range 1 GHz to 70 GHz, the antenna gain G (dBi):

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda}\varphi\right)^2 \quad \text{for} \quad 0^\circ < \varphi < \varphi_m$$

$$G(\varphi) = G_1 \quad \text{for} \quad \varphi_m \le \varphi < \max(\varphi_m, \varphi_r)$$

$$G(\varphi) = 29 - 25 \log \varphi \quad \text{for} \quad \max(\varphi_m, \varphi_r) \le \varphi < 48^\circ$$

$$G(\varphi) = -13 \quad \text{for} \quad 48^\circ \le \varphi \le 180^\circ$$

2.1.2 for frequencies in the range 70 GHz to 86 GHz, the antenna gain G (dBi):

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda}\varphi\right)^2 \quad \text{for} \quad 0^\circ < \varphi < \varphi_m$$

$$G(\varphi) = G_1 \quad \text{for} \quad \varphi_m \le \varphi < \max(\varphi_m, \varphi_r)$$

$$G(\varphi) = 29 - 25 \log \varphi \quad \text{for} \quad \max(\varphi_m, \varphi_r) \le \varphi < 120^\circ$$

$$G(\varphi) = -23 \quad \text{for} \quad 120^\circ \le \varphi \le 180^\circ$$

where:

G_{max}: maximum antenna gain (dBi) (see Note 2)

$G(\varphi)$: gain (dBi) relative to an isotropic antenna

φ: off-axis angle (degrees)

D: antenna diameter }
λ: wavelength } expressed in the same unit

G_1: gain of the first side lobe

= $2 + 15 \log (D/\lambda)$

$$\varphi_m = \frac{20\lambda}{D}\sqrt{G_{max} - G_1} \qquad \text{degrees}$$

$$\varphi_r = 12.02\,(D/\lambda)^{-0.6} \qquad \text{degrees}$$

2.2 in cases where the ratio between the antenna diameter and the wavelength is less than or equal to 100 ($D/\lambda \leq 100$), the following equations should be used (see Notes 3 and 7):

2.2.1 for frequencies in the range 1 GHz to 70 GHz, the antenna gain G (dBi):

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3}\left(\frac{D}{\lambda}\varphi\right)^2 \qquad \text{for} \quad 0^\circ < \varphi < \varphi_m$$

$$G(\varphi) = 39 - 5\log(D/\lambda) - 25\log\varphi \qquad \text{for} \quad \varphi_m \leq \varphi < 48^\circ$$

$$G(\varphi) = -3 - 5\log(D/\lambda) \qquad \text{for} \quad 48^\circ \leq \varphi \leq 180^\circ$$

2.2.2 for frequencies in the range 70 GHz to 86 GHz, the antenna gain G (dBi):

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3}\left(\frac{D}{\lambda}\varphi\right)^2 \qquad \text{for} \quad 0^\circ < \varphi < \varphi_m$$

$$G(\varphi) = 39 - 5\log(D/\lambda) - 25\log\varphi \qquad \text{for} \quad \varphi_m \leq \varphi < 120^\circ$$

$$G(\varphi) = -13 - 5\log(D/\lambda) \qquad \text{for} \quad 120^\circ \leq \varphi \leq 180^\circ;$$

3 that Annex 1 may be provisionally referred to for generalized radiation patterns of point-to-point FWS antennas which may be used in statistical interference analyses involving a few interference entries such as from geostationary satellites into systems in the fixed service (see Note 9);

4 that the following Notes should be regarded as part of this Recommendation.

NOTE 1 – The average side-lobe levels in § 2.1 are 3 dB lower than peak envelope side-lobe levels in § 2.1 of Recommendation ITU-R F.699.

NOTE 2 – The relationship between G_{max} and D/λ is $20\log\frac{D}{\lambda} \approx G_{max} - 7.7$; see Recommendation ITU-R F.699, *recommends* 3.

NOTE 3 – The mathematical model in § 2.2 was derived from the condition that the total power emitted from the antenna should not exceed the total power fed into the antenna.

NOTE 4 – The radiation pattern in § 2 is only applicable for one co-polarization.

NOTE 5 – The radiation pattern included in this Recommendation is only for antennas which are rotationally symmetrical. It can be applied also to square/polygonal reflectors and flat panel antennas, provided that their equivalent D/λ ratio is derived from the maximum gain, using the formula in Recommendation ITU-R F.699, *recommends* 3.

NOTE 6 – The average radiation pattern in this Recommendation may be somewhat different from radiation patterns of actual antennas. The purpose of this Recommendation is solely to provide a mathematical model for use in interference assessment for the applications referred to in *considering* c).

NOTE 7 – Radio-relay antennas generally employ linear polarization. Therefore, when the interference from a system employing single circular polarization, such as in the mainbeam-to-mainbeam coupling from space stations, is evaluated, the effective radio-relay antenna gain, $G_{eff}(\varphi)$, taking account of polarization advantage, may be estimated by using the following formula within 3 dB of the boresight direction in the main-lobe region ($0 < \varphi < \varphi_{3\,dB}$) instead of the first formula in §§ 2.1 or 2.2 as demonstrated in Annex 2:

$$G_{eff}(\varphi) = G(\varphi) - 1.7 \qquad \text{dBi}$$

where $G(\varphi)$ is the gain according to the first formula in §§ 2.1 and 2.2.

The above formula assumes that the cross-polarized antenna gain for $0° < \varphi < \varphi_{3\ dB}$ is 20 dB lower than G_{max}. The polarization advantage should not be expected for $\varphi > \varphi_{3\ dB}$ or when the radio-relay station is outside the main beam of the antenna of the other service.

The angle $\varphi_{3\ dB}$ (i.e. half of the 3 dB beamwidth) at which the co-polarized gain is 3 dB below the maximum gain G_{max}, can be calculated by replacing $G(\varphi)$ with $G_{max} - 3\ \text{dB}$ in the expression for $G(\varphi)$ for $0° < \varphi < \varphi_m$:

$$\phi_{3dB} \approx \frac{35}{\left(\frac{D}{\lambda}\right)}$$

NOTE 8 – ITU-R membership is encouraged to provide information comparing the average side-lobe levels and the generalized radiation patterns given in this Recommendation with those obtained by radiation pattern measurements on real antennas. This information may assist in the further development of this Recommendation.

NOTE 9 – ITU-R membership is encouraged to examine the feasibility of expanding the application of the model in Annex 1.

Annex 1

Mathematical model of generalized radiation patterns of point-to-point fixed-service antennas for use in statistical interference assessment

1 Introduction

Recommendation ITU-R F.699 gives the reference radiation patterns of point-to-point fixed service antennas, based on the peak envelope of side-lobe levels. Therefore, the interference assessment using this Recommendation may inevitably lead to overestimation of interference.

On the other hand, the main text of this Recommendation gives a mathematical model for average radiation patterns of point-to-point fixed service antennas, representing average side-lobe levels. However, this can be applied only in the case of multiple interference entries or time-varying interference entries.

A mathematical model is required for generalized radiation patterns of antennas for use only in spatial statistical analysis such as deriving the probability distribution function (pdf) of interference from a few GSO satellite systems into a large number of interfered with fixed service systems or stations.

2 Antennas with D/λ greater than 100

The reference radiation pattern of antennas with D/λ greater than 100 representing peak envelope side-lobe levels is given by *recommends* 2.1 of Recommendation ITU-R F.699. According to *recommends* 2.1 of Recommendation F.699; the average side-lobe level is 3 dB below the peak envelope side-lobe level. It seems reasonable to assume that the actual side-lobe levels vary sinusoidally. Therefore, the actual radiation pattern will be expressed as follows:

For frequencies in the range 1 GHz to 70 GHz, the antenna gain G (dBi):

$$G(\varphi) = \max\,[G_a(\varphi), G_b(\varphi)] \qquad \text{for} \quad 0 \leq \varphi < \varphi_r \tag{1a}$$

$$G(\varphi) = 32 - 25 \log \varphi + F(\varphi) \qquad \text{for} \quad \varphi_r \leq \varphi < 48^\circ \tag{1b}$$

$$G(\varphi) = -10 + F(\varphi) \qquad \text{for} \quad 48^\circ \leq \varphi \leq 180^\circ \tag{1c}$$

For frequencies in the range 70 GHz to 86 GHz, the antenna gain G (dBi):

$$G(\varphi) = \max\,[G_a(\varphi), G_b(\varphi)] \qquad \text{for} \quad 0 \leq \varphi < \varphi_r \tag{1a1}$$

$$G(\varphi) = 32 - 25 \log \varphi + F(\varphi) \qquad \text{for} \quad \varphi_r \leq \varphi < 120^\circ \tag{1b1}$$

$$G(\varphi) = -20 + F(\varphi) \qquad \text{for} \quad 120^\circ \leq \varphi \leq 180^\circ \tag{1c1}$$

where:

$$G_a(\varphi) = G_{max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda} \varphi \right)^2 \tag{1d}$$

$$G_b(\varphi) = G_1 + F(\varphi) \tag{1e}$$

$$G_1 = 2 + 15 \log (D/\lambda) \qquad \text{dB} \tag{2a}$$

$$\varphi_r = 15.85 \left(\frac{D}{\lambda} \right)^{-0.6} \qquad \text{degrees} \tag{2b}$$

$$F(\varphi) = 10 \log \left(0.9 \sin^2 \left(\frac{3 \pi \varphi}{2 \varphi_r} \right) + 0.1 \right) \qquad \text{dB} \tag{2c}$$

where φ_r is assumed to correspond to the off-axis angle of the peak of the first side-lobe and the phase at $\varphi = \varphi_r$ is assumed to be 1.5π. It should be noted that the argument of sin function in equation (2c) is expressed in radians and that the value of $F(\varphi)$ is nearly zero or negative. $F(\varphi) = 0$ corresponds to side-lobe peaks. The parameter 0.1 is introduced in equation (2c) in order to avoid the situation that $F(\varphi)$ falls below −10 dB.

3 Antennas with *D*/λ equal to or smaller than 100

In the case of antennas with D/λ equal to or smaller than 100, it will be assumed again that side-lobe peak levels are 3 dB higher than the average side-lobe level given in the main text of this Recommendation.

Thus, the following pattern is presented as a generalized radiation pattern of the antenna with D/λ equal to or smaller than 100:

For frequencies in the range 1 GHz to 70 GHz, the antenna gain G (dBi):

$$G(\varphi) = \max\,[G_a(\varphi), G_b(\varphi)] \qquad \text{for} \quad 0 \leq \varphi < \varphi_r \tag{3a}$$

$$G(\varphi) = 42 - 5 \log (D/\lambda) - 25 \log \varphi + F(\varphi) \qquad \text{for} \quad \varphi_r \leq \varphi < 48^\circ \tag{3b}$$

$$G(\varphi) = -5 \log (D/\lambda) + F(\varphi) \qquad \text{for} \quad 48^\circ \leq \varphi \leq 180^\circ \tag{3c}$$

For frequencies in the range 70 GHz to 86 GHz, the antenna gain G (dBi):

$$G(\varphi) = \max\,[G_a(\varphi), G_b(\varphi)] \qquad \text{for } 0 \le \varphi < \varphi_r \qquad (3a1)$$

$$G(\varphi) = 42 - 5\log(D/\lambda) - 25\log\varphi + F(\varphi) \qquad \text{for } \varphi_r \le \varphi < 120^\circ \qquad (3b1)$$

$$G(\varphi) = -10 - 5\log(D/\lambda) + F(\varphi) \qquad \text{for } 120^\circ \le \varphi \le 180^\circ \qquad (3c1)$$

where:

$$G_a(\varphi) = G_{max} - 2{,}5 \times 10^{-3}\left(\frac{D}{\lambda}\varphi\right)^2 \qquad (3d)$$

$$G_b(\varphi) = G_1 + F(\varphi) \qquad (3e)$$

$$G_1 = 2 + 15\log(D/\lambda) \qquad \text{dB} \qquad (4a)$$

$$\varphi_r = 39.8\left(\frac{D}{\lambda}\right)^{-0.8} \qquad \text{degrees} \qquad (4b)$$

$$F(\varphi) = 10\log\left(0.9\sin^2\left(\frac{3\pi\varphi}{2\varphi_r}\right) + 0.1\right) \qquad \text{dB} \qquad (4c)$$

Again, it should be noted that the argument of sin function in equation (4c) is expressed in radians and that the value of $F(\varphi)$ is nearly zero or negative and $F(\varphi) = 0$ corresponds to side-lobe peaks. The reason for introducing the parameter 0.1 in equation (4c) is the same as that for equation (2c).

4 Conclusion

Equations (1a) to (1e) (together with (2a) to (2c)) and (3a) to (3e) (together with (4a) to (4c)) are presented as mathematical models of generalized radiation patterns of point-to-point fixed service antennas for use only in spatial statistical interference assessment.

Annex 2

Derivation of $G_{eff}(\varphi)$ in Note 7 regarding polarization advantage between linear-polarized and circular-polarized systems

1 Introduction

Radio-relay antennas generally employ linear polarization. Therefore, when the interference from a system employing single circular polarization comes into the radio-relay antennas, it is important to evaluate the circular to linear polarization loss, or polarization advantage between linear-polarized and circular-polarized systems. In the ideal case, the circular to linear polarization loss will be 3 dB. Practical systems will achieve somewhat less polarization discrimination than in the ideal situation.

This Annex discusses the derivation of a circular to linear polarization loss in practical cases.

2 Equation for polarization loss for non-ideal antennas

The polarization loss (in dB) for non-ideal antennas is generally given by the following:

$$L_p = -10\log\left(\frac{1}{2} + \frac{4R_w R_a + \left(R_w^{\;2} - 1\right)\left(R_a^{\;2} - 1\right)\cos 2\Delta\tau}{2\left(R_w^{\;2} + 1\right)\left(R_a^{\;2} + 1\right)}\right)$$

where:

L_p: polarization loss

R_w: voltage axial ratio of the radio wave

R_a: voltage axial ratio of the antenna

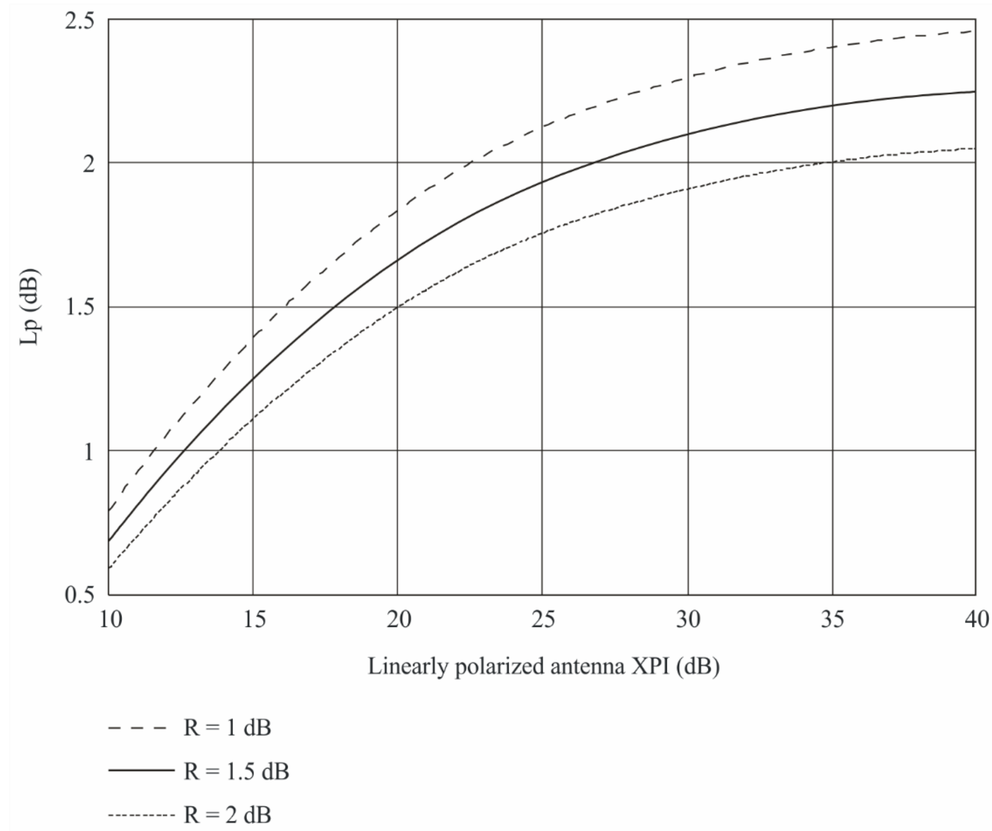
$\Delta\tau$: angle between the tilt angle of the antenna polarization ellipse and the tilt angle of the incident wave polarization ellipse, both referred to horizontal at the Earth's surface. For the purposes of this analysis, it is assumed that $\Delta\tau = 0$, which is the most conservative case.

For a circularly polarized antenna, the voltage axial ratio is usually specified in decibels. These terms are related by the relationship: $R(\text{dB}) = 20\log\left(\left|R_w\right|\right)$. For a linearly polarized antenna, the decibel axial ratio is equivalent in magnitude to the antenna cross-polarization isolation as in the following relationship: $XPI(\text{dB}) = 20\log\left(\left|R_a\right|\right)$.

Figure 1 below shows a plot of polarization loss, L_p, versus cross-polarization isolation (XPI) for three values of circular polarization axial ratio, R. This plot is independent of frequency.

FIGURE 1

Polarization loss vs. XPI for various values of *R*



F.1245-01

The appropriate value of L_p will depend on the characteristics of circularly and linearly polarized antennas through the frequency range from 1 to 86 GHz.

3 Examples of the XPI data

Examples of the XPI data of fixed service antennas from two administrations are shown in Tables 1 and 2. Table 1 contains a summary of information from one administration's licensing database for a range of frequency bands from around 1 GHz up to 40 GHz; and Table 2 shows another XPI data based on different antenna types used in another administration for frequency bands from about 6 GHz up to 22 GHz.

TABLE 1

Example of the XPI data in one administration

Band (GHz)	Number of antenna records	5th percentile XPI (dB)	10th percentile XPI (dB)	Median XPI (dB)
0.953-1.525	484	12	20	30
1.7-2.7	698	20	20	30
3.4-5.0	280	15	20	30
5.85-7.125	532	20	28	30
7.125-7.725	403	24	28	30
7.725-8.5	213	30	30	30
10.5-10.68	151	28	30	30
10.7-11.7	202	20	25	30
12.7-13.25	209	25	25	30
14.5-15.35	172	28	30	30
17.7-19.7	181	27	30	30
21.2-23.6	164	25	28	30
24.25-25.25	8	30	30	32
24.35-28.35	4	30	30	32
28.6-40.0	30	23	26	30

TABLE 2

Example of the XPI data in another administration

Band (GHz)	Number of antenna types	Number of deployed antennas	10th percentile XPI (dB)	Average XPI (dB)
5.925-7.75	11	600	25	29
10.7-15.23	27	5 700	32	35
17.85-23.2	13	2 806	26	28

According to this data, an assumption of a minimum XPI of 20 dB would seem to be appropriate at the frequencies up to 40 GHz.

Above 40 GHz a better cross-polarization performance is expected as frequency and gains increase. Therefore, consistent with *recommends* 2, it can tentatively be concluded that a minimum XPI of more than 20 dB may also be used between 40 GHz and 86 GHz.

4 Co polarization and XPI equations versus measurements

Figure 2 compares the XPI (dB) at 72 GHz:

1) ETSI classes 2 and 4;

2) Measured dish 2 feet, MT-799001 71-76 GHz, 50 dBi, 0.45^0, Vertical/Horizontal.

FIGURE 2

Polarization loss XPI: standards versus measurement

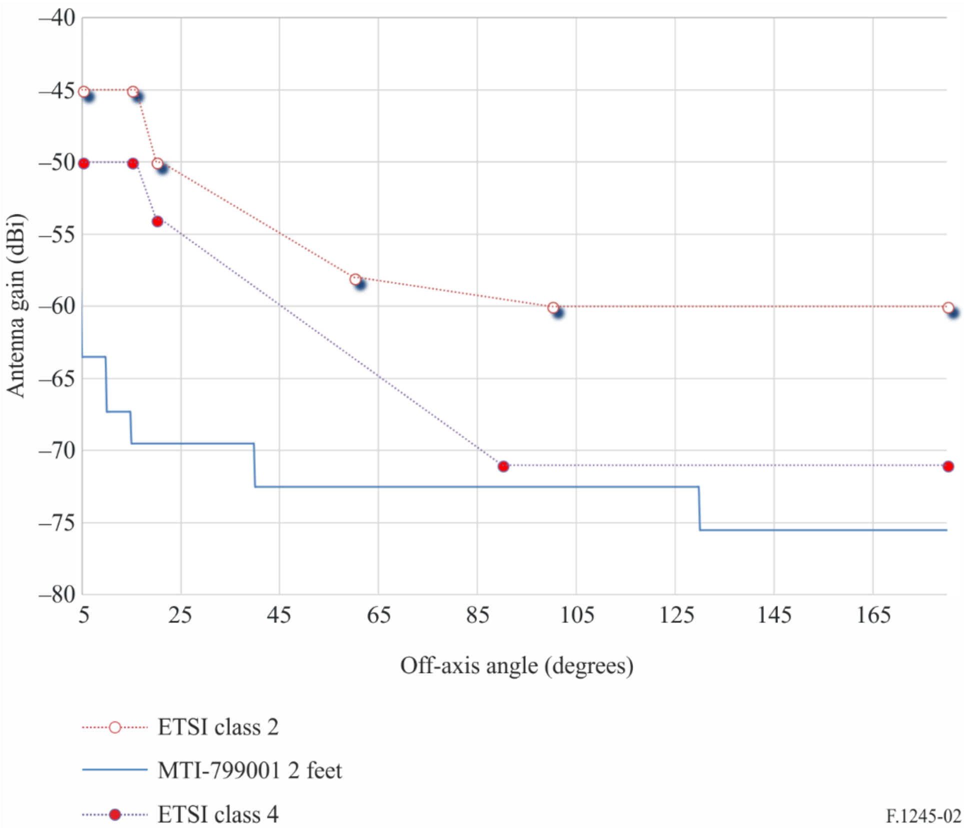
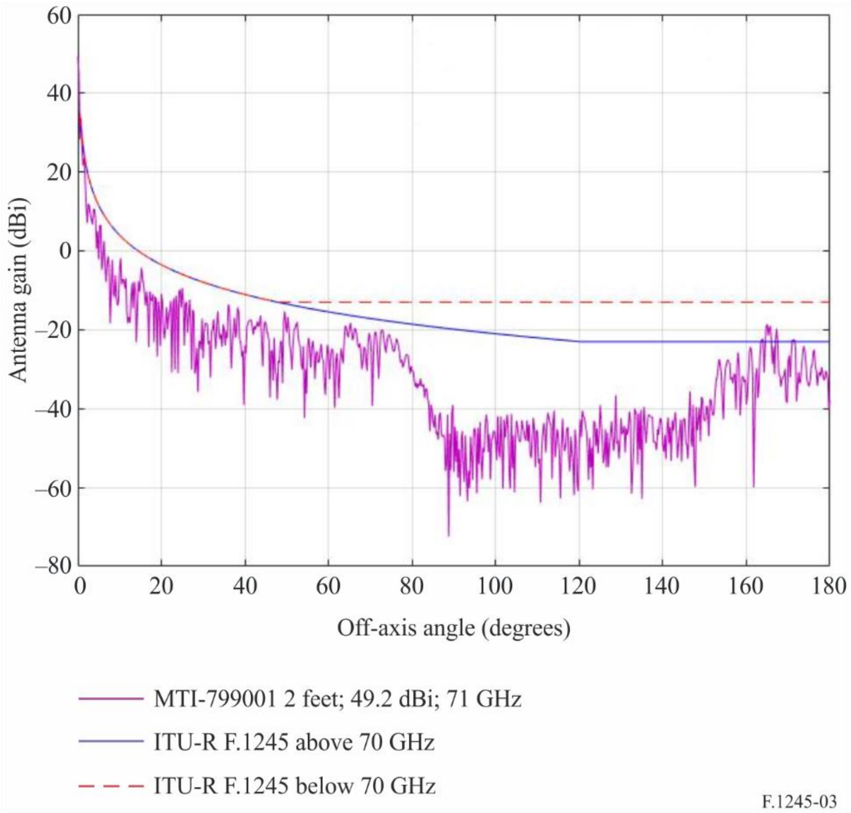


Figure 3 depicts measured pattern at 71 GHz of antenna D/λ = 140, compared to the equation in *recommends* 2.1.1 for frequencies below 70 GHz and 2.1.2 for frequencies above 70 GHz.

FIGURE 3

Measured antenna pattern at 71 GHz, compared to the equations below/above 70 GHz



5 Conclusion

Taking into account Tables 1 and 2, an XPI of 20 dB of radio-relay antenna seems appropriate below 40 GHz. However, modern antennas provide higher XPI. Taking into account Fig. 1, for an XPI of 20 dB and a tentative interfering antenna maximum circular polarization axial ratio (R) of 1.5 dB, which is applicable around the boresight direction of space stations antenna not practicing frequency reuse by orthogonal polarization operated at around 2 to 30 GHz frequency bands, the polarization loss would be 1.7 dB. This value would be applicable only within the antenna 3 dB beamwidth of radio-relay antenna and around the boresight direction of space stations antenna and should be applicable between 1 and 86 GHz.